Reconstruction of Propagating and Confined Microwaves States

Bachelor Thesis
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München, July 2015

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Chapter 1

Introduction

The reconstruction of quantum states is essential to study fundamental properties of light. Especially light in the microwave regime has been proven a promising candidate for these studies. The quantum nature of light has recently been demonstrated by analyzing quantized states confined in a cavity [1] and the entanglement of propagating microwaves [2]. However, the latter experiment examined only steady state configurations. To advance in this field of study, characterizing the time-evolution of quantum states is a desirable goal [3, 4]. To achieve this goal, an experimental detection setup capable of broadband signal analysis is necessary.

In this work, we modify an existing time-domain measurement setup to allow for time-resolved state reconstruction on a nanosecond timescale. The analyzed quantum states are generated using electromagnetic fields in the microwave regime. To suppress thermal excitations, we work with superconducting circuits placed inside a dilution refrigerator. Data acquisition is performed using a heterodyne detection setup with a digital homodyning stage. In this setup, we concentrate on the implementation of real-time data processing including additional calculations in the software part. We characterize this modified detection scheme by performing necessary calibration measurements. We compare two different calibration methods (Planck spectroscopy and ac-Stark-shift) and find good agreement. Using results obtained from these calibrations, we reconstruct several Gaussian states.

This thesis is structured as follows: In Chap. 2, we start by introducing the quantization of the electromagnetic field (2.1), give a short overview of the theoretical background of state reconstruction (2.2), present relevant states of light (2.3), and explain basic light-matter-interaction (2.4). We continue by describing the hardware (3.1) and software part (3.2) of the experimental setup. Finally, we present the main results of our work, a comparison of two calibration techniques (4.1) and a reconstruction of thermal and coherent states of light. (4.2).
Chapter 2

Theory

The description of classical and quantum states of light is essential for the study of fundamental light-matter-interaction. Therefore, in this chapter, we discuss how light can be described classically and quantum mechanically and how the state of a quantum system can be retrieved in an experiment. Furthermore, we examine different states of light and the interaction of a quantum mechanical two-level-system with a coherent light state within a resonator.

2.1 Quantization of the electromagnetic field

At first, we characterize some general quantum properties of the quantized electromagnetic field and afterwards treat in particular the field within a coplanar waveguide resonator (CPW).

We start by explaining the most important aspects of field quantization at the example of the electromagnetic field in a finite volume $V$ [5–7]. In classical mechanics, light, or to be more precisely, electromagnetic radiation, is completely described by the Maxwell-equations. Due to boundary conditions in a finite volume, for example a cavity, only certain modes of the field are valid. So we can expand the electric field in the eigenmodes of the volume: $E_x(z,t) = \sum_j A_j q_j \sin(k_j z - \omega_j t)$ with $q_j$ the amplitude and $k_j$ the wave vector of the eigenmode $j$. $A_j = \sqrt{\omega_j^2 m_j/V \varepsilon_0}$ is a constant with the dimension of $V m^{-2}$, $\omega_j/2\pi$ is the $j$th-eigenfrequency. We find a similar result for the magnetic field $B_y = \sum_j A_j p_j (\varepsilon_0 \mu_0/k_j) \cos(k_j z - \omega_j t)$ with an amplitude $p \propto \dot{q}$. The classical Hamiltonian of the system can therefore be written as

$$H_{em} = \frac{1}{2} \int_V d^3 x \left( \varepsilon_0 E^2 + \frac{B^2}{\mu_0} \right).$$

(2.1)
2.1 Quantization of the electromagnetic field

For pedagogical reasons, we rewrite the radiation field from Eq. (2.1) in the form of an infinite sum over classical harmonic oscillators

\[ H_{em} = \frac{1}{2} \sum_j \left( \frac{p_j^2 \cos(k_j z - \omega_j t)^2}{m_j} + m_j \omega_j^2 q_j^2 \sin(k_j z - \omega_j t)^2 \right) . \quad (2.2) \]

Here, the Cartesian coordinate \( q_j \) and the canonical momentum \( p_j = m_j \dot{q}_j \) represent the position and momentum of each oscillator. We call \( q_j \) and \( p_j \) the quadratures of the field.

In the following, we quantize the Hamiltonian from Eq. (2.2) by promoting the quadratures to operators \( \hat{q}_j \) and \( \hat{p}_j \) and postulating the commutation relations \([\hat{q}_j, \hat{p}_{j'}] = i \hbar \delta_{jj'}\) and \([\hat{q}_j, \hat{q}_{j'}] = [\hat{p}_j, \hat{p}_{j'}] = 0\). Combining Eq. (2.2) and the common creation and annihilation operators \([6]\),

\[
\hat{a}^\dagger_j = \sqrt{\frac{\omega_j}{2\hbar}} \hat{q}_j - i \frac{1}{\sqrt{2\hbar \omega_j}} \hat{p}_j ,
\hat{a}_j = \sqrt{\frac{\omega_j}{2\hbar}} \hat{q}_j + i \frac{1}{\sqrt{2\hbar \omega_j}} \hat{p}_j \quad (2.3)
\]

the Hamiltonian can be written as

\[ \hat{H}_{em} = \sum_j \hbar \omega_j \left( \hat{a}^\dagger_j \hat{a}_j + \frac{1}{2} \right) . \quad (2.4) \]

Here, the average occupation number of each mode \( j \) is \( \langle \hat{n}_j \rangle = \langle \hat{a}_j^\dagger \hat{a}_j \rangle \) and the term \( \hbar \omega_j/2 \) is the energy due to the vacuum fluctuations. Although we restrict ourselves to a finite volume in this derivation, they can be shown to be valid in the continuum limit, i.e., for free space \([5]\). Thus, the electromagnetic field is quantized in general.

**Electromagnetic field in a single mode resonator**

In our work, we use a coplanar waveguide resonator (CPW), which has an analogon in quantum optics, the optical cavity. As shown in Fig. 2.1 (a), the CPW is a planar structure consisting of a center conductor with two groundplanes on each side. It can be thought of as a cross section of a coaxial cable. When inserting two capacitors into the center conductor, the segment defined in this way becomes a resonator. As discussed in the previous section, the finite volume gives rise to a discrete set of standing-wave excitations called modes. The shape of these modes is defined by the boundary conditions due to the capacitors. The first and second harmonic mode of
Figure 2.1: Schematic layout of a half-wavelength CPW resonator. It consists of a center conductor (green), which is capacitively coupled to a transmission line (red) for readout purposes. The line is enclosed by two ground planes (light grey), the whole structure is fabricated on silicon substrate covered with a thin layer of silicon dioxide (dark grey). (a) 3D view (b) Top view. White: First harmonic mode ($\lambda/2$). Black: Second harmonic mode ($\lambda$) of current distribution.

The current distribution in the resonator are visualized in Fig. 2.1 (b). The resonance frequency depends on the length of the resonator, which is designed at will. As we are working with frequencies around 5 GHz, the resonator is about 10 mm long. Since the length of the resonator is half of the wavelength of the first harmonic, this resonator is called half-wavelength resonator. In the remainder of this work, we can ignore all other modes except for this one. For the resulting effective single mode resonator, we can identify the flux $\hat{\Phi}$ and charge $\hat{Q}$ with the conjugate variables $\hat{q}$ and $\hat{p}$ and write down the Hamiltonian

$$H_{\text{res}} = \hbar \omega_{\text{res}} \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right).$$

(2.5)

### 2.2 Wigner function representation

In classical mechanics, we can characterize a system by its components position $q$ and momentum $p$. Thus, the state of the system at a particular time is represented by a point in phase space. This state can be described by the phase space probability distribution $W(q, p)$. For a particular point $q_0, p_0$ in phase space, $W(q_0, p_0)$ denotes the probability of finding the system at this point. In quantum mechanics, there is
a particular uncertainty inherent to the system, the Heisenberg uncertainty \( \Delta q \Delta p \geq \hbar / 2 \). Therefore, a simultaneous measurement of conjugate variables such as position and momentum is not possible. Searching for a quantum mechanical analogon of the phase space probability distribution leads to quasiprobability distributions, which, for example, can lack positivity. In this work, we will concentrate especially on the Wigner function defined by the expression [8–10]

\[
W(q,p) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp(ipx) \left\langle q - \frac{x}{2} | \hat{\rho} | q + \frac{x}{2} \right\rangle dx .
\]  

(2.6)

Here, \( \hat{\rho} = \sum_l p_l |\Psi_l\rangle \langle \Psi_l| \) is the density operator, a general description of a quantum state [6, 11], \(|\Psi_l\rangle\) refers to the \( l \)-th eigenstate and \( p_l \) to the probability of finding this particular state. While the quasiprobabilities can assume negative values, actual probability distributions can be extracted from the Wigner function by integrating along any specific quadrature direction.

