Probing Quantum States of Josephson Junctions with Ferromagnetic Barriers by Microwaves

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Introduction

At the beginning of the 20th century, Max Planck studied the distribution function of black body radiation, which could not be explained by the classical assumption of a continuous energy distribution. In order to explain the distribution function, he proposed discrete energy quanta. Other physicists, like Einstein, de Broglie or Schrödinger, used this quantum hypothesis as fundamental law and opened a new field of physics, the quantum physics. They microscopically explained the photo effect and discussed new concepts like particle-wave duality.

Usually, quantum physics only plays a role for microscopic objects. Nevertheless, several macroscopic systems exist, for which quantum effects can be observed. Tunneling phenomena of the phase particle in Josephson junctions are one of the most prominent examples. This so called macroscopic quantum tunneling (MQT) was experimentally seen for the first time in 1981 by Voss and Webb (cf. ref. [1]).

Another quantum phenomenon is the existence of quantised energy levels in the Josephson junction potential. These energy levels can be probed spectroscopically. By exciting the phase particle from the ground state to higher energy levels by microwave irradiation. The microwave-induced population of higher energy levels can be observed by a change of the switching current of Josephson junctions and was experimentally seen by Martinis et. al. in 1987 for conventional Josephson junctions (cf. ref. [2]) and recently for Josephson junctions with High-Tc superconductors by Bauch et. al. (cf. ref [3]). Until now, for Josephson junctions with ferromagnetic barriers, it is unclear whether they exhibit macroscopic quantum behaviours.

Furthermore, the observation of macroscopic quantum phenomena is a promising sign for good coherence properties of Josephson junctions. The possibility to realise artificial quantum mechanical two-level systems, so called quantum bits (qubits), based on Josephson junctions, the Josephson physics became of special interest in the last years. Moreover, Josephson junctions are fabricated by standard lithographic processes, which provides scal-
ability and high manufacturing variability.

The German Science Foundation (DFG) has set a cooperative research center in Munich with the goal of developing solid-state based systems suitable for quantum information processing. This work has been conducted within the scope of this research initiative.

**Outline**

In the first chapter, we will introduce the physics of small Josephson junctions, which will be the basis for the interpretation and discussion of the measurements. Starting with a brief overview of superconductivity, we will introduce the Josephson effect and derivate the first and second Josephson equations. Next, a commonly used model (RCSJ model) for Josephson junctions will be discussed, where the tilted washboard potential will be established, followed by a description of the different escape mechanisms out of one of the potential wells. Moreover, we will introduce the quantisation of the energy levels for a virtual "phase particle" trapped in one of the metastable wells and the effect of microwave irradiation on the escape of this particle both for small and large microwave signals. Finally, the additional properties of Josephson junctions with ferromagnetic barriers will be described.

The main topic of the second chapter will be the application of Josephson junctions in superconducting qubits. At the beginning, we will introduce the mathematical model of a quantum mechanical two-level system, followed by a description of influence of a perturbation on the two basic states. The second chapter concludes with an overview over the different types of qubits based on Josephson junctions.

Starting in the third chapter with a description of the measurement principle, we will introduce the experimental setup and the methods used for data evaluation. Next, the results of escape temperature measurements will be presented and discussed, which were performed on SINFS$^1$ "0"- and "$\pi$"-Josephson junctions. Finally, experiments based on microwave spectroscopy are discussed, completed on a SINFS "$\pi$" junction.

A summary concludes this thesis.

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$^1$Supercondutor/Insulator/Normal-conducting metal/Ferromagnet/Supercondutor
Chapter 1

The Physics of small Josephson Junctions

In this chapter, we will introduce the physics of small Josephson junctions, which will be the basis for the interpretation and discussion of the measurements. Starting with superconductivity in section 1.1, we will give a short introduction into the Josephson effect in section 1.2. A commonly used model (RCSJ\(^1\) model) for a Josephson tunnel junction will be discussed in section 1.3, where the tilted washboard potential will be established. The escape rates out of the tilted washboard potential in the classical and quantum mechanical limit will be introduced in section 1.4. In the local minima of the potential, the energy is quantised, see section 1.5, which enables excitation of higher energy levels by microwave radiation, (section 1.6). We will discuss the effect of a ferromagnetic layer separating two superconductors (SFS Josephson junction) in section 1.7.

\(^{1}\)Resistively and Capacitively Shunted Junction
1.1 Superconductivity

At room temperature, the electrons in metals are fermions and obey the Fermi-Dirac statistics as particles with spin $\frac{1}{2}$. That means that every energy level, which is distinguished from all other levels by quantum numbers, can only be populated by two electrons with opposite spin. At temperature $T = 0$ the electrons fill the all energy levels up to the Fermi energy $E_F$.

In contrast, if we decrease the temperature of a superconductor under its critical temperature $T_c$, two electrons with opposite spin and momentum ($-\vec{k}, \vec{k}$) interact via phonons and form a Cooper pair, which can be seen experimentally by a sudden drop of the resistance. These Cooper pairs have total spin zero and can be described by the Bose-Einstein statistics. They all can populate one ground state, while single unpaired electrons, the so called quasiparticles are separated by an energy gap $\Delta$ from this ground state of the Cooper pairs. The diameter of a Cooper pair is approximately 10 to 1,000 nm, and thus large compared to the mean distance of the electrons in the superconductor, which leads to a strong overlap of the Cooper pairs. Because of this highly correlated behaviour all Cooper pairs have the same phase $\Theta$ and can be described with a single, complex wave function $\Psi$, which is called macroscopic wave function or superconducting order parameter, in the form

$$\Psi(\vec{r}, t) = |\Psi_0(\vec{r}, t)| \exp(i\Theta(\vec{r}, t)), \quad (1.1)$$

with the amplitude $\Psi_0$ and the density $|\Psi_0|^2$ of the Cooper pairs. As a consequence the macroscopic effect of superconductivity is determined by the macroscopic wave function $\Psi$.

![Figure 1.1:](image)

Figure 1.1: The density of the Cooper pairs in a SIS Josephson junction, where coupling of the two superconductors is enabled by the overlap of $|\Psi_1|^2$ and $|\Psi_2|^2$.

$^2$Superconductivity was first observed by Kammerlingh-Onnes in 1911 [4].
1.1 Superconductivity

In the case of two superconductors which are connected via a weak link, like a thin insulating layer, the macroscopic wave function of superconductor 1 ($S_1$) couples to the macroscopic wave function of superconductor 2 ($S_2$), see figure 1.1. Therefore, a so called Josephson supercurrent can occur, caused by the tunneling of Cooper pairs through the barrier. To simplify matters we will describe in the following chapters the physics of a superconductor-insulator-superconductor (SIS) junction, also called the Josephson tunnel junction. Nevertheless, the formulas to describe macroscopic quantum tunneling (MQT) can also be applied to SNS junctions, where the insulator in a SIS Josephson junction is replaced by a normal-conducting metal, and SFS junctions, where the insulator was replaced by a ferromagnet. A more detailed description of SFS junctions will be given in section 1.7.
1.2 The Josephson Effect

In 1962, B. D. Josephson showed theoretically, that for two superconductors separated by a thin insulator, the tunneling of Cooper pairs across the barrier has the same probability as of single electrons (see [5]). Therefore, the macroscopic wave function, which describes the entire ensemble of Cooper pairs, is involved in the tunneling process, instead of two single electrons. The density of the Cooper pairs $|\Psi_0|^2$ is constant within the superconductor and decays exponentially in the insulator. Both, the density $|\Psi_0|^2$ and the critical current $I_{c0}$ are depending on the material and on the thickness of the insulator. $I_{c0}$ is the maximum supercurrent, that can flow across the Josephson junction.

The weak coupling of the wave function $\Psi_1$ of superconductor 1 and the wave function $\Psi_2$ of superconductor 2 leads to an additional coupling Hamiltonian $\mathcal{H}_c$ and thus the Hamiltonian $\mathcal{H}_{tot}$ to describe the system of weakly coupled wave functions is given by

$$\mathcal{H}_{tot} = \mathcal{H}_0 + \mathcal{H}_c = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} + \begin{pmatrix} 0 & K \\ K & 0 \end{pmatrix},$$

(1.2)

where $K$ is the coupling strength and $\mathcal{H}_0$ is the Hamiltonian of the uncoupled system, with the eigenenergies $E_1$ and $E_2$. The Schrödinger equations of this Hamiltonian is given by

$$ih \frac{d\Psi_1}{dt} = E_1\Psi_1 + K\Psi_2,$$

(1.3)

$$ih \frac{d\Psi_2}{dt} = K\Psi_1 + E_2\Psi_2,$$

(1.4)

where this linear differential equation system can be solved with the following ansatz

$$\Psi_1 = \sqrt{n_1} e^{i\theta_1},$$

(1.5)

$$\Psi_2 = \sqrt{n_2} e^{i\theta_2}.$$  

(1.6)

Separating the coupled wave functions into their imaginary and real parts yields two equations, the first and the second Josephson equation\(^3\).

The first Josephson equation is given by

$$I_s = I_{c0} \sin (\varphi) + \sum_{m=2}^{\infty} I_m \sin (m\varphi).$$

(1.7)

---

\(^3\)See [6] for a detailed derivation.
Hence the supercurrent $I_s$ depends only on the gauge invariant phase difference $\varphi = \Theta_2 - \Theta_1$. Where $\Theta_{1,2}$ are the phases of the macroscopic wave functions of superconductor 1 and 2, respectively. Equation (1.7) is also called the current-phase relation. In weakly coupled superconductors the terms of higher order can generally be neglected, leading to

$$I_s = I_{c0} \sin(\varphi). \quad (1.8)$$

For bias currents $I_b$ below the critical current, no voltage will drop across the junction (dc Josephson effect) and the phase difference will remain constant at $\varphi = \arcsin(I_b/I_{c0}) + 2\pi n$ ($n = 0, 1, 2 \ldots$) for constant currents.

A phase-voltage relation is given by the second Josephson equation

$$\dot{\varphi} = \frac{d\varphi}{dt} = \frac{2\pi}{\Phi_0} V, \quad (1.9)$$

with the flux quantum $\Phi_0 = \frac{\hbar}{2e} = 2.068 \cdot 10^{-15}$ Vs and with the Planck constant $\hbar = \frac{\hbar}{2\pi} = 1.05457266 \cdot 10^{-34}$ Js. Due to (1.9), a constant voltage applied to the junction, changes the phase difference in time (ac Josephson effect) and according to the first Josephson equation (1.7) a change of the supercurrent

$$I_s = I_{c0} \sin\left(\varphi_0 + \frac{2\pi}{\Phi_0} V t\right), \quad (1.10)$$

with the frequency-to-voltage relation given by

$$\frac{\nu}{V} = \frac{1}{\Phi_0} = 483.6 \frac{\text{MHz}}{\mu\text{V}}. \quad (1.11)$$

Since frequencies can be measured with very high accuracy, the ac Josephson effect is used to define the voltage standard in the SI system. On the other hand, frequencies in the GHz regime can be generated even at low voltages.
Chapter 1  The Physics of small Josephson Junctions

1.3 The RCSJ Model

In 1968, W. C. Stewart [7] and D. E. McCumber [8] introduced a simple model to describe the I-V characteristics of a Josephson junction for currents below and above the critical current with high accuracy. They extended the ideal Josephson junction, which is determined by equations (1.7) and (1.9), with a shunt resistor and a shunt capacitor. The shunt resistor models the presence of quasiparticles for $T > 0$. The capacitor is formed by the two superconducting electrodes with the barrier as dielectric. Therefore, the model is called Resistively and Capacitively Shunted Junction model (RCSJ model).

![Figure 1.2: Equivalent circuit of a Josephson junction in the RCSJ model. $R$ and $C$ denote the shunt resistor and capacitor, respectively. The ideal Josephson junction is depicted by the cross.](image)

According to Kirchhoff’s law, the bias current is given by the sum of the currents at a node, see figure 1.2,

$$I_b = I_s + I_R + I_C = I_0 \sin \varphi + \frac{V}{R} + C \dot{V}.$$  \hspace{1cm} (1.12)

Eliminating the voltage $V$ using the second Josephson equation (1.9), we obtain a non-linear differential equation of second order for the phase difference

$$I_b = I_0 \sin \varphi + \frac{\Phi_0}{2\pi R} \dot{\varphi} + C \frac{\Phi_0}{2\pi} \ddot{\varphi}.$$ \hspace{1cm} (1.13)

This equation is equal to the equation of motion of a classical particle with ”mass” $m = C \left( \frac{\Phi_0}{2\pi} \right)^2$ and ”damping” $\eta = \left( \frac{\Phi_0}{2\pi} \right)^2 \cdot R^{-1}$ in a potential $U$:

$$m \ddot{\varphi} + \eta \dot{\varphi} + \frac{\partial U}{\partial \varphi} = 0,$$ \hspace{1cm} (1.14)

where $\varphi$ is the generalized coordinate. Comparing equation (1.13) and (1.14), we obtain
the potential in the form,

\[ U(I_b, \phi) = \frac{\Phi_0}{2\pi} I_b \phi - \frac{\Phi_0}{2\pi} I_{c0} \cos \phi = -E_J \left( \frac{I_b}{I_{c0}} \phi + \cos \phi \right), \quad (1.15) \]

with the Josephson coupling energy \( E_J = I_{c0} \Phi_0 / 2\pi \). \( U(I_b, \phi) \) is the so called tilted washboard potential. A current through the Josephson junction tilts the potential, where the slope depends on the normalized bias current \( \gamma = I_b / I_{c0} \). Therefore, the potential barrier height, which separates two local minima, decreases with increasing \( \gamma \), see figure 1.3.

