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Torque Magnetometry With Quartz Tuning Forks

Drehmoment-Magnetometrie mit Stimmgabeln aus Quartz

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Introduction

The magnetic properties of materials are of fundamental importance for, e.g., the development of ever smaller magnetic digital storage devices. Torque magnetometry has proven to be an efficient and very sensitive way to investigate magnetic properties, in particular magnetic anisotropy [1], [2]. Moreover, torque magnetometry also connects magnetic with mechanical degrees of freedom. With the advent of nanomechanics, spin-mechanics coupling is increasingly interesting.

The idea in this bachelor thesis is to establish torque magnetometry, using a tuning fork. Instead of a cantilever which is used in most other approaches.

Figure 1: Panel A: a quartz tuning fork (black arrow) is the key element in many time-keeping applications, in particular in wrist watches. Upon opening the DIP package, the tuning fork is visible (panel B). The photograph is taken from [3].

Due to this very stable resonance frequency of 32.768 kHz, these oscillators can be used as a time-keeping element in wrist watches or micro processors (Fig. 1). Their stiffness of a few kilonewtons per meter and their high quality factor of about $10^5$ makes them predestined for atomic force microscopy (AFM) [1], [3], [5], scanning tunneling microscopy (STM) [6] and magnetic force microscopy [7]. Due to an annual production volume of more than $2 \times 10^9$ the costs for the tuning forks are at a range of a few cents per piece which is another reason to use them as torque magnetometry sensors. In the area of AFM, quartz tuning forks are well established and often used by fixing one prong of the fork to a substrate and attaching a tip on the other prong which is highly sensible for small forces. This design was invented by F.Giessibl and called „Qplus“-sensor [8],[9],[6].
Introduction

For torque magnetometry experiments, the tuning fork is loaded with a magnetic specimen like an iron or nickel wire and placed into an external magnetic field. In analogy to an external magnetic field exerting a torque on a compass needle, the ferromagnet attached on the oscillating tuning fork experiences a torque as well. This torque is dependent on the anisotropy constant of the wire. Since the ferromagnet is rigidly mounted on the prong of the tuning fork, this torque shifts the resonance frequency of the oscillating fork. Measuring the corresponding frequency shift $\Delta f$ enables the determination of the anisotropy constant of the ferromagnet.

In this bachelor thesis the possibility of quartz tuning fork based-torque magnetometry shall be established and tested.

At the beginning of this thesis, theoretical aspects concerning the quartz tuning forks and the estimation of the anisotropy will be laid out. Afterwards, the experimental set-up will be presented in detail, as well as different techniques that we used to measure $\Delta f$. A brief description of the loading procedure developed to attach samples to a tuning fork will be given in this chapter, too. In the experimental part of this thesis, the tuning forks will then be investigated in terms of their resonance behaviour in an external magnetic field. Finally we will show that torque magnetometry is indeed possible using a nickel wire (1 mm long and 250 $\mu$m in diameter) attached to a commercially available tuning fork.

This thesis forms the basis for further torque experiments with quartz tuning forks using magnetic thin layers. Therefore, an estimation of the minimal thickness of a nickel layer, at which it is still possible to measure anisotropy will be given.
Chapter 1

Theoretical concepts

This chapter addresses the extraction of anisotropy constants from torque magnetometry data. Since we use torque magnetometry based on quartz tuning forks, an electrical model describing the oscillation behaviour of these tuning forks is also given.
1.1 Properties of quartz tuning forks

1.1.1 Design and functional principle

Quartz tuning forks are mainly used as frequency standards in wristwatches or microprocessors [10]. In particular, the resonance frequency of the tuning fork is used as a time standard. Therefore it is essential to adjust the resonance frequency as precisely as possible. The tuning forks used in the experiments in this thesis, for example, have a resonance frequency of $2^{15} = 32768.0 \pm 1.6$ Hz [11]. This corresponds to a relative frequency uncertainty of roughly 50 ppm. As a result a wristwatch would have an accuracy of about 100 s per month. Because of the stable resonance frequency and the fact that tuning forks have a large quality factor of $\sim 10^5$ [6] they can also be used as sensors in different force measurements [?, 8].

Figure 1.1 shows a photograph of a tuning fork. The blackish of the tuning fork are the quartz while the light grey parts correspond to the electrodes which are deposited on the quartz crystal. As obvious from Fig. 1.1 the upper part of the two prongs are not perfectly rectangular in shape. In the production process, removing material from the edges allows to fine-tune the resonance frequency.

On each prong two electrodes are deposited as indicated by the numbers in Fig. 1.1.

![Image of quartz tuning fork](image)

Figure 1.1: Top and side view of a quartz tuning fork without vacuum casing. The blackish area corresponds to the quartz material on which electrodes (light grey) are deposited. The tuning fork has an inversion symmetry around the $x$ axis so that two opposite sides have the same design. The numbers indicate the two electrodes of opposite polarity.

These electrodes are deposited in such a way that adjacent sides of a prong always carry opposite charges. The electrodes thus allow to apply an electric field to the
1.1 Properties of quartz tuning forks

quartz. Due to the piezoelectric effect, the quartz prongs will deform in the pressure of a finite electric field, such that potential energy is stored. Removing the electric field results in a damped oscillation of the prongs. A constant oscillation amplitude can be reached by applying an alternating electric field which drives the tuning fork at a certain frequency.

For a more quantitative treatment, the spring constant $k$ of a prong is of very importance. For a single beam, the following expression holds [10]:

$$k = \frac{E}{4} W \left(\frac{T}{L}\right)^3$$

(1.1)

where $W$, $T$ and $L$ are the dimensions of the tuning fork given in Fig. 1.1 and $E = 7.78 \times 10^{10}$ N/m$^2$ [10] is Young’s modulus for quartz. For the tuning fork of type Buerklin 78D202 which is mainly used in our experiments the dimensions are $L = 3.85 \pm 0.20$ mm, $T = 0.58 \pm 0.05$ mm and $W = 0.33 \pm 0.05$ mm. With Eq. (1.1) the spring constant is calculated to $k = (2.22 \pm 0.67) \times 10^4$ N/m.

1.1.2 Electrical model

Mechanical harmonic oscillators are closely analogous to electrical resonant circuits. A quartz tuning fork for example can be modelled by an electrical $LRC$-circuit called Butterworth-Van Dyke circuit [12]. It consists of an inductance $L$, a resistor $R$, a capacitance $C$ and another capacitance $C_0$ which is connected in parallel as shown in Fig. 1.2.

