Circularly polarized microwaves for magnetic resonance experiments

Master’s Thesis
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Introduction

Magnetic resonance is a well established method for assessing the spin degree of freedom of paramagnetic centers. Electron paramagnetic resonance (EPR) has been traditionally used to investigate the response of materials with unpaired electrons to applied microwave magnetic fields in physics, chemistry, biology, medicine, etc. Up to the present, much efforts have been made to develop and extend this technique to higher frequencies, increased sensitivity and circularly polarized microwaves. High frequency EPR is used for the detection of subtle spectroscopic details. Buldil et al. conducted EPR experiments with frequencies as high as 250 GHz and were able to reveal the microscopic motion of spin probes [1]. Recently, even frequencies in the quasi optical Terahertz range have been accessible that allow EPR signals from slightly different g-factors to be separated. Boero et al. used a microcoil–based probe for EPR on a micrometer sized sample and achieved a high spin sensitivity of $10^{10}$ spins/GHz$^{1/2}$ at the relatively low operating frequency of 1.4 GHz [2]. This higher sensitivity can be used for EPR hardware and biomolecules with atomic scale resolution.

Circularly polarized microwaves are also used for spin-based structures in quantum information and computation applications. For example, the controlling the microwave stimuli could be used to efficiently select different spin transitions. However, most EPR experiments to date are conducted with linearly polarized rf–magnetic fields. These rf–fields are composed of one circularly polarized microwave field inducing spin state transitions from ground state to excited state, and its oppositely rotating field component which is ineffective for these transitions. In other words, the sign of the g–factor, which indicates the precessional direction of the spins, can be determined. For the typical ferromagnetic material, the gyromagnetic ratio $\gamma$, which is proportional to the g–factor, is a constant and substance–specific value. For a ferrimagnetic material composed of multiple magnetic elements and thus multiple magnetic sublattices (spin species), the effective g–factor contributed by the magnetization in each sublattice varies depending on the temperature, and thus the investigation of the g–factor is critical for understanding of the temperature–dependent magnetization behavior. Eshbach and Stranberg [3] discussed several types of extrinsic circularly polarized microwaves. Hollow metallic waveguides, whose shape is either square or circular, allow two types of the microwaves to propagate, which are orthogonal degenerate but identical modes. When two microwaves are 90$^\circ$ out of phase in time with each other, then the superimposed microwaves can be circularly polarized. Diaz et al. [4] designed resonator cavities, which excite such modes, and thus
attained a high degree of circular polarization at a fixed frequency. These schemes for creating the circularly polarized microwaves are called extrinsic since one linearly polarized microwave component is physically separated from the other. On the other hand, in the intrinsic polarization scheme, parts of the propagating microwaves are already circularly polarized: At a position off center of the waveguide axis through its broad face, the magnetic fields of traveling microwaves appear circularly polarized [5]. Henderson et al. [6] constructed a microstrip cross resonator, composed of two half–wavelength microstrip line resonators. The two independent microwave stimuli were applied to the cross resonator orthogonally but were 90° out of phase, such that circularly polarized microwaves were generated on top of the crossing point. At low temperatures, an 82% fidelity circularly polarized microwave radiation field was achieved.

In this thesis, we create circularly polarized microwaves using two approaches, (i) with a rectangular waveguide and (ii) with a cross resonator. We compare these intrinsic and extrinsic schemes by studying the microwave properties. The key goal is to establish circularly polarized microwaves in order to enable the sign of the g–factor to be determined in our institute. The thesis is organized as follows: To begin with, we give overviews of the theoretical explanation about the electron spin resonance and ferromagnetic resonance in Chapt. 1. For understanding the motion of the magnetization under influence of microwave magnetic fields, the Landau–Lifshitz–Gilbert equation is introduced, and the response of the magnetization vector to polarized microwaves is considered. In Chapt. 2, we start with an explanation of the device dimensions which determine the microwave properties for both methods. In this thesis, the magnetic field component of the polarized microwaves is important to determine the sign of the g–factor, hence we illustrate the experimental set–up with defining the microwave polarity. With a finite element method software, transmission and reflection of microwave radiation are simulated for the cross resonator, in order to distinguish the resonant modes of the microwaves, and for the rectangular waveguide in order to investigate the polarity of the microwaves in the transverse electric mode. In Chapt. 3, we concentrate on the analysis of the susceptibility which describes the magnetization response to the polarized microwaves. For the cross resonator, the microwave polarity dependence of the susceptibility is investigated. For the rectangular waveguide, the magnetization response with varying the frequency is surveyed at a fixed sample position. With utilizing the frequency dependence of the line width of the magnetic resonance, the damping constant of the sample is also calculated using transmission and reflection of the microwaves. In order to characterize the cross resonator and the rectangular waveguide for the creation of the circularly polarized microwaves, the polarity, accuracy, sensitivity, etc. for both devices are compared in Chapt. 4. We also propose possible improvements from simulations and measurement results. As a conclusion, we summarize our studies and give an outlook for the future experiments to utilize the devices investigated in this thesis effectively in Chapt. 5.
Chapter 1

Theory

This chapter presents a description of magnetic resonance. We aim at clarifying the role of circularly polarized radiation and at providing a model for the analysis of the complex resonant microwave absorption. We first present the quantum mechanical single spin model usually employed for electron paramagnetic resonance. For the paramagnet, the resonance is explained by the Zeeman splitting of a single electron spin in an external magnetic field. To excite such a resonance, not only the energy of the applied microwaves needs to be equal to the Zeeman energy splitting but also the radiation polarization needs to satisfy the angular momentum conservation. We then turn to a classical model of the macroscopic magnetization dynamics in an external field and introduce the Landau–Lifshitz–Gilbert (LLG) equation for ferromagnetic resonance (FMR). There, we also find similar selection rules as for the quantum mechanical approach and relate the susceptibility to the induced voltage (i.e., the FMR detection signal). Finally, we deduce the resonance frequency for ferromagnets which needs to include magnetic anisotropies.

1.1 Electron paramagnetic resonance (EPR)

Electron paramagnetic resonance [7] is a widely used spectroscopy technique for the characterization of magnetic systems. In a paramagnetic material, the magnetic moments of two nearby atoms do not interact with each other strongly. This justifies a simple quantum mechanical model which is only concerned with a single isolated spin. In paramagnets the spin of an atom is due to unpaired electrons. Every electron has spin quantum number $s = 1/2$, and thus a magnetic moment. For simplicity, we here only consider atoms with one unpaired spin, such that the total spin is $S = \hbar/2$ where $\hbar$ is the reduced Plank constant. By application of an external static magnetic field $\mu_0H_0$, the two degenerated states $m_s = +1/2$ and $m_s = -1/2$ split into a Zeeman doublet as illustrated in Fig. 1.1a. For a positive $g$–factor $g$, the applied magnetic field increases the energy of the spin–up ($m_s = +1/2$) quantum state where the spin is aligned parallel to the magnetic field, and lowers the spin–down ($m_s = -1/2$) quantum state [7]. The energy
difference is given by
\[
\Delta E = E(m_s = +1/2) - E(m_s = -1/2) = g\mu_B \Delta m_s \mu_0 H_0, \tag{1.1}
\]
where \(\mu_B\) is the Bohr magneton. \(\Delta m_s\) is the difference between the magnetic quantum numbers of the states in a multiplet. According to the selection rules \([8]\) for dipole radiation, allowed transitions are described by
\[
\Delta m_s = 0, \pm 1. \tag{1.2}
\]
In order to introduce such a transition, microwaves, whose energy is equal to the energy splitting \(\Delta E\) in Eq. 1.1, needs to be applied. The resonance condition for EPR thus is
\[
h\omega = g\mu_B \Delta m_s \mu_0 H_0, \tag{1.3}
\]
where \(\omega\) is the angular frequency of the microwaves. For the single spin 1/2, \(\Delta m_s = +1\) \(^1\), so that the frequency of the microwaves has to fulfill the condition \(\omega_L = g\mu_B\mu_0 H_{res}/\hbar\), called Larmor frequency, where \(\mu_0 H_{res}\) is resonant magnetic field. During the transition from the ground states to the excited states, the angular momentum of the electron changes from \(m_s = -1/2\) to \(m_s = +1/2\). In order to conserve the total angular momentum of the system, this angular momentum needs to be provided by the photon, the quanta of the microwave. The resulting angular momentum of the spin is then equal to the vector sum of its initial angular momentum and the angular momentum of the absorbed photon (see Fig. 1.1b). Note that photons transfer the spin angular momentum \(|S| = \hbar\), hence the transitions described in Eq. 1.2 is restricted to \(\Delta m_s = +1\), and thus only either of the spin states \(m_s = \pm 1/2\) of photons can excite the spin system.