![Figure 1.3: The tilt of the washboard potential for four different \( \gamma \) values. The phase particle is indicated by a circle.](image)

The height is given by

\[ \Delta U = 2E_J \left[ \sqrt{1 - \gamma^2} - \gamma \arccos \gamma \right] \quad \text{for } 1 < \gamma \]

\[ \approx E_J \frac{4\sqrt{2}}{3} (1 - \gamma)^{3/2} \quad \text{for } \gamma \rightarrow 1, \quad (1.16) \]

(see [9]). For currents \( \gamma < 1 \) across the junction, the phase particle is in the so called locked state, where it is trapped in a local minimum and oscillates with the plasma frequency

\[ \omega_p = \omega_{p0} \left( 1 - \gamma^2 \right)^{1/4}, \quad (1.18) \]

where \( \omega_{p0} \) is designated as the oscillation frequency of the particle in the well at zero bias current

\[ \omega_{p0} = \sqrt{\frac{2eI_{c0}}{\hbar C}} = \sqrt{\frac{2\pi I_{c0}}{\Phi_0 C}}. \quad (1.19) \]

\( E_J \) results from the overlap of the wave functions of superconductor 1 and 2.
Chapter 1 The Physics of small Josephson Junctions

Increasing the applied current causes the potential barrier to vanish at \( \gamma = 1 \). Therefore, the phase particle can escape the local minimum and roll down along the washboard potential (cf. fig. 1.3). This yields a change of the phase and, as a result of the second Josephson equation (1.9), a finite voltage will drop across the Josephson junction. Thus, the Josephson junction switches from the zero-voltage state, where no voltage occurs, to the voltage state. This transition is observable by a voltage step in the \( I-V \) curve, compare figure 1.5.

If we decrease the current, another characteristic parameter of the Josephson junction has to be taken into account, namely the quality factor\(^5\) \( Q = \omega_p RC \), which can be derived from equation (1.13), using the dimensionless parameter \( \tau = \omega_p \nu t \).

In the low damping limit (\( Q \gg 1 \)), the Josephson junction is still in the voltage state for bias currents \( I_b < I_{c0} \) and switches back to the zero-voltage state at the retrapping current \( I_r \), which yields a hysteretic \( I-V \) curve, see figure 1.5 (b). Such a Josephson junction is called underdamped. The quality factor can be determined from the retrapping current and the critical current, given by

\[
Q = \frac{4 I_{c0}}{\pi I_r}. \tag{1.20}
\]

For an overdamped Josephson junction (\( Q \ll 1 \)), the virtual phase particle is retrapped at \( I = I_{c0} \) and the \( I-V \) curve is not hysteretic. Figure 1.4 shows an estimate for the states of the phase particle, depending on \( Q \) and \( \gamma \).

\(^5\)Instead of the quality factor, the Stewart-McCumber parameter \( \beta_c = Q^2 \) is often used.
1.3 The RCSJ Model

Figure 1.4: Phase diagram of a Josephson junction. The ratio $I/I_c$ and the quality factor $Q$, determine the switching behaviour of the Josephson junction.

Figure 1.5: $I$-$V$ dependence for (a) an overdamped and (b) underdamped Josephson junction [6].
1.4 Escape Mechanisms

So far we showed, that the phase particle moving in the tilted washboard potential is an accurate model to describe the dynamics of a Josephson junction, defined by characteristic parameters like a capacitance $C$, a resistance $R$ and a critical current $I_c$. Consequently, we can derive these parameters and information about the physics of Josephson junctions and superconductors from the escape mechanisms of the phase particle out of the well.

Two escape mechanisms can be distinguished. Due to the thermal energy, the particle escapes from the well over the barrier, which is the so called classical or thermal escape. Including quantum mechanics we can derive the so called quantum escape. The particle has a finite probability distribution in the barrier, hence tunneling through the barrier becomes possible.

1.4.1 Thermal Escape

In the classical limit for $k_B T/h\omega_p \gg 1$, the escape rate is determined by the Nyquist noise of the shunt resistor. H.A. Kramers [10] investigated the problem of the escape of a classical particle out of a potential well, which is important for various problems in physics, like the dynamics of chemical reactions. The escape rate\(^6\) at temperature $T$ is given by

$$\Gamma_{th} = a_t \frac{\omega_p}{2\pi} \exp \left[ -\frac{\Delta U}{k_B T} \right], \quad (1.21)$$

where, according to Büttiker et. al. (see [11]), for an underdamped Josephson junction $(Q \gg 1)$ the prefactor $a_t$ is

$$a_t = \frac{4\alpha}{\left( 1 + \frac{\alpha Q k_B T}{1.86 U} \right)^{1/2} + 1}. \quad (1.22)$$

$\Delta U$ is the barrier height, $\omega_p$ the plasma frequency and $\alpha = 1 \pm 0.05$ a fitting parameter. According to (1.21), the escape rate depends exponentially on the temperature of the junction.

1.4.2 Quantum Escape

In a quantum mechanical model, depending on the height and the width of the barrier, the phase particle penetrates into the potential barrier and can tunnel through it. According to the RCSJ model, the phase particle describes the ensemble of Cooper pairs but not single

\(^6\)Instead of the escape rate we can use the lifetime $\tau = \Gamma^{-1}$ of the zero-voltage state.
1.4 Escape Mechanisms

Figure 1.6: Shown is the trapped phase particle in the ground state of a local minimum, the barrier height $\Delta U$ and the three regions (I-III) used for the solution of the Schrödinger equation [6].

Cooper pairs or electrons, so that we denominate this tunneling process in the case of a Josephson junction as macroscopic quantum tunneling (MQT). Without dissipative effects the tunneling problem can be solved using the quasi classical WKB method. The quantum characteristic of the phase particle is included, by solving the Schrödinger equation for the regions I, II and III (cf. fig. 1.6) and matching the solutions at the boundaries. Due to the finite probability distribution of the wave function in the barrier, which is forbidden in the classical consideration, tunneling is enabled. The escape rate is obtained by relating $|\Psi_I|^2$ to $|\Psi_{III}|^2$, hence it follows (see [12], page 79)

$$\Gamma_q = \frac{\omega_p}{2\pi} \left[ 120\pi \left( \frac{7.2\Delta U}{\hbar\omega_p} \right)^{\frac{1}{2}} \exp \left( -7.2 \frac{\Delta U}{\hbar\omega_p} \right) \right].$$

(1.23)

Caldeira and Leggett [13, 14] described dissipative effects by including an infinite set of coupled harmonic oscillators to include dissipative effects. In this model, the quantum escape rate for a cubic potential is given by

$$\Gamma_q = a_q \frac{\omega_p}{2\pi} \exp \left[ -7.2 \frac{\Delta U}{\hbar\omega_p} \left[ 1 + \frac{0.87}{Q} + O(Q^{-2}) \right] \right],$$

(1.24)

where

$$a_q \approx \left[ 120\pi \left( \frac{7.2\Delta U}{\hbar\omega_p} \right)^{\frac{1}{2}} \right].$$

(1.25)

Compared to the thermal escape rate $\Gamma_{th}$, the quantum escape is not directly proportional to the temperature $T$ but depends on the quality factor $Q$. Therefore, thermal escape is the
dominating process for higher temperatures and quantum escape for lower temperatures.

1.4.3 Crossover and Escape Temperature

Even though both processes appear at any temperature $T > 0$, the crossover from the thermal to the quantum regime is determined by a characteristic temperature, the crossover temperature $T^*$, given by

$$T^* = \frac{\hbar \omega_P}{2 \pi k_B} \sqrt{1 + \left(\frac{1}{2Q}\right)^2 - \frac{1}{2Q}},$$

(1.26)

(see ref. [15]). For a Josephson junction with weak damping ($Q \gg 1$), the crossover temperature can be approximated by $T^* = \frac{\hbar \omega_P}{(2\pi k_B)}$. In the case of strong damping ($Q \ll 1$), the expression in square brackets can be approximated by $Q$, therefore $T^* = Q\frac{\hbar \omega_P}{(2\pi k_B)}$ follows, yielding a reduction of $T^*$.

A common method to compare experimental results from escape temperature measure-

![Figure 1.7: Temperature dependence of the escape temperature $T_{esc}$ for a quantum and a classical junction. The arrows indicate the theoretically predicted crossover temperature $T^*$. Data are from reference [6]](image)
ments with theoretical predictions was first introduced by M. H. Devoret [16] by using the so called escape temperature $T_{\text{esc}}$. It is defined by

$$\Gamma_t = a_t \frac{\omega_p}{2\pi} \exp \left[ - \frac{\Delta U}{k_B T_{\text{esc}}} \right]. \quad (1.27)$$

In the thermal regime ($k_B T \gg \hbar \omega_p$), the escape temperature follows, by comparing equation (1.27) with (1.21),

$$T_{\text{esc}} = \frac{T}{1 - p_t}, \quad (1.28)$$

where

$$p_t = \frac{k_B T}{\Delta U} \ln a_t. \quad (1.29)$$

A good approximation is to neglect $p_t$, since $a_t$ is close to unity. This almost linear behaviour can be determined by plotting the escape temperature against the bath temperature, see figure 1.7, where $T_{\text{esc}}$ approaches the straight line given by $T_{\text{esc}} = T$. For $T \rightarrow 0$, the influence of the thermal escape vanishes and according to (1.24) and (1.27), the escape temperature in the quantum regime is given by

$$T_{\text{esc}} = \frac{\hbar \omega_p}{7.2 k_B} \cdot \frac{1}{1 + \frac{0.87}{Q}} \cdot \frac{1}{1 - p_q}, \quad (1.30)$$

where

$$p_q = \frac{\ln a_q}{7.2 \frac{\Delta U}{\hbar \omega_p} \left(1 + \frac{0.87}{Q}\right)}. \quad (1.31)$$

$T_{\text{esc}}$ becomes independent of temperature and $p_q$ can not be neglected, since $a_q$ is large. Figure 1.7 shows a plot of $T_{\text{esc}}$ versus the physical temperatures of SIS Josephson junctions for small damping (red circles) and for high damping (blue circles). In the quantum regime, $T_{\text{esc}}$ becomes constant. Hence $T_{\text{esc}}$ does not correspond to the actual temperature of the sample, but the value is equivalent to a temperature value in the thermal regime, where thermal escape leads to the same escape rate as quantum tunneling does in the quantum regime.
1.5 Energy Level Quantisation

As we introduced the quantum escape mechanism, the analogon of a classical particle is no longer suitable and it has to be replaced by a quantum particle. Therefore, quantum escape by tunneling is possible. According to the quantum mechanical behaviour the quantisation of the energy in the potential well becomes relevant. The local minimum in the tilted washboard potential can be approximated by a harmonic potential to estimate the positions of the energy levels, see figure 1.8.

Figure 1.8: Quantised energy levels in the harmonic approximation of a local minimum of the tilted washboard potential.

These energy levels are given by

\[ E_n = \left(n + \frac{1}{2}\right) \frac{\hbar \omega_p}{2} \quad \text{for} \quad n = 0, 1, 2, \ldots \]  

(1.32)

\( n \) is the quantum number of the state and \( E_0 = \frac{\hbar \omega_p}{2} \) the ground state energy in the well. Even if this approximation is simple, it has a high accuracy for currents just below the critical current [17]. Due to the anharmonicity of the washboard potential, the level distance decreases for levels close to the potential barrier edge. The tilt of the washboard potential is taken into account by the plasma frequency \( \omega_p \), leading to a decreasing number of the levels for decreasing barrier height \( \Delta U \). The total number of levels in the well can be estimated by

\[ N \approx \frac{\Delta U}{\hbar \omega_p}. \]  

(1.33)
1.5 Energy Level Quantisation

Methods to calculate the exact number of levels are described in reference [17]. The observation of these quantum states is only possible at low temperatures, where thermal fluctuations can be neglected. Due to the reduced potential barrier for excited levels, \( U_{\text{eff}} = \Delta U - E_n \) and according to equation (1.24), the escape rate will increase exponentially for excited states, yielding a smaller bias current where the junction switches to the voltage state.