Figure 1.2: Butterworth-Van Dyke equivalent circuit for a quartz tuning fork. This figure stems from a paper of Nanonis GmbH [12]

The inductance $L \propto m$ represents the kinetic energy contained in an oscillating system. It is proportional to the effective mass $m$ of the quartz tuning fork. In contrast, the capacitance $C$ models the potential energy stored in the system. It
is inversely proportional to the spring constant of the tuning fork, $C \propto 1/k$. The resistor $R \propto \gamma$ stands for the dissipative processes like damping with the damping factor $\gamma$. The capacitance $C_0$ is the geometrical capacitance between the electrodes.

### 1.1.3 Resonance behaviour

The aim of modelling a quartz tuning fork is to describe its resonance behaviour. To this end, the admittance $Y(f) = I(f)/U(f)$ can be used with the electrical current $I$ and the voltage $U$. $Y(f)$ is the inverse impedance called the transfer function of a system and can be experimentally measured. According to the circuit in Fig. 1.2,

$$Y(f) = \frac{1}{R + 2\pi i f L + \frac{1}{2\pi i f C}} + 2\pi i f C_0 = \frac{2\pi i f C}{2\pi i f CR + u} + 2\pi i f C_0 \quad (1.2)$$

Here, $u := 1 - 4\pi^2 f^2 LC$ is a substitution variable. The resonance frequency of a LRC-circuit is $f_0 = 1/2\pi \sqrt{LC}$. Hence,

$$u = 1 - \frac{f^2}{f_0^2} \quad (1.3)$$

With this substitution one can easily separate real and imaginary part. Since we are interested in the resonance curve of our oscillator, one can furthermore assume that $f \approx f_0$:

$$Y(f) = \frac{2\pi f^2 C^2 R + if Cu f_0^4}{2\pi f^2 C^2 R^2 + u^2} + 2\pi i f C_0 \approx \frac{4\pi^2 f_0^4 C^2 R + 2\pi i f_0^4 C(f_0^2 - f^2)}{4\pi^2 f_0^4 C^2 R^2 + (f_0^2 - f^2)^2} + 2\pi i f_0 C_0 \quad (1.4)$$

With $f \approx f_0$, $f - f_0 \approx \delta$ is a small number and $f^2 \approx f_0^2 + 2f_0\delta$. It follows that $f^2 - f_0^2 \approx 2f_0\delta$ and hence $f^2 - f_0^2 \approx 2f_0(f - f_0)$. With these approximations, Eq. (1.4) can be written as a complex Lorentzian:

$$Y(f) = \frac{2\pi f_0^4 C(2\pi f_0^4 C R - 2i(f - f_0))}{4\pi^2 f_0^4 C^2 R^2 + (f - f_0)^2} + 2\pi i f_0 C_0 \quad (1.5)$$

Figure 1.3 shows the absolute value of $|Y(f)|$. Because of the capacitance $C_0$ in parallel, the admittance shows a characteristic minimum for frequencies somewhat larger than $f_0$. 

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**Chapter 1 Theoretical concepts**


As usual in resonant circuits \cite{13} the phase $\Phi$ between the excitation voltage and the measured current shifts by $180^\circ$ around the resonance frequency. However the capacitance $C_0$ causes a second phase shift opposite to the first one so that the phase reaches its original level for large $f$. With $|Y|$ it is possible to fit the measured resonance data and thereby extract the values for $L, C, R$ and $C_0$ of the electrical model. By means of a comparison of Eq. (1.5) to a typical complex Lorentzian it is possible to associate these values with the properties of the tuning fork as follows:

$$A_0 \propto \frac{1}{\gamma} \quad (1.6)$$

$$\Delta f_B \propto \frac{\gamma}{m} \quad (1.7)$$

$$f_0 \propto \sqrt{\frac{k}{m}} \quad (1.8)$$

Here, $A_0$ is the amplitude of the resonance curve at the resonance frequency $f_0$ and...
\( \Delta f_B \) is the bandwidth of the resonance curve\(^1\). The calculation is given in more detail in Appendix A. From \( f_0 \) and \( \Delta f_B \) the quality factor \(^{15}\)

\[
Q = \frac{f_0}{\Delta f_B}
\]  

(1.9)

of the oscillator can be determined.

\(^1\)While the bandwidth of a power signal \( P(f) \) represents the distance between two points in the frequency domain, where the signal is at the half maximum, the bandwidth for an amplitude signal \( A(f) \propto \sqrt{P(f)} \) has to be measured at the \( 1/\sqrt{2} \)-fold of the signal’s maximum amplitude \(^{14},\,^{15}\).
1.2 Determination of magnetic anisotropy from torque experiments

Figure 1.4 shows a sketch of an oscillating tuning fork in an external magnetic field $H \parallel \hat{z}$. The tuning fork is loaded with a ferromagnetic specimen. Due to the magnetic field the ferromagnet exerts a torque on the prong which shifts the resonance frequency of the tuning fork.

In the following, we derive an expression for this magnetic torque-induced resonance frequency shift $\Delta f$ in the case where $H$ is parallel to the long side of the tuning fork (see Fig. 1.4). In particular, the magnetic anisotropy of the ferromagnet shall be inferred from $\Delta f$. The considerations are based on the work of Stipe et al. [1].

In our model approach, we assume that the two prongs do not interact with each other. While oscillating, the prongs form an angle $\beta$ to their equilibrium position. Since the easy axis of the ferromagnet is not parallel to $H$, the magnetization aligns along the effective magnetic field which forms an angle $\Theta$ to the easy axis. We assume that the time the magnetisation $M$ of the magnet needs to align is of the order of $< 1 \mu s$ [16] and thus much shorter than the oscillating period of the tuning fork of about $30 \mu s$. Therefore it is reasonable to assume that the magnetisation is in a quasi equilibrium for each angle $\beta$.