1.2 Motion of the magnetization

For a single spin, the absorption of the microwaves gives rise to a flip of the spin. In EPR, not only one but a whole ensemble of spins is under investigation. In order to describe a many body system, the magnetization \(\mathbf{M}\), the vector sum of the magnetic moments per unit volume, is defined. When a constant magnetic field \(\mu_0 \mathbf{H}_0\) is applied to the magnetization at an angle \(\theta\), the magnetization will experience a torque and start precessing. This so called Larmor precession can be described in terms of the Landau–Lifshitz equation \([9]\) (Fig. 1.2a), described by
\[
\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mu_0 \mathbf{H}_0, \tag{1.4}
\]
\(^1\)\(\Delta m_s = 0\) viz. \(\Delta m_s = -1\) are not taken into account since in general the state of the lower energy has the greater population of spins.
1.2 Motion of the magnetization

![Diagram](image)

**Figure 1.1:** (a) Zeeman energy splitting of a spin $1/2$ with respect to the magnetic field. At the resonant field $\mu_0H_{\text{res}}$, the spin on the ground state is excited by the microwaves whose energy is exactly matching with the energy split $\Delta E = g\mu_B \Delta m \mu_0 H_{\text{res}} = g\mu_B \mu_0 H_{\text{res}}$. (b) Conservation of angular momentum during the absorption of a photon in this system. The resulting angular momentum of the spin is then equal to the vector sum of its initial angular momentum plus the angular momentum of the absorbed photon.

where $\gamma$ is the gyromagnetic ratio:

$$\gamma = \frac{g\mu_B}{\hbar}.$$  

(1.5)

Note that the value of $g$ is substance-specific, for example, the g–value of electron is $g = -2.0023$ [7]. The sign of the g-factor determines the precession direction. For $g > 0$, the magnetization precesses in the counter-clockwise direction and $g < 0$, the clockwise direction when viewed anti–along the magnetic field direction [5] (Fig. 1.2b).

![Diagram](image)

**Figure 1.2:** (a) Schematic image of Larmor precession. (b) Magnetization precesses around the magnetic field. The magnetization rotates in the counter counter-clockwise direction for $g > 0$ and clockwise for $g < 0$

In Eq. 1.4, the magnetization (once excited) precesses infinitely around the magnetic field. However, these oscillations of the magnetization are inevitably accompanied by energy relaxation. To take into account the relaxation, Gilbert [10] added a phenomenological
damping to the Landau–Lifshitz equation, which then is referred to as the Landau–Lifshitz–
Gilbert (LLG) equation [11]:

$$\frac{dM}{dt} = -\gamma (M \times \mu_0 H) + \alpha \frac{M}{|M_0|} (M \times \frac{dM}{dt}),$$  

(1.6)

where $\alpha$ is the dimensionless (Gilbert) damping constant and $M_0$ is the equilibrium magnetization. In Eq. (1.6) the damping constant $\alpha$ describes how precession in a system decays (see Fig. 1.3a).

Now, in order to solve the LLG equation analytically, let us consider that the rf–magnetic field $\mu_0 h(t)$ acts as a time dependent small perturbation on the equilibrium magnetization $M$, resulting in dynamic components $m(t)$. Magnetic field and magnetization are separated in static and time dependent components

$$M = \begin{pmatrix} m_x(t) \\ m_y(t) \\ M_0 \end{pmatrix} = m(t) + M_0, \quad \mu_0 H = \mu_0 \begin{pmatrix} h_x(t) \\ h_y(t) \\ H_0 \end{pmatrix} = \mu_0 h(t) + \mu_0 H_0. $$  

(1.7)

where $M_0$ is the static magnetization and we have assumed $|m(t)| \ll |M_0|$ and $|h(t)| \ll |H_0|$. For a harmonic precession $m(t) \propto e^{\omega_0 t}$, one has $d_m(t)/dt = i\omega m(t)$, and inserting Eq. (1.7) into Eq. (1.6) results in

$$i\omega m_x + (\omega_0 + \frac{M_0}{|M_0|} \alpha \omega) m_y = \gamma M_0 \mu_0 h_y,$$

$$-(\omega_0 + \frac{M_0}{|M_0|} \alpha \omega) m_x + i\omega m_y = \gamma M_0 \mu_0 h_x$$  

(1.8)

where $\omega_0 = \gamma \mu_0 H_0$. In the paramagnets, the magnetization points along the external magnetic field. With the sign factor $\eta_H$, which formally corresponds to the sign of the external magnetic field, we define $M_0/|M_0| = \eta_H$. In matrix form, the equations above are written as

$$m(t) = \frac{\gamma M_0}{(\omega_0 + \frac{M_0}{|M_0|} \alpha \omega)^2 - \omega^2} \begin{pmatrix} \omega_0 + \frac{M_0}{|M_0|} \alpha \omega & i\omega \\ -i\omega & \omega_0 + \frac{M_0}{|M_0|} \alpha \omega \end{pmatrix} \mu_0 h(t)$$

$$= \begin{pmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{pmatrix} \mu_0 h(t) = \tilde{\chi} \mu_0 h(t).$$  

(1.9)

where the $\tilde{\chi}$ is the complex susceptibility tensor. Note that the matrix components of the susceptibility fulfill the relations $\chi_{11} = \chi_{22}$ and $\chi_{12} = -\chi_{21}$. $\chi_{11}$ and $\chi_{12}$ separated in real
and imaginary part are described as

\[ \chi_{11} = \frac{\gamma M_0 \omega_0 (\omega_0^2 - \omega^2 (1 - \alpha^2))}{(\omega_0^2 - \omega^2 (1 + \alpha^2))^2 + 4\omega_0^2 \omega^2 \alpha^2} - i \frac{\gamma M_0 \eta_H \omega (\omega_0^2 + \omega^2 (1 + \alpha^2) \alpha)}{(\omega_0^2 - \omega^2 (1 + \alpha^2))^2 + 4\omega_0^2 \omega^2 \alpha^2} \]

\[ \chi_{12} = \frac{2\gamma M_0 \eta_H \omega \omega^2 \alpha}{(\omega_0^2 - \omega^2 (1 + \alpha^2))^2 + 4\omega_0^2 \omega^2 \alpha^2} + i \frac{\gamma M_0 \omega (\omega_0^2 - \omega^2 (1 + \alpha^2))}{(\omega_0^2 - \omega^2 (1 + \alpha^2))^2 + 4\omega_0^2 \omega^2 \alpha^2}. \] (1.10)

The real part and imaginary part of \( \chi_{11} \) and \( \chi_{12} \) are depicted for \( g > 0 \) and \( g < 0 \) in Fig. 1.3b. Note that \( \omega_0 = \gamma \mu_0 H_0 \) is used to describe the susceptibility as a function of external magnetic field \( \mu_0 H_0 \). The susceptibility is the dimensionless proportionality constant between the magnetization and magnetic field. It contains the information about the response of a magnetization to an external time varying magnetic field. Around the resonant field \( \mu_0 H_{\text{res}} = \pm |\omega/\gamma| \) expressed by Larmor precession frequency, the real part of \( \chi_{11} \) and the imaginary part of \( \chi_{12} \) consist of a dispersion-like function \( D(\mu_0 H_0) \), while the real part of \( \chi_{11} \) and the imaginary part of \( \chi_{12} \) are composed of an absorption-like function \( A(\mu_0 H_0) \). For reversal of the magnetic field \( \mu_0 H_0 (\propto \omega_0) \), the sign of static magnetization \( M_0 \) is also inverted, and when the sign of \( g \)-factor is changed, the sign of the gyromagnetic ratio \( \gamma \) and \( \omega_0 (\propto \gamma) \) proportional to the \( g \)-factor are also changed. Note that the angular frequency \( \omega \) is positive fixed value and its sign is not influenced by the sign change of the magnetic field and \( g \)-factor. With respect to the magnetic field, \( \chi_{11} \) is symmetric while \( \chi_{12} \) is anti-symmetric. When the sign of \( g \)-factor is changed, the sign of imaginary part of both susceptibility is inverted. With \( D(\mu_0 H_0) \) and \( A(\mu_0 H_0) \), and taking into account the sign, the susceptibility is represented as

\[ \chi_{11} = D(\mu_0 H_0) - \eta_\theta A(\mu_0 H_0), \]

\[ \chi_{12} = \eta_H A(\mu_0 H_0) + i \eta_H \eta_\theta D(\mu_0 H_0), \] (1.11)

where \( \eta_\theta \) is the sign factor corresponding to the sign of \( g \)-factor. In consideration with this sign factor, the relation between \( \chi_{11} \) and \( \chi_{12} \) is described as

\[ \chi_{12} = i \eta_\theta \eta_H \chi_{11} = i \eta \chi_{11}, \] (1.12)

where \( \eta = \eta_\theta \eta_H \). For a given \( g \)-factor, the direction of the external magnetic field \( \mu_0 H_0 \) (viz. the sign of \( \mu_0 H_0 \)) determines the sign of \( \eta \). The sign of \( \eta \) thus implicitly indicates the polarity of the magnetization precession corresponding to the magnetic field direction and the sign of the \( g \)-factor. The sign of \( \eta \) can be directly inferred via the application of polarized microwaves as explained later - this is the key idea behind this thesis.