Experimentally, the transition to higher energy levels can be achieved by thermal activation, a rapid change of the bias current or by applying microwaves, as we used in our experiments.
1.6 Microwave Interactions

As seen in the previous section, the local minima in the washboard potential have quantised energy levels which can be populated by applying microwave radiation that matches the energy difference of the levels. It follows from equation (1.24) that the escape rate increases exponentially. This gives us a tool to observe and to analyse these energy levels. For microwave signals, we have to consider a small and a large limit. In the small-signal limit, we have to choose the microwave frequency to find a balance between a proper excitation rate without changing the original potential given by eq. (1.15). Whereas in the large-signal limit the washboard potential is modified, which yields an effective suppression of the barrier [18].
For both limits, the microwave modulates the bias current by a small current $I_{rf}$, according to

$$I_{rf} = \eta \sin(\omega_{rf} t + \varphi_0), \quad (1.34)$$

where $\eta$ is the microwave amplitude, $\omega_{rf}$ its frequency and $\varphi_0$ an arbitrary phase.

1.6.1 Small-Signal Limit

The energy of microwave photons is given by $\hbar \omega_{rf}$ with frequency $\omega_{rf}$, while two adjacent energy levels have an energy difference $\Delta E = \hbar \omega_p$. Therefore, the resonance condition for a high transition probability between two nearest-neighbour energy levels is fulfilled, when these two energies are equal. This leads with equation (1.18) to

$$\omega_{rf} = \omega_p \left(1 - \gamma^2\right)^{\frac{1}{4}}. \quad (1.35)$$

For bias currents close to the critical current, $1 - \gamma^2$ can be approximated by $1 - \gamma$. According to equation (1.24), the exponent should be of the order of unity, to receive an optimal escape rate, with a proper excitation rate without changing the original potential. Using for the barrier height equation (1.17) and for the plasma frequency (1.24) to get an estimate for a microwave-plasma frequency ratio

$$1 - \gamma \leq \left(\frac{\hbar \omega_p}{E_J}\right)^{\frac{1}{4}}. \quad (1.36)$$
1.6 Microwave Interactions

Therefore, a criterion for the small-signal limit can be derived from equations (1.35) and (1.36), see reference [18]

\[
\left(\frac{\omega_{rf}}{\omega_{p0}}\right)^5 \leq \frac{\hbar}{E_J} .
\]

(1.37)

Microwave frequencies which fulfill equation (1.37) cause a high excitation rate to higher energy levels, while the potential (1.15) remains almost unchanged. Due to the smaller potential barrier height for excited levels, the escape rate can be orders of magnitude larger than in the ground state. Thus, this process is called resonant activation, where the enhancement rate is defined as

\[
\tilde{\Gamma} = \frac{\Gamma \left( P, \omega_{rf} \right)}{\Gamma (0)} ,
\]

(1.38)

where \( \Gamma (0) \) is the escape rate without and \( \Gamma \left( P, \omega_{rf} \right) \) the escape rate with frequency \( \omega_{rf} \) and power \( P \) of the applied microwave.

Due to the anharmonicity of the potential, a transition from the ground state to the second excited level is allowed, which is forbidden in the case of harmonic oscillators. Moreover, so called multi-photon transitions are allowed for an anharmonic oscillator. The microwave frequency is given by

\[
\omega_{rf} = \frac{1}{q} \omega_p ,
\]

(1.39)

where \( q \) is an integer. Whereas the energy difference \( \Delta E \) for the excitation is fulfilled by the simultaneous absorption of \( q \) photons with a total energy of

\[
\Delta E = q \hbar \omega_{rf} .
\]

(1.40)

Experimental proof for this behaviour was first given in reference [19].

1.6.2 Large-Signal Limit

As mentioned above, microwave irradiation causes a small current modulation of the bias current, cf. eq. (1.34), which can be neglected for frequencies in the small signal limit. If the microwave frequency \( \omega_{rf} \) is increased steadily, so that condition (1.37) is no longer fulfilled, the influence of (1.34) has to be taken into account. This yields a potential energy of the Josephson junction given by

\[
U(\varphi) = U_0(\varphi) - E_J \eta \frac{\hbar}{I_0} \sin \left( \omega_{rf} t \right) \varphi ,
\]

(1.41)
where $U_0(\varphi)$ is the potential energy in the absence of microwave irradiation, cf. eq. (1.15). Equation (1.41) is used in the so called large-signal limit, where a condition for the microwave frequency $\omega_{rf}$ is defined by

$$\left(\frac{\omega_{rf}}{\omega_p}\right)^5 \gg \frac{\hbar \omega_p}{E_J}.$$ (1.42)

For microwave frequencies which fulfill condition (1.42), the potential barrier is effectively suppressed. A detailed discussion is given in reference [18]. In the following, we will briefly retrace this discussion.

The phase difference $\varphi$ is separated into a fast oscillating resonant term $\xi(t)$ and a slowly varying term $\varphi_0(t)$. Using the average $\langle \xi(t) \rangle$ in equation (1.41) yields an effective potential energy

$$U_{eff}(\varphi_0) = -E_J \left[ \frac{I_b}{I_{c0}} \cos \varphi_0 \left( 1 - \frac{\pi^2}{2} \sum_{nm} \frac{f_{nm}^4}{\hbar^{-1}E_{nm}(I_b) - \omega_{rf}^2 + \alpha^2} \right) \right],$$ (1.43)

where $E_{nm}(I_b) = E_n(I_b) - E_m(I_b)$ is the energy difference between the levels $n$ and $m$ at the bias current $I_b$, which are calculated from the Schrödinger equation, and $f_{nm} = \langle n | \hat{\varphi} | m \rangle$ are the matrix elements. The damping of the junction $Q$ is considered by the damping factor $\alpha = (\omega_p^2/Q)^{1/2} = (\omega_p / RC)^{1/2}$, which determines also the strength of the resonant interaction.

The microwave radiation with power $P = k \eta^2 / 2$ leads, with equation (1.43) to a shift of the switching currents $I_{sw}(P) = (I_{c0} - I_{sw}(P)) / I_{c0}$, which is determined by the equation

$$\delta I_{sw}(P) = k^{-1} P \sum_{nm} \frac{f_{nm}^4}{\left( \hbar^{-1}E_{nm}(I) - \omega_{rf} \right)^2 + \alpha^2},$$ (1.44)

with the microwave coupling coefficient $k$.

As a consequence of equation (1.44), a sharp drop occurs in the switching current distribution at a critical microwave power $P_{cr}$. Due to the coupling term $k$, the resonant interaction for $P < P_{cr}$ is too weak to reduce the potential barrier, while it becomes sufficient at $P_{cr}$. Furthermore, in a small microwave power range around $P_{cr}$, a double peak appears in the switching current distribution. This means that a bistable state exists for the switching current. The damping factor $\alpha$, which we introduced above, also determines the width of the resonantly activated peak at lower current, while the width of the peak at higher current de-
1.6 Microwave Interactions

Figure 1.9: Microwave power dependence of the switching current. Arrows and the dashed line for $\alpha = 0.06$ bound the region in which a double peak in the switching current distributions should be observed. The microwave frequency is $\omega_{rf} = 0.4\omega_{p0}$ and $\alpha$ is varied from $0.4\omega_{p0}$ to $0.8\omega_{p0}$, corresponding to the quality factor of the junction $Q = \alpha^{-2}\omega_{rf}^{-2}$ ranging from 100 to 25. Data are from reference [18].

depends on the presence of fluctuations. Figure 1.9 shows the switching current for different damping factors $\alpha$ and the bistable state for $\alpha = 0.06$ at $P_{cr}$.

Since thermal fluctuations cause premature switching of the junction, we have to extend equation (1.44) by the fluctuation-induced shift of the switching current. Hence the mean switching current shift is given by

$$
\langle \delta I_{sw}(P) \rangle = \langle \delta I_{sw}(0) \rangle + k^{-1}P \sum_{nm} \frac{f_{nm}^4}{\left(\hbar^{-1}E_{nm}(\langle \delta I_{sw}(P) \rangle) - \omega_{rf}\right)^2 + \alpha^2}, \quad (1.45)
$$

where for temperatures above the crossover temperature the fluctuation induced shift $\langle \delta I_{sw}(0) \rangle$ is given by

$$
\langle \delta I_{sw}(0) \rangle = \left[\frac{k_B T}{2E_f} \ln \left(\frac{\omega_{p0}I_{c0}}{2\pi I}\right)\right]^{\frac{1}{2}}, \quad (1.46)
$$
Chapter 1  The Physics of small Josephson Junctions

with the bias current ramp speed $I$. The thermal fluctuations lead to a smearing of the drop of the switching current.
In a SIS Josephson junction, the Cooper pairs tunnel through the insulating barrier, leading to a supercurrent \( I_s = I_{c0} \sin(\varphi) \) (see section 1.2). If the insulator in the Josephson junction is replaced by a normal-conducting metal layer\(^7\) N, the Cooper pair density from the superconductor S decays in N up to a certain distance, the coherence length \( \xi_{F1} \). Therefore, superconducting properties are induced in the metal. This is the so called \textit{proximity effect}. Simultaneously, superconductivity is weakened in the superconductor (\textit{inverse proximity effect}). See references [20, 21] for a detailed discussion. According to non-vanishing \( \Delta^8 \) close to the N/S interface, the electrons and holes generate excited states, the so called \textit{quasiparticles}, where the strength of the correlation depends on \( \Delta \). Therefore, close to the N/S interface there are no electrons or holes, but electron-like states and hole-like states. These excited quasiparticle states enable, that particles, which are far from N/S interface electrons, with their energy \( E_F + \epsilon_0 \) are reflected at the N/S interface as holes with the energy \( E_F - \epsilon_0 \).\(^9\) Simultaneously, during the reflection process, a Cooper pair is condensed in the superconductor due to charge conservation. This charge transport is called \textit{Andreev reflection} (see [22]).

Figure 1.10 shows the charge transport between two superconductors. Since in the superconductor, between \( E_F \) and \( E_F \pm \Delta \) no quasiparticle states exist, electrons with energy

\[
\begin{align*}
E_s + \Delta \\
E_s + \epsilon_0 \\
E_s - \epsilon_0 \\
E_s - \Delta
\end{align*}
\]

The reflection of the electrons and the reflection of the holes at the N/S interfaces lead, due to constructive interference, to the Andreev bound states.

\[^7\]To simplify matters, the proximity effect and the Andreev bound states will be described in the case of a normal-conducting metal.

\[^8\]\( \Delta \) is the pair energy of a Cooper pair.

\[^9\]For \( \epsilon_0 < \Delta \), electrons cannot enter into the superconductor.
Chapter 1 The Physics of small Josephson Junctions

$E_F + \epsilon_0$ and holes with energy $E_F - \epsilon_0$ (with $\epsilon_0 < \Delta$) are reflected on the N/S interfaces. The constructive interference of the electrons and the holes gives rise to the Andreev bound states in N, which are carrying the supercurrent.

Cooper pairs are pairs of two electrons with opposite spin and momentum ($-\vec{k}, \vec{k}$). If the normal conducting metal is replaced by a ferromagnetic layer (F), we have to consider the spin splitting of the spin-up and spin-down electron bands in the ferromagnet, caused by its exchange field $E_{ex}$. Therefore, the electrons and the holes of the Andreev bound states are in opposite spin subbands, since the Andreev bound states include the spins of the electrons in the Cooper pairs. The part of the Andreev bound state with spin direction in energetically favourable direction decreases its energy by $E_{ex}$. The part of the Andreev bound state with spin direction opposite to the energetically favourable direction increases its energy by $E_{ex}$, yielding a total momentum of the Andreev bound state in the exchange field of $2q$ or $-2q$ with $q = E_{ex}/v_F$, where $v_F$ is the Fermi velocity. Combination of the two possibilities leads to an oscillating macroscopic wave function $\Psi$ in the junction along the direction normal to the S/F interface. Due to the uncertainty principle $\Delta p \Delta x \approx 2\pi \hbar$ and $p = \hbar k$, it follows that $\Delta x$ can be replaced by the thickness $d_F$ of the ferromagnetic layer. Hence the phase shift is given by

$$\Delta \varphi = 2 \frac{\epsilon \pm E_{ex}}{\hbar v_F} d_F,$$

(1.47)

where a $\pi$ shift can be attained by a certain thickness, leading with the first Josephson equation to

$$I_s = I_{c0} \sin (\varphi + \Delta \varphi) = I_{c0} \sin (\varphi + \pi) = -I_{c0} \sin (\varphi),$$

(1.48)

(see [23]). The critical current becomes negative for a change of the state from "0" to "\pi", which is observable by a sharp cusp in the critical current, depending on the ferromagnetic layer thickness $d_F$, see figure 1.11. Such a Josephson junction is a so called "\pi"-junction.

Another method to get a relation between the critical current, the temperature and the thickness, was introduced by Ryazanov [24], the order parameter $\Psi \propto e^{-i/\xi_F}$ is a function of the complex coherence length $\xi_F$, which is given by

$$\xi_F = \sqrt{\frac{\hbar D}{2(\pi k_B T + iE_{ex})}},$$

(1.49)
Figure 1.11: Dependence of the Josephson coupling $I_c R_n$ on the ferromagnetic layer thickness for SIFS junctions. The cusp at $d_F \approx 6.5$ nm indicates the transition from a "0" to a "$\pi$" coupling. Data are from reference [25].