We here consider the simple case of a ferromagnet with uniaxial anisotropy. Thus, the magnetic free energy [1] consists of the anisotropy energy and the Zeeman energy

$$E = K_u V \sin^2 \Theta - \frac{V M \cdot B}{\text{anisotropy energy}} - \frac{\mu_0 H V M_S}{\text{Zeeman energy}} \sin \beta \cos \Theta,$$  \hspace{1cm} (1.10)

where $K_u$ is the uniaxial anisotropy constant, $M_S$ the saturation magnetisation of the ferromagnet and $V$ the volume. For a quasi equilibrium state the free energy has to be minimal:

$$\frac{\partial E}{\partial \Theta} = 2K_u \sin(\Theta) \cos(\Theta) - \mu_0 H M_S \sin(\beta - \Theta) = 0$$  \hspace{1cm} (1.11)

In the small angle approximation, this can be approximated by

$$2K_u \Theta = \mu_0 H M_S (\beta - \Theta)$$  \hspace{1cm} (1.12)

Solving this equation for $\Theta$ one obtains

$$\Theta = \frac{M_S \mu_0 H \beta}{M_S \mu_0 H + 2K_u}$$  \hspace{1cm} (1.13)
As already mentioned, the magnetisation which is rotated by the angle $\Theta$ exerts a torque $\tau_{\text{mag}}$ on the prong of the tuning fork. In the special orientation shown in Fig. 1.4 this torque can be determined using Eq. (1.13) and small angle approximation:

$$
\tau_{\text{mag}} \equiv V |\mathbf{M} \times \mathbf{B}| = VM_5\mu_0 H \sin(\beta - \Theta) \approx 2V\beta \frac{\mu_0 H K_u}{\mu_0 H + \frac{2K_u}{M_5}} \quad (1.14)
$$

while the torque exerted by the stiffness of the prong itself can be calculated with

$$
\tau_{\text{prong}} = L_e \cdot \mathbf{F} = k \cdot \mathbf{x} \cdot L_e = k \beta L_e^2. \quad (1.15)
$$

Here, $L_e$ is the effective length of the oscillating prong and $k$ its spring constant. Thus $\tau_{\text{mag}}$ effectively adds a spring constant $\Delta k$ on the prong, given by

$$
\tau_{\text{mag}} = \Delta k \beta L_e^2 \quad (1.16)
$$
1.2 Determination of magnetic anisotropy from torque experiments

\[ \Delta k = \frac{M_S V}{2L_e^2} \cdot \frac{\mu_0 H \cdot \frac{2K_u}{M_S}}{\mu_0 H + \frac{2K_u}{M_S}}. \]  

(1.17)

As the tuning fork without loading is a harmonic oscillator, the resonance frequency is given by \( \omega_0 = \sqrt{k/m_e} \), where \( m_e \) is the effective mass of the prong. The magnetic torque adds a spring constant \( \Delta k \) whereby the resonance frequency shift can be calculated as

\[ \frac{f(H)}{f_0} = \sqrt{\frac{k + \Delta k(H)}{k}} \approx 1 + \frac{\Delta k(H)}{2k} \]  

(1.18)

The frequency shift due to the magnetic torque thus is

\[ \Delta f(H) = \frac{1}{2} \frac{\Delta k(H)}{k} f_0 = \frac{M_S V f_0}{2L_e^2} \cdot \frac{\mu_0 H \cdot \frac{2K_u}{M_S}}{\mu_0 H + \frac{2K_u}{M_S}} \equiv A \]  

(1.19)

where \( A \) is a constant.

With the experimental data from torque magnetometry it is now possible to determine the anisotropy constant \( K_u \) with a fitting curve.

\[ \Delta f(H) = A \cdot \frac{B \cdot B_k}{B + B_k} \]  

(1.20)

where \( B_k = \frac{2K_u}{M_S} \) is the anisotropy field. Furthermore, if \( L_e \) and \( k \) are given, the saturation magnetisation \( M_S \) of the ferromagnet can be derived from the fitting coefficient \( A \).
Chapter 2

Experimental techniques

This chapter describes our experimental approach to measure the resonance frequency shift of a tuning fork loaded with ferromagnetic material as a function of an applied magnetic field. We first present the measurement set-up and discuss three different techniques to evaluate the frequency shift $\Delta f$. Secondly, the technique we developed to load the ferromagnet onto the tuning fork is outlined.
2.1 Measurement set-up

Figure 2.1: Schematic of the experimental set-up. In the right part one can see two coils with iron cores which produce a magnetic field $H$ in the gap between them. In this gap a Hall probe measures the magnetic flux density. Via a PID control it is possible to tune the current flowing through the coils in order to precisely adjust $H$. The quartz tuning fork, electrically driven by a frequency generator is located in the gap between the coils. The current flowing through the fork is detected via a lock-in amplifier.

The measurement set-up consists of a 2D-vector magnet with power supplies and Hall-probes (Lakeshore DSP 475) with PID control, a Stanford Research SR830 lock-in amplifier and a frequency generator (Agilent 33250A). For our experiments, magnetic field had to be only in one direction such that one pair of coils was used.

Figure 2.1 shows this pair of coils, generating a homogeneous field $H$ in the 1.9 cm wide air gap between them.

To be able to measure at magnetic fields of up to 90 mT it is important to make this air gap as small as possible, so that the tuning fork on a sample holder and the Hall probe just fit in. The Hall probe measures the magnetic flux density $B$ and regulates the current flowing through the coils. Therefore the tuning fork and the Hall probe should be positioned in the gap as close as possible to ensure a field measurement right at the position of the fork.

We detect the frequency response of the tuning fork by applying an alternating voltage. A lock-in connected to the other electrode of the tuning fork amplifies and measures the current flowing through it.
The lock-in technique yields a good signal-to-noise ratio, allowing to detect small currents (nA range) in short measurement times. To understand how current flows through the fork and is detected by the lock-in amplifier it is helpful to consider the set-up in the electrical model given in Sect. 1.1.2. In Fig. 2.2, the electrical model of the measuring arrangement is shown. The tuning fork can be replaced by the Butterworth-Van-Dyke circuit, the frequency generator by an alternating voltage source and the lock-in amplifier can be seen as a current measurement device. As discussed in Sect. 1.1.2, the impedance of the tuning fork $Z(f) = 1/Y(f)$ is a function of $f$. Assuming that the resistance $R$ in a current measurement device is small, the current $I$ flowing through the fork is given by

$$I(f) = \frac{U(f)}{R} = \frac{U_F}{Z(f) + R} \approx \frac{U_F}{Z(f)} = Y(f) \cdot U_F$$  \hspace{1cm} (2.1)$$

where $U_F$ is the voltage applied by the frequency generator as shown in Fig. 2.2. Since every current measurement device measures a voltage and divide it by a small internal resistance, the lock-in amplifier measures a voltage $U(f)$ and divides it by $R$ to derive the current. Equation (2.1) shows that the current measured in the experiments behaves like the admittance discussed in Sect. 1.1.3 and will have its resonance peak at the eigenfrequency $f_0$ of the Butterworth-Van-Dyke circuit.

---

**Figure 2.2:** Measurement set-up with electrical model. The frequency generator is shown as a low voltage oscillator while the lock-in can be seen as a current measurement device. The tuning fork is replaced by its Butterworth-Van-Dyke equivalent circuit.
Chapter 2 Experimental techniques

2.2 Measurement of the frequency shift

As described in Sect. 1.2, we aim to extract the anisotropy constant of the ferromagnetic specimen by fitting torque magnetometry data with Eq. (1.20). To their end, the frequency shift \( \Delta f(H) \) has to be measured as a function of magnetic field \( H \). We have tested three different techniques of measuring this frequency shift.