**Reconstruction of a Wigner function**

Eq. 2.6 suggests that we can equivalently describe a quantum state with the density operator or the Wigner function. It can be shown that having full knowledge of the density operator or the Wigner function is also equivalent to knowing all so-called moments \( \langle (\hat{a}^\dagger)^m \hat{a}^n \rangle \) with \( m, n \in \mathbb{N} \) [12, 13]. These moments are the products of the photon creation and annihilation operators, as they appear in the displacement operator \( \hat{D}(\alpha) \) (see Eq. (2.7)). To express the Wigner function with those moments, we introduce the displacement operator

\[
\hat{D}(\alpha) = \exp (\alpha \hat{a}^\dagger - \alpha^* \hat{a})
\]

\[
= 1 + (\alpha \hat{a}^\dagger - \alpha^* \hat{a}) + \frac{1}{2} \left( (\alpha^*)^2 (\hat{a}^\dagger \hat{a})^2 + (\alpha^*)^2 \hat{a}^2 \right) - \alpha \alpha^* \hat{a}^\dagger \hat{a} + \ldots ,
\]  

(2.7)

where \( \alpha = q + ip \) is a complex number referring to a point \((q, p)\) in phase space. The displacement operator creates a coherent state \(|\alpha\rangle = \hat{D}(\alpha) |0\rangle\), if we apply it to a vacuum state. The expectation value of the displacement operator in Eq. (2.7) is called characteristic function \( \chi(\xi) = \text{Tr}[\hat{\rho} \hat{D}(\xi)] \), where \( \xi \) is a point in phase space. The Fourier-transformation of the characteristic function allows one to calculate the Wigner function [12, 14] via the expression

\[
W(\alpha) = \int \exp (\alpha \xi^* - \alpha^* \xi) \chi(\xi) d^2\xi .
\]  

(2.8)
The integration volume is the complex plane. While theoretically an infinite number of moments must be known, the reconstruction of a state is possible from a finite number of moments in many practical scenarios [12, 13, 15]. In particular, Gaussian states are completely described by the moments up to the second order \((m+n \leq 2)\). Also, showing the negativity of the Wigner function for quantum states requires at least moments up to the fourth order [2, 16].

The representation of a quantum state via creation and annihilation operator \(\hat{a}^\dagger\) and \(\hat{a}\) is theoretical correct, but not feasible experimentally. Hence, we need variables, which are accessible in an experiment. We find the in-phase and out-of-phase components of the electric field, which we call \(I\) and \(Q\), respectively [9]. We can express both \(I\) and \(Q\) with \(\hat{a}^\dagger\) and \(\hat{a}\) as

\[
\hat{I} = \frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{a}) \quad \text{and} \quad \hat{Q} = \frac{i}{\sqrt{2}}(\hat{a}^\dagger - \hat{a}).
\]

\[ (2.9) \]

### 2.3 States of light

In this section, we discuss important quantum mechanical states of the electromagnetic field. We examine quantum mechanical states and how classical states relate to them.

#### 2.3.1 Fock States

At first, we treat the Fock states. These quantum states are important, because they are the most convenient set of states used as an orthonormal basis for the Hilbert-space in quantum mechanics. For us, these non-classical states motivate the need for the measurement of many moments [2, 9]. Fock states \(|n\rangle\) are the eigenstates of the number operator \(\hat{n} = \hat{a}^\dagger\hat{a}\) [5, 6]

\[
\hat{n} |n\rangle = n |n\rangle.
\]

\[ (2.10) \]

Different Fock states can be obtained by applying the creation and annihilation operators to an initial Fock state

\[
\hat{a} |n\rangle = \sqrt{n} |n-1\rangle, \quad |n\rangle \leq 1\]

\[
\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle.
\]

\[ (2.11) \quad (2.12) \]
2.3 States of light

Figure 2.2: Wigner function of the Fock state $|1\rangle$, visualized (a) from the top and (b) from the bottom.

Regarding the Hamiltonian of the electromagnetic field in Eq. (2.4), we see that every eigenstate $|n\rangle$ corresponds to a certain eigenenergy $\hbar \omega (\hat{a}^\dagger \hat{a} + 1/2) |n\rangle = E_n |n\rangle$. The operators $\hat{a}^\dagger$ and $\hat{a}$ create or annihilate photon states with energy $E_n$ by raising or lowering the (photon) number in each eigenstate and thus are named creation and annihilation operators.

The Wigner function of Fock states can be derived from the wave function of the state [9]. For Fock state $|1\rangle$, we get $W(q, p) = \exp \left(-q^2 - p^2\right) \left(2q^2 + 2p^2 - 1\right) / \pi$, which is shown in Fig. 2.2. In this figure, the presence of negative quasiprobabilities is clearly visible and therefore reveals the quantum character of a Fock state.

2.3.2 Vacuum state

When we apply Eq. (2.11) $n$-times, meaning we remove all $n$ photons from the Fock state $|n\rangle$, we reach a state without any photons, the so-called vacuum state $|0\rangle$. Classically, this state with no photons would have no energy and no physical relevance. But in quantum mechanics, the Heisenberg uncertainty principle holds. Because two quadrature components (e.g. position and momentum or amplitude and phase) cannot be measured simultaneously, they fluctuate within certain borders around the zero point, resulting in a finite energy $\hbar \omega / 2$, the zero point energy. The vacuum fluctuations around the origin are illustrated in the Gaussian shape of the Wigner function $W(q, p) = \exp(-q^2 - p^2) / \pi$, see Fig. 2.3 (a). The vacuum state corresponding to the vacuum fluctuations inherent to any quantum system is a state, which fulfills the Heisenberg principle with a minimum uncertainty. Although, no
2.3 States of light

Figure 2.3: Wigner function of: (a) Vacuum state $|0\rangle$. (b) Coherent state $|\alpha\rangle$ for $\alpha = q_0 + ip_0 = 1.5 + i1.5$. The Gaussian shape is equivalent to a vacuum state displaced to the point $(q_0 = 1.5, p_0 = 1.5)$ [9].

photons are existent, the vacuum state has physical consequences and measurable effects, such as the Casimir effect [17].

2.3.3 Coherent states

We now turn to the eigenstates of the annihilation operator

$$\hat{a} |\alpha\rangle = \alpha |\alpha\rangle .$$

These states are called coherent states. The complex eigenvalue $\alpha = q_0 + ip_0$ of the states corresponds to the amplitude $\sqrt{q_0^2 + p_0^2}$ and phase $\arctan(p_0/q_0)$ in classical optics. Therefore, a coherent state is a state with well-defined amplitude and phase. Coherent states are the most classical states provided by quantum mechanics in the sense that in the equations of motion of the quantum harmonic oscillator, the operators can be replaced by their expectation values at all times. An example for such a state is the light emitted by a laser. We can create a coherent state by applying the displacement operator, given in Eq. (2.7), on the vacuum state

$$|\alpha\rangle = D(\alpha) |0\rangle = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a}) |0\rangle$$

A coherent state is often referred to as a displaced vacuum, as can be seen in the Wigner function $W(q, p) = \exp(-(q - q_0)^2 - (p - p_0)^2)/\pi$ shown in Fig. 2.3 (b).
This designation is slightly misleading, as both vacuum and coherent state only correspond in their uncertainty properties. Both fulfill the Heisenberg principle with a minimum quantum noise, but while the vacuum states consists of no photons, the coherent state represents a certain amount of photons with dedicated relations.

2.3.4 Squeezed states

Another class of states with minimum uncertainty are the so-called squeezed states [2, 5, 9]. In contrast to a vacuum or coherent state, the Wigner function of the squeezed state is no longer rotationally symmetric. Specifically, the fluctuations along a specific quadrature direction can be „squeezed“ below those of the vacuum state at the cost of increased fluctuations in the orthogonal quadrature. A squeezed state can be produced by applying the squeeze operator to the vacuum

\[
|\xi\rangle = \hat{S}(\xi)|0\rangle = \exp\left(\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi(\hat{a}^\dagger)^2\right)|0\rangle.
\]

(2.15)

Here \(\xi\) is a complex number, the squeezing parameter. The Wigner function of a squeezed state can be written as \(W(q, p) = \exp(-q^2/\sigma^2_q - p^2/\sigma^2_p)/(\sigma_q\sigma_p)\). An example is shown in Fig. 2.4 (a). The squeezed states belong to the class of Gaussian states.

2.3.5 Thermal states

As a last example, we consider thermal states [2, 5, 18]. They represent chaotic light with no coherence at all and can be generated by black body emitters of finite temperature \(T\). All photons forming a thermal state, underlie the Bose-Einstein-distribution [19], so the mean photon number depends on the temperature \(T\) of the emitter

\[
\langle n \rangle = \langle \hat{a}^\dagger \hat{a} \rangle = \frac{1}{\exp(h\omega/k_B T) - 1}.
\]

(2.16)

Here \(k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}\) is the Boltzmann constant. The higher the system temperature is, the more probable are many photons with high energy. The Wigner function \(W(q, p) = \exp(-q^2 - p^2/(\langle n \rangle + 1/2))/\pi(\langle n \rangle + 1/2)\) is Gaussian and broadens with increasing temperature, as can seen in Fig. 2.4 (b). If we compare the thermal and the vacuum state, we see that the vacuum state represents the limit of thermal states at \(T = 0\) K.