Separating $\frac{1}{\xi_F} = \frac{1}{\xi_{F1}} + \frac{1}{i\xi_{F2}}$ into its real $\xi_{F1}$ and imaginary part $\xi_{F2}$ yields

$$\xi_{F1,2} = \frac{\hbar D}{\sqrt{E_{ex}^2 + (\pi k_B T)^2}^{1/2} \pm k_B T},$$  \hspace{1cm} (1.50)

with the electron diffusion coefficient $D$ in the ferromagnet. $\xi_{F1}$ is the decay length of the order parameter and $\xi_{F2}$ the oscillation length. Accordingly $\xi_{F1}$ and $\xi_{F2}$ will become equal for small temperatures, $\xi_{F1,2} = \sqrt{\hbar D/E_{ex}}$. The $\pi$ state in a SFS junction can be accomplished, if the ferromagnetic layer thickness is close to half the period of the order parameter spatial oscillations $\lambda_{ex} = 2\pi \xi_{F2}$, see ref. [26].

Since the ferromagnetic layer is an alloy\textsuperscript{10}, the magnetic scattering becomes important for $\xi_{F1}$ and $\xi_{F2}$. To get an idea of the influence of the magnetic scattering on the proximity effect, we use the linearised Usadel equation for the anomalous Green’s function $F_f$ in a ferromagnet

$$\left(\omega + iE_{ex} \text{sgn} (\omega) + \frac{\hbar}{\tau_f}\right) F_f - \frac{\hbar D}{2} \frac{\partial^2 F_f}{\partial x^2} = 0 ,$$  \hspace{1cm} (1.51)

\textsuperscript{10}A alloy is used, to get not to high exchange fields.
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with the inverse spin-flip scattering time $\hbar/\tau_s$. The exponentially decaying solution has the form

$$F_f(x,\omega > 0) = A \exp \left(-x (k_1 + ik_2)\right), \quad (1.52)$$

with

$$k = k_1 + ik_2 = \sqrt{\frac{|\omega| + iE_{ex} \text{sgn} (\omega) + \frac{\hbar}{\tau_s}}{\hbar D}}. \quad (1.53)$$

For a good S/F interface transparency and $d_F \gg \xi_{F1}$, the critical current is given by

$$I_{c0} \propto \exp \left(-\frac{d_F}{\xi_{F1}}\right) \left[\cos \left(\frac{d_F}{\xi_{F2}}\right) + \frac{\xi_{F1}}{\xi_{F2}} \sin \left(\frac{d_F}{\xi_{F2}}\right)\right]. \quad (1.54)$$

Figure 1.12 shows the dependence of the critical current on the temperature for different ferromagnetic layer thicknesses [21].

Figure 1.12: Temperature dependence of the SFS junction critical current density with different ferromagnetic layer thicknesses. The cusps at $T \approx 3$ K for $d_F = 11$ nm and $T \approx 1.7$ K for $d_F = 22$ nm indicate a temperature induced crossover between the "0" and "$\pi$" states. Data are from reference [26].
Chapter 2

The Josephson Qubit

In this chapter, we will give some reasons why we are so much interested in Josephson junctions. In 1965 Moore predicted that the number of transistors on a chip will increase exponentially. But this law has a natural limit. The more transistors are on a chip, the narrower the lines, until the distribution probability of the electrons in the lines overlap and tunneling from one line to the other is possible. One way out of the dead end is the idea of quantum computing, which has become very popular in the past few years. The basic idea of quantum computing is that information is computed by the controlled time evolution of quantum mechanical systems. Instead of the two stable states (0 and 1) of a conventional computer, we use the quantum mechanical superposition of a two-level quantum system, which is a so called quantum bit (qubit). Theoretical papers showed that some calculations can be solved faster if such a quantum mechanical superposition of two states is used. The best known example is the factorization of large numbers. The time to factorize a number by the best classical algorithms scales exponentially with the length of the number, while for a quantum algorithm the time scales polynomially (cf. ref. [27]).

In section 2.1, we will introduce the mathematical model of a quantum mechanical two-level system. The influence of a perturbation on the energy levels will be discussed in section 2.2. In section 2.3, we will introduce different types of qubits based on Josephson junctions.
Chapter 2 The Josephson Qubit

2.1 The Qubit Ground State

In a conventional computer, a bit is either "0" or "1". This is different for a quantum bit (qubit), which is a two-level quantum system, with the eigenstates $|0\rangle$ and $|1\rangle$. The most famous example of such a two-level system is a spin $1/2$ particle in an external magnetic field. Based on the mathematical description of this system, we have an equivalent technique to describe qubits, particularly superconducting qubits based on Josephson junctions, if only the two lowest energy levels are taken into account. The eigenstates are given by

$$
|\phi_0\rangle = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |\phi_1\rangle = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
$$

(2.1)

The qubit state is a superposition of these states

$$
|\psi(t)\rangle = a(t)|0\rangle + b(t)|1\rangle,
$$

(2.2)

with the complex amplitudes $a(t)$ and $b(t)$. The probability to measure $|0\rangle$ is given by $|a(t)|^2$ and the probability to measure $|1\rangle$ by $|b(t)|^2$. The total probability to find the qubit in either the $|0\rangle$ state or the $|1\rangle$ state is unity

$$
\langle \psi(t)|\psi(t)\rangle = |a(t)|^2 + |b(t)|^2 = 1.
$$

(2.3)

Therefore, in the two-dimensional Hilbert space $\mathcal{H}_2$, the qubit states are represented by a unitary vector, which is spanned by the basis states $|0\rangle$ and $|1\rangle$. A possible way to represent the state vector $|\psi\rangle$, which obeys equation (2.3), is geometrically by an unit vector on the Bloch sphere $S^2$

$$
|\psi\rangle = \cos \left( \frac{\theta}{2} \right) |0\rangle + \sin \left( \frac{\theta}{2} \right) e^{i\phi} |1\rangle,
$$

(2.4)

with the angles $\theta$ and $\phi$, see figure 2.1. Logical operations on a qubit are transformations of the state, where every point on the Bloch sphere can be reached by unitary rotation operations around the $x$-, $y$- and $z$-axis, given by the operator

$$
\hat{R}_n(\alpha) = \exp \left( -i \frac{\alpha}{2} \vec{n} \cdot \vec{\sigma} \right),
$$

(2.5)

with the Pauli spin matrices $\vec{\sigma}$. Hence, $\hat{R}_n(\alpha)$ performs a rotation of $|\psi\rangle$ around the unit vector $\vec{n}$ with an angle $\alpha$. 
2.1 The Qubit Ground State

Figure 2.1: The points on the Bloch sphere represent the qubit states $|\psi\rangle$. In particular, the north pole correspond to the basis state $|0\rangle$ and the south pole to $|1\rangle$.

In the case of two qubits, the states can couple and form new entangled states, the so called Bell states,

\begin{align*}
|\psi\rangle &= (|00\rangle + |11\rangle) / 2 \quad (2.6) \\
|\phi\rangle &= (|00\rangle - |11\rangle) / 2 \quad (2.7) \\
|\psi\rangle &= (|01\rangle + |10\rangle) / 2 \quad (2.8) \\
|\phi\rangle &= (|01\rangle - |10\rangle) / 2 \quad (2.9)
\end{align*}

where the latter is the singlet state. As a result of the entanglement, the state of qubit one will be determined by reading out the state of qubit two.
Chapter 2  The Josephson Qubit

2.2 Perturbed two-level Systems

The quantum mechanical two-level system is described by eigenstates and eigenvalues of the Hamiltonian \( \mathcal{H}_0 \). In this section we will take a look at a two-level system under the influence of a perturbation [28], whereas no difference between an external perturbation and an internal interaction is made. The two-dimensional state space is provided by the two orthonormal eigenstates \(| \phi_0 \rangle \) and \(| \phi_1 \rangle \) of the Hamiltonian \( \mathcal{H}_0 \). The eigenvalues \( E_0 \) and \( E_1 \) are derived from the stationary Schrödinger equations

\[
\mathcal{H}_0 | \phi_0 \rangle = E_0 | \phi_0 \rangle , \\
\mathcal{H}_0 | \phi_1 \rangle = E_1 | \phi_1 \rangle .
\] (2.10)

If perturbation should be considered, a coupling operator \( \mathcal{W} \) must be added to the Hamiltonian \( \mathcal{H}_0 \), yielding

\[
\mathcal{H} = \mathcal{H}_0 + \mathcal{W} = \begin{pmatrix} E_0 + W_{00} & W_{01} \\ W_{10} & E_1 + W_{11} \end{pmatrix} .
\] (2.11)

The two new eigenstates \( | \Psi_+ \rangle \) and \( | \Psi_- \rangle \) with eigenvalues \( E_+ \) and \( E_- \) of the Hamiltonian \( \mathcal{H} \) are given by

\[
\mathcal{H} | \Psi_+ \rangle = E_+ | \Psi_+ \rangle , \\
\mathcal{H} | \Psi_- \rangle = E_- | \Psi_- \rangle .
\] (2.12)

By the diagonalisation of the Hamiltonian operator, the new eigenstates are given by

\[
E_\pm = \frac{1}{2} (E_0 + W_{00} + E_1 + W_{11}) \pm \frac{1}{2} \sqrt{(E_0 + W_{00} - E_1 - W_{11})^2 + 4 |W_{01}|^2} .
\] (2.13)

The corresponding eigenvectors are given by

\[
| \Psi_+ \rangle = \cos \left( \frac{\theta}{2} \right) e^{-i\frac{\phi}{2}} | \Psi_0 \rangle + \sin \left( \frac{\theta}{2} \right) e^{i\frac{\phi}{2}} | \Psi_1 \rangle , \\
| \Psi_- \rangle = -\sin \left( \frac{\theta}{2} \right) e^{-i\frac{\phi}{2}} | \Psi_0 \rangle + \cos \left( \frac{\theta}{2} \right) e^{i\frac{\phi}{2}} | \Psi_1 \rangle .
\] (2.14)
2.2 Perturbed two-level Systems

Figure 2.2: Perturbed energies $E_\pm$ and unperturbed energies $E_{0,1}$ versus the energy difference $\Delta$. For $\Delta = 0$, the perturbed energies are split, where the difference depends on the coupling $W_{01}$.

The angles $\theta$ and $\phi$ are derived from

\[
\tan \theta = \frac{2|W_{01}|}{E_0 + W_{00} - E_1 + W_{11}} \quad \text{with } 0 \leq \theta < \pi , \quad (2.15)
\]

\[
W_{10} = |W_{10}| e^{i\phi} . \quad (2.16)
\]

If we suppose, that the matrix elements $W_{10}$ and $W_{01}$ are close to zero but are not vanishing and $W_{00} = W_{11} = 0$, we can simplify the eigenstates $E_\pm$ and $\tan \theta$ to

\[
E_\pm = E_m \pm \sqrt{\Delta^2 + |W_{01}|^2} , \quad (2.17)
\]

\[
\tan \theta = \frac{|W_{01}|}{\Delta} , \quad (2.18)
\]

with the two new parameters

\[
E_m = \frac{1}{2} (E_0 + E_1) , \quad (2.19)
\]

\[
\Delta = \frac{1}{2} (E_0 - E_1) . \quad (2.20)
\]

In figure 2.2, the unperturbed energies $E_0$ and $E_1$ as well as the energies of the perturbed two-level system $E_\pm$ are plotted versus the energy difference $\Delta$. According to equation (2.17), changing $E_m$ leads to an energy shift of the intersect point from the unperturbed energies $E_0$ and $E_1$, while the eigenstates $|\Psi_\pm\rangle$ remain unchanged. The coupling causes an
anticrossing behaviour of the energies and, according to equations (2.14) and (2.18), also changes the eigenstates $|\Psi_\pm\rangle$.

For energies $E_0 = E_1$, the energy difference $\Delta$ is 0, while the perturbed energies differ by an amount of $2|W_{01}|$. Therefore, the stronger the coupling $W_{01}$, the bigger the difference between $E_+$ and $E_-$. In the case $|\Delta| \gg |W_{01}|$, we can approximate equation (2.17) by

$$E_{\pm} = E_m \pm \Delta \left(1 + \frac{1}{2} \left|\frac{W_{01}}{\Delta}\right|^2 + \ldots\right).$$

(2.21)
2.3 Josephson Qubits

There are many different possibilities, to realise qubits, which have all different advantages and disadvantages. The major challenge is always the same: The quantum states have to be manipulated from outside to prepare the system in a well defined state and to initiate the computation. Otherwise the device should be maximally decoupled from the environment to avoid decoherence.

On a microscopic scale, quantum two level systems have been implemented in quantum cavity electrodynamics, trapped atoms and nuclear spins (see references [29, 30, 31]). These have, due to the microscopic scale, long coherence times, but the scaling to $10^4$ qubits to a useful computer seems to be difficult. Therefore, mesoscopic and macroscopic solid-state implementations of qubits have been investigated like spin systems in quantum dots (see reference [32]). Due to the highly developed nanoscale fabrication technology, these qubits provide a large scalability and high manufacturing variability. Unfortunately, the large quantum number of degrees of freedom reduces the coherence time.