The magnetic resonance line width \( \mu_0 \Delta H_0 \), given by the full width at half maximum (FWHM) of the absorption dip, allows the Gilbert damping to be extracted. To establish the relationship between the line width and the damping parameter \( \alpha \), the LLG equation
\[
\chi_{ij} \propto g > 0 - \mu_0 H_{\text{res}} \mu_0 H_0
\]

\[
\chi_{12} \propto g < 0 - \mu_0 H_{\text{res}} \mu_0 H_0
\]

Re \((\chi_{ij})\)

Im \((\chi_{ij})\)

Figure 1.3: (a) Schematic image of the magnetization precession with the damping term expressed in Eq. (1.6). (b) Susceptibility \(\chi_{11}\) and \(\chi_{12}\) as a function of magnetic field for the real part (red) and imaginary part (blue) for \(g > 0\) and \(g < 0\). Both functions are composed of \(D(\mu_0 H_0)\) and \(A(\mu_0 H_0)\). \(\chi_{11}\) does not change the sign of the functions upon reversal of the direction of the magnetic field, while \(\chi_{12}\) does. When the sign of \(g\)-factor is inverted, only the sign of imaginary part of susceptibility is changed.

is solved [12]:

\[
\mu_0 \Delta H_0 = \mu_0 \Delta H_0|_{H_0=0} + \left|\frac{2\alpha}{\gamma}\right| \omega. \tag{1.13}
\]

The inhomogeneous broadening \(\mu_0 \Delta H_0|_{H_0=0}\), defined as the zero–magnetic–field–intercept of line width, originates from extrinsic damping effects and is not included in the Gilbert formalism. The inhomogeneous broadening arises from structural defects, scattering with magnetization excitations (magnons) etc. The Gilbert damping parameter \(\alpha\) is determined by the slope of 2nd term in Eq. (1.13). In general, the damping constant \(\alpha\) can be composed of both intrinsic damping, involving electron scattering on phonons and thermally excited spin waves, and extrinsic damping (inhomogeneous broadening). A reliable determination of \(\alpha\) is thus possible only when the magnetic resonance experiments at different frequencies are conducted.

**Magnetization precession induced by polarized microwaves**

The LLG equation gives the resonance frequency that matches to the Zeeman energy gap in paramagnetic systems. For the resonance excitation of the magnetization, the conservation of angular momentum must be fulfilled as well. The rf–components of the magnetic field are expressed as \(\mu_0 h(t) = \mu_0 (h_x, h_y) = \mu_0 (h_0 e^{i\omega t}, h_0 e^{i(\omega t + \beta)})\). The phase difference \(\beta\) between the \(h_x\) and \(h_y\) components determines the polarity of the microwaves. For instance, microwaves are linearly polarized at \(\beta = 0^\circ\), and for \(\beta = 90^\circ\) they are purely circularly polarized. Inserting the equation above into Eq. (1.9) results in

\[
m(t) = \chi_{11} \mu_0 h_0 \begin{pmatrix} 1 & i\eta \\ -i\eta & 1 \end{pmatrix} \begin{pmatrix} 1 \\ e^{i\beta} \end{pmatrix} e^{i\omega t} = \chi_{11} \mu_0 h_0 \begin{pmatrix} 1 + i\eta e^{i\beta} \\ -i\eta + e^{i\beta} \end{pmatrix} e^{i\omega t}. \tag{1.14}
\]
When the magnetic field is applied in the positive direction and the g–factor is positive, \( \eta = +1 \). For \( \beta = 90^\circ \) viz. \( \beta = 270^\circ \), the dynamic component of magnetization is calculated as

\[
m(t) = \chi_{11} \mu_0 h_0 \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} e^{i\omega t} \quad \text{for} \quad \beta = 90^\circ,
\]

\[
m(t) = \chi_{11} \mu_0 h_0 \begin{pmatrix} 2 \\ -2i \\ 0 \end{pmatrix} e^{i\omega t} \quad \text{for} \quad \beta = 270^\circ.
\]

(1.15)

Thus, when the microwaves are circularly polarized in the clockwise direction at \( \beta = 90^\circ \) viewed anti–along the magnetic field direction, the magnetization does not precess \( (m(t) = 0) \). On the other hand, microwaves rotating in counter–clockwise direction \( (\beta = 270^\circ) \) excite the magnetization, i.e., drive magnetic resonance. This explanation coincides with the single spin model that only one of the spin states of the photon can excite the magnetization precession and the other cannot. This means, the LLG equation indeed correctly takes into account the conservation of angular momentum between the microwaves and the spin system.

1.3 Ferromagnetic resonance

In Sect. 1.1 and 1.2, the dynamic component of magnetization were derived considering a paramagnetic spin ensemble. However, in ferromagnets, the interaction between neighboring spins is relevant [13]. In this section, we consider the internal field generated by the magnetization in an exchange–coupled magnet to investigate ferromagnetic resonance [11].

The total magnetic field in a region containing magnetic moments is the sum of the demagnetization \( \mu_0 H_d \) [14] and the external magnetic field. \( \mu_0 H_d \) is proportional to \( M \) and sensitive to the shape of the specimen. Demagnetization is described as \( \mu_0 H_d = -\mu_0 \vec{N} M \), where \( \vec{N} \) is the demagnetization 3 \times 3 tensor. It is symmetric and becomes diagonal in the axes coinciding with the axes of the ellipsoid (assuming that the sample shape can be approximated with an ellipsoid). The components of \( \vec{N} \) in these axes, \( N_x, N_y \) and \( N_z \) are called demagnetization factors with the relation \( N_x + N_y + N_z = 1 \) [11].

We consider the undamped system \( \alpha = 0 \) to calculate the precession frequency. For that, we project the magnetization onto the axes of a Cartesian coordinate system in which the z–axis coincides with the direction of the external magnetic field and the static magnetization. In this scenario, the effective magnetic field \( \mu_0 H_{\text{eff}} \) is described as

\[
\mu_0 H = \mu_0 H + \mu_0 H_d = \mu_0 \begin{pmatrix} -N_x m_x(t) \\ -N_y m_y(t) \\ H_0 - N_z M_0 \end{pmatrix},
\]

(1.16)
and with Eq. 1.4 one gets the two linear equations,

\[
\begin{align*}
  i\omega m_x + \gamma \mu_0 [H_0 + (N_y - N_z)M_0]m_y &= 0, \\
  -\gamma \mu_0 [H_0 + (N_x - N_z)M_0]m_x + i\omega m_y &= 0.
\end{align*}
\] (1.17)

The condition of the compatibility of these equations gives the expression for the eigen frequency (ferromagnetic resonance frequency):

\[
\omega = \gamma \mu_0 \sqrt{|H_0 + (N_x - N_z)M_0| |(H_0 + (N_y - N_z)M_0)|}.
\] (1.18)

Equation (1.18) is a famous formula obtained by Kittel [15]. The ferromagnetic resonance frequency can be simplified for the limiting case of an ellipsoid. Now we consider the case of a thin film sample in the \(xy\) plane and the magnetic field is applied into the positive \(z\)–direction as shown in Fig. 1.4. Only on the surface of the film, the magnetic poles appear, so that the demagnetization factor is expressed as \(N_x = N_y = 0, N_z = 1\) and we obtain the resonance frequency:

\[
\omega_{\text{res}} = \gamma \mu_0 (H_0 - M_0). 
\] (1.19)

The resonance frequency is simply proportional to the magnetic field but shifted by \(\gamma \mu_0 M_0\) when compared to the paramagnetic case \((\omega_{\text{res}} = \gamma \mu_0 H_0)\).

With the LLG equation and Eq. (1.16), one obtains the susceptibility composed of a dispersion-like function \(D(\mu_0 H_0)\) and an absorption function \(A(\mu_0 H_0)\) with the sign factor \(\eta\) including the demagnetization anisotropy, and thus the response of the dynamic components of the magnetization to the circularly polarized microwaves as explained in Sect. 1.2 can be applied for FMR. Note that the sign factor \(\eta\) corresponds to the sign of \(gH_0\) under the condition \(|H_0| > |M_0|\). From the line width of the FMR absorption dip, the damping constant \(\alpha\) is calculated as expressed in Eq. (1.13).

\[\text{Figure 1.4: Schematic image for the calculation of FMR in a thin magnetic film. The magnetic field is applied in \(z\)–direction parallel to the normal of the thin film.}\]
Chapter 2

Experimental set–up and simulations

2.1 Cross resonator

Coplanar waveguides are 2D–patterned structures which allow propagation of microwaves in an electro–magnetic mode. The electromagnetic field created on top of the waveguides by the propagating microwaves can be used to drive EPR or FMR. This concept is employed extensively for on–chip EPR [16] or FMR [17] and quantum information processing applications [17] [18].