In our experiments, we use the temperature dependence of the mean photon number
2.4 Interaction between light and a two-level-system

So far, we have encountered quantization and different states of light and how they are represented. In this chapter, we shortly introduce quantum two-level-systems (qubits) and then examine their interaction with the formerly established resonators.

2.4.1 Qubits as quantum two-level-systems

The use of superconducting circuits as qubits has been in the focus of numerous studies recently [20, 21]. There are several different realizations of qubits for both possible control variables flux and charge. We use a gradiometric tunable flux qubit [22–24] fabricated and analyzed in Ref. [23]. Here, we only concentrate on the property of this qubit as a two-level-system.

As shown in Fig. 2.5 (a), the gradiometric tunable flux qubit consists of an eight-shaped superconducting loop interrupted by three Josephson junctions on the center line. One of the three junctions is realized in a dc-SQUID geometry [23–25]. The potential of this qubit can be reduced to a double-well, see Fig. 2.5 (b). Classically, the two minima are associated to states of a clockwise and counterclockwise persistent current in each gradiometer loop. For a suitable flux bias the potential is symmetric and the persistent current states are degenerate. Quantum mechanically, this de-
2.4 Interaction between light and a two-level-system

Figure 2.5: (a) Left: Sketch of a gradiometric tunable qubit (black lines) inductively coupled to the center conductor of a CPW resonator (green lines). Black: The superconducting circuit in gradiometric tunable design. Orange: Josephson junctions. Yellow: The antenna for adjusting the qubit gap $\Delta$.
(b) Potential of our flux qubit (left) coupled to a quantum harmonic oscillator (right) with coupling constant $g$. The blue and red circular arrows indicate the persistent current states, the blue-red arrow the superposition state forming the qubit.

generacy is lifted due to tunneling through the small barrier of the double-well. The coupling energy $\Delta$ lifts the degeneracy, giving rise to symmetric and antisymmetric superpositions of the persistent current states. These superposition states form the ground and the excited state of a two level system, our qubit. We can describe this two-level-system with the Hamiltonian

$$H_q = \frac{\Delta}{2} \hat{\sigma}_z = \frac{1}{2} \begin{pmatrix} \Delta & 0 \\ 0 & -\Delta \end{pmatrix}$$  \hspace{1cm} \text{(2.17)}$$

where $\hat{\sigma}_z$ is the diagonal Pauli operator.

2.4.2 Interaction between resonator and qubit

Having introduced a resonator in Sec. 2.1 as well as the flux qubit as two-level-system, we are now able to combine both and study their interaction. This whole
field of study is known as circuit quantum electrodynamics (QED), emphasizing a close relationship to cavity QED in quantum optics. In circuit QED, the qubit acts as a box for microwave quantum states and the qubit as an artificial two-level atom \[23, 26\].

The interaction between qubit and resonator is of dipolar nature and, in our case, well described by the Jaynes-Cummings model \[27\]. We combine Hamiltonian of the resonator \( H_{\text{res}} \) (2.5), the qubit \( H_q \) (2.17) and the interaction \( H_{\text{int}} \) \[5, 23\] to get

\[
H_{\text{tot}} = H_{\text{res}} + H_q + H_{\text{int}}
\]

\[
= \hbar \omega_{\text{res}} \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \frac{\Delta}{2} \hat{\sigma}_z + \hbar g \left( \hat{a}^\dagger \hat{\sigma}_- + \hat{\sigma}_+ \hat{a} \right). \tag{2.18}
\]

Here, \( g \) is the coupling constant, and \( \hat{\sigma}_+ = \left\vert e \right\rangle \left\langle g \right\vert \) and \( \hat{\sigma}_- = \left\vert g \right\rangle \left\langle e \right\vert \) are the raising and lowering operators for the two-level-system.

We have to distinguish between two regimes. First, the resonant regime, where the qubit level splitting \( \Delta \) is approximately equal to the energy of the resonator \( \hbar \omega_{\text{res}} \). In this regime, qubit and resonator can coherently exchange excitations. Second, the dispersive regime, where resonator and qubit are far detuned and cannot exchange excitations. In this regime, the resonator can be used as a readout device for the qubit \[20\].

When the detuning \( \delta \equiv \omega_{\text{res}} - \omega_q \) is much larger than \( g \), the Hamiltonian of Eq. (2.18) can be approximately diagonalized by applying a particular unitary transformation \[26\] and omitting terms on the order of \( (g/\delta)^2 \) or higher. We then obtain \[5, 23, 28, 29\]

\[
H_{\text{disp}} = \hbar \omega_{\text{res}} \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \frac{\hbar}{2} \left( \omega_q + \frac{2g^2}{\delta} \hat{a}^\dagger \hat{a} + \frac{g^2}{\delta} \right) \hat{\sigma}_z. \tag{2.19}
\]

We see that both qubit and resonator undergo frequency shifts, which depend on each other’s state. The qubit frequency is shifted by \( g^2/\delta \) due to the coupling to vacuum fluctuations (Lamb shift) and the resonator gives rise to a shift depending on its photon number \( \hat{a}^\dagger \hat{a} \) (as-Stark shift). We use the latter to calibrate the number of photons populating the resonator, see Sec. 4.1.2 \[28\].
Chapter 3

Experimental setup

The goal of this work is to build an experimental setup that enables us to measure the moments of a quantum state and to reconstruct this state. In the past years, there has already been much effort in developing such detecting schemes at the Walther-Meißner-Institut (WMI) [2, 30]. These experimental setups were successfully used to examine the entanglement of quantum states [31] or to characterize the quantum properties of microwave beam splitters [32].

In these experiments, only steady quantum states were analyzed, while the time evolution was not surveyed. Our aim in this work is to build a setup suitable to analyze the time evolution of quantum states. Hence, we use a detection scheme directly connected to an existing time-domain setup [25]. However, as this setup is only capable of measuring the first moments, we modify the existing structure to calculate higher moments for state reconstruction. In this work, we have successfully implemented the detection of moments up to the second order, which is sufficient for reconstruction of Gaussian quantum states.

In Sec. 3.1, we describe the preexisting measurement setup used for state preparation and readout as well as detection. We then move on the modified part, especially the software used for averaging, moment calculation and signal demodulation in Sec. 3.2.

3.1 Measurement setup

In this section, we introduce the hardware part of the experimental setup. We describe the generation of readout pulses in Sec. 3.1.1, the cryogenic part in Sec. 3.1.2 and the detection setup in Sec. 3.1.3.
3.1 Measurement setup

Figure 3.1: Sketch of (a) pulse generating setup and (b) detection setup.

3.1.1 Pulse generation setup

In this section, we describe the pulse generation for readout purposes. In Fig. 3.1 (a), we show a sketch of this setup. As we perform pulsed measurements, we need to generate pulses on a microsecond timescale. We create such a pulse by applying a continuous microwave signal generated by a microwave source to a mixer (Marki M1-0218LA). This mixer can be saturated with square pulses from an arbitrary function generator (Agilent 81160A). For a better on-off-ratio of the AFG, we include two mixers into the signal line. As we need a readout (RO) pulse as well as a drive pulse, we have two independent pulse generation chains. The microwave tone for readout is generated by an Agilent PSG E8267D and the drive tone by a Rohde & Schwarz SMF100A. Because we have only one input line for the two signals, we combine both signals before entering the cryostat. Both microwave sources and both arbitrary function generators are connected to a 10 MHz-reference (SRS FS725).

3.1.2 Cryogenic setup

In this section, we describe the cryogenic part of the measurement setup. To suppress thermal excitations, which would cover the quantum properties of the system, and to bring our sample into a superconducting state, we cool our sample to approximately
30 mK using a home-made wet dilution refrigerator build at the WMI [33]. As we work with signals on the single photon level in the resonator, which correspond to extremely low energies in the attowatt range, a complex combination of attenuation and amplification is used to ensure a good signal to noise ratio, schematically shown in Fig. 3.2. For our experiments, we use two rf-input and one rf-output line of this cryostat. The cables used to transmit signals to the sample are heavily attenuated to remove thermal noise from the signals generated at room temperature. One of these lines, the so-called input line, is used for Planck spectroscopy in Sec. 4.1.1. The other input line, the so-called antenna line is used for the ac-Stark-shift measurements presented in Sec. 4.1.2. We can switch between both configurations using a five-port switch (Agilent N1812UL) at the sample stage, shown by different circuits A and B in Fig. 3.2. To generate the thermal states for the Planck-spectroscopy, we equip a 30 dB attenuator at the sample stage with a 100 Ω heater and a thermometer. This device is visualized as the orange attenuator in Fig. 3.2. We have found out during the experiments that the connection between the switch and the input line of the sample is broken. Therefore, we use the antenna to probe experiments with the resonator.

To perform the ac-Stark-shift calibration, we use a tunable gradiometric flux qubit in a half-wavelength resonator. This sample was fabricated and characterized by M. Schwarz in Ref. [23]. This sample is filtered on the input and output line with a low-pass (Stycast) and a band-pass (Mini-Circuits VBFZ 5500+) filter at the 30 mK stage. In the output line, we use cryogenic circulators at the sample stage (Pamtech) and at the dilution niveau (Raditek) to suppress the backflow of amplifier noise to our sample. At 4 K, a high electron mobility transistor (HEMT) amplifier (Low Noise Factory LNC 4-8A) with a gain of 40 dB is used to amplify the outgoing signal. The noise temperature of the amplifier is expected to be approximately 2 K.