Superconducting qubits, based on electrical circuits containing Josephson junctions are promising candidates, since the energy gap between the Cooper pairs and the quasiparticles causes long decoherence times and since they can be fabricated by using standard lithographic techniques.

For the number of Cooper pairs $N$ and the phase difference $\varphi$, an uncertainty principle exists

$$\Delta N \cdot \Delta \varphi \geq 1.$$  \hspace{1cm} (2.22)

Hence, one variable can be determined, while the other variable stays completely uncertain. Therefore, two classes of Josephson junction based qubits can be designated.

The "charge" devices, like the single Cooper box (SCB), use the number of Cooper pairs $N$ as the distinguished (relevant) parameter, while for "phase" or "flux" devices (persistent current qubits and flux qubits) the phase $\varphi$ is the relevant variable. In the following we will give a brief introduction to the different types of qubits based on Josephson junctions, given in references [33, 34].

2.3.1 Charge Devices

Charge qubits use the excess number of Cooper pair as a degree of freedom and operate in the regime $E_C \gg E_J$, where $E_C = (2e)^2/(2C)$ is the Coulomb energy and $E_J = I_c \Phi_0 / (2\pi)$
is the Josephson coupling energy. The simplest experimental realisation of a charge qubit is the so called single Cooper box, where a small superconducting island is connected via a Josephson tunnel junction (with a capacitance $C_J$) to a large superconducting reservoir. The island is capacitively (capacitance $C_g$) coupled to another massive electrode, which acts as an electrostatic gate. The voltage $V_g$ controls the gate potential (see figure 2.3 a)).

\[ H = E_C \left( N + N_g \right)^2 - E_J \cos \Theta , \quad (2.23) \]

where $N_g = -C_g V_g / (2e)$ is the charge on the gate capacitor in units of Cooper pairs $(2e)$. The required energy to change the number of Cooper pairs on the island by one is given by the Coulomb energy $E_C = (2e)^2 / \left( 2 \left( C_J + C_g \right) \right)$. $\Theta$ is the phase difference operator. Since $N$ and $\varphi$ have to be treated as operators, it follows $[\Theta, N] = i$.

The two ground states $\ket{0}$ and $\ket{1}$ of the SCB are denoted by the states, where either zero or one excess Cooper pair is on the island. According to the Hamiltonian in equation (2.23), the qubit level energies are given by

\[ E_{1,2} = \pm \frac{1}{2} \sqrt{E_C^2 \left( 1 - 2N_g \right)^2 + E_J^2} , \quad (2.24) \]

where at the degeneracy point $N_g = 1/2$, the levels are separated by the Josephson coupling energy $E_J$.

Theoretically, the charge qubit was first described by Büttiker [35] and the SCB was realised experimentally by Lafarge et. al. [36].
2.3.2 Flux Devices

Flux devices consist of superconducting loops interrupted by one or more Josephson junctions. In flux devices, the qubit states are distinguished by the direction of a circulating current.

The simplest example for a flux qubit is a superconducting loop, with inductance $L$, interrupted by one Josephson junction (cf. fig. 2.3 b)), operating in the phase regime, $E_J \gg E_C$. This is the so called radio-frequency superconducting quantum interference device, short rf-SQUID. Its Hamiltonian is given by

$$
\mathcal{H} = \frac{Q^2}{2C_J} + \frac{(\phi - \phi_x)^2}{2L} - E_J \cos\left(\frac{2\pi\phi}{\phi_0}\right),
$$

(2.25)

where $\phi_x$ is the externally applied flux (cf. ref. [37]). The magnetic flux in the superconducting loop $\phi$ and the charge $Q$ on the capacitance $C_J$ obey $[\phi, Q] = i\hbar$. For an externally applied flux near half the flux quantum $\phi_0$, the two lowest states are symmetric and antisymmetric superpositions of two classical states with clockwise and anticlockwise circulating current, see figure 2.4. Therefore, a flux qubit is also called a persistent current qubit. The main drawback of the rf-SQUID is the need of the relatively large loop, which causes a strong magnetic coupling to the environment, leading to decoherence. Mooij et.al. proposed in 1999 a persistent current qubit containing a superconducting loop with three Josephson junctions, two identical junctions and one with a smaller area than the others. For these qubits the size of the loop can be reduced, since the missing inductance is replaced by the Josephson inductance of the additional tunnel junction.

In contrast to flux qubits, which have large inductance loops, leading to a high sensitivity to external magnetic field noise, the phase qubits have a very small inductance and hence are magnetically inactive. Blatter et.al. suggested so called "quiet" qubits, containing four Josephson junctions and one "$\pi$"-junction, which are storing the information in form of a phase drop across the junction (see reference [38]). Because of this "$\pi$" - junction, no external magnetic flux has to be applied. The other four Josephson junctions have two lowest energy states with a phase difference of $\pm \frac{\pi}{2}$. 
Chapter 2 The Josephson Qubit

Figure 2.4: Double well potential of the rf-SQUID. Data are from reference [34].
Chapter 3

Current ramp experiments

In the following chapter, we will present and discuss the results of the escape temperature measurements (section 3.3), which were performed on SINFS "0"- and "\pi"-junctions, and the microwave spectroscopy (section 3.4), completed on a SINFS "\pi"-junction. Starting with the introduction of the general measurement principle, we will give an overview of the experimental setup in section 3.2.
Chapter 3  Current ramp experiments

3.1 Measurement Principle

Fulton and Dunkleberger [9] were the first, who used the experimental technique suitable for our measurements. Starting with the junction in the zero-voltage state, we increased the bias current steadily from zero, while simultaneously the voltage across the junction was measured. For bias currents close to the critical current $I_c$, the junction switched from the zero-voltage to the voltage state, where a voltage step occurred. The switching current was measured and the junction was reset to the zero-voltage state by ramping the current back to zero. To ensure that the junction was in the zero-voltage state again, a small current in opposite direction was applied. Since the escape of the phase particle (see section 1.4) is a statistical process caused by MQT, thermal activation and current noise in the circuits, we had to repeat this measurement usually several thousand times. By plotting a histogram of the switching currents (cf. sec. 3.2.6), we could extract the fluctuation free critical current. The probability that the junction switched into the voltage stage in the current range from $I$ to $I + \Delta I$ is given by $P(I) dI$. Since the junction switched sooner or later, $P(I)$ is normalized over the entire current range

$$\int_0^\infty P(I) dI = 1 .$$  \hspace{1cm} (3.1)

The relation between the probability distribution and the escape rate is given by [9]

$$P(I) = \Gamma(I) \left( \frac{dI}{dt} \right)^{-1} \left( 1 - \int_0^I P(I') dI' \right) ,$$  \hspace{1cm} (3.2)

where $dI/dt$ is the current ramping speed. The last factor on the right hand side is the probability, that the junction has not switched to the voltage state yet.

Another method to measure the switching probability is to apply a constant current across the junction and measure the time until it switches to the voltage state.
3.2 Experimental Setup

In the following, we will give a short description to the experimental setup used for the current ramp experiments. We will cover low temperature equipment, measurement electronics, shielding and filtering, and thermal anchoring. We highlight especially the advancements we did during this diploma thesis and give a short outlook of further possible improvements for the experimental setup.

3.2.1 Low Temperature Equipment

For the experiments, a home made $^3$He/$^4$He dilution refrigerator has been used, with a $^3$He circulation rate of 25 - 30 $\text{mmol s}^{-1}$, which provides a base temperature of about 40 mK\textsuperscript{1}. Due to its compact size, a cooldown and warmup time of approximately one day were accomplished. A battery-operated heater (still heater in figure 3.1) was used to adjust the optimal vapour pressure for reaching the minimum temperature. The cryostat is placed in a glass fiber reinforced dewar, that is mounted on a damping base to reduce vibrations. The vacuum pump (diffusion pump), the gas handling system and the $^3$He pump, which is the main source for vibrations, are installed in a rack. A common way for vibration damping of the $^3$He pumping line is to guide a part of it through a box filled with sand. We obtain an even better damping by demounting the $^3$He pump from the rack and placing it on four damped logs.

3.2.2 Electronics

Figure 3.1 shows a schematic diagram of the measurement setup. It can be separated into three main parts. The temperature control adjusts the temperature of the junction, the measurement electronics control the current ramp experiment and the microwave source enables the variation of the microwave frequency and power. All instruments inside the shielded room are battery powered to prevent external noise from the electrical network. For temperature readout at the sample stage, the resistance bridge Picowatt AVS-47A has been used\textsuperscript{2}. A 1 kΩ heater is mounted close to the sample stage thermometer. Using the temperature measured by the resistance bridge, the current through the heater was controlled using a Picowatt TS-530A temperature controller. This temperature control setup enabled an ac-

\textsuperscript{1}The minimum base temperature increased from 20 mK [39] to approximately 40 mK due to the installation of the microwave line [40].

\textsuperscript{2}There are five thermometers at different temperatures, which were monitored during cooldown.
accuracy of \( \pm 2 \text{ mK} \). The still heater has to be switched off during heating the sample stage. Otherwise the vapour pressure would reach a critical range of more than 700 mbars at the exhaust of the \(^3\)He pump. A known problem is that the bias current lines of the junction induce a voltage pulse in the thermometer lines, depending on the current ramping speed \( dI/dt \) (see section 3.2.3), causing an overload at the input of the resistance bridge. With an excitation voltage of 100 \( \mu \text{V} \) the overload vanished. The switching current did not shift and this the higher excitation voltage did not change the measurements.

The current ramp experiments were performed in a four-point setup with two pairs of twisted wires, one pair for the current lines and one for the voltage lines. A sawtooth current signal was applied to the junction by a home made analog current source with variable ramping speed \( dI/dt \) and amplitude. If a voltage step occurred at the junction, which was amplified with a SR560 preamplifier by a factor of 1,000 to 10,000\(^3\), a trigger signal initiate two processes. The actual current was stored in a sample and hold unit, which sent the current value to a multimeter (Agilent 3458A) and the current was ramped into the opposite direction to zero. For the readout, a Labview program has been used, which controlled the multimeter via GPIB and displayed a histogram of the switching currents \( I_b \) and

\(^3\)1,000 for the SIS Josephson junction and the SINFS "π"-junction; 10,000 for the SINFS "0"-junction.
3.2 Experimental Setup

Figure 3.2: Switching currents recorded in the current ramp experiments for a ”π”-junction at 60 mK. The red line is a continuous average of 50 data points.

a time trace (see figure 3.2). The computer was decoupled optically via glass fibres from the measurement electronics. After an adjustable number of switching events, the measured values of $I_b$ were saved to a file. The delay between two ramping signals was tunable using a pulse generator (Agilent 33250A). We will discuss the influence of the delay on the measurements in 3.2.4.

The microwave spectroscopy has been performed with the microwave generator Rohde & Schwarz SMP 04, which provides a frequency range of 10 MHz to 40 GHz with controllable microwave power. A coaxial cable guided the microwave down to the sample stage, ending in an antenna 2-3 mm above the junction. Due to the small size of the sample and the small distance between antenna and junction, a homogeneous irradiation of the sample was ensured. The advantage of the antenna is the possibility to analyse even samples without on-chip microwave structures spectroscopically. A disadvantage is the frequency depending coupling between the microwave and the junction, which might even be enhanced by the silver housing around the sample holder, causing standing electromagnetic waves. The microwave line was thermally anchored with a 20 dB attenuator at 4.2 K and a 10 dB attenuator at the still with $\approx$ 700 mK. A detailed description of the microwave implementation is given in reference [40].
3.2.3 Shielding and Filtering

Several precautions have to be taken in order to reduce the influence of extrinsic noise. Otherwise MQT could not be observed. Therefore, the cryostat, the dewar, the current source and the preamplifier (see figure 3.1) were installed in a shielded room to prevent high frequency noise up to 20 GHz. For preventing ground loops between the cryostat and different grounds, like the gas handling system, the shielded room has been galvanically disconnected from the gas handling system by using plastic centering rings. The shielded room also served as the main ground for the experimental setup. The sample stage was shielded by a brass cylinder and the sample itself was hermetically enclosed by a silver pot, providing a good rf shielding. Two concentric mu-metal shields and one cryoperm shield were integrated into the dewar for magnetic shielding. Three different filter stages have been installed to reduce the noise and radio-frequency (rf) radiation of the measurement lines and the lines for the thermometers and heaters. Starting at room temperature, low pass $LCR$-filters are mounted on top of the cryostat, with a cut-off frequency of about 100 kHz. In the next filter stage at 4.2 K, an $RC$ low-pass filter has been installed, including a current divider of $1/16.9$ to increase the signal to noise ratio. Only the measurement lines and the heater for the sample stage are fed through the second filter stage. For a detailed description of the filters see [39] and [40]. To reduce noise in the frequency region above 3 GHz, which is not attenuated by the $LCR$ and $RC$ filter, a copper powder filter has been used in the 40 mK region. The powder filter was first introduced by Martinis [2]. In our case six twisted pairs of copper wires (diameter 200 $\mu$m/length 1.2 m) pass through a stainless steel powder with a grain size of 0.83 $\mu$m. Since the wires are surrounded by numerous grains, the surface of the powder is large and thus supports the skin effect, yielding a good damping of more than 50 dB for frequencies above 0.5 GHz.