2.2.1 Phase tracking

One possibility to measure the resonance frequency shift \( \Delta f(H) \) is to determine the phase \( \Phi(f) \) at the resonance frequency for a given magnetic field \( H_a \). The characteristic phase diagram \( \Phi(f) \) is shown in Fig. 2.3. Upon changing the magnetic field to \( H_b \), the resonance frequency and thus the phase \( \Phi(f_b) > \Phi(f_a) \) also will change. As shown in Fig. 2.4, an algorithm is used to systematically change the frequency in such a way that the phase \( \Phi_b = \Phi_a \) reaches its previous level. Now \( \Delta f \) can be determined by subtracting \( f_a \) from \( f_b \). Repeating this procedure for different fields leads to the required data of \( \Delta f(H) \).
2.2 Measurement of the frequency shift

Accurcay and time of this method is mainly limited by the step size of the tracking algorithm. Increasing the step size leads to a shorter time but also to a worse accuracy. In our measurement a step size of 0.2 Hz is used. This technique takes about 20 to 30 seconds per magnetic field value taking into account that the settling time of the system depends mainly on the time constant of $TC = 300$ ms chosen in the lock-in amplifier. For our measurements 9 time constants and the settling time of the tuning fork itself results in roughly 5 seconds per frequency step.

2.2.2 Sweeping over field and frequency

A second technique to determine the resonance frequency shift is to record $|Y(f)|$ over the full frequency range for every magnetic field value. One frequency sweep with a step size of 0.5 Hz takes about 13 minutes which is more than an order of magnitude larger than the phase tracking approach. The accuracy of this method of about 0.2 Hz can be increased by choosing smaller steps and estimating the resonance frequencies by fitting the single resonant responses with Eq. (1.5).
2.2.3 Measuring the phase shift

The simplest and fastest method for the determination of \( \Delta f \) is to measure the phase shift \( \Delta \Phi \) at a fixed frequency. In Fig. 2.5, the phase shift occurring upon increasing the magnetic field is shown again. However, instead of tracking the frequency, one now simply measures the phase \( \Phi(H_b) \), which yields \( \Delta \Phi = \Phi(H_b) - \Phi(H_a) \).

![Figure 2.5: Exaggerated illustration of \( \Phi(f) \) upon increasing the magnetic field. Instead of tracking the frequency, the phase shift \( \Delta \Phi \) can be measured directly to determine \( \Delta f \). The proportionality of \( \Delta \Phi \) and \( \Delta f \) in the linear region yields \( \Delta f \) as a function of the field \( H \).](image)

To extract the frequency shift \( \Delta f(H) \) from the measured phase shift \( \Delta \Phi(H) \) we take advantage of the linear relation \( f(H) \propto \Phi(H) \) around the fixed frequency \( f_{\text{fix}} \). Fitting the linear region (cf. Fig. 2.5) yields then the slope \( a \) which is the proportionality constant between phase shift and frequency shift:

\[
\Delta f(H) = \frac{\Delta \Phi(H)}{a} \quad (2.2)
\]

The advantage of this method is that the measuring system has to settle only once. Hence, the waiting time per magnetic field value is about 5 seconds. At higher magnetic fields or strong phase drifts due to temperature fluctuations, a non-linear
fitting curve is however required to determine the correlation between phase and frequency shift.
Chapter 2 Experimental techniques

2.3 Technique of loading the tuning fork

For our experiments the tuning forks have to be loaded with different materials. First, the vacuum casing has to be removed. This is done by a lathe or by squeezing the base of the vacuum casing softly with pliers. The latter breaks the material which seals the bottom of the casing and the tuning fork can be pulled out easily. This removes the whole casing. In contrast the lathe only removes the upper part of the casing (Fig. 2.6).

For magnetic materials it is important to mount them on the top of the prong to make sure that the torque exerted by the materials has the strongest possible effect. It is also important to fix them in a rigid way so that the force can be transmitted directly to the prong. A good way to do this is to use a thin layer of UHU instant glue gel. The material is mounted on the side of the prong as shown in Fig. 2.6. It is fixed right next to the gap between the two electrodes (see Fig. 2.7). However the electrodes must not be shorted.

This mounting procedure is done under a microscope by adding a thin layer of glue with a toothpick on top of the prong. After that the sample of interest (in our case mostly iron and nickel but also nonmagnetic materials like copper, wood or glass) can be fixed on the prong.

![Figure 2.6: Different stages of tuning fork preparation and sample mounting while loading it. The rightmost image shows a tuning fork from which the entire magnetic casing has been removed.](image)

![Figure 2.7: Side view of a tuning fork. The area in which the sample will be glued is indicated by the dashed rectangle.](image)
Chapter 3

Results and discussion

In this chapter, the experimental results obtained in this thesis are presented. We first address the impact of sample preparation procedures onto the tuning fork properties. After that we describe how the tuning forks behave in an external magnetic field and address the issues encountered in these experiments. Finally, the anisotropy constant of a nickel wire is examined in torque magnetometry experiments.

We would like to emphasize that the experiments, the loading of different samples and the measurement strategy were developed in close collaboration with Akashdeep Kamra.

3.1 Resonance behaviour of quartz tuning forks

In the following a comparison of the resonance behaviour between the three different tuning fork samples shown in the first three photographs of Fig. 2.6 is given: An undisturbed tuning fork in its vacuum casing (sample #1), a tuning fork whose casing is removed (sample #2) and a tuning fork whose casing is removed and a copper wire is attached on one prong (sample #3). The copper wire is 1 mm long and has a diameter of 0.25 mm. All three tuning forks are of the type 78D202 by Bürklin. While the casing of the latter two samples was removed using a lathe, the first one is still in its original as-purchased state.

Figure 3.1 shows the resonant response of the three samples. With the Lorentzian fitting curve, Eq. (1.5), the resonance frequency $f_0$, the resonance bandwidth of the peak $\Delta f_B$ and therefore the quality factor $Q$ of the oscillator can be determined. These values for the three different samples are summarized in Tab. 3.1.

It is remarkable that the resonance frequency $f_0 = 32763$ Hz of the as-purchased tuning fork is not within the uncertainty range of $32768$ Hz $\pm 50$ ppm quoted by the manufacturer [11]. This deviation could stem the temperature gradients applied to the sample while soldering it on the sample holder.