Often, to increase sensitivity, the waveguide is disrupted by two coupling gaps creating a structure that exhibits a resonant transverse electromagnetic (TEM) mode. The created electromagnetic field on top of such a coplanar waveguide resonator (see Fig. 2.1) is linearly polarized. Our goal, circularly polarized microwaves, can be achieved by superimposing two perpendicular linearly polarized microwaves with well defined relative phase. We achieve this by combining two coplanar waveguide (CPW) resonators into a cross like structure and adjusting the phase of both microwave stimuli appropriately.

In the following, the geometry of the cross resonator and the experimental set–up is explained. In addition to that, the rf–magnetic field configuration is simulated using a finite element method (FEM) software.

2.1.1 Geometrical parameters of the cross resonator

Coplanar waveguides [19] are a type of electrical transmission line and allow the propagation of microwaves in a TEM mode where both electric and magnetic fields are perpendicular to the direction of the propagation. A CPW is a 2D structure patterned in a conductor on a substrate. The substrate needs to exhibit low dielectric loss and a constant permeability in the desired operating frequency range. The CPW structure is made up of a center strip line, called signal line, isolated by a narrow gap on both sides from two ground planes. In our case, we use a grounded CPW which means that also the backside of the substrate is plated with conductor. The width of the signal line $w$, the gap $g_{CPW}$, the thickness $t$ and the dielectric constant $\varepsilon$ of the substrate determine the impedance characteristics of the CPW.
Figure 2.1a shows the geometry of the CPW resonator. The resonator is set on the substrate RT/duroid 5870 (Rogers) with a dielectric constant \( \varepsilon = 2.33 \), and a thickness of \( t = 0.508 \text{ mm} \). The dielectric is coated with 35\( \mu \)m thick copper as a conductive material on both sides. The length of the resonator \( L = 10.8 \text{ mm} \) determines the resonance frequency of the resonator, which is described as \( f \sim nc/(2L\sqrt{\varepsilon}) \), where \( n \) is the resonant mode, \( c \) is the speed of light. Our half-wavelength resonator is designed to work for the frequency \( f \sim 10 \text{ GHz} \) in its fundamental standing wave mode \( (L = \lambda/2) \). The gap between signal line and feed line called coupling gap of \( g_C = 0.1 \text{ mm} \) primarily affects the resonance amplitude. An impedance matching to 50\( \Omega \) is important to obtain the maximum broadband power transfer into the CPW and ensure a transmission which does not show any parasitic modes and thus is flat in frequency. In order to obtain the impedance of 50\( \Omega \) around 10 GHz, the width of signal line \( w = 1.08 \text{ mm} \) and the CPW gap of \( g_{\text{CPW}} = 0.4 \text{ mm} \) between the signal line and ground line are selected.

In order to create the circularly polarized microwaves, we introduce the cross resonator, composed of two half-wavelength CPW resonators (see Fig. 2.1b). This approach to generate the circularly polarized microwaves by superimposing two orthogonal microwaves has already proposed by Barco’s group. In his group, properties of a half-wavelength microstrip line were investigate to utilize for EPR experiments in detail [20] and then, Henderson et al. [6] assembled the two half-wavelength microstrip resonators together to form a cross microstrip resonator. This idea of superimposing two microwaves were also considered by Diaz et al. [4]. They used the square microwave cavity, which had the two orthogonal degenerated and independent modes, and excitation of two modes separately with 90° phase difference resulted in circularly polarized microwaves at the center of the cavity. In our experiment, two CPW resonators are placed together forming the square cross section positioned in the center of both resonators as proposed by Henderson et al. [6]. The two microwave stimuli can transmit perpendicular to each other and at the cross section, the two microwaves are superposed. Owing to technical restrictions in the preparation of the printed circuit board such that maximum size of the chip is 21 mm \( \times \) 29.7 mm, the length of the feed lines \( L_x \) and \( L_y \) which are normal to each other differ by 4.5 mm.

### 2.1.2 Experimental set-up

In order to be able to excite the coplanar structure, we attach Southwest Microwave SMA end launch connectors to the end of each feed line as depicted in Fig. 2.2a. The external magnetic field is applied anti-parallel to the normal vector of the cross resonator surface. Each of the end launches is connected using same-length microwave cables to the four ports of a N5242A PNA-X Vector Network Analyser (VNA). A VNA is a test system that enables the transmission and reflection of microwave devices to be characterized. Scattering parameters (S-parameters) are used to describe the electrical behavior of a
2.1 Cross resonator

Figure 2.1: (a) Basic geometry of a CPW resonator. The values $g_{\text{CPW}} = 0.4\text{ mm}$, $t = 0.508\text{ mm}$ and $w = 1.08\text{ mm}$ are selected to achieve $50\Omega$ matching of the feed lines and thus reduce reflections at the ends. The length $L = 10.8\text{ mm}$ is chosen to obtain a resonance frequency $10\text{ GHz}$ for the CPW resonator fundamental mode. (b) Basic geometry of a CPW cross resonator composed of two half-wavelength CPW resonators. On the center of the cross point, circularly polarized microwaves can be generated using two microwave stimuli.

high frequency device. For example, the transmission parameter $S_{21}$ is the ratio of the voltage detected at port 2 ($B_2 e^{i\phi_2}$) of the VNA to the voltage emitted at port 1 ($A_1 e^{i\phi_1}$), and then the complex number $S_{21}$ is defined as

\[ S_{21} = \frac{B_2 e^{i\phi_2}}{A_1 e^{i\phi_1}} = \frac{B_2}{A_1} e^{i(\phi_2 - \phi_1)}. \tag{2.1} \]

As opposed to a Scalar Network Analyzer (SNA) which measures the amplitude $B_2/A_1$ only, the VNA can measure not only $B_2/A_1$ but also the phase $\phi_2 - \phi_1$. The Agilent PNA–X can provide output microwaves with a defined phase difference at two ports simultaneously, in a so-called phase control mode. One source is selected as the controlled source and the other is the referenced source. The phase difference can be set quickly between two arbitrary phase values. As a result, a phase sweep measurement, in which for example the phase difference between the two microwave drives is swept from $0^\circ$ to $360^\circ$, can be quickly performed. In this mode, the relative power of the controlled port as compared to referenced power amplitude can be set in addition. Under phase control, the stimuli from port 1 and port 3 ($A_3 e^{i\phi_3}$) are applied simultaneously and detected at port 2. In our set up, the microwaves from port 1 are referenced by port 3, so that the phase difference $\phi_3 - \phi_1$ and the relative amplitude $A_3/A_1$ can be set. The remaining port 4 is unused but connected to the VNA as well and thus terminated with $50\Omega$ in order to avoid any reflections. The phase difference is important for the polarity of the microwaves at the center of the resonator. If the phase difference at the resonator is $0^\circ$ and $90^\circ$, the microwaves are linearly and circularly polarized, respectively (see Fig 2.2b and c).

In order to reduce cross talk and suppress any excitations outside of the signal line, a via shield is introduced along to the signal lines. A via is an electrical connection, usually a metalized hole of small diameter, between the top and bottom conducting layers in a circuit board. Because no facilities are available in–house to create vias, we introduce a quasi via shield: The ground line 1 mm away from the side of CPW gap is cut off and
silver paste is applied to the side instead of via holes to connect the front and back side of the ground metal electrically. If there is no via shield, the microwaves radiate to the edge and reflect, and thus the energy loss of microwaves increases. Via shields stop the surface wave inside the structure from being created.

The sample used in our experiments is diphenyl-picryl-hydrazyl (DPPH). It is widely used as a paramagnetic calibration standard in EPR experiment because of its narrow line width and very well known g–factor. DPPH contains a single unpaired electron, whose orbital angular momentum is zero. Therefore, the electron has only the spin angular momentum \( S = \hbar/2 \). The g-factor of DPPH is \( g = +2.0036 \) [21]. The DPPH is usually available as a dark-colored (crystalline) solid [22]. The DPPH is pressed into a pellet to obtain a large EPR signal and coated by the plastic (Bondic) in order not to be unintentionally scattered across the cross resonator. Our DPPH test sample has dimensions 1 mm × 1 mm × 1 mm. DPPH sample is mounted by rubber cement on the center of cross resonator.

![Diagram](image)

**Figure 2.2:** (a) Set up of the experiment with the cross resonator. Silver paste is applied to the edge of the circuit board to connect the top and bottom of ground metal plates. The magnetic field is collinear with the \( z \)-axis. We define the magnetic field as pointing along \(-z\) direction as positive. The DPPH sample is mounted on the center of the cross resonator. When the microwaves are applied from port 1 and port 3 with the same phase simultaneously, the microwaves are (b) linearly polarized, whereas the microwaves are (c) circularly polarized when the stimuli are 90° out of phase.