3.2.4 Thermal Anchoring

For the actability of the experiment and to assure to get reproducible results, we had to ensure that the sample was in thermal equilibrium during the experiment. Therefore, it was important to localise and, if possible, to minimise heat sources. We distinguish between two main sources: heat, which is fed in by the lines, and energy dissipation by resistors, like in wires or the Josephson junction in the voltage stage. The measurement lines, the lines for the thermometer and heater at the sample stage and the microwave line passed
3.2 Experimental Setup

Figure 3.3: Original wafer layout (left) and new wafer layout with wider gold structures (right).

from room temperature to the mK region. Therefore, we had to assure a good thermal contact of the lines, which was realised at 4.2 K and at the still with \( \approx 700 \) mK by feedthroughs. We replaced the connector (see original design in [39], page 48) by a silver feedthrough\(^4\), which has been mounted to the still and provided a better thermal anchoring. Because of the low thermal conductivity in the superconducting state, superconducting wires\(^5\) have been used between the 700 mK region and the sample stage at 40 mK. The thermal anchoring of the microwave line was obtained by mounting a 20 dB attenuator at 4.2 K and a 10 dB attenuator at 700 mK.

Because of the good thermal anchoring in the 4.2 K and 700 mK region, the dissipation in the lines and the \( RC \) filters can be neglected. A crucial problem was the heating of the sample everytime it switched from the zero-voltage to the voltage state for bias currents larger than the critical current. Hence, a good thermal coupling of the junction was essential. In our setup the Josephson junction\(^6\) was glued on a one inch thermally oxidised Si wafer with sputtered gold structures\(^7\) for the supply of measurement lines, see figure 3.3. The measurement lines and the junction were connected by aluminium bond wires. The wafer was clamped between the silver disc and the gold-plated spring loaded pins. Coming from the powder filter, the measurement lines were glued on the wafer with silver glue. For better thermal contact, we also installed a new silver feedthrough between the powder filter and the sample.

The thermal conductivity of Phonons decreases for decreasing temperatures with \( T^3 \), whereas

\(^4\) Made by a silver block, where degussit pipes copper wires, were glued.

\(^5\) Nb in CuNi matrix.

\(^6\) For testing the thermal coupling, an SIS Nb/AlO\(_x\)/Al junction has been used.

\(^7\) First we sputtered 4 nm chromium as bottom layer and then a 200 nm gold layer.
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<table>
<thead>
<tr>
<th></th>
<th>temperature</th>
<th>original structure</th>
<th>new structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>not tempered</td>
<td>295 K</td>
<td>12.413 Ω</td>
<td>0.597 Ω</td>
</tr>
<tr>
<td></td>
<td>77 K</td>
<td>8.819 Ω</td>
<td>0.354 Ω</td>
</tr>
<tr>
<td>tempered</td>
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<td>5.37 Ω</td>
<td>0.334 Ω</td>
</tr>
<tr>
<td></td>
<td>77 K</td>
<td>1.957 Ω</td>
<td>0.114 Ω</td>
</tr>
</tbody>
</table>

Table 3.1: Resistance of one line from the wafer structure, see figure 3.3.

the thermal conductivity of conducting electrons decreases with $T$. Therefore, the thermal anchoring of the junction has to be established by the measurement lines.

The unwanted heating power of the lines is proportional to $R I^2$. Hence, the resistance of the gold lines on the wafer, on which the junction was glued, was problematical. A new structure design with wider gold lines and tempering of the wafer with the gold structures at 450 °C decreased the resistance as measured in a four point measurement (see table 3.1). Figure 3.3 shows the original and the new wafer structure. A known bottleneck is formed by the aluminium bond wires, since aluminium becomes superconducting at 1.19 K. Hence, we tried to replace them by gold wires but this turned out to be problematical, since the gold did not stick to the wafer. Both, replacing the chromium layer by a titanium layer and different thicknesses of the layers have been tried, but the bond wires still did not stick to the

![Figure 3.4: Procedure steps for fabricating a SINFS junction. (a) Three level photo mask layer procedure, followed by ion-etching (b) and Nb-anodization (c) [41].](image)

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wafer. With a trick we managed to realise gold bond wires. We stucked the gold wires to rests of the aluminium bond wires left on the wafer structure. Thermal conductivity was ensured by silver glue.

The time trace of the switching current $I_{sw}$ is a good method to observe insufficient cooling of the junctions, since $I_{sw}$ decreases with increasing temperature. Due to the improvements of thermal anchoring as described above, we could reduce the measurement period from 500 ms to 400 ms for a SIS Josephson junction with a critical current of about 160 $\mu$A and the shift of the switching current was less than 1%. For measurement periods faster than 50 ms, the shift of $I_{sw}$ with aluminium bond wires was reduced by about 50%. Due to these improvements and since the dissipated energy in the junction is proportional $V \cdot I$, we were able to reduce the measurement period to 90 ms, because of the lower critical current of the SINFS junctions of about 14.2 $\mu$A.

Figure 3.5: Topview of a SINFS junction we used for our measurements. The area of the junction is 100 $\mu$m $\times$ 100 $\mu$m [42].

3.2.5 Sample Design

The SINFS junctions\(^8\) on which the experiments were performed, were produced by Martin Weides at the CNI - Center of Nanoelectronic Systems for Information Technology in Jülich, Germany, cf. [41]. Figure 3.4 illustrates the fabrication process and the involved layers but without the normal conducting copper layer. On a thermally oxidised Si wafer, first a 120 nm niobium bottom electrode was sputtered, then a 5 nm aluminium layer on

\(^8\)Superconductor/Insulator/Normal-conducting metal/Ferromagnet/Superconductor
top. Aluminium has been used due to the fact that it forms a homogeneous oxide $\text{Al}_2\text{O}_3$ whereas niobium forms inhomogeneous oxides. A known problem of this method is that the aluminium oxide surface is non-uniform, leading to a variation of the layer thickness. This yields a strongly irregular dependence of the critical current on magnetic field, which deviates from the ideal Fraunhofer pattern. Furthermore, the fabrication process is not reproducible. To reduce the roughness, a thin copper layer ($\approx 2 \text{ nm}$) was sputtered between the insulator $\text{Al}_2\text{O}_3$ and the ferromagnetic layer (Ni$_{60}$Cu$_{40}$), only causing a negligible pair breaking due to the strong proximity effect. According to section 1.7 the phase shift of the macroscopic wave function depends on the thickness of the ferromagnetic layer, leading to a ”0”-junction for $d_F = 5 \text{ nm}$ and a ”π”-junction for $d_F = 5.43 \text{ nm}$. Figure 3.5 is a picture of the topview on one of these SINFS junctions.

### 3.2.6 Data Evaluation

From the current-ramp experiment we get several thousand current values, at which the junction switched from the zero-voltage state to the voltage state. In the following, we will introduce a common method, which we used to calculate the fluctuation free critical current $I_{c0}$ and the escape temperature $T_{esc}$, see section 1.4. First, the current values were sorted into a histogram. The current range from the maximum to the minimum current was divided into usually 100 bins of size $\Delta I$, see figure 3.6. $n_i$ is the number of switching events, that fit into the current range from $I$ to $I + \Delta I$. From the histogram, the switching probability was determined by

$$P(I) = \frac{n_i}{N \Delta I},$$

where $N$ is the total number of current values. According to equation (3.2), the escape rate was calculated with the switching probability,

$$\Gamma(I) = \frac{dI}{dt} \frac{1}{\Delta I} \ln \left( \frac{\sum_{i=1}^{N} n_i}{\sum_{i=1+\Delta I} n_i} \right),$$

where $dI/dt$ is the current ramping speed. The integral $\int_{I}^{\infty} P(I')dI'$ was approximated by a series of exponential fits between adjacent points of $\sum_{i=1}^{N} P(i)$. For temperatures $T > T^*$ the escape is dominated by thermal activation and the escape rate is given by equation (1.21). It follows

$$\ln \left( \frac{a_p \omega_p}{2\pi \Gamma(I_b)} \right)^{2/3} = - \left( \frac{E_f A \sqrt{2}}{3k_B T} \right)^{2/3} \frac{1}{I_{c0}} I_b + \left( \frac{E_f A \sqrt{2}}{3k_B T} \right)^{2/3},$$

(3.5)
3.2 Experimental Setup

where we used eq. (1.17) for the potential barrier height. From plotting $\ln \left( \frac{\omega_p}{2\pi(I_b)} \right)^{2/3}$ versus the bias current and performing a least squares linear fit, latter has the slope $\propto c_{\text{lin}} I_b + c_{\text{const}}$, the critical current and the escape temperature were calculated:

$$I_{c0} = \frac{c_{\text{const}}}{c_{\text{lin}}}$$  \hspace{1cm} (3.6)

$$T_{\text{esc}} = \Phi_0 \frac{2}{3k_B \pi} \frac{1}{c_{\text{const}}^{3/2}}$$  \hspace{1cm} (3.7)

Since the plasma frequency $\omega_p$ and the Josephson coupling energy $E_J$ depend on the critical current, the fit procedure had to be repeated iteratively until the error was minimised. Figure 3.7 shows the escape rate calculated from equation (3.4), with a linear fit (dashed line). The intersection of the linear fit with the $x$-axis (current) is determined as the critical current $I_{c0}$.

**Figure 3.6:** Switching current histogram of the "$\pi$"-junction at 60 mK. On the ordinate the number of switching events $n_i$ and on the right ordinate the switching probability $P(I_b)$ is plotted versus the bias current $I_b$. 

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Figure 3.7: Plot of $\ln \left( \frac{\alpha_{\pi}}{2\pi I_b} \right)^{2/3}$ versus the bias current $I_b$ for the "\pi"-junction at 60 mK. The red dashed line is the linear fit of the data.
3.3 Escape Temperature Measurements

3.3.1 Experimental Procedure

In the following section we will introduce and discuss the results of the escape temperature measurements of the SINFS Josephson junctions, namely a "0"- and a "π"-junction. Escape rate measurements are a capable method to determine the influence of external noise on the experiment, since macroscopic quantum tunneling is only observable if premature switching caused by external noise is negligible. Therefore, the temperature of the sample holder $T_{bath}$ was adjusted with the temperature controller Picowatt TS-530A in a range from 39 mK up to 1,050 mK, which controls the current through the heater at the sample stage. To ensure that the Josephson junctions are in thermal equilibrium with the sample holder, we waited 15 minutes before starting the measurements. At each temperature 15,000 (20,000) switching events were recorded with a measurement period of 90 (50) ms for the "π" ("0") junction. Since the switching current distribution of the "0"-junction develops a double peak structure, we will present the results of these escape temperature measurements in the section 3.3.4.

To check, if Josephson junction was in the thermal regime or the quantum regime, we have

![Figure 3.8: Temperature dependence of the escape temperature of a SIS (Nb/Al₂O₃/Nb) Josephson junction. The read circles are data obtained in measurements with our experimental setup (cf. ref. [40]) and compared with reference data of the same SIS junction determined by T.Bauch [43].]
to determine the crossover temperature $T^*$. Hence, the escape temperature $T_{esc}$ was calculated for each temperature value $T_{bath}$ as described in section 3.2.6. In figure 3.8, $T_{esc}$ is plotted versus $T_{bath}$ and shows the temperature dependence of the escape temperature for a SIS Josephson junction [40]. According to section 1.4, macroscopic quantum tunneling is the dominating escape process for $T < T^*$. The crossover temperature $T^*$ can be determined from figure 3.8 as the temperature, where the linear behaviour of the escape temperature $T_{esc} \propto T$ changes to a temperature independent behaviour. Applying a magnetic field to the junction yields a reduction of the critical current and the crossover temperature $T^*$. If the reduction of $T^*$ is not observed, then the change in the slope is caused by an insufficient cooling of the sample or external noise. In references [39, 40], G. Wild and K. Madek performed escape temperature measurements on a SIS Josephson junction with the experimental setup we used and showed that the external noise is sufficiently reduced to observe macroscopic quantum tunneling.

Since escape rate measurements and microwave spectroscopy were not yet performed on junctions with a ferromagnetic layer, the results will be compared with measurements accomplished on SIS Josephson tunnel junctions, since these measurements are well-known methods to observe the quantum behaviour of SIS Josephson junctions, like the existence of quantised energy levels in the potential well and macroscopic quantum tunneling (cf. ref. [2]). We can also determine with both measurements characteristic parameters of the Josephson junction, like the fluctuation-free critical current $I_{c0}$ and the plasma frequency $\omega_{p0}$ with high accuracy.