As one can see in Tab. 3.1, $f_0$ changes by merely 4.1 Hz when the casing is removed. However, since the tuning fork is now oscillating in air, the damping is increased. Accordingly, the amplitude of oscillation changes significantly comparing $f_{0,\#1}$ and $f_{0,\#2}$ to the resonance frequency of the loaded tuning fork, one
Figure 3.1: Comparison of the resonance behaviour of three different tuning fork samples at room temperature. An as-purchased tuning fork in its vacuum casing (black triangles), a tuning fork whose casing is removed (blue squares) and a tuning fork whose casing is removed and a copper wire is mounted on one prong (red circles). The bandwidth $\Delta f_B$ of each resonance behaviour is determined at the $1/\sqrt{2}$-fold of the maximum.

observes that $f_{0,3}$ is shifted. According to Eq. (1.8), $f_0$ is proportional to $\sqrt{k/m}$. Loading one prong with a copper wire of mass $\Delta m$ yields a resonance frequency of $f_1 \propto \sqrt{k/(m + \Delta m)}$. Assuming that the spring constant remains unchanged, a comparison of these frequencies leads to

$$\frac{f_1}{f_0} = \sqrt{\frac{m}{m + \Delta m}}. \tag{3.1}$$

With $m = 0.664 \times 10^{-6}$ kg taken over from [15], $\Delta m = 43.8 \times 10^{-9}$ kg of the copper wire and $f_0 = 32759$ Hz the frequency shift results in

$$\Delta f \approx f_0 \left(1 - \sqrt{\frac{m}{m + \Delta m}}\right) \approx 7 \text{ kHz}. \tag{3.2}$$
3.2 Quartz tuning forks in an external magnetic field

<table>
<thead>
<tr>
<th></th>
<th># 1 as purchased</th>
<th># 2 without casing, unloaded</th>
<th># 3 without casing, loaded</th>
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</thead>
<tbody>
<tr>
<td>$f_0$ (Hz)</td>
<td>32763.3</td>
<td>32759.2</td>
<td>30845.3</td>
</tr>
<tr>
<td>$\Delta f_B$ (Hz)</td>
<td>1.42</td>
<td>3.37</td>
<td>4.34</td>
</tr>
<tr>
<td>Q-factor</td>
<td>22997</td>
<td>9725.7</td>
<td>7113.8</td>
</tr>
<tr>
<td>$A_0$ (nA)</td>
<td>178.9</td>
<td>72.3</td>
<td>44.5</td>
</tr>
</tbody>
</table>

Table 3.1: The relevant values describing the tuning fork’s resonant response. $f_0$ is the resonance frequency, $\Delta f_B$ is the width of the resonant response at the $1/\sqrt{2}$-fold of the maximum, $Q$ is the resulting quality factor described in Sect. 1.1.3 and $A_0$ is the current amplitude at $f_0$.

This is in the same order of magnitude as the measured frequency shift of 1.9 kHz. The bandwidth $\Delta f_B$ increases upon opening or loading the tuning fork. However the greatest bandwidth change by more than a factor of two appears upon removing the vacuum casing. This is explained by enhanced damping attributed to air viscosity. According to Rychen an additional mass of 50 $\mu$g on one prong leads to a quality factor of 839 [15]. For nearly the same mass ($\Delta m = 43.8 \mu$g) a 10 times better quality factor was achieved in our experiments. This can be the slightly different mass of the prong, since different tuning forks are used, but the main effect decreasing the quality factor, is due to the technique of attaching the additional mass onto the prong.

3.2 Quartz tuning forks in an external magnetic field

In order to measure magnetic anisotropy in a ferromagnetic specimen, a magnetic field must be applied. As described above this is realized by inserting a tuning fork into an electromagnet and recording $f_0$ as a function of magnetic field strength. To verify that an observed dependence $f_0(B)$ stems from the magnetic specimen attached on the fork, it is useful to estimate the expected $\Delta f(B)$.

3.2.1 Estimation of the expected frequency shift

With Eq. (1.19), the frequency shift to be expected in the following experiments can be estimated. Using the values for the shape anisotropy of $K_u = 130$ kJ/m$^3$ [18] and $M_S = 1.7 \times 10^6$ A/m [18] for an iron wire of length $L_e = 2.8 \times 10^{-3}$ m [11], volume $V = 0.1$ mm$^3$ and the properties of the tuning fork $k = 2.22 \times 10^4$ N/m and $f_0 = 32768$ Hz, one obtains the $f_0(B)$ curve shown in Fig. 3.2 a).
At a magnetic field of $B = 90$ mT, the maximum field available in our experiments, the shift of the tuning fork resonance frequency is $\Delta f \approx 0.9$ Hz, which should be easily detectable.

While Eq. (1.20) is true for magnetic fields for which the magnetisation is saturated, a description of the hysteresis behaviour at low fields can become complicated due to domains formations. Figure 3.2 b) shows a torque magnetometry measurement using a cantilever taken from Stipe et al. [1]. In this experiment a hysteresis behaviour of a magnetic nanowire at low fields was observed. Since our set-up is analogous to Stipe’s with a tuning fork instead of a cantilever, our measurements should behave in a similar manner and show a hysteresis at lower fields and a saturation at higher fields.

Figure 3.2: a) Calculation of the expected tuning fork resonance frequency shift for an iron wire. The calculation is based on Eq. (1.19) using values quoted in the text. b) Measured frequency shift in a torque magnetometry experiment using a cantilever by Stipe et al. [1], showing the magnetic hysteresis behaviour at low fields.

3.2.2 Tuning fork loaded with iron wire

Now, the tuning fork resonance frequency shift shall be measured with a magnetic specimen. For this a iron wire is glued on one prong of the tuning fork (sample #4). The wire is about 2 mm long and has a diameter of 0.25 mm. The tuning fork is then soldered to a sample holder and inserted into the electromagnet. In the experiments, the magnetic field is directed along the z-axis as it can be seen in Fig. 1.4 and the technique derived in Sect. 2.2.2 is used, where the frequency and the magnetic field are tuned. Figure 3.3 shows the results of this experiment in a false color plot in which the magnetic field sweep is shown on the x-axis and the frequency sweep around the resonance frequency on the y-axis. The current through the tuning fork measured by the lock-in is colour coded.
3.2 Quartz tuning forks in an external magnetic field

Around 28837 Hz one can see a red area, corresponding to the resonance peak of the tuning fork. Upon changing the magnetic field, the resonance peak shifts. Note that the resonance frequency is higher for large magnetic fields, while it has a minimum for small fields. At $B = 0 \text{ mT}$, however there is a local frequency maximum. A drift due to temperature variations is furthermore evident from Fig. 3.3. In order to take data more efficiently and faster, we therefore use the technique detailed in Sect. 2.2.2 in which $\Delta f$ is inferred from $\Delta \Phi$.