### 2.1.3 EPR detection

Conventionally, the EPR spectra are analyzed with the susceptibility function \( \chi \) considering one microwave stimulus. In our case, two stimuli with the phase difference \( \beta \) are simultaneously applied to the cross resonator, and thus the detected signal may change depending on \( \beta \).

To model the magnetic resonance response in our cross resonator, we introduce a Cartesian coordinate system on the cross resonator described in Fig. 2.2a. At the resonance
2.1 Cross resonator

frequency, standing waves appear in the resonator lines. In view of the microwaves in TEM mode, the rf-magnetic fields $\mu_0 h(t)$ under phase control are represented as

$$\mu_0 h(t) = \mu_0 \left( \begin{array}{c} h_x \\ h_y \end{array} \right) = \mu_0 \left( \begin{array}{c} h_0^x e^{i(\omega t + \beta)} \\ h_0^y e^{i\omega t} \end{array} \right) = \mu_0 h_0 e^{i\omega t} \left( \begin{array}{c} ke^{i\beta} \\ 1 \end{array} \right),$$

(2.2)

where $h_0^x$ and $h_0^y (= h_0)$ are the amplitude of $h(t)$ for $x$– and $y$–component. $k$ is the proportionality factor, defined as $k = h_0^y/h_0^x$, which characterizes the amplitude ratio of the two microwave stimuli at the sample position. In the following, the case $k = 1$ where the amplitude of both microwave components is the same, is considered. At $\beta = 90^\circ$ and $270^\circ$, the real part of the rf–magnetic fields are calculated as

$$\text{Re} (\mu_0 h(\omega t)) = \text{Re} (\mu_0 h_0) \left( \begin{array}{c} \cos(\omega t + 90^\circ) \\ \cos(\omega t) \end{array} \right) = \text{Re} (\mu_0 h_0) \left( \begin{array}{c} -\sin(\omega t) \\ \cos(\omega t) \end{array} \right),$$

(2.3)

$$\text{Re} (\mu_0 h(t)) = \text{Re} (\mu_0 h_0) \left( \begin{array}{c} \cos(\omega t + 270^\circ) \\ \cos(\omega t) \end{array} \right) = \text{Re} (\mu_0 h_0) \left( \begin{array}{c} \sin(\omega t) \\ \cos(\omega t) \end{array} \right),$$

(2.4)

Therefore, the microwaves on the cross resonator rotate in the counter-clockwise direction at $\beta = 90^\circ$ and in the clockwise direction at $\beta = 270^\circ$ as viewed from the top of the cross resonator.

The polarity of the microwaves exciting the EPR described in Eq. (1.14) is determined by the sign of the g–factor and the direction of the external field (cf. the discussion in Sect. 1.2). Now the case $g > 0$ is considered for the DPPH sample. When $\mu_0 H_0 > 0$, the magnetization precesses in the clockwise direction according to Eq. (1.4), and thus the EPR is hardly excited when the microwaves are circularly polarized in the counter clockwise direction ($\beta = 90^\circ$) (see Fig. 2.3a). Note that $\mu_0 H_0$ is applied into the paper plane ($-z$ direction) and the polarity of microwaves is viewed from the top of the cross resonator, which is not the same definition of relationship between magnetic field and microwave direction in the previous chapter. On the other hand, when $\beta = 270^\circ$, the microwaves are circularly polarized in the clockwise direction (Fig. 2.3b), and EPR is excited. When the direction of the external magnetic field is reversed, also the magnetization precession reverses and in order to excite EPR, so that the microwave polarization also needs to be opposite (see Fig. 2.3c and d).

The precessing magnetization arising in EPR induces the voltage $V = V_{\text{ind}}$ in the cross resonator and this induced voltage is detected by VNA. The microwave radiation emitted owing to EPR can transmit in all four directions (into all 4 ports). Due to the nature of the TEM mode supported by the CPW, microwave magnetic fields which are perpendicular to the propagation direction can be transmitted. In consideration of reciprocity, the voltage induced by the magnetization precession of the $m_y$ component, is detected dominantly at port 2. With the assumption, $\frac{d m(t)}{dt} = i \omega m(t)$ used in Sect. 1.2, $V_{\text{ind}}$ detected by $S_{21}$
Figure 2.3: (a) EPR is not excited for positive magnetic field for circularly polarized microwaves which rotate (are circularly polarized) in counter–clockwise direction ($\beta = 90^\circ$), (b) On the other hand, EPR will be excited upon applying microwaves which rotate clockwise ($\beta = 270^\circ$). When a negative magnetic field is applied, (c) counter-clockwise polarized microwaves excite EPR but (d) clockwise polarized microwaves cannot.

is described as

$$V_{\text{ind}}^{21} \propto \frac{dm_y}{dt} = i\omega m_y \propto m_y.$$  (2.5)

At port 4 the voltage induced by $m_x$ is sensitively detected. With Eq. (1.9), (1.12) and Eq. (2.2), the EPR signal in $S_{21}$ is calculated as

$$S_{21} \propto \frac{(\chi_{21}\mu_0 h_x + \chi_{22}\mu_0 h_y)}{e^{i\omega t}}
= \mu_0 h_0 \chi_{11} (-i\eta ke^{i\beta} + 1).$$  (2.6)

Note that $\eta$ contains the sign of the magnetic field and g–factor of the sample material (see Sect. 1.2 and Eq. (1.12)). The magnitude of the EPR signal is calculated as

$$S_{21} \propto |\mu_0 h_0||\chi_{11}|| - i\eta ke^{i\beta} + 1|$$
$$= |\mu_0 h_0||\chi_{11}| \sqrt{(-i\eta ke^{i\beta} + 1)(i\eta ke^{-i\beta} + 1)}$$
$$= |\mu_0 h_0||\chi_{11}| \sqrt{k^2 + 1 + 2k\eta \sin \beta},$$
$$S_{21}|_{k=1} \propto |\mu_0 h_0||\chi_{11}| \sqrt{2(1 + \eta \sin \beta)},$$  (2.7)

where $|\mu_0 h_0|$ is constant and $|\chi_{11}|$ is the magnitude of the susceptibility, which is the field dependent value at fixed frequency. Note that $\eta = -1$ for positive magnetic field and $\eta = +1$ for negative magnetic field in our set–up for the cross resonator. In case $k = 1$ and $\beta = 90^\circ$, the microwaves on the cross resonator are circularly polarized in the counter-clockwise direction, $S_{21}$ is nearly zero for the positive magnetic field ($\eta = -1$) and $S_{21}$ has the maximum value for the negative field ($\eta = +1$).

Under phase control mode, the same detected signal is shown in $S_{21}$ and $S_{23}$ since the
microwave radiation detected at port 2 is identical. For the same reason, $S_{43}$ and $S_{41}$ show the same result, and due to the characteristic of TEM mode the rf–magnetic field which is linearly polarized in $x$–direction is detected at port 4 so that $m_x$ oscillation is dominantly observed. Note that S–parameters do not have reciprocal properties any more under phase control mode, so that $S_{ij} \neq S_{ji}$. At port 1 ($S_{12}$ and $S_{14}$), the microwave radiation contributed by the $m_y$ oscillation responded to linearly polarized microwaves is detected, and in an analogous way the microwave radiation contributed by $m_x$ oscillation is detected at port 3 ($S_{32}$ and $S_{34}$).

### 2.1.4 Distinguishing the modes by FEM simulations

In order to predict the microwave characteristics of the patterned structure, we employed a FEM simulation (CST Studio Suite). Therefore we create an exact 3D CAD model of the structure. Apart from variations due to the production, only the modelled vias (0.2 mm plated holes at 0.3 mm distance of the CPW gap) differ from the actually used CPW structure (silver paste via shield). We first turn to an analysis of the Scattering parameters (see Fig. 2.4a). At $f_0 = 9.67$ GHz and $f_0 = 10.72$ GHz, transmission peaks are observed in which S–parameters indicate resonant modes. In reflection the dips corresponding to the peaks in transmission is also observed. To identify the nature of the resonant modes, the 3D dimensional magnetic field distribution is simulated and its value on top of the cross resonator is displayed in Fig. 2.4b and c. At 9.67 GHz the resonance has only a current density in one resonator, which we label dipole mode in the following. This is the fundamental mode of the resonator, and therefore the anti–node of the standing wave is located at the center of the cross resonator. In this mode, the amplitude of the rf–magnetic field above the crossing point of the resonator is calculated to 3.6 $\mu$T with the stimulus power 0.0 dBm. At 10.72 GHz the resonant excitation of all four arms of the cross resonator is developed, called quadrupole mode. The quadrupole mode can be excited and detected at all four ports. More specifically it also couples two neighboring ports (e.g., 2 and 3) and thus it can also be observed in $S_{23}$. The node of the standing wave is located on the center of the cross resonator for the quadrupole mode, so that the microwave amplitude there will also be small. For the creation of circularly polarized microwaves, only the dipole mode can be used because of its TEM characteristics at the crossing point.