### 3.1.3 Detection setup

After leaving the cryostat, the signal passes a band-pass filter (Mini-Circuits VBFZ 5500+) and an amplifier (AMT-A0019) with a gain of 25 dB at room temperature. In addition, a power divider (MCLI PS2-11) has been interposed with one end leading to the ADCs and one to a VNA, in order to allow parallel experiments using the VNA, while the setup with the ADCs has been under construction. The setup is shown in Fig. 3.1 (b).

The signals $U(t) = A(t) \exp(i \omega_{RF} t + \phi(t)) = I(t) \cos(\omega_{RF} t) + Q(t) \sin(\omega_{RF} t)$ [30] pass
Figure 3.2: Cryogenic setup. Shown is a sketch of the main devices interposed in the transmission line. The circuit symbols are labeled in Fig. 3.1. The orange attenuator is used as a black body emitter to generate thermal states. Right: Photograph of the fridge.
### Table 3.1: Estimated losses of devices in amplified output line before being detected in the ADCs.

![Table with device names and their corresponding losses](image)

We estimate the overall gain in the output line before the signal enters the ADCs. We use three amplifiers with $G_{\text{HEMT}} = 40\, \text{dB}$, $G_{\text{AMT-A0019}} = 25\, \text{dB}$ and $G_{\text{AU 1525}} = 65.5\, \text{dB}$.

### 3.1.4 Gain and losses in output line

We estimate the overall gain in the output line before the signal enters the ADCs. We use three amplifiers with $G_{\text{HEMT}} = 40\, \text{dB}$, $G_{\text{AMT-A0019}} = 25\, \text{dB}$ and $G_{\text{AU 1525}} = 65.5\, \text{dB}$.
The losses for the detection chain in the cryostat and at room temperature are shown in Tab. 3.1. The values are taken from data sheets or estimated based on measurements. Altogether, we have a total estimated loss of $L_{\text{tot}} = -31.1 \, \text{dB}$, and, thus, we have a total gain in the output line of $G_{\text{out}} = 99.4 \, \text{dB}$.

### 3.2 Software

In the previous sections, we have described the physical setup to produce and measure quantum states. In this section, we show the conversion from the incoming IF signals to dc moments suitable for state reconstruction. For this, we have to do several additional tasks in post-processing:

1. We transfer the data from the ADCs to the evaluation software.

2. We calculate the moments $I_{\text{IF}}^2$, $Q_{\text{IF}}^2$, $I_{\text{IF}} Q_{\text{IF}}$, ... of $I_{\text{IF}}$ and $Q_{\text{IF}}$.

3. We average the data to minimize statistical errors.

4. We demodulate the sinusoidal signals to get quasi dc amplitudes.

These tasks will be discussed in detail in the following section.

#### 3.2.1 Implementation of software tasks

We now discuss the different tasks of the post-processing sequence. The processing scheme is shown in Fig. 3.3. The main difference between the setup for steady state measurements used by Menzel et al. [2], and our version is the stage, where the signals are demodulated. While Menzel et al. used already demodulated data to calculate the moments, we use the IF data to calculate the moments and then demodulate. This change makes our approach more compatible with the current time domain setup, but has effects on the application and the implementation.

1. **Data transfer** We use two analog-digital-converters from an 8-channel X6-250M measurement card by Innovative Integration to detect $I_{\text{IF}}$ and $Q_{\text{IF}}$. These 14-bit ADCs sample with 250 MSPS. The data acquisition is triggered with 100 kHz. This means that data recording, transfer, as well as moment calculation and averaging as explained in Step 2 and 3, are done in a 10 µs time frame. The ADCs typically record 512 data points per frame, corresponding...
3.2 Software

Figure 3.3: Flowchart of the software procedure. After the data retrieval from the ADCs, the Snap-tool is responsible for calculation and averaging moments. After that, LabVIEW is responsible for filtering and demodulating the data for final processing. The numbers are the step numbers used in the main text.

to a time span of 2.048 µs. This data block together with a header including channel information is sent to the control computer via a one-lane PCIe 2.0 connection. Using this protocol, we reach streaming rates of 240 MByte s⁻¹, which is approximately half the maximum rate of this PCIe connection. On the control computer, we use a C++-coded software, the so-called Snap-tool. This program acts as a communication bridge between ADCs and the measurement computer.

2. Calculation of moments During this work, we modify the existing Snap-tool to implement the calculation of moments up to the fourth order. When a data package with 512 data points for each $I_{IF}$ and $Q_{IF}$ arrives at the control computer, we read out the header. We extract the information, whether the data is valid and assign a channel number to $I_{IF}$ and $Q_{IF}$. Then, the data packages are stored in a temporary buffer, before we calculate the moments for the valid data via cross-multiplying the value of $I_{IF}$ and $Q_{IF}$ with each other for each data point separately. To ensure sufficient speed, our program uses multiple CPU cores in parallel.

3. Averaging A single shot data trace typically consists of noise with an amplitude several times higher than the encoded signal, see Sec. 4.1.1.2. For the first moments $I_{IF}$ and $Q_{IF}$, the noise is uncorrelated, thus we can retrieve the signal by simple averaging, as the mean value of the noise is zero. For the square
moments $I_{IF}^2$ and $Q_{IF}^2$ however, the averaged noise does not vanish and we need a calibration measurement to retrieve the signal, see Sec. 3.2.2.

The averaging is implemented as follows: We store the cross-multiplicated moments in a buffer array, each of the 512 data points separately. For each incoming data frame, we add the values to the existing ones in the buffer, until the desired number of averages is reached, which is normally one million. Finally, we divide each value in the buffer by the number of averages. This data buffer is then sent via a TCP/IP-based server functionality to the measurement computer, which performs the demodulation. The server handles 32-bit integers. As this data type is not capable of storing the large values of the third and fourth moments for many averages, we implement a scaling factor.

4. Demodulation and optimizing The LabVIEW measurement program, which also controls the measurement devices of the setup, receives the averaged data from the Snap-tool. The incoming signals are of the form

\[
I_{IF}(t) = I(t) \cos(\omega_{IF} t + \phi(t)) + \delta I
\]
\[
Q_{IF}(t) = Q(t) \sin(\omega_{IF} t + \phi(t) + \gamma) + \delta Q, \tag{3.1}
\]

where $I_{IF}(t)$ and $Q_{IF}(t)$ are the detected signals, $I(t)$ and $Q(t)$ the time dependent amplitudes and $\omega_{IF} = \omega_{RF} - \omega_{LO}$ is the down converted intermediate frequency. Furthermore, $\phi(t)$ is a global phase, while $\gamma$ is the deviation of 90° between $I_{IF}$ and $Q_{IF}$ due to the imperfect I-Q-mixer. The values $\delta I$ and $\delta Q$ are dc-offsets resulting from an imperfect calibration of the ADCs. We demodulate the incoming signals $I_{IF}$ and $Q_{IF}$ to retrieve physical information from the incoming signal. Therefore, we calculate the amplitude $A(t) = \sqrt{I(t)^2 + Q(t)^2}$ and phase $\phi(t) = \arctan(Q(t)/I(t))$. Because the signals are of the form in Eq. (3.1) and Eq. (3.2), we subtract the dc-offsets $\delta I$ and $\delta Q$ and find for the phase $\phi(t)$

\[
\phi(t) = \arctan \left[ \frac{Q_{IF}/Q_{IF}(t)}{I_{IF}/I_{IF}(t)} - \sin(\gamma) \right] - \omega_{IF} t - \gamma \tag{3.3}
\]
and, using this information, for the amplitudes $I(t)$ and $Q(t)$

$$
\frac{I(t)}{\langle I \rangle} = \frac{I_{\text{IF}}(t)}{\cos(\omega_{\text{IF}} + \phi(t))} \quad (3.4)
$$

$$
\frac{Q(t)}{\langle Q \rangle} = \frac{Q_{\text{IF}}(t)}{\cos(\omega_{\text{IF}} + \phi(t) + \gamma)} \quad . \quad (3.5)
$$

Here, $\langle I \rangle$ and $\langle Q \rangle$ are the mean amplitudes. Both values as well as $\gamma$ are calculated by fitting a sine and a cosine to the incoming signals $I_{\text{IF}}$ and $Q_{\text{IF}}$ and afterwards optimizing these parameters for a minimized variance in the demodulated signals. Thus, we can extract the information for the first moment.