Figure 3.3: Tuning fork resonance $I(f)$ as function of external magnetic field strength $B$. The current $I$ through the tuning fork is recorded upon excitation at frequency $f$, and shown in false colour.

Figure 3.4 a) shows the result of this method for the same iron wire-loaded tuning fork. To reduce noise 30 frequency sweeps have been taken and averaged for Fig. 3.4 a). Furthermore a correction for temperature drift was done by assuming the drift to be linear upon one magnetic field sweep and subtracting it from the measured data. In contrast to the data in Fig. 3.3, a magnetic field up and down sweep was measured to investigate whether a hysteresis behaviour is present in the iron wire-loaded tuning fork.

As shown in Fig. 3.4 a), there is a clear magnetic field dependence of the resonance
Chapter 3 Results and discussion

frequency. Like in Fig. 3.3, a strong frequency shift can be observed for large fields whereas a minimum for lower fields and a local maximum for zero field is found. There is also a hysteresis which saturates at 60 mT. However this behaviour differs from the expected one in which no maximum should occur at zero field and the slope for higher fields should decrease.

3.2.3 Tuning fork loaded with non-magnetic wire

To cross-check the data recorded on the iron wire-loaded tuning fork, in a more detailed way, we also have studied tuning forks with non-magnetic materials on the prong. It is appropriate to take a copper wire instead of an iron wire because it is diamagnetic, stiff and can be mounted in an identical way. Because of the higher density of copper ($\rho_{\text{Cu}} = 8.92 \text{ g/cm}^3$ [19] as compared to $\rho_{\text{Fe}} = 7.87 \text{ g/cm}^3$ [19]) a 1 mm piece with a diameter of 0.25 mm was mounted on the prong (sample #3). Figure 3.4 shows a comparison between the copper and iron wire-loaded tuning fork is made. Both behave similarly for high magnetic fields while sample #4 has a more than two times stronger magnetic field dependence of $f_0(B)$ than sample #3. In addition the iron sample shows a hysteresis, while the hysteresis for the copper sample is much weaker but still there.

![Graph showing frequency shift vs. magnetic field for tuning forks with Fe and Cu wires](image)

Figure 3.4: A comparison between the frequency behaviour of (a) the iron wire-loaded tuning fork (sample #4) and (b) the copper wire-loaded tuning fork (sample #3) is shown. The resonance frequencies of both tuning forks are depending on the magnetic field.

This is remarkable since copper is diamagnetic and should not exert a torque on the prong of the tuning fork. Therefore sample #3 should exhibit no $B$ dependence and also no hysteresis behaviour. The fact that sample #3 shows a similar behaviour
as sample #4 suggests that not the magnetic torque of magnetic samples, but something else causes the ‘W’-shaped frequency shift behaviour of the tuning fork. Summarizing, one can say that there are two contributions to the measured frequency shift data. Firstly, the ‘W’-shaped magnetic field dependence which is there for tuning forks loaded with both magnetic and non-magnetic material. Secondly, the hysteresis behaviour, which appears less pronounced in sample #3.

3.2.4 Reasons for magnetic field dependent tuning fork resonance frequency

3.2.4.1 The role of asymmetric loading

One possible reason for the ‘W’-behaviour evident from Fig. 3.4 is an asymmetric loading of the tuning fork: one prong is loaded with wire, the other not. To verify this hypothesis we tried to load the tuning fork as asymmetrically as possible. One way to do that is to remove one prong, e.g. using a saw with a diamond blade. The „one-prong“tuning fork was then inserted into electromagnet as before and \( f(B) \) recorded using the technique of Sect 2.2.3. Another simple way to increase asymmetry is to prevent one prong from vibrating freely, by pushing it softly to a rigid material such as the Hall probe or a magnetic pole shoe.
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Figure 3.5: Comparison of the strength of the ‘W’-behaviour for different samples. The highly asymmetric samples (red circles and blue squares) show the strongest magnetic field dependence. The slightly asymmetric iron wire-loaded tuning fork (sample #4, black triangles) which was discussed above is also shown and has a clearly weaker magnetic field dependence. The most symmetrical sample, an unloaded tuning fork which is represented in orange diamonds shows nearly no field dependence. For a better visualization, the sample #4 data are offset by -10 Hz, while the unloaded sample is offset by -12.5 Hz.

In Fig. 3.5 the influence of asymmetric loading is shown. For this a comparison is made between the highly asymmetric samples (the “one-prong” sample and the sample with a fixed prong), sample #4 and an unloaded tuning fork. The latter shows a small magnetic field dependence in the order of 10 mHz. In contrast, the two asymmetric samples show a pronounced ‘W’-behaviour, about 1 order of magnitude larger than sample #4. The asymmetric samples moreover show a hysteresis behaviour which is more pronounced for the sample with only one prong. Thus, the ‘W’-behaviour clearly is due to asymmetric loading.

One possibility to reduce the asymmetry but still load a tuning fork with magnetic material is to load both prongs equally. Figure 3.6 shows a comparison of the already discussed Cu wire-loaded tuning fork (sample #3) and another tuning fork (sample #5) loaded with two 2 mm-long Cu wires which have a diameter of 0.13 mm and thus has the same additional mass \( \Delta m \) as sample #3.
3.2 Quartz tuning forks in an external magnetic field

The magnetic field dependence is clearly smaller for the symmetrical sample #5, but is still discernible. The reason for this is probably some left-over asymmetrical loading, stemming from two wires with slightly different mass and the different amount of glue used to fix the wires on the prongs. The two prongs are therefore not loaded equally, which may cause a small magnetic field dependence. In summary, these experiments show that loading the tuning fork in an asymmetrical way leads to an unwanted magnetic field dependence, which is not related to the torque exerted by a magnet on the tuning fork. Until now it is not understood why an asymmetry causes the observed 'W'-behaviour. However, the latter can be avoided by loading the fork in a symmetrical way. Unfortunately, this technique has only an influence on the 'W'-behaviour and not on the hysteresis observed in tuning forks loaded with non-magnetic samples.

3.2.4.2 The role of the magnetic casing

We now address the hysteresis in the $f_0(B)$ behaviour of tuning forks loaded with non-magnetic material. To this end an unloaded tuning fork is inserted into a strongly inhomogeneous magnetic field of a permanent magnet. This shows that the tuning fork itself is non-magnetic, while the base and the legs (see Fig. 3.7) are magnetic.
Figure 3.7: Tuning fork without casing. Only the base and the legs are made of a magnetic material.