### 2.2 Rectangular waveguide

Conventionally, rectangular waveguides are used to transmit microwaves when an exceptionally low loss or high power is required. Here, we use the fact that at a certain fixed position in the waveguide, the propagating microwave mode can exhibit a polarized character allowing to drive EPR or FMR with circularly polarized microwave radiation.
Figure 2.4: (a) S-parameters of the cross resonator as a function of frequency. The magnetic field distribution in absolute value (b) at $f_0 = 9.67$ GHz and (c) at $f_0 = 10.72$ GHz are shown. At the first resonance frequency (9.67 GHz), there is the resonance on dipole mode in $S_{21}$, and at the second resonance frequency (10.72 GHz), there is the resonance on quadrupole mode in $S_{21}$ and $S_{23}$. The dipole mode is used for the creation of circularly polarized microwaves.

The approach is inherently different to the crossed coplanar waveguide resonators described in the previous section. In the rectangular waveguide, the frequency can be freely chosen above the waveguide’s cut-off frequency and thus, broadband FMR can be performed.

In the following we describe the basic properties of the rectangular waveguide and the propagating fundamental mode. We furthermore describe the experimental set-up and calculate the polarization in dependence of the frequency and the position.

### 2.2.1 Microwave modes in rectangular waveguides

A rectangular waveguide is a hollow metallic conductor with rectangular cross section in which several electro–magnetic wave modes can propagate. The inner dimensions of the waveguide’s cross section, $a$ and $b$ (see Fig. 2.5a) and the dielectric constant of the filling material mainly determine which frequency it can support. Unlike coplanar waveguides, the rectangular waveguide does not allow the microwave to be transmitted as TEM modes but as transverse electric (TE) modes and transverse magnetic (TM) modes. The basic common feature of TE modes is that all electric fields are perpendicular to the propagation direction, while magnetic fields are transverse to the direction of the microwave propagation in TM modes. A certain TE or TM mode can be excited above its cut-off frequency, which is given by [23]

$$f_{n,m} = \frac{1}{2 \sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 } \quad m, n = 0,1,\ldots, \text{TE mode,}$$

$$m, n = 1,2,\ldots, \text{TM mode},$$

(2.8)

where $m$ and $n$ are the number of a half–wavelength variation of fields in the $a$–direction and $b$–direction with $mn \neq 0$, and $\mu$ and $\varepsilon$ are the permeability and the dielectric constant of the dielectric inside the waveguide (typically air or a gas).
mode of the waveguide, the TE$_{10}$ mode, has the lowest cut-off frequency of all modes. Until the onset of the next higher mode, only the TE$_{10}$ mode can propagate. Its cut-off frequency is given by:

$$f_{1,0} = \frac{1}{2 \sqrt{\mu \varepsilon}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = \frac{1}{2a \sqrt{\mu \varepsilon}}.$$  

(2.9)

The recommended frequency band of operation of an air filled waveguide WR-90 is from 8.2 GHz to 12.4 GHz [23]. From Eq. (2.8), the cut-off frequency of the lowest order mode of WR-90 is 6.6 GHz and of the next mode is 13.1 GHz. In order to avoid dispersion, where the velocity of microwave propagation is a function of $f$, the lower edge of the recommended transmission band is set approximately 30% higher than the cut-off frequency. The upper range of the band is approximately 5% lower than the cut-off frequency of next mode in order to avoid the coupling via higher order modes.

Figure 2.5a shows the schematic image of electric field in TE$_{10}$ mode at a certain cross section with the given dimensions [24]. The electric field shows no longitudinal component. The magnetic field loops of the propagating microwaves at different distinct times are shown in Fig. 2.5b. Views are displayed for every quarter of a period, described as the mode’s phase $\phi$. At a position off the center axis of the waveguide marked with a black circle, the direction of the magnetic field rotates by 90° for every quarter of the period of the microwaves [5] [25]. This means, viewed at this exact positions the microwave magnetic field is circularly polarized. Except for this position, the microwaves will be elliptically polarized since the vector components of the microwaves vary as a function of position. In Sect. 2.2.3, we determine the sample position where circularly polarized microwaves are generated using numerical simulations with CST.

![Figure 2.5](image-url)

**Figure 2.5:** (a) The dimensions of rectangular waveguides ($a > b$). The electric field configuration shows no longitudinal components. (b) The rf-magnetic field of TE$_{10}$ mode as viewed from the top. The direction of the magnetic field around the area highlighted by black circle changes for every quarter of a period of the microwaves.
2.2.2 Experimental set–up

In order to excite FMR, a magnetic field is required at the sample position. Therefore, we mount a rectangular waveguide between the pole shoes of an electromagnet in such a way that the magnetic field is applied vertical to the broad face of the waveguide (along the $b$–direction) (Fig. 2.6a). The waveguide standard used in the experiment is WR-90, its inside dimensions are $22.86 \text{ mm} \times 10.16 \text{ mm}$, which cover the X–band frequency range from $8.2 \text{ GHz}$ to $12.4 \text{ GHz}$ [23].

The microwaves are driven and detected by a VNA. SMA connectors are mounted to both ends of the waveguide which are connected using SMA cables to the VNA. Before the measurement, a 2 port calibration is performed in order to remove the influence of the microwave cables on the results. We use a Yttrium Iron Garnet ($\text{Y}_3\text{Fe}_5\text{O}_{12}$, YIG), grown by liquid phase epitaxy (LPE) by Innvonent Jena, with a thickness of $5 \mu \text{m}$ and lateral dimensions of $2 \text{ mm} \times 2 \text{ mm}$. The measured sample is mounted by Fixogum, a non–magnetic rubber cement, on a rectangular shape Teflon $22 \text{ mm} \times 5 \text{ mm} \times 14 \text{ mm}$ sample holder inserted in the rectangular waveguide. The Teflon sample holder is designed to fix the sample at $z = 0$, the center in the $b$–axis, while it can be positioned in $y$–direction (along the $a$–direction) (see Fig.2.6b). The center of the waveguide in $a$–direction is chosen as $y = 0$. This way, the sample’s surface is perpendicular to the static magnetic field which is positive when it is applied in $+z$ direction. In our experiment, the sample is placed at $y = -2 \text{ mm}$.

The visualized magnetic field at $8 \text{ GHz}$ for a $S_{21}$ VNA measurement is shown in Fig. 2.6c. The microwaves excited at port 1 propagate in the positive $x$–direction, and therefore clockwise rotating microwaves are generated for the sample position lower than center in $y$–direction. In $S_{12}$, just the propagation direction is inverted hence the microwaves are circularly polarized in counter–clockwise direction [26].

![Figure 2.6](image)

**Figure 2.6:** (a) Schematic image of set–up with the rectangular waveguide. The sample is inserted on a Teflon holder that fits the inner dimensions of the waveguide (dashed lines) in order to mount it (b) at a specific position in the waveguide. (c) Off the waveguide center ($y \neq 0$), there is circularly polarized magnetic field.

In case of $g > 0$, the magnetization precesses in the clockwise direction for positive
magnetic field from Eq. (1.4), hence in $S_{21}$ the FMR is barely excited (Fig. 2.7a). On the other hand, microwaves in a $S_{12}$ VNA measurement, in which the microwaves are transmitted in the negative $x$–direction and are counter-clockwise polarized therefore, the FMR is strongly excited (Fig. 2.7b). When negative magnetic field is applied, the FMR is detected by $S_{21}$ but not by $S_{12}$ (Fig. 2.7c and d). The relation of FMR to the sign of the magnetic field and microwave direction is inverted for $g < 0$.

![Figure 2.7](image)

**Figure 2.7:** (a) The FMR is very weakly excited for the positive magnetic field with $S_{21}$. (b) On the other hand, the FMR is detected by $S_{21}$ with the negative magnetic field. With $S_{12}$, the FMR is detected with (c) the positive magnetic field but not with (d) the negative magnetic field.

### 2.2.3 Simulating the polarization of the TE$_{10}$ mode

In order to characterize the modes polarization we again employ a FEM simulation using CST Studio Suite. In CST the air filled WR–90 waveguide is modelled and only the TE$_{10}$ is excited using the appropriate setting at the virtual port. The transmission and reflection are shown in Fig. 2.8a as a function of frequency. Clear to see is the low frequency cut–off of the mode at 6.6 GHz which is in good agreement with the analytic solution by Eq. (2.9). In CST, the TE mode cannot be excited below the cut–off frequency at the virtual port and no signal is calculated. In reality, the microwaves transmitted through the SMA connector cannot propagate into the waveguide and thus a reflection close to one is expected instead below the cut–off frequency.