Ideally, the second moments are of the form $I(t)^2 \sin^2(\omega_{\text{IF}} t)$, $Q(t)^2 \cos^2(\omega_{\text{IF}} t)$ and $I(t)Q(t) \sin(\omega_{\text{IF}} t) \cos(\omega_{\text{IF}} t)$, which we call pure terms. However, with our setup, we calculate the square of Eq. (3.1) and Eq. (3.2) and their cross product. This introduces mixed terms such as $2 \delta I \cdot I \sin(\omega_{\text{IF}} t)$, which have to be subtracted from the calculated square terms. We calculate the pure terms to

$$
I_p^2 = I(t)^2 \sin^2(\omega_{\text{IF}} t)
$$

$$
= I_{\text{IF}}(t)^2 - 2 I(t) \delta I \cos(x) + \delta I^2 \quad (3.6)
$$

$$
Q_p^2 = Q(t)^2 \cos^2(\omega_{\text{IF}} t)
$$

$$
= Q_{\text{IF}}(t)^2 - 2 \delta Q^2 - Q(t)^2
$$

$$
- 4 Q(t) \delta Q \{ \sin(x) \cos(\phi) + \cos(x) \sin(\phi) \}
$$

$$
- 4 Q(t)^2 \sin(x) \cos(x) \sin(\phi) \cos(\phi)
$$

$$
+ Q(t)^2 \cos^2(x) \{ \cos^2(\phi) - \sin^2(\phi) \} \}
$$

$$
/ [\cos^2(\phi) - \sin^2(\phi)] \quad (3.7)
$$

$$
(IQ)_p = I(t)Q(t) \sin(\omega_{\text{IF}} t) \cos(\omega_{\text{IF}} t)
$$

$$
= 2 I_{\text{IF}}(t) Q_{\text{IF}}(t) \sec(\phi)
$$

$$
- 2 \delta I \delta Q \sec(\phi) - 2 I(t) \delta Q \cos(x) \sec(\phi)
$$

$$
- 2 Q(t) \delta I \{ \sin(x) + \cos(x) \tan(\phi) \}
$$

$$
- I(t) Q(t) \tan(\phi) [1 - \sin^2(x) + \cos^2(x)] \} . \quad (3.8)
$$

In the LabVIEW code, we first subtract a constant offset generated by the amplifier noise. We define this offset as the vacuum or thermal noise used in the Planck spectroscopy. Then, we demodulate the second moments using Eq. (3.6), Eq. (3.7), and Eq. (3.8). In order to do this in a consistent way, we
use $\gamma$, $\phi(t)$, $I(t)$, $Q(t)$, $\delta I$ and $\delta Q$ evaluated from the first moments. The demodulation of higher moments requires different formulas. Therefore, we show in the App. A the theoretical calculations for demodulating moments of third order.

For our modified setup and analysis, proof-of-principle experiments are an important benchmark. By measuring thermal and coherent states and reconstructing the Wigner function, we show that our setup is working. These measurements are discussed in Chap. 4.

### 3.2.2 Structure of readout time trace

In this section, we discuss, how the physical information describing a quantum state can be extracted from a data trace. We make use of the fact that we can switch the readout and preparation pulses on and off on a ns-timescale. This allows us to split one data trace into several parts, as depicted in Fig. 3.4.

As explained in the previous section, the data acquisition is triggered every 10 $\mu$s to record the data points for approximately 2 $\mu$s. Part A: The first 0.5 $\mu$s of a trace, we apply strong coherent readout tone. We use this data to calculate the two values necessary for demodulation, the deviation $\gamma = \Phi_I - \Phi_Q + \pi/2$ in phase, as well as the difference in gain $\Delta G$ for $I$ and $Q$ amplification lines. Part B: The next 0.5 $\mu$s, we do not apply any signal and assume that we detect a vacuum state or thermal state. This is important to calculate the noise contribution of our amplification chain to the higher moments. Part C: During the last 1 $\mu$s, we apply the signal to generate the actual quantum state, which we want to analyze.

### 3.2.3 Discussion of our setup

In this section, we discuss advantages and disadvantages of the setup introduced in Sec 3.2.1. An advantage of our setup is the in-situ calibration. As we have seen in the previous section, our time trace structure allows us to calculate slight changes in gain and phase as well as the vacuum noise within each single data frame. Another advantage is the usability for the detection of changes of the quantum state over a large bandwidth up to approximately 100 MHz. This includes, for example, the analysis of the time evolution of quantum states. The limiting factor is the speed of the ADCs used for data acquisition, which is limited to 250 MHz. Therefore, according to the Nyquist-Shannon sampling theorem [34], the maximum frequency of the analyzed signal is 125 MHz.
So far, we have implemented the calculation of moments up to the fourth order for a single channel measurement. This leads to a moderate decrease in speed of approximately 10% compared to the case, where no higher moments are calculated. Because the amount of calculations grows rapidly with higher ordered moments and more channels, the speed of the data acquisition puts a limit to this method. Another negative aspect of our method is the complexity of the demodulation software for higher moments. So far, we are able to implement the demodulation of moments up to the second order, which is sufficient for the reconstruction of Gaussian states.

In this chapter, we have described the experimental setup developed for the purpose of quantum state reconstruction and discussed advantages and disadvantages. In the following chapter, we demonstrate the functionality of this setup by conducting proof-of-principle experiments.

Figure 3.4: Sketch of time trace for in-situ calibration. The applied signal as well as the measured value and the purpose of this part of the time trace is shown for each time segment respectively.
Chapter 4

Results

In this chapter, we benchmark the setup introduced in the previous section by analyzing its response to known states of light. To this end, we generate thermal and coherent states and measure the corresponding moments with our setup. From the information gathered in this way, we are able to calibrate relation between number of photons and detected voltage, the so-called photon number conversion factor (PNCF). In Sec. 4.1, we use two independent approaches to determine the PNCF, while in Sec. 4.2, we reconstruct the Wigner function of several Gaussian states.

4.1 Photon number calibration

The number of photons populating a state of light is an important quantity in circuit QED. Here, we compare two different situations. In Sec. 4.1.1, we use the Planck spectrum of a black body emitter to calibrate the photon number of propagating microwave signals in an open-line [2, 32]. Then, in Sec. 4.1.2, we use the ac-Stark-shift of a superconducting qubit coupled to a CPW resonator to determine the population of the resonant structure [28, 29].

4.1.1 Calibration via Planck spectroscopy

For microwave signals propagating in an open transmission line, one can perform a so-called Planck spectroscopy experiment based on a broadband microwave black-body radiation to calibrate the photon number [2, 32]. In this approach, we take advantage of the fact that the output power of a thermal state is related to its temperature.
4.1 Photon number calibration

Figure 4.1: Detected power as a function of attenuator temperature $T$. The red line is a fit of Eq. (4.1) to the data. The analyzed frequency is (a) $\omega_1/2\pi = 6.986 \text{ GHz}$ and (b) $\omega_2/2\pi = 6.972 \text{ GHz}$.

4.1.1.1 Temperature sweeps

We use the experimental setup introduced in Sec. 3.1 and the time trace protocol explained in Sec. 3.2.2 to detect the power during part B in Fig. 3.4, where no signal is applied. As broadband microwave blackbody emitter, we use a heatable 30 dB-attenuator in the transmission line, which is thermally weakly coupled to the base plate of the fridge. That way, we can generate thermal states and measure the corresponding output power. During our measurements, we stabilize the temperature of the base plate to $T_{\text{Base}} = (50.0 \pm 0.1) \text{ mK}$. In our specific setup, the minimal temperature of the emitter is approximately 70 mK. To perform the calibration via Planck-spectroscopy, we vary the temperature of the attenuator between 70 mK and 250 mK with a step size of 1 mK and an accuracy of $\pm 0.25 \text{ mK}$, while the other components included in the transmission line remain stable at their temperature. We measure the corresponding output power for each temperature step. We perform two independent temperature sweeps with slightly different probe tone frequencies $\omega_1/2\pi = 6.986 \text{ GHz}$ and $\omega_2/2\pi = 6.972 \text{ GHz}$. Fig. 4.1 shows the detected power $P_{\text{det}}$ as a function of attenuator temperature $T$.

For thermal photons, we expect the detected power to follow Eq. (2.16). Besides the Planck-distributed thermal voltage fluctuations [19], we also have to consider vacuum fluctuations $\hbar \omega/2$. However, the use of amplifiers in the transmission line introduces an amplification factor $G$ as well as additional $T$-independent fluctuations.
4.1 Photon number calibration

The detected power at the output of the amplification chain is given by

\[
P_{\text{det}} = \frac{\langle I^2 \rangle + \langle Q^2 \rangle}{R} = 4GB \left( \frac{\hbar \omega}{\exp \left( \frac{\hbar \omega}{k_B(T + \delta T)} \right) - 1} + \frac{\hbar \omega}{2} + k_B T_{\text{Noise}} \right) 
= 4GB \left( \frac{\hbar \omega}{2} \coth \left( \frac{\hbar \omega}{2k_B(T + \delta T)} \right) + k_B T_{\text{Noise}} \right). \tag{4.1}
\]

Here, the bandwidth \( B = 100 \text{ MHz} \) is determined by the bandwidth of the ADCs, \( R = 50 \Omega \) is the input resistance of the ADCs. With \( \delta T \), we take a possible difference between the temperature \( T \) of the attenuator and the effective electron temperature in the transmission line into account, and \( \omega \) is the frequency, at which we analyze the thermal distribution.