To check whether the hysteresis behaviour stems from the magnetic material of the base and the legs, both parts are removed using the method described in Sect. 2.3. Instead of the original legs, two copper wires with approximately equal stiffness are soldered on the electrodes of the tuning fork to allow for electrical measurements. In Fig. 3.8 a) one can see the Cu wire-loaded tuning fork (sample #3) already discussed in context of Fig. 3.4 compared to an equally loaded tuning fork (sample #6) without magnetic base and legs. Clearly, the base and legs of the original TF casing are responsible for hysteresis and also the 'W'-behaviour. This shows that the base and the legs of the tuning fork have a strong influence on the $f_0(B)$ behaviour in magnetic field. Why the resonance frequency shifts as a function of $B$ is not understood. It is conceivable that the base exerts stress on the fork which is dependent on the magnetic field. A simulation of the tuning fork would be helpful to give more insight into these effects.
Figure 3.8: Comparison between samples with and without magnetic base and legs. Panel a) shows the already discussed sample #3 (tuning fork loaded with copper wire) in comparison to a similarly loaded sample without base and magnetic legs. In panel b) sample #4 (tuning fork loaded with iron wire) is compared to a similar loaded sample without base and magnetic legs. The ‘W’-behaviour and stray hysteresis are due to the magnetic base.

In Fig. 3.8 b) the iron wire-loaded tuning fork without magnetic base is compared to the equally loaded sample #4 with base. Like for the copper-loaded tuning forks the magnetic field dependence changes significantly when the original base and legs are removed. In the tuning fork without base and legs, the measured shift at 90 mT is about $\Delta f = 0.4$ Hz, which compares well to the frequency shift calculated in Sect. 3.2.1 of $\Delta f = 0.55$ Hz at 90 mT.

Note also that for tuning forks without base, the experimental data is more noisy due to the reduction in quality factor. In fact, the base is specially designed to reduce energy dissipation. For instance the resonant response of a copper wire-loaded tuning fork without base has a small current amplitude of about $A_0 = 18.5$ nA, compared to the amplitude of a similarly loaded sample #3 with base with $A_0 = 44.5$ nA. As a result the quality factor of the tuning fork without base decreases by a factor of about 3, which shows that the base is important for reducing energy dissipation (see Fig. 3.9).
Figure 3.9: Comparison of the resonance behaviour between asymmetrically loaded tuning forks without a) and with b) magnetic base. For the sample without base (sample #5) the quality factor decreases by a factor of about 3 compared to the equally loaded sample with base (sample #3).

A custom made tuning fork with a base consisting of non-magnetic material would lead to better energy trapping and thus to a higher quality factor. To our knowledge, such tuning forks are not available yet. However, since the sensitivity of tuning forks without base is high enough, removing the base and minimizing the asymmetry is the easiest way to torque magnetometry, as discussed in the following.

### 3.3 Torque magnetometry on a nickel wire

After having established the tuning fork preparation and sample mounting procedure, we now present torque magnetometry on a nickel wire. We use nickel since this material has a lower $M_S$ and lower anisotropy than iron, such that it should be easier to saturate nickel in the magnetic field range available in experiment. To achieve as symmetrical loading as possible, two nickel wires (2 mm in length and 0.25 mm in diameter) are mounted on one tuning fork, one wire on each prong. Obviously, we use a tuning fork from which the base and the legs are removed and two non-magnetic copper wires are soldered to the base instead.
3.3 Torque magnetometry on a nickel wire

Figure 3.10: Torque magnetometry on a tuning fork loaded symmetrically with Ni wire. The fitting curve according to Eq. (1.20) is shown as a black line.

Figure 3.10 shows the magnetic field dependence of the resonance frequency shift $\Delta f_0(B)$ of this sample. The frequency shift from 0 mT to 90 mT is $\Delta f = 1.06 \pm 0.02$ Hz. Moreover $\Delta f_0(B)$ shows hysteresis between about $-30$ mT and $30$ mT. As described in Sect. 1.2, the data can be fitted with Eq. (1.20). Since this fitting curve is applicable only in the saturated magnetisation regime, we only fit Eq. (1.20) in the field range from 30 mT until 90 mT and from $-30$ mT to $-90$ mT. In this regime the hysteresis behaviour has a negligible influence. Since we recorded both up and down sweep, it is possible to determine four independent values for the anisotropy field which can be averaged afterwards. The four values determined by the fit are given in Tab. 3.2. The average value of the anisotropy field is $B_k = 0.15 \pm 0.02$ T whereby the uncertainty corresponds to error propagation. As a result the uniaxial anisotropy constant can be determined using the saturation magnetisation $\mu_0 M_S = 0.61$ T \cite{18} of bulk nickel, which yields $K_u = B_k M_S / 2 = 35 \pm 4$ kJ/m$^3$.

Since the nickel wire is poly-crystalline there is no magnetocrystalline anisotropy and thus for wires with infinite length the demagnetisation field $B_{\text{demag}}$ is the anisotropy field $B_k$. The demagnetisation field is given by \cite{20}

$$B_k \approx B_{\text{demag}} = \frac{1}{2} \mu_0 M_S N = 0.153 \text{ T.} \tag{3.3}$$

Here, $N = 0.5$ is the demagnetization factor perpendicular to the long wire axis. The measured anisotropy field corresponds well to the the demagnetization field.
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<table>
<thead>
<tr>
<th>$B_k$ (T)</th>
<th>up sweep, negative $B$</th>
<th>up sweep, positive $B$</th>
<th>down sweep, negative $B$</th>
<th>down sweep, positive $B$</th>
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<td></td>
<td>$0.12 \pm 0.03$</td>
<td>$0.15 \pm 0.03$</td>
<td>$0.18 \pm 0.05$</td>
<td>$0.13 \pm 0.03$</td>
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Table 3.2: Values for the anisotropy field $B_k$ of nickel wire. For positive and negative fields, up and down sweeps are fitted with Eq. (1.20).

Nevertheless, it should be noted that for fitting only 30 data points were available and the area in which hysteresis takes place can only be estimated. To get more reliable results it is indispensable to measure up to higher magnetic fields, up to the range of Tesla, in order to obtain more data points and a quantitative value for the anisotropy.
Chapter 4
Conclusions and Outlook

This thesis examines the possibility of torque magnetometry based on commercially available quartz tuning forks. In a first set of experiments the vacuum casing in which the as-purchased tuning fork are enclosed was removed. It was observed that the quality factor of the tuning fork decreases by a factor of 2 from about 22997 to 9726 due to enhanced damping attributed to air. Attaching a copper wire with a mass of about $44\,\mu g$ to one of the tuning forks prongs decreases the quality factor once more to 7114. Compared to another tuning fork experiment of Rychen et al. [15], attaching a similar mass on one prong, the quality factor of our loaded tuning fork is about 1 order of magnitude larger.