To determine the position where circularly polarized microwaves are generated, the strength of microwave’s $x$– and $y$– component is calculated as a function of the position in the waveguide, where the strength of both components is equal, the microwaves are ideally circularly polarized. In the ideal case no change is expected along the waveguide’s long axis ($x$) and the narrow face axis ($z$). In Fig. 2.8b, the variation along the $a$–axis of the waveguide is shown. The amplitude of the rf–magnetic field $x$–component $|\mu_0 h_x|$ at $y = 0$ nearly vanishes while $|\mu_0 h_y|$ has a peak. On the other hand, at $y = \pm a/2$, $|\mu_0 h_x|$ has its maximal value and $|\mu_0 h_y|$ is almost zero. The position, where the magnetic field of both components is the same, is $y = \pm 4.5$ mm. CST can calculate not only the
magnitude of magnetic field but also the phase of microwaves for both components, $\phi_x$ and $\phi_y$. As specified in Eq. (2.2), phase difference $\beta = \phi_x - \phi_y$ determines the polarity of the microwaves, for instance, when $\phi_x - \phi_y = 0^{\circ}$ and $90^{\circ}$, the microwaves are linearly and circularly polarized, respectively. The orbit of magnetic field with the real part of $\mu_0 h_x = |\mu_0 h_x| e^{i(\omega t + \phi_x)}$ and $\mu_0 h_y = |\mu_0 h_y| e^{i(\omega t + \phi_y)}$ for several $y$ positions is shown in Fig. 2.8c. At $y = 0$ and $y = a/2$, the microwaves are nearly linearly polarized (see red line and yellow line). The closer to $y = -4.5 \text{ mm}$ the position is, the more circularly polarized the microwaves are. At $y = -4.5 \text{ mm}$ (see black line), the phase difference $|\phi_x - \phi_y| = 90.1^{\circ}$ and the microwaves are almost purely circularly polarized.

Figure 2.8: Microwave properties of a rectangular WR–90 waveguide as simulated with CST. (a) $S_{11}$ (blue) and $S_{21}$ (green) as a function of frequency. The cut-off frequency is 6.6 GHz which is in good agreement with analytic solution. (b) $|\mu_0 h_x|$ (red) and $|\mu_0 h_y|$ (blue) as a function of $y$–positions. (c) The orbit of microwaves for several $y$–positions. At $y = \pm 4.5 \text{ mm}$ (black), the microwaves are circularly polarized.

Figure 2.9a shows the optimal position where circularly polarized microwaves are created as a function of frequency, as calculated with CST. The optimal position does change with increasing frequency, and this indicates that the polarity of the microwaves at a fixed position will vary with the frequency. This variation is shown in Fig. 2.9b for the phase difference $\beta$ of the two components and the polarization at $y = -2 \text{ mm}$ in Fig. 2.9c for reference with the measurements in Chapt. 3 later. If the sample holder is not taken into account, the deviation of $\beta$ from the optimal value of $90^{\circ}$ is only minor. Still the polarization decreases strongly with frequency which can be attributed to the difference in microwave strength of the two linear components as the optimal position shifts outwards. If the sample holder is considered, the phase difference at low frequencies is, however, substantial and thus the total polarization at low frequencies is already lower than if it is not considered. The Teflon holder, whose dielectric constant is 2.1 and thus more than twice of the dielectric constant of the air, gives rise to an inhomogeneous field, and thus purely circularly polarized microwaves are not generated anymore. To compare with experimental data, the polarity as a function of frequency is calculated. The polarization depends strongly on the holder design and simulations show that a thin holder is preferable.
(cf. discussed in Chapt. 4).

In our experiment, the polarity we measure using the rectangular waveguide does not show the polarity of the microwaves generated directly since microwaves radiated by the precessing magnetic moment travel preferably either forward or backward depending on the sense of precession. The detected polarity therefore includes the excitation efficiency for the radiated microwaves. When taking into account this directivity (see Appendix B), the polarity \( P \) is described as

\[
P = \frac{|2k \sin \beta \sqrt{1 + k^2 \cos^2 \beta}|}{1 + k^2}
\]

(2.10)

where \( k \) and \( \beta \) are the results of the FEM simulation.

Figure 2.9: (a) The optimal position and (b) phase difference \( \beta \) as a function of frequency. Green dots and yellow dots indicate non-insertion and insertion of the Teflon holder, respectively. At the optimal position, \( \beta \) is not 90° any more with the Teflon holder, it is no longer possible to obtain perfect circular polarization. (c) The polarization \( P \) at a fixed position \((y = -2 \text{ mm})\) decreases with increasing frequency.
Chapter 3

Experimental results

3.1 Cross resonator

One way to create circularly polarized microwave radiation is by superimposing two orthogonal linearly polarized microwave fields. This principle is used in the cross resonator design described in Chapt. 2 when both resonators are driven simultaneously with a fixed phase relationship. Depending on the amplitude and phase difference between both stimuli, the cross resonator creates linearly, elliptically or circularly polarized microwaves. In this section, we experimentally characterize the cross resonator’s transmission under excitation from one port and its behaviour when excited simultaneously from two ports. Furthermore, we analyze the polarization of the generated microwaves using EPR.

3.1.1 Microwave characteristics

The reflection and transmission parameters of the cross resonator structure (see Fig. 2.1 and Fig. 2.2) are measured without a mounted DPPH sample in a frequency range from 8 GHz to 12 GHz at room temperature. The cross resonator is connected to the 4 ports of the N5242A PNA–X VNA. The two clear resonances at 9.5 GHz and 10.58 GHz expected from the simulations (see Fig. 2.4) are indeed observed in this cross resonator. From the simulation in Sect. 2.1.4, the dipole mode is observed in $S_{11}$ and $S_{21}$ at first resonant frequency $f_0 = 9.5$ GHz, whereas the quadrupole mode can be observed at the second resonant frequency $f_0 = 10.58$ GHz in $S_{11}$, $S_{21}$ and $S_{23}$ (see Fig. 3.1a). On dipole mode, the resonant excitation develops along one of the arms of the cross resonator, whereas on the quadrupole mode, the resonance occurs in both arms of the cross resonator, hence the resonant dip is also seen in $S_{23}$. Q–factors, i.e. the ratio of stored energy and dissipated energy of the resonator, can be deduced by

$$Q = \frac{f_0}{\Delta f_0},$$  

(3.1)

where $\Delta f_0$ is the full width at half maximum of the resonance. The Q–factor on the dipole mode shown in Fig. 3.1a is 34 and on the quadrupole mode is 46.
Figure 3.1: (a) Reflection and transmission parameters of the cross resonator measured at room temperature. The dipole mode resonance at 9.5 GHz and the quadrupole mode resonance at 10.58 GHz are evident as peaks and dips. (b) The transmission parameter $|S_{21}|$ as a function of the phase difference $\beta$ applied between port 1 and port 3. The observed $|S_{21}|$ is shown in false color. On quadrupole mode, the resonance vanishes at $\beta = 195^\circ$ while the dipolar mode intensity only very slightly changes (see panel (c)). (c) and (d) show $|S_{21}|$ at $f_0 = 9.5$ GHz and $f_0 = 10.58$ GHz.

The two different modes of the cross resonator can also be observed when applying microwave stimulus at port 1 and 3 simultaneously with a fixed phase relationship $\beta$. We denote $S_{21}$ as the voltage applied at port 1 divided by the voltage received at port 2. Note that the voltage received at port 2 is transmitted from both port 1 and port 3. Our VNA is equipped with a phase control feature which provides a specific phase difference between its two sources. The phase difference can be swept between two arbitrary phase values, for example, from 0° to 360°. For phase control, a full 3 port calibration is performed before the experiment [27].

Figure 3.1b shows the magnitude of the transmission parameter, denoted here as $|S_{21}|$, as a function of microwave frequency $f$ and phase difference $\beta$ between port 1 and port 3 under phase control. On the quadrupole mode, the strength of the resonance characteristically changes when both microwave stimuli are in–phase ($0^\circ$ phase difference viz. $\beta = 0^\circ$) and out of phase ($\beta = 180^\circ$) on the cross resonator. Figure 3.1d shows the detail of $|S_{21}|$ on the quadrupole mode. At $\beta = 15^\circ$ the microwaves interfere constructively while nearly cancel each other out at $\beta = 195^\circ$. On the dipole mode, the anti–node of the standing waves are located on the center of the cross resonator while on the quadrupole mode, the node is on the center on quadrupole mode, so the rf–magnetic field is not generated on the center of the cross resonator. That is why the circularly polarized microwaves on the quadrupole mode are not generated.