4.1.1.2 Calibration results using Planck spectroscopy

We fit Eq. (4.1) to the data shown in Fig. 4.1, resulting in values given in Tab. 4.1. For the frequencies \( \omega_1 \) and \( \omega_2 \), we find a total gain of the amplification chain, \( G_1 = 97.12 \text{ dB} \) and \( G_2 = 98.17 \text{ dB} \), respectively. These values are in very good agreement with the estimation done in \( G_{\text{theo}} = 99.4 \text{ dB} \) done in Sec. 3.1.4. The small difference of approximately 1 dB between both measurements arises from external influences, especially the room temperature. The room temperature during the first measurement was higher than during the second, resulting in a temperature difference of approximately 1 K between the IF-amplifiers. The amplification of this device is temperature dependent with a difference of approximately 0.5 dB K\(^{-1}\). We attribute the remaining difference to a small temperature dependence of the gains and losses of the other components of the room temperature receiver.

For both measurements, we find that \( \delta T \) is approximately 30% of the absolute temperature and negative, which means, the actual attenuator has a lower temperature than the thermometer. A possible explanation is the geometry of the clamp used to fix attenuator, heater and thermometer. On this clamp, the heater is nearer to

<table>
<thead>
<tr>
<th>( G ) (dB)</th>
<th>( T_{\text{Noise}} ) (K)</th>
<th>( \delta T ) (mK)</th>
<th>( \mu \text{W} )</th>
<th>( \text{nW} ) (( \mu \text{W}^{-1} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>97.12</td>
<td>2.75</td>
<td>-25.54</td>
<td>9.54</td>
</tr>
<tr>
<td>2</td>
<td>98.17</td>
<td>2.35</td>
<td>-34.92</td>
<td>12.13</td>
</tr>
</tbody>
</table>

Table 4.1: Fitting values of calibration via Planck spectroscopy, following Eq. (4.1) for two different measurements.
the thermometer than the attenuator and thus could heat the thermometer more than the attenuator. We also tried fitting without considering $\delta T$ in Eq. (4.1), which leads to a decrease in gain of approximately 1.5 dB as well as an increase in noise temperature of approximately 0.6 K. A quantitative evaluation of this term is part of contemporary and future discussions, but is beyond the scope of this work.

The fits also reveal the noise temperature $T_{\text{Noise},1} = 2.75 \text{ K}$ and $T_{\text{Noise},2} = 2.35 \text{ K}$, corresponding to a number of noise photons $N_{\text{Noise},1} = k_B T_{\text{Noise},1}/\hbar \omega = 8.20$ and $N_{\text{Noise},2} = 7.02$ added by the amplification chain. This noise temperature is very close to the one specified in the data sheet of the HEMT amplifier of $T_{\text{Noise, HEMT}} = 2 \text{ K}$ at a temperature $T_{\text{HEMT}} = 4 \text{ K}$. The fact that the measured noise temperature corresponds with the noise temperature of just the HEMT amplifier, shows that this device is mainly responsible for the noise of the setup. Due to the large amplification of 40 dB of the HEMT amplifier, the noise added by the room temperature amplifiers of approximately 70 K plays a minor role to the total noise.

The noise photons outnumber the photons of a thermal state. For this reason, we have to calibrate our data and subtract the vacuum level from the measured data, when we want to reconstruct a Wigner function. This can be done within every time trace, see Sec. 3.2.2. Therefore, the fit of the gain is important, as the gain is resulting from the slope of a temperature sweep and cannot be calibrated for every time trace in an actual reconstruction measurement.

Using the fit results $G_1$ and $G_2$, we calculate the so-called photon number conversion factor (PNCF) [2]

$$ p_{\text{therm}} = 4 G B \hbar \omega. \quad (4.2) $$

This figure relates the number of photons in the open transmission line to the detected power at the ADCs. For our data, the fit yields $p_{\text{therm},1} = 9.54 \mu W$ and $p_{\text{therm},2} = 12.13 \mu W$ per photon, which corresponds to $n_{\text{therm},1} = 1/p_{\text{therm},1} = 0.11 \mu W^{-1}$ and $n_{\text{therm},2} = 0.08 \mu W^{-1}$ photons per $\mu W$ detected power. The difference between $p_{\text{therm},1}$ and $p_{\text{therm},2}$ cannot be neglected. The fit used for the Planck spectroscopy includes many variables, due to which slight deviations are possible. In Fig. 4.1 (b), the data show some deviation from the fit at approximately 200 mK. This could have been caused by external disturbances, because the room temperature in the used laboratory is not stabilized.

For state reconstruction, it is necessary to calibrate the vacuum for $I^2$ and $Q^2$ separately. In Fig. 4.2, we show the separate data for both components of the measurement shown in Fig. 4.1 (b). Here, $R = 50 \Omega$ relates the measured voltage to measured
4.1 Photon number calibration

4.1.2 Calibration via ac-Stark-shift

In the previous section, we calibrate the photon number conversion factor in an open-line, where the emitted photons of the black body emitter are directly transmitted to the cold amplifiers. In contrast, we now determine the number of photons residing in a CPW resonator. For this, we use the dependence of the qubit transition frequency $\tilde{\omega}_q$ on the photon number $N = \hat{a}^\dagger \hat{a}$, as given in Eq. (2.19).

In the following, we calibrate the input power entering the cryostat in Sec. 4.1.2.1, describe the experimental method two-tone spectroscopy and analyze the qubit frequency in Sec. 4.1.2.2 and calculate the PNCF using these results in Sec. 4.1.2.3.

Figure 4.2: Detected power at $\omega_2/2\pi = 6.972$ GHz for (a) $I^2$ and (b) $Q^2$ as a function of the attenuator temperature. The red line is a fit of Eq. (4.1) to the data.

In the following section, we calculate the PNCF with another independent method in order to compare both approaches.
4.1 Photon number calibration

4.1.2.1 Power calibration

Precise knowledge of the power $P_{\text{in}}$ entering the cryostat is necessary to calculate the mean photon number $N$. Therefore, we start by calibrating the power entering the input line at the top of the cryostat. The need for this arises from the mixers in the input line, which have a non-linear characteristic, and the microwave source, which is not perfectly leveled. In order to do this, we mount a powermeter to the microwave cable entering the cryostat and measure the power $P_{\text{in}}$ as a function of the microwave source power $P_{\text{source}}$. The results are shown in Fig. 4.3 (a).

The data shows a linear characteristic up to $P_{\text{source}} = 6$ dBm. For higher powers, the mixers saturate. Thus, we relate to the calibrated power in further calculations. The difference in magnitude between $P_{\text{in}}$ and $P_{\text{source}}$ of approximately 40 dBm results from attenuators as well as the mixer and splitter at room temperature placed between the microwave source and the entrance of the cryostat.

4.1.2.2 Two-tone-spectroscopy

Two-tone-spectroscopy is an experimental technique to determine the qubit transition frequency based on the dispersive interaction of the qubit-resonator system. Here, the eigenfrequency of the coupled system in the dispersive limit changes de-

Figure 4.3: (a) Power calibration: Detected power at the entrance of the cryostat versus set readout power at the microwave source. (b) Two-tone-spectroscopy: Detected signal amplitude versus drive frequency exemplary for three selected source powers (black: 9 dBm, red: 3 dBm, green: $-3$ dBm). The dip reveals the qubit frequency $\tilde{\omega}_q$. 
pending on the qubit being excited or not. We can rewrite Eq. (2.19) to an effective resonator frequency \( \tilde{\omega}_{\text{res}} = \omega_{\text{res}} \pm g^2/\delta \), where the qubit is in the excited (+) or ground (−) state, respectively [5, 23].

In order to determine the qubit frequency, we use two different microwave frequencies. We apply a probe tone, which is tuned to match the eigenfrequency of the resonator with the qubit in the ground state. Additionally, we apply a drive tone. The drive tone is swept systematically while the transmission of the probe tone is detected. When we hit the qubit transition frequency with the drive tone, we see a decrease in transmission. Due to a continuous drive, the qubit state and thus the effective resonator frequency changes. Thus, the resonator frequency is detuned from the constant probe tone, which manifests as a dip in the transmission. The center of this dip reveals the effective qubit frequency \( \tilde{\omega}_q \). When we vary the readout power \( P_{\text{source}} \), we find the dip and thus \( \tilde{\omega}_q \) at different frequencies.

In Fig. 4.3 (b), we show data for three exemplary two-tone measurements with different probe powers. The qubit dip broadens, as we expect theoretically [28, 35]. There is a linear trend on the detected amplitude towards higher frequencies. This effect comes from the fact that the drive frequency approaches the eigenfrequency of the resonator, which is around \( \omega_{\text{res}} = 6.79 \text{ GHz} \). Thus, we generate photons inside the resonator with the drive tone leading to higher detected amplitudes. This effect strengthens for higher power, thus increasing the magnitude difference.

We will now use the data from the two-tone-measurement to calibrate the photon number in the experimental setup.