### 3.3.2 Results and Discussion

Two important parameters can be determined from these measurements, the mean switching current $\langle I_{sw} \rangle$ and the standard deviation $\sigma$ of the switching current distribution. The latter is correlated with the width of the histogram and $\langle I_{sw} \rangle$ is correlated with the position of the barycentre of the histogram. Due to premature switching to the voltage state, the histogram broadens and the position of the mean switching current decreases with increasing temperature. Figure 3.9 shows the switching current distribution for different temperatures of the "$\pi$"-junction. Therefore, using $\langle I_{sw} \rangle$ (cf. fig. 3.13) and $\sigma$ enable another suitable way to observe the influence of external noise, even if the switching current distribution is slightly asymmetric. Similar to the temperature independence of the escape temperature for $T < T^*$, the relative width $\sigma / \langle I_{sw} \rangle$ becomes minimal for temperatures smaller than the crossover temperature.
3.3 Escape Temperature Measurements

In figure 3.11, $T_{esc}$ is plotted versus $T_{bath}$ for the "\pi"-junction, with and without considering the thermal prefactor $\alpha_i$ in equation (3.5). Both measurements deviate from $T_{esc} = T_{bath}$, whereas the data points, which consider $\alpha_i$, should lie in a small temperature range ($\pm 2$ mK) around the line with $T_{esc} = T$, see reference [44]. This deviation is caused by the readout of the temperature at the sample stage, which was performed in a two-point measurement. The disadvantage of the two-point setup is that the measured resistance of the thermometer includes the resistance of the wires and the LCR filters, leading to a total resistance of approximately 140\,\Omega. If we compare the temperature difference between the measured $T_{esc}$ and $T_{esc} = T$, we could determined the resistance of the LCR filters and the thermometer lines using the calibration curve of the temperature. Our determined value $R \approx 140\,\Omega$, is in good agreement with the resistance we expected. Therefore, the actual temperature of the junction is higher than the measured one, yielding a displacement of the curve to lower $T_{bath}$ values. The increasing slope in figure 3.11 for higher temperatures can also be explained by the additional resistance. Since for semiconductor thermometers, which we used in the mK-region, the resistance increases non-linearly with decreasing temperature and thus the relative error caused by the additional resistance increases with increasing $T$. Figure 3.12 shows $T_{esc}$ versus $T_{bath}$, where the recalibrated $T_{bath}$ values were determined from the cali-

Figure 3.9: Evolution of the switching current distribution for different temperatures of the "\pi"-junction.

![Graph showing the evolution of switching current distribution for different temperatures.](image_url)
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Figure 3.10: Shown is the standard deviation $\sigma$ versus the bath temperature.

Since neither the escape temperature becomes independent of the temperature, nor the standard deviation sets to a constant level, macroscopic quantum tunneling is not assured in the "$\pi$"-junction. The small change in the slope for temperatures below $\approx 250$ mK in $T_{esc}$ (fig. 3.12) and $\sigma$ (fig. 3.10) can be explained by an intermediate state, which is observable for temperatures $1.4T^* \leq T \leq 3T^*$ (see ref. [2]). To verify this, we determine the crossover temperature and the escape temperature with measured parameters, like the critical current $I_{c0}$ and the plasma frequency $\omega_{p0}$, which are determined from the escape temperature measurements and the microwave spectroscopy.

3.3.3 Crossover Temperature

In the following section, we will redraw the procedure of the calculations performed during this thesis. We started with a first rough approximation for the capacitance $C$ and without a quality factor $Q$ to determine $T^*$ and repeated the calculations with the parameters determined from the microwave spectroscopy.

A suitable way to determine the crossover temperature is the intersection of two fit lines, first the fit of the temperature independent data points for $T < T^*$ and second the fit of the
3.3 Escape Temperature Measurements

Figure 3.11: Escape temperature versus bath temperature for the "π"-junction. Shown is a comparison between data fitted with and without considering the prefactor $a_t$.

Linear range where $T_{esc}$ is linear to $T$, see reference [44].

The crossover temperature $T^*$ can be calculated from equation (1.26). For a first estimation of the crossover temperature of the junction, we assumed that the junction is in the low damping limit ($Q \gg 1$), which leads to

$$T^* = \frac{\hbar \omega_p}{2\pi k_B} \quad \text{with} \quad \omega_p = \sqrt{\frac{2\pi I_{c0}}{\Phi_0 C} \left(1 - \left(\frac{I_b}{I_{c0}}\right)^2\right)^{\frac{3}{2}}}.$$  \hspace{1cm} (3.8)

For a first rough approximation, we used the capacitance of $C = 400 \text{ pF}$, which was determined by M. Weides for SINFS junctions with similar materials and size [42]. In section 3.4.3 we will compare this capacitance with the spectroscopically determined capacitance. $I_b$ was estimated by calculating the mean switching current $\langle I_{sw} \rangle$ of the "π"-junction at 40 mK. The critical current was calculated by using a linear fit, which is explained in section 3.2.6. Thus,

$$T^* \approx \frac{\hbar}{2\pi k_B} \sqrt{\frac{2\pi 14.22 \mu \text{A}}{\Phi_0 400 \text{ pF}}} \left(1 - \left(\frac{13.92 \mu \text{A}}{14.22 \mu \text{A}}\right)^2\right)^{\frac{3}{2}} = 5.7 \text{ mK},$$ \hspace{1cm} (3.9)
Figure 3.12: Recalibrated escape temperature versus the bath temperature for the "\(\pi\)"-junction.

which is about a factor seven lower than the base temperature of our setup. We repeat the calculation with the parameters determined from the microwave spectroscopy from the next section. In section 3.4.3 the quality factor will be calculated with \(Q \approx 173\). Therefore, the assumption that the "\(\pi\)"-junction is in low damping limit was accurate. The difference between the approximation for \(T^*\) and equation (1.26) is for \(Q \approx 173\) less than one percent. For \(\omega_{\mu_0} = (55.384 \pm 0.219)\) GHz and \(I_{c_0} = (14.24 \pm 0.02)\) mA (which will be determined in section 3.4.3), for the crossover temperature follows \(T^* = (31.0 \pm 0.7)\) mK, which is still about 8 mK lower than the base temperature of our setup. Since the escape temperature is nearly independent of the bias current, we used \(I_b = \langle I_{sw} \rangle = (40\) mK) in equation (1.30) to calculate the escape temperature \(T_{esc} \approx 28.56\) mK.

In the intermediate temperature range, the thermal escape (see equation (1.21)) is far enough suppressed and the thermal broadening of the quantised energy levels is far enough reduced, so that they did not overlap and thus quantised energy levels occur. The phase particle is excited to higher energy levels by thermal activation. This yields an exponential increase of the tunnel probability, since the relative barrier height decreases for higher energy levels. The existence of quantised energy levels above the crossover temperature has been proved experimentally by Silvestrini et. al. in reference [45]. For the escape rate, the intermediate
3.3 Escape Temperature Measurements

state can be considered by using the prefactor

\[
a_i = \frac{\sinh \left( \frac{\hbar \omega_p}{2 k_B T} \right)}{\sin \left( \frac{\hbar \omega_p}{2 k_B T} \right)}
\]  

(3.10)

instead of \( a_t \) in equation (1.21). We did not include \( a_t \) in our calculations.

According to section 3.2.6, we determined the fluctuation-free critical current as the current, where a linear fit of \( \ln \left( \frac{a_t \omega_p}{2 k_B T} \right)^{2/3} \) intersects the \( x \)-axis (current). In figure 3.13, the critical current \( I_{c0} \) is plotted versus the bath temperature. The measurements were repeated, therefore the deviation between the two measurements can be explained by a slight change of the settings of the measurement electronics. To avoid the effect of thermally excited levels, \( I_{c0} \) was estimated from temperatures \( T > 200 \text{ mK} \) of the second measurement cycle. This yields \( I_{c0} = (14.24 \pm 0.02) \mu \text{A} \) without considering \( a_t \) and \( I_{c0} = (14.18 \pm 0.01) \mu \text{A} \) with \( a_t \). The evolution of the prefactor with the temperature \( T \) is shown in figure 3.14.

![Figure 3.13](image-url)

**Figure 3.13:** Mean switching current \( \langle I_{sw} \rangle \) and critical current, determined with considering \( a_t \), versus the bath temperature for the "\( \pi \)"-junction. The filled symbols accord to the first measurement cycle and the symbols with a cross accord to the second cycle.
3.3.4 "0"-Junction

Since two peaks occurred in the switching current distribution of the "0"-junction, without applied microwaves, we will present and discuss the results of the measurements in this separate section. The existence of the second peak can be explained by trapped flux in the layers in one or more layers. These fluxes can be trapped in so called pinning centers, caused by impurities or defects and thus leading to relative stable states. Another explanation for the developing of the double peak structure could be the existence of so called fluctuators. Charge trapping centers in the layer materials cause local changes in distribution of Cooper pairs, yielding a modulation in the potential of the Josephson junction. The ferromagnetic layer can also be the source for fluctuations, since changes in the domain boundaries lead to a modulation of the potential. Due to the double peaks in the switching current distribution, the linear fit described in section 3.2.6 is no longer suitable (see figure 3.15) and as a consequence the data evaluation method has to be modified. According to section 3.1, the switching probability of a Josephson junction is given by equation (3.2). If we assume, that each peak can be described by a Josephson junction with
3.3 Escape Temperature Measurements

slightly different parameters, then the switching probability is given by

\[ P(I) = \alpha_1 \Gamma_1(I) \left( \frac{dI}{dt} \right)^{-1} \left( 1 - \int_0^\infty p_1(I') dI' \right) + \alpha_2 \Gamma_2(I) \left( \frac{dI}{dt} \right)^{-1} \left( 1 - \int_0^\infty p_2(I') dI' \right), \tag{3.11} \]

where \( p_{1,2} \) are the switching probabilities of the junctions 1 and 2. \( \alpha_{1,2} \) denote the probabilities, whether junction 1 or junction 2 switched to the voltage state, where the normalisation condition for \( P(I) \) has to be fulfilled (cf. eq. (3.1)). The escape temperatures and the critical currents were determined by fitting the measured histogram with equation (3.11).

\[ \text{Figure 3.15: Switching current distribution and escape rate } \Gamma \text{ for the "0"-junction at 60mK.} \]

For all temperatures, the current differences between the two peaks were almost equal (cf. fig. 3.17). Changing the measurement period from 50 ms to 25 ms did not change the determined critical current of the peaks, as shown in figure 3.16. Current ramp measurements with a measurement period of 500 ms were also realised, where these critical currents had almost same values as \( I_{c0} \) from the 50 ms and 25 ms measurements, but were not included in the plots. Accordingly, the cooling of the "0"-junction was sufficient and for the fol-
lowing calculations we will confine on considering the data from the 50 ms measurement period. The deviation from $T_{esc} = T$ may be explained by the fact that we neglected the thermal prefactor $a_t$. The temperatures were recalibrated with the resistance value of the LCR filters and temperature lines, we determined from the escape temperature measurements performed on the "π"-junction.

For all temperatures, the peak at lower currents was usually smaller than the peak at higher currents. Both peaks move to lower current values and the width broadens for increasing temperatures. This leads to an overlap of the two peaks for temperatures above 400 mK, until they are nearly undistinguishable at 863 mK.

From the $I$-$V$ curve, we determined the retrapping current $I_r \approx 0.23 \mu A$ and calculated with equation (3.14) the quality factor $Q$ of the "0"-junction. Instead of the switching current from the $I$-$V$ curve, which is always $I_{c0}$ of the peak at lower current, we used the mean critical current determined from the critical currents for $T > 200$ mK ($I_{c0} = (5.67 \pm 0.04) \mu A$) (cf. fig. 3.16), leading to $Q \approx 32$.

Since the area and the materials of the different layers of the "0"- and the "π"-junctions are equal and just differ in the layer thickness, the capacitance of the "0"-junction was approximated from the capacitance $C = 14.11$ pF of the "π"-junction, which will be determined from the microwave spectroscopy (cf. sec. 3.4.3) and thus $C \approx 15$ pF for the capacitance of the "0"-junction. The plasma frequency is given by equation (1.19), leading to $\omega_{p0} \approx 34$ GHz. To determine the crossover temperature we used for $I_b$ (cf. eq. (1.26)) the current from the maximum of the switching current distribution, thus $T^* \approx 22$ mK. Because of the scatter of the determined escape temperature, as shown in figure 3.18, even a crossover to the intermediate state was not observable.

As mentioned in section 1.7, a temperature induced crossover between the "0" and the "π" state exists for "0"-junctions with layer thicknesses close to layer thicknesses, where the "π" state occurs. Since the "0"-junction we used in our measurements fulfil this condition, there is a chance to observe a temperature induced crossover from the "0" to the "π" state. Therefore, for further experiments the critical current should be determined in a temperature range from 39 mK to 9.2 K, where the latter is the critical temperature of the niobium electrodes. A second cooldown also checks, if the double peak characteristics in the switching current distribution is caused by a trapped flux. If it is caused by a trapped flux, the second peak at lower currents should not exist or at a different current value, otherwise the second peak could be a characteristic behaviour of this "0"-junction.
3.3 Escape Temperature Measurements

**Figure 3.16:** The measured critical current of the higher and the lower peaks for both, 25 ms and 50 ms, measurement periods.