In a second series of measurements tuning forks loaded with ferromagnetic specimens were inserted into an external magnetic field $H$. The ferromagnet (vibrating in $H$ since the tuning fork is driven into resonant oscillations) exerts a magnetic torque onto the tuning fork, which depends on the magnitude of $H$. It thus induces a shift $\Delta f$ of the resonance frequency of the tuning fork depending on $B = \mu_0 H$. We experimentally recorded $\Delta f$ and showed that not only the magnetic torque, but two other properties of the tuning fork have a significant influence on the measured $\Delta f$.

Firstly, by loading the tuning fork asymmetrically i.e., one prong differently from the other, a strong magnetic field dependence even for a non-magnetic specimen was observed. By loading the tuning fork in a symmetrical way, this effect could be reduced by a factor of 8.

Secondly, a spurious magnetic field dependence was found which is related to the magnetic material onto which the as-purchased tuning forks are mounted. By removing this base we showed that the magnetic field dependence disappears for tuning forks loaded with non-magnetic material while tuning forks loaded with ferromagnetic material exhibit a magnetic field dependence of the resonance frequency. However, the tuning fork’s quality factor decreases by a factor of 3 when the commercial base is removed.

Using the established method of loading the tuning fork symmetrically and removing the magnetic base, the torque magnetometry signal of a 2 mm long and 250 $\mu m$ diameter nickel wire was investigated. Fitting the magnetic field dependent
resonance frequency to the theoretical expectation for uniaxial anisotropy yields an anisotropy field of $B_k = 0.15 \pm 0.02$ T which corresponds well to the demagnetization field $B_{\text{demag}} = 0.153$ T. However, more detailed experiments will be required to corroborate this first result. As we measured from $B = -90$ mT to $B = 90$ mT we would propose to increase the magnetic field range up to $B = \pm 1$ T in order to enable an accurate fit.

For the future, several interesting questions remain: Since we measured the torque only for one magnetic field direction ($H \parallel$ long wire axis, see Fig. 4.1), determining the anisotropy as a function of the angle between tuning fork and magnetic field will be another important future step. Furthermore it is conceivable to attach thin ferromagnetic films instead of macroscopic ferromagnetic wires onto the tuning fork.

![Figure 4.1: Theoretical estimation of the frequency shift behaviour versus thickness of a thin nickel film attached on the prong of a tuning fork at a constant magnetic field of $B = 1$ T. The red line illustrates the nickel layer mounted on the tuning fork Buerklin 78D202 while a smaller type (Micro Crystal MS1V-10) was used to calculate the blue line. A minimum detectable frequency shift of $\Delta f = 1$ mHz is considered. Thus, magnetic properties can be determined for nickel layers of a minimal thickness of 150 nm for the Buerklin and 25 nm for the Micro Crystal tuning fork.](image)

For this, a rough estimation of the minimum thickness of a nickel layer, at which it is still possible to measure anisotropy is given. According to Eq. (1.19), $\Delta f$ is proportional to the volume $V$ and hence to the thickness $d_{\text{Ni}}$ of the nickel layer. This dependence $\Delta f \propto d_{\text{Ni}}$ at a magnetic field of $B = 1$ T is illustrated in Fig. 4.1.
where the red line indicates the estimate of the tuning fork mainly used in this thesis (Bürklin 78D202) and the blue line shows the calculation for a smaller tuning fork (Micro Crystal MS1V-10).

According to measurements with an unloaded tuning fork, a frequency shift of 1 mHz has a signal to noise ratio (S/N ratio) of roughly 4 to 5 at a magnetic field of 90 mT. Assuming that a thin layer does not affect this S/N ratio, and that averaging over more data points improves it, the minimal thickness of the nickel film on a Bürklin tuning fork is about $d_{\text{Ni}} = 150$ nm. Using the smaller Micro Crystal tuning fork, $\Delta f$ gets larger and thus measuring the anisotropy of layers of $d_{\text{Ni}} = 25$ nm should be possible. In this estimation a saturation magnetisation of $\mu_0 M_S = 0.153$ T \cite{18} and a uniaxial anisotropy constant of $K_u = 37$ kJ/m$^3$ \cite{20} of nickel was used. Furthermore the areas of the thin layers are assumed to be $A_{\text{Buerklin}} = 1.1$ mm$^2$ and $A_{\text{Micro Crystal}} = 0.3$ mm$^2$.

As a conclusion of these considerations, torque magnetometry based on quartz tuning forks as already now introduced in this thesis enables a simple and fast way to determine magnetic properties of macroscopic ferromagnets such as $0.1$ mm$^3$ of nickel. According to our estimations, it should be also possible to investigate thin films down to few hundred nanometers in thickness, such that this technique should be applicable in research on thin film and nano structure physics.
Appendix A

Comparison of the admittance to a typical Lorentzian

The following equation shows a typical Lorentzian.

\[ H(f) = \frac{A_0 \cdot \Delta f_B (\Delta f_B - 2i(f - f_0))}{\Delta f_B^2 + 4(f - f_0)^2} + iP \]  

(A.1)

Comparing this Lorentzian to Eq. (1.5) leads to

\[ A_0 = \frac{2\pi f_0^2 C}{\Delta f_B} = \frac{1}{R} \]  

(A.2)

\[ \Delta f_B = 2\pi f_0^2 C R = \frac{R}{2\pi L} \]  

(A.3)

\[ f_0 = \frac{1}{2\pi \cdot \sqrt{LC}}. \]  

(A.4)

Taking into account that \( L \propto m \), \( C \propto 1/k \) and \( R \propto \gamma \) this yields

\[ A_0 \propto \frac{1}{\gamma} \]  

(A.5)

\[ \Delta f_B \propto \frac{\gamma}{m} \]  

(A.6)

\[ f_0 \propto \sqrt{\frac{k}{m}} \]  

(A.7)
Bibliography


Bibliography

[10] KARRAI, K.: *Lecture notes on shear and friction force detection with quartz tuning forks*. Center for NanoScience, Sektion Physik der Ludwigs-Maximilians-Universität München, 2000 [http://www.nano.physik.uni-muenchen.de/publikationen/Preprints/p-00-03_Karrai.pdf](http://www.nano.physik.uni-muenchen.de/publikationen/Preprints/p-00-03_Karrai.pdf)


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