### 4.1.2.3 Calibration results using ac-Stark-shift

The qubit transition frequency \( \tilde{\omega}_q \) changes with the photon number \( N = \hat{a}^\dagger \hat{a} \) in the resonator, see Eq. (2.19). Because the photon number is proportional to the input power, \( N \propto P_{\text{in}} \), we see a linear shift in \( \tilde{\omega}_q \) for varying input powers

\[
\tilde{\omega}_q = \tilde{\omega}_{q,0} + \frac{2g^2}{\delta} \hat{a}^\dagger \hat{a} = \tilde{\omega}_{q,0} + \alpha P_{\text{in}}.
\]  

(4.3)

This shift is called ac-Stark-shift [28, 29]. Here, \( \tilde{\omega}_{q,0} = \omega_q + g^2/(\omega_q - \omega_{\text{res}}) \) is the qubit frequency, when no power is applied, \( \alpha = \partial \tilde{\omega}_q / \partial P_{\text{in}} \) is a power to qubit frequency conversion factor, \( \delta \) is the detuning and \( g \) the coupling between resonator and qubit. Using these values, we can determine the power conversion \( n_{\text{ac,in}} = N/P_{\text{in}} \),
the number of photons per detected power as

\[ n_{\text{ac, in}} = \alpha \frac{\delta}{2g^2}. \] (4.4)

In the following, we use the data from two-tone-spectroscopy to calibrate the photon number inside the resonator. The qubit transition frequency \( \tilde{\omega}_q \) is given by the minimum of the dip in the transmission, shown in Fig. 4.3 (b). We show the qubit frequency as a function of readout power in Fig. 4.4 (a). The data points show a linear behavior, as expected from Eq. (4.3). Because the bare qubit frequency is lower than the resonator frequency, the qubit transition frequency decreases with higher power and the ac-Stark-shift is negative. Fitting Eq. (4.3) to the data points, we obtain a slope of \( \alpha = -41.2 \text{ GHz mW}^{-1} \) and an offset of \( \tilde{\omega}_q,0/2\pi = 6.468 \text{ GHz} \).

Following Eq. (4.4), we calculate the photon number conversion factor \( n_{\text{ac, in}} \), the number of photons per detected power. The coupling constant \( g = 65 \text{ MHz} \) is determined by a measurement of the vacuum Rabi mode splitting [21, 23]. In order to calculate the detuning \( \delta = \omega_q - \omega_{\text{res}} \), we determine the bare qubit frequency

\[ \omega_q = \frac{1}{2}\left(\omega_{\text{res}} + \tilde{\omega}_q,0 - \sqrt{(\tilde{\omega}_q,0 - \omega_{\text{res}})^2 - 4g^2}\right). \] (4.5)

The resonator frequency at the chosen working point is \( \omega_{\text{res}}/2\pi = 6.976 \text{ GHz} \). Inserting \( \tilde{\omega}_q,0/2\pi = 6.468 \text{ GHz} \), we retrieve \( \omega_q/2\pi = 6.477 \text{ GHz} \). Using Eq. (4.4), we find a photon number conversion factor per input power of \( n_{\text{ac, in}} = 2432 \mu\text{W}^{-1} \) (Fig. 4.4 (a)).

So far, we have determined the relation \( n_{\text{ac, in}} \) between photon number and the power \( P_{\text{in}} \) entering the cryostat. To determine the relation \( n_{\text{ac, det}} \) between the intraresonator photons and the detected power \( P_{\text{det}} \), we need a conversion from detected to input power \( \alpha_{\text{det, in}} = P_{\text{det}}/P_{\text{in}} \), so that

\[ n_{\text{ac, det}} = n_{\text{ac, in}}/\alpha_{\text{det, in}}. \] (4.6)

We determine \( \alpha_{\text{det, in}} \) by averaging the amplitude \( A \) of the two-tone-spectroscopy over the whole range of the drive frequency, when the drive tone is off. The power is given by \( P_{\text{det}} = A^2/50 \Omega \), where \( A = \sqrt{T^2 + Q^2} \) is the amplitude of the detected signal at the center frequency of the ac-Stark-shifted resonator. The detected power \( P_{\text{det}} \) as a function of the input power \( P_{\text{in}} \) is shown in Fig. 4.4,b. The data points show a linear behavior up to \( P_{\text{in}} = 0.2 \mu\text{W} \). For higher powers, the IF amplifier go into compression, thus the flattening out of the function. Therefore, we exclude
4.1 Photon number calibration

Figure 4.4: (a) Qubit frequency versus drive powers from 10 dBm to −3 dBm.Calibrated power after −40 dB attenuators. Red line: linear fit. Also shown is the photon number. (b) Detected power versus input power to determine the conversion factor $\alpha_{\text{det,in}}$. Red line: linear fit.

those data points from the fit. The linear fit shows a slope of $\alpha_{\text{det,in}} = 65.76$ and a y-offset of approximately 0.2 mV. This offset is reasonable, as it is on the order of the noise of the ADCs.

We now calculate the power conversion factor, photons per detected power with Eq. (4.6), yielding $n_{\text{ac,det}} = 36.99 \mu W^{-1}$. This corresponds to a photon number conversion factor of $p_{\text{ac,det}} = 0.027 \mu W$.

4.1.3 Comparison

In this section, we compare the two calibrations for the power conversion factor $n$, the photon number per detected power, which have been introduced in Sec. 4.1.1 (a) and Sec. 4.1.2. The average value of the thermal calibrations is $n_{\text{therm}} \simeq 0.1 \mu W^{-1}$ and for the ac-Stark-shift calibration measurement $n_{\text{ac,det}} \simeq 37 \mu W^{-1}$, thus the ratio between both is approximately 25.7 dB. In order to understand this ratio, we have to consider the so-called quality factors of the resonator. In general, the transmission through a resonator is described by the internal $Q_i$ and the external quality factor $Q_x$ [36]. The first one describes the ratio, at which the photons within the resonator decay into the environment. The second factor describes the ratio, at which a photon in the resonator is transmitted to the transmission line and can be measured. Due to this mechanisms, not all photons in the resonator, are measured at the output,
resulting in the so-called insertion loss [36]. For our resonator, we find \( Q_I = 1420 \) and \( Q_X = 13000 \), thus the insertion loss can be calculated to be \( IL = 21.3 \text{ dB} \). This means, there is a difference of 4 dB between the relation of the conversion factors and the insertion loss. To explain this difference, one has to consider the uncertainty of the insertion loss estimation, which is typically in the order of several dB. Another explanation could be that as we probe the resonator via the antenna, we do not have forward transmission and might lose photons to the input capacity of the resonator.

We show in this section that our experimental setup is capable of two different calibrations. Both calibrations allow the link of detected power to single photons, which is essentially for state reconstruction. Differences between both methods can be explained by deviations in room temperature and loss estimation. Thus, both techniques are in the limits of these deviations, feasible for calibration. After calibrating our system, we prove in the following section that our experimental setup is capable of reconstructing Wigner functions.

### 4.2 Wigner function reconstruction of thermal and coherent states

In this section, we show proof of principle experiments, where we reconstruct the Wigner function for simple Gaussian states. We use the experimental setup described in Chap. 3 to generate quantum states and measure the first and second moments in order to reconstruct the Wigner function. In Sec. 4.2.1, we use the calibration via Planck spectroscopy to reconstruct thermal states and in Sec. 4.2.2 the calibration via ac-Stark-shift for coherent states. As still some improvements to the setup and further data analysis have to be done, only qualitative results are shown in this section.

#### 4.2.1 Reconstruction of thermal states

In this section, we show the reconstruction of Wigner functions for two thermal states. We select the second temperature sweep introduced in Sec. 4.1.1 and reconstruct the Wigner function for two different temperature states of the attenuator, 70 mK and 206 mK. We start by calculating the photon number conversion factor \( n_{\text{therm}} \) for both \( I \) and \( Q \) separately, using the Planck spectroscopy in Sec. 4.1.1.2. To reconstruct thermal states, we isolate the thermal contribution of the measured
4.2 Wigner function reconstruction of thermal and coherent states

Figure 4.5: (a) Reconstructed Wigner function of 70 mK thermal state. (b) \(1/e\)-contour for reconstructed 70 mK (blue circle) and 206 mK (red circle) thermal states as well as for the ideal vacuum (black circle).

second moments. Therefore, we subtract the amplifier noise, which we also obtain from the Planck spectroscopy. Using the photon number conversion factor to relate all moments to photons, we are able to reconstruct the Wigner function for both thermal states. In Fig. 4.5 (a), we exemplary show the reconstructed Wigner function of the 70 mK thermal state. To visualize the expected broadening of the Wigner function for higher temperatures, we show the \(1/e\)-contour for 70 mK and 206 mK and for comparison also the contour of the ideal vacuum, in Fig. 4.5 (b).