**Figure 3.17:** The current distance between the measured critical currents $I_{c0}$ of higher and the lower peaks.
Figure 3.18: Shown is the escape temperature versus the bath temperature of the peak at higher currents and a measurement period of 50 ms.
3.4 Microwave Spectroscopy

According to section 1.5, the energy of the phase particle in the tilted washboard potential is quantised and it can be excited to higher energy levels by applying microwaves with suitable frequencies $\nu_{rf}$ and powers $P_{rf}$ at the Josephson junction. During the irradiation, the current was swept with a constant rate $dI/dt$ from zero, until a voltage step occurs at the switching current $I_{sw}$. The applied bias current $I_b$ tilted the washboard potential, which caused a change of the energy level spacing, according to equation (1.18) and (1.32). If the energy of microwave photons $E_{rf} = h\omega_{rf}$, where $\omega_{rf} = 2\pi\nu_{rf}$, fits an energy distance between two levels, the phase particle was excited to higher levels. Due to the reduced potential barrier $\Delta U$ of higher energy levels, the escape rate $\Gamma$ increased exponentially, compare section 1.4. Therefore, a second peak occurs at lower current and is designated as the resonance excited state. The current of the maximum of the induced peak is the so-called *resonance current* $I_{res}$.

We confined the microwave spectroscopy measurements on the "$\pi$"-junction, due to the low critical current and crossover temperature of the "0"-junction. The microwave frequency and power was adjusted from 770 MHz to 7.46 GHz with the microwave generator Rohde & Schwarz SMP 04, while the switching current was measured simultaneously. For suitable microwaves a double peak occurs in the switching current distribution, which is also observable in the time trace of the switching current. Figure 3.19 is a plot of the data for $\nu_{rf} = 2.3$ GHz and $P_{rf} = -20.5$ dBm. For each frequency $\nu_{rf}$ 10,000 switching events were recorded with a measurement period of 90 ms, whereas the frequency $\nu_{rf}$ and the power $P_{rf}$ of the microwave were kept constant.

The temperature was set by the temperature controller Picowatt TS-530A to 50 mK, which is above the calculated crossover temperature of the "$\pi$"-junction, but allowed us to observe quantised energy levels, as will be shown in this section.

Since the microwave generator provides a broad frequency range of 10 MHz to 40 GHz, we tried to narrow down the frequency range, by finding an estimate for the plasma frequency. Due to equation (1.19) with the fluctuation-free critical current, which was determined from the escape temperature measurements, and with $C = 400$ pF, we calculated $\omega_{p0} = (10.38 \pm 0.05)$ GHz. After the first measurements, resonance currents for microwave frequencies above $\nu_{p0} = \omega_{p0}/(2\pi) \approx 1.65$ GHz were found, which confirmed the assumption that the capacitance was less than 400 pF. The calculation of the capacitance from the spectroscopically determined parameters $I_{c0}$ and $\omega_{p0}$ is described in detail in section 3.4.3.
Chapter 3  Current ramp experiments

Figure 3.19: Double peak structure in the switching current distribution for a microwave with $\nu_{rf} = 2.3$ GHz and $P_{rf} = -20.3$ dBm. The inset shows the time dependent switching current behaviour of the same data.

3.4.1 Power Dependence of the Switching Current Distribution

For different microwave powers $P_{rf}$, the switching current distribution $P(I)$ is plotted in figure 3.20, where $\nu_{rf} = 2.2$ GHz was kept constant and is compared with $P(I)$ without microwave. For increasing microwave power, the width of $P(I)$ broadened. Simultaneously, the peak at higher currents vanished and a second peak developed, until the peak at lower currents remained and shifted further to lower currents. Figure 3.21 shows the corresponding escape rates. The bulge in the slope indicates the beginning excitation to higher energy levels. Even if the escape rates with applied microwave $\Gamma(P)$ (where $P$ is the microwave power $P_{rf}$) were close to the escape rates without microwave $\Gamma(0)$, we could not determined the enhancement factor $\tilde{\Gamma} = \Gamma(P)/\Gamma(0)$ for the small-signal limit, due to the fact, that the shift of the resonant peak to lower current was to big. Hence we confirmed the suggestion that the applied microwave was in the large-signal limit as described in section 1.6. According to equation (1.44) microwave frequencies in the large signal limit shift the switching current distribution to lower currents for increasing microwave power, until the distribution develops a double peak structure at the critical power $P_{cr}$. With the
3.4 Microwave Spectroscopy

Figure 3.20: Switching current distribution $P(I_b)$ for 2.2 GHz at different microwave power $P_{rf}$.

determined parameters for $I_{c0}$ and $\omega_{p0}$ from the microwave spectroscopy, we can verify the criterion for the large-signal limit, given by equation (1.42). This yields a lower boundary of $\omega_{rf}/(2\pi) = 2.314$ GHz. Frequencies above the lower boundary are leading to a suppression of the potential barrier, as described in section 1.6. Since for multi-photon transitions the frequency has to be multiplied with the number of involved photons $q$, all microwaves we have applied were above the lower boundary for the large-signal limit.

The disadvantage of applying the microwave by an antenna to the junction, as used in our setup, is the frequency dependence of the coupling, due to the formation of standing waves in the silver housing. Thus, the prefactor $k$ in equation (1.44) is frequency dependent, where $k$ designates the coupling between the microwave and the junction. Therefore, we can not estimate $k$ and thus we can also not determine the shift of the switching current $\delta I_{sw}(P)$.

3.4.2 Multi-Photon Transitions

In section 1.6, we introduced the possibility of multi-photon transitions of the phase particle from the ground state to higher energy levels, due to the anharmonicity of the junction.
potential. The energy difference between the ground and higher levels has to be provided by $q$ photons, with a total energy $\Delta E = qh\nu_{rf}$, where $\nu_{rf}$ is the applied microwave frequency.

Multi-photon transitions can be observed experimentally by plotting $\nu_{rf}$ versus the resonance current $I_{res}$, which is the current of the maximum of the resonance activated peak in the switching current distribution. According to equation (1.39) the resonance condition is given by

$$\nu_{rf} = \frac{\nu_{p0}}{q} \left( 1 - \left( \frac{I_{res}}{I_{c0}} \right)^2 \right)^{\frac{1}{2}}, \quad (3.12)$$

where the plasma frequency $\nu_{p0} = \omega_{p0}/(2\pi)$ was replaced by $\nu_{p0}/q$ and $q$ is the number of photons involved in the transition. Figure 3.22 shows data of our measurements, where different branches, which correspond to different number of photons, are distinguishable. The dependence of the resonance currents on the applied frequency of our measured data are in good agreement with the theoretical prediction (cf. eq. 3.12). The circles are data with low statistics (less than 100 counts in a 10,000 point measurement) or peaks caused by very high microwave power $P_{rf}$.

**Figure 3.21:** Escape rate for the two-photon escape process at 2.2 GHz with different microwave power $P_{rf}$.
3.4 Microwave Spectroscopy

3.4.3 Spectroscopic Determination of $\omega_{p0}$ and $I_{c0}$

From the microwave induced escape rate measurements, the critical current $I_{c0}$ and the plasma frequency $\omega_{p0}$ were determined by fitting equation (1.35) to the obtained data. Since the single branches in figure 3.22 designate multi-photon transitions with $q$ photons, we multiplied, according to equation (1.39) the microwave frequencies with the number of photons $q$, to obtain the one photon curve, see figure 3.23. In this way, the plasma frequency was found to be $\nu_{p0} = (8.315 \pm 0.035)\, \text{GHz}$, and the fluctuation-free critical current $I_{c0} = (14.244 \pm 0.023)\, \mu\text{A}$. Latter shows a good agreement with the critical current we determined from the escape temperature measurements in section 3.3.3, yielding $I_{c0} = (14.183 \pm 0.014)\, \mu\text{A}$. We calculated with these values the lines in figure 3.22.

According to equation (1.19), the capacitor is then given by

$$C = \frac{2eI_{c0}}{\hbar \omega_{p0}^2} \approx (14.11 \pm 0.09)\, \text{pF}. \quad (3.13)$$

The difference between the capacitance we received from M.Weides ($C \approx 400\, \text{pF}$) and the value calculated above ($C = 14.11\, \text{pF}$) can be explained by the fact, that layers in the "$\pi$"-junction we used, were thicker than the junction from which M.Weides determined its value.

The quality factor $Q$ can be determined with the retrapping current, which we extracted from a $I$-$V$ curve, where $I_r = 0.11\, \mu\text{A}$. This leads to

$$Q = \frac{4}{\pi} \cdot \frac{I_{c0}}{I_r} \approx 172.64 \pm 0.28. \quad (3.14)$$

In summary, spectroscopic measurements in the large-signal limit have been performed on a SINFS "$\pi$"-junction and agreed well with the theoretical predictions for microwave spectroscopy of SIS Josephson junctions. We could observe the escape of the phase particle from the first excited state in the potential well, by the occurrence of microwave induced resonance peaks in the switching current distribution. In future experiments, microwave spectroscopy should be performed in a cryostat with a lower base temperature, to observe macroscopic quantum tunneling from the ground state and the first excited state.
Chapter 3 Current ramp experiments

Figure 3.22: Multi photon transition. The circles with the cross indicate microwaves, which were not considered for the fit, due to high microwave power or low statistics.

Figure 3.23: The same data as in figure 3.22, but multiplied by the number of photons $q$. 
Summary

During this thesis, we probed the quantum states of SINFS Josephson junctions with microwaves. We successfully proved the existence of quantized energy levels in the potential of a SINFS "π"-Josephson junction. Previously, improvements on an existing setup for current ramp experiments have to be made.

To ensure, that the junction was in thermal equilibrium during the experiment, we localized and minimized existing heat sources. Two new silver feedthroughs, one at the still and one at the sample stage, were installed, leading to a better thermal contact of the lines.

Another main goal was the efficient cooling of the sample by the measurement lines. By designing a new wafer with wider gold lines on which the junction was glued for measuring, the resistance was decreased by a factor of 40.

Another attempt improving of the thermal anchoring was made by replacing the aluminium bond wires with gold bond wires, which connected the junction with the gold lines on the wafer. These improvements on thermal anchoring were seen on the faster measurements.

Moreover, we could reduce external vibrations and external noise by a more efficient damping of the $He^3$ pump in the low temperature equipment.

This improved setup was used for escape temperature measurements on SINFS "0"- and "π"-junctions and microwave spectroscopy measurements on a "π"-junction. For both junctions, the fluctuation free critical current was determined in the thermal regime with high accuracy. Moreover, for both junctions the escape temperature and the crossover temperature, where latter designates the transition from the thermal regime to the quantum regime, could be determined. Unfortunately, both crossover temperatures were less than the base temperature of the experimental setup and hence no macroscopic quantum tunneling (MQT) was observable. Nevertheless, we observed a change in the slope of the escape temperature for the "π"-junction, which indicates the existence of MQT. In this so called intermediate state, between the quantum regime and the thermal regime, microwave spectroscopy was performed on the "π"-junction at 50 mK.
SUMMARY

For several microwave frequencies a double peak structure in the switching current distribution $P(I)$ exists, which was also observable by a sharp drop of the switching current at a critical microwave power $P_{cr}$. This was in good agreement with the theoretical prediction for the quantum escape in the large-signal limit. The second peak, which is caused by the escape of the phase particle from the first excited state, indicates the existence of quantised energy levels in the potential well. That was shown here for the first time. Therefore, we could proof that the macroscopic quantum behaviour of the phase $\varphi$ is not destroyed (suppressed) by the ferromagnetic layer.

Furthermore, multi-photon induced transitions of the phase particle from the ground state to the first excite state and the good agreement of the measurements with the frequency-current relation underlines, that theoretical predictions for the potential in Josephson junctions can also be used to describe the behaviour of the potential in SINFS-junctions.

The fluctuation free critical current, the plasma frequency and the capacitance of the ”$\pi$”-junction were determined with high accuracy. The value of the critical current was in good agreement with the results from the escape temperature measurements. For further experiments escape temperature measurements should be performed with the ”$\pi$”-junction in a cryostat with a base temperature of $\approx 20$ mK, to verify the existence of MQT.

For ”0”-junctions with ferromagnetic layer thicknesses close to the ”$\pi$” state, a temperature induced crossover between the ”0” and the ”$\pi$” state is possible. Therefore, for further experiments with the ”0”-junction, the critical current should be measured in a temperature range from 39 mK to 9.2 K, to check the existence of the temperature induced crossover in SINFS-junctions.

In summary quantised energy levels in the intermediate state for a SINFS ”$\pi$”-junction was shown on an improved experimental setup.
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