4.2.2 Reconstruction of coherent states

After reconstructing thermal states in the previous section, we show the Wigner function of coherent states in this section. The first coherent state has been generated during the second temperature sweep measurement in Sec. 4.1.1.1 by applying a coherent signal in part C of the time trace, introduced in Sec. 3.2.2. We analyze the coherent state generated at 70 mK. Therefore, we calibrate the second moments by subtracting the noise level of the amplifier, which we measured during part B of the time trace. The remaining signal is then demodulated as introduced in Sec. 3.2. The reconstructed Wigner function is shown in Fig. 4.6 (a). The center of the sphere is displaced compared to a thermal state, which is characteristic for a coherent state, see Sec. 2.3.3. When we compare the coherent state to the thermal state 70 mK in the previous section, we see that the \(1/e\)-contour of the coherent state is approxi-
4.2 Wigner function reconstruction of thermal and coherent states

Figure 4.6: Wigner function reconstruction: (a) Coherent state in open line at 70 mK. (b) Coherent state inside a resonator.

mately 15% narrower than the thermal state. For an ideal setup, we would expect that the shape of the Wigner function and thus the radius of the contour is identical for both states. A plausible explanation for the deviation when considering a coherent state, is the demodulation. In order to demodulate thermal states, we have to separate the thermal fluctuations caused by the thermal state from white noise. Because the thermal fluctuations in our analyzed bandwidth act similar to white noise, the technical realization is complex. Whereas for a coherent state, we have to separate a coherent signal with known frequency from constant white noise, which is easier to achieve technically. Quantitative analysis and improvement of the setup considering this difference is a goal of future works.

In Fig. 4.6 (b), we see the Wigner function of a coherent state inside a resonator. We apply a resonant probe tone to the sample, thus exciting photons inside the resonator. We see that the center of the Wigner function is displaced stronger than the coherent state shown in Fig. 4.6 (a). This indicates that we generate more photons inside the resonator than in the open line.

In this chapter, we have performed two independent calibration techniques to determine technical properties of our experimental setup, such as gain, amplifier noise or the photon number conversion factor. Using those, we are able to reconstruct the Wigner function for both thermal and coherent states. That way, we show qualitatively, that our method is feasible for state reconstruction in future experiments.
Chapter 5

Conclusion and outlook

In this work, we successfully develop a measurement setup capable of time-resolved reconstruction of quantum states.
To fulfill this purpose, we modify an existing experimental setup used for time-domain measurements of qubit-resonator structures. The main modification is done in the software part. Here, we implement the calculation of quadrature moments up to the fourth order without suffering from significant speed losses. Additionally, we include the demodulation of moments up to the second order, which is sufficient for the reconstruction of Gaussian states. By performing the calculation step in data processing before demodulating the signals, we are, in future experiments, able to analyze time-evolution of these quantum states.
There are several prerequisites for demodulation of sinusoidal signals, e.g. the phase deviation $\gamma$ between $I_{IF}$ and $Q_{IF}$. Therefore, we measure with a triggered time trace separated into three independent parts. Each section serves the purpose of providing different information necessary for demodulation. Also essential is the photon number conversion factor (PNCF), relating single photons to measured signals. We calibrate the PNCF using two different techniques. First, we perform Planck spectroscopy experiments, where we analyze the photon number inside an open-line. We then compare these measurements to a calibration via the ac-Stark-shift, which is generated by a flux qubit coupled to a coplanar waveguide resonator. Considering insertion loss of the resonator, we find good agreement between both methods. Using the information provided by the structure of the time trace as well as the photon number conversion factor, we are able to reconstruct the Wigner function of microwave states.
We find, that our setup is capable of measuring and demodulating Gaussian states. However, experiments for quantitative data analysis and further improvements of our setup are part of future work. In order to reconstruct certain quantum states such
as Fock states, higher ordered moments are required. Also, showing that measured states are really Gaussian, requires moments of at least fourth order. Therefore, further developments of this setup should include the demodulation of such higher moments. Anyways, future experiments could proceed in testing the setup by measuring and reconstructing squeezed states. Or, when the demodulation of higher moments is implemented, quantum states such as Fock states are an interesting object of study. And finally, because of its large Bandwidth up to 100 MHz, our setup can be tested with its original purpose, to reconstruct and characterize time-evolving quantum states.
Appendix A

Demodulation of third order moments

We have introduced the demodulation of first and second order moments in Sec. 3.2.1. However, there are several reasons to demodulate higher moments. For example, for a Gaussian state, the third and fourth moments should vanish. Therefore, we can prove, that a state is Gaussian by analyzing the higher ordered moments.

In this section, we now introduce the demodulation of third moments. To this end, we follow the idea of demodulating second moments. We have to extract the pure terms $I_p^3$, $(I^2 Q)_p$, $(IQ^2)_p$ and $Q_p^3$ from the corresponding multiplication of the incoming signals $I_{IF}$ in Eq. 3.1 and $Q_{IF}$ in Eq. 3.2. We follow the idea of demodulating second moments:

$$I_{IF}^3(t) = I(t)^3 \cos^3(x) + 3I(t)^2 \cos^2(x) \delta I$$
$$+ 3I(t) \cos(x) \delta I^2 + \delta I^3.$$  \hspace{1cm} (A.1)

Here, $x = \omega_{IF} t + \phi(t)$. We can solve this equation for the pure term

$$I_p^3(t) = I(t)^3 \cos^3(x)$$
$$= I_{IF}^3(t) - 3I(t)^2 \cos^2(x) \delta I$$
$$- 3I(t) \cos(x) \delta I^2 - \delta I^3.$$  \hspace{1cm} (A.2)
We follow this idea for the next moments of third order. For $I(t)^2Q$

$$(I^2Q)_{IF}(t) = I(t)^2\delta Q \cos^2(x)$$

$$+ 2I(t)\delta I \cos(x) (\delta Q + Q(t) \sin(\phi) \cos(x) + Q(t) \cos(\phi) \sin(x))$$

$$+ \delta I^2 (\delta Q + Q(t) \cos(x) \sin(\phi) + Q(t) \cos(\phi) \sin(x))$$

$$+ I(t)^2Q(t) (\cos^3(x) \sin(\phi) + \cos^2(x) \sin(x) \cos(\phi)) \quad , \quad (A.3)$$

we get the solution:

$$\quad (I^2Q)_p(t) = I(t)^2Q(t)(t) \cos^2(x) \sin(x)$$

$$= \{ (I^2Q)_{IF}(t) - I(t)^2\delta Q \cos^2(x)$$

$$- 2I(t)\delta I \cos(x) (\delta Q + Q(t) \sin(\phi) \cos(x) + Q(t) \cos(\phi) \sin(x))$$

$$- \delta I^2 (\delta Q + Q(t) \cos(x) \sin(\phi) + Q(t) \cos(\phi) \sin(x)) \} /$$

$$\quad (\cot(x) \sin(\phi) + \cos(\phi)) \quad . \quad (A.4)$$

For $IQ^2(t)$, we find

$$\quad (IQ)^2_p(t) = I(t)Q(t)(t) \cos(x) \sin^2(x)$$

$$= \{ (IQ)^2_{IF}(t)$$

$$- Q(t)^2\delta I (\cos^2(\phi) \sin^2(x) + \sin^2(\phi) \cos^2(x)$$

$$+ 2 \cos(\phi) \sin(\phi) \cos(x) \sin(x))$$

$$- 2Q(t)\delta Q(\delta I + I(t) \cos(x))(\cos(x) \sin(\phi) + \cos(\phi) \sin(x))$$

$$- \delta Q^2(\delta I + I(t) \cos(x)) \} /$$

$$\quad (\cos^2(\phi) + \sin^2(\phi) \cot^2(x) + 2 \cot(x) \sin(\phi) \cos(\phi)) \quad . \quad (A.5)$$

And for $Q^3(t)$, the demodulation is achieved using

$$\quad Q^3_p(t) = Q^3(t) \sin^3(x)$$

$$= \{ Q^3_{IF}(t)$$

$$- 3Q(t)\delta Q^2 (\cos(x) \sin(\phi) + \cos(\phi) \sin(x))$$

$$- 3Q(t)^2\delta Q (\cos(x) \sin(\phi) + \cos(\phi) \sin(x))^2 - \delta Q^3 \} /$$

$$\quad (\sin^3(\phi) \cot^3(x) + 3 \cot^2(x) \cos(\phi) \sin^2(\phi)$$

$$+ 3 \cot(x) \sin(\phi) \cos^2(\phi) + \cos^3(\phi)) \quad \quad (A.6)$$
Bibliography


Danksagung

An dieser Stelle möchte ich mich bei allen Menschen bedanken, die irgendwie zu dieser Bachelor-Arbeit beigetragen haben.


Ebenfalls danke ich Dr. Gönnewein, der mich durch seine überragende Vorlesung der Kondensierten Materie davon überzeugt hat, Festkörperphysiker zu werden.


Ebenfalls bedanken möchte ich mich bei allen anderen am WMI, die ich jetzt nicht namentlich nennen kann, für eine gute Arbeitsatmosphäre. Besonders alle Bacheloranden, Masteranden und Doktoranden, wie Stefan, Peter, Max, Friedrich, Michael... seien hier erwähnt, da sie immer die passende Ablenkung parat haben, wenn man sie gerade braucht.

Vielen Dank an euch alle und alle, die ich vergessen habe. Ohne euch wäre diese Bachelorarbeit nicht zustande gekommen.