For the dipole mode, the configuration of the microwaves is determined by the amplitude of $S_{21}$. The received voltage at port 2 originates mainly from the microwave voltage $A_1 e^{i\omega t}$ at port 1 but a small contribution $A_3 e^{i(\omega t + \Delta \beta)}$ from port 3 is detected. Note that
$\Delta \beta$ is the relative phase of two microwaves at the center of the cross resonator. Due to the length difference $L_x - L_y$ of the feed line which are orthogonal to each other (see Fig. 2.1b), the phase difference $\Delta \beta$ at the center of the cross resonator is different from the phase difference $\beta$ between port 1 and 3. If the contribution of the microwave at port 3 is as $\delta$ times smaller as at port 1 ($\delta \ll 1$), $S_{21}$ on the dipole mode is calculated as

$$S_{21} \propto \frac{e^{i\omega t} + \delta e^{(i\omega t + \Delta \beta)}}{e^{i\omega t}} = 1 + \delta \cos \Delta \beta + i\delta \sin \Delta \beta$$

$$|S_{21}| \propto \sqrt{1 + 2\delta \cos \Delta \beta + \delta^2} \simeq (1 + \delta \cos \Delta \beta).$$

Equation (3.2) denotes that $|S_{21}|$ is expected to vary as $\cos(\Delta \beta)$. Even if the contribution of the microwave stimulus from port 3 to $|S_{21}|$ is very low, we still can infer the polarization states of the resulting, compound microwaves from the dependence of $S_{21}$ on $\Delta \beta$. Figure 3.2 shows the corresponding relationship between $|S_{21}|$ and the polarization state. The maximum absorption at $\Delta \beta = 0^\circ$ and the minimum absorption at $\Delta \beta = 180^\circ$ correspond to linearly polarized microwaves in the diagonal direction whereas the intermediate absorption at $\Delta \beta = 90^\circ$ and $270^\circ$ indicate the circularly polarized microwaves. From the phase difference $\Delta \beta$, we can decide the polarity of microwaves. For example, when $\Delta \beta = 90^\circ$, the microwaves rotate in the counter-clockwise direction.

![Figure 3.2: $|S_{21}|$ as a function of $\beta$ on the dipole mode under phase control. The phase determines the polarity of microwave radiation.](image)

Figure 3.1c shows $|S_{21}|$ on dipole mode. At $\beta = 80^\circ$ and $260^\circ$, $|S_{21}|$ is slightly smaller and larger than the average value. This shows that at these phase difference values $\beta$ the microwave on top of the cross resonator is linearly polarized. Microwaves are circularly polarized in a counter-clockwise direction at $\beta = 170^\circ$ and a clockwise direction at $\beta = 350^\circ$. 
3.1.2 Electron paramagnetic resonance using the cross resonator in the dipole mode

In order to characterize the polarization of the microwave magnetic fields, a small DPPH sample is mounted on top of the cross resonator. Using the phase control approach described in the previous section, $S_{21}$ is measured while sweeping the magnetic field $\mu_0 H_0$ and the phase difference $\beta$ at the dipole mode frequency (9.5 GHz). At that frequency, the EPR of DPPH is observed for $\mu_0 H_{\text{res}} = 340\,\text{mT}$. The resonance field $\mu_0 H_{\text{res}}$ is given by the resonance condition, $hf = g\mu_B\mu_0 H_{\text{res}}$, and $g = +2.0036$ is the $g$–value of DPPH. The magnetic field is swept up and down in the range of 336 mT to 344 mT in 0.05 mT steps. The microwave power of port 1 is set to 0 dBm while port 3 is driven with 0 dBc relative to port 1.

$S_{21}$ magnitude

To investigate the contribution of EPR to $S_{21}$, one usually considers the relative change of the magnitude of $S_{21}$, described as

$$\Delta_{\text{Mag}}|S_{21}|(f,\mu_0 H_0) = |S_{21}|(f, \mu_0 H_0) - |S_{21}^{\text{BG}}|$$

(3.3)

where $S_{21}^{\text{BG}} = S_{21}(f, \mu_0 H_0)|_{H_0 \neq H_{\text{res}}}$ is the off resonant field independent background. We would like to stress here that Eq. (3.3) is problematic, since only the magnitude of the complex–valued S–parameter are considered and even subtracted. However, this approach appears to be commonplace in the literature. In particular, we believe that the data shown in Ref. [6] were analyzed in this way. We therefore here also first use Eq. (3.3), only to show that ignoring the phase does not allow to draw robust conclusions on the degree of circular polarization.

Figure 3.3 shows $\Delta_{\text{Mag}}|S_{21}|$ for positive and negative magnetic field. The resonance dip shown in $S_{21}$ at positive magnetic field is shifted by $180^\circ$ with respect to the negative magnetic field. At $\beta = 0^\circ$, the strength of EPR is larger for positive magnetic field direction than for negative magnetic field direction, and vice versa at $\beta = 180^\circ$ (see Fig. 3.4a and c). On the other hand, at $\beta = 90^\circ$ and $270^\circ$ in Fig. 3.4b and d, $\Delta_{\text{Mag}}|S_{21}|$ on the resonance field of EPR signal is nearly the same for each magnetic field direction.

This general behavior can be explained by the polarization of the created microwaves on top of the cross resonator as discussed in Sect. 2.1. For circularly polarized microwaves radiation, the EPR is excited for either positive or negative magnetic field. On the other hand, the linearly polarized microwaves consist of circularly polarized microwaves rotating clockwise and counter-clockwise and hence EPR is excited for both field directions. Generally, when the EPR is detected, only an absorption-like signal is seen while we can observe an absorption-like signal at $\beta = 0^\circ$ and $180^\circ$ and a dispersion-like signal at $\beta = 90^\circ$ and $270^\circ$ for both directions of $\mu_0 H_0$. These dispersion-like signals appear due to the
3.1 Cross resonator

Figure 3.3: False colour plot of the relative change $\Delta_{Mag}|S_{21}|$ as a function of magnetic field $\mu_0 H$ and phase difference $\beta$, for a DPPH sample on top of the cross resonator. The EPR of DPPH is clearly seen around $\mu_0 H_{res} = \pm 340$ mT. The EPR in positive magnetic field is shifted by 180° in the relative phase $\beta$ as compared to the negative magnetic field.

Figure 3.4: The relative change $\Delta_{Mag}|S_{21}|$ as a function of magnetic field $\mu_0 H$ (a) at $\beta = 0^\circ$, (b) 90°, (c) 180° and (d) 270°. A larger (stronger) absorption dip is seen in positive field as compared to negative field at $\beta = 0^\circ$ and vice versa at $\beta = 180^\circ$. At $\beta = 90^\circ$ and 270°, dispersion-like magnetic resonance signal is observed.
background subtraction in $|S_{21}|$, which is dependent on phase difference $\beta$.

Figure 3.5a shows $\Delta_{\text{Mag}}|S_{21}|$ at the resonance field $\mu_0 H_{\text{res}} = \pm 340 \text{ mT}$. The 180° characteristic shift of the EPR between both field directions is also clearly observed. To evaluate the polarization of EPR, we define the degree of phase dependent polarization $P$ as

$$P = \left| \frac{|A^+| - |A^-|}{|A^+| + |A^-|} \right|$$

where $A^+$ and $A^-$ are the amplitude of EPR in positive and negative magnetic field, respectively. Note that $A^\pm = \Delta_{\text{Mag}}|S_{21}|$ at $\mu_0 H_{\text{res}} = \pm 340 \text{ mT}$ is used. Figure 3.5b shows $P$ as a function of $\beta$, and no correlation can be observed. In Fig. 3.4, the peak and dip of EPR is seen. In addition to that, the appearance of a dispersion-like signal indicates the EPR at certain $\beta$, however, due to the shape of the dispersion-like signal, $\Delta_{\text{Mag}}|S_{21}|$ can be 0 at the resonance field even if the EPR is excited. This small peak and dispersion–like function are artifacts originating from the magnitude background subtraction (see Appendix A), so that detecting the amplitude with magnitude background subtraction of this signal does not help to define the polarization of the microwaves. Therefore, though circular polarization is indicated, quantitative analysis is not possible in a set–up using phase insensitive detection.

Figure 3.5: (a) The amplitude of $\Delta_{\text{Mag}}|S_{21}|$ in positive (red) and negative (blue) magnetic field and (b) $P$ as a function of $\beta$ at the resonance field $\mu_0 H_{\text{res}} = \pm 340 \text{ mT}$. Considering only the amplitude of $\Delta_{\text{Mag}}|S_{21}|$, $P$ cannot be calculated accurately.

**Real and imaginary part of $S_{21}$**

Until now, as well as in previous experiments on crossed coplanar resonators in Ref. [6], only the magnitude of the $S_{21}$ signal has been taken into account. However, in order to employ a model that is physically more accurate and to analyse the polarization precisely in our set–up, the phase information is paramount. Instead of Eq. (3.3), we therefore use

$$\Delta_{\text{Comp}} S_{21}(f, \mu_0 H_0) = S_{21}(f, \mu_0 H_0) - \text{Re}(S_{21}^{BG}) - i \text{ Im}(S_{21}^{BG})$$

(3.5)
for the data analysis. $\Delta_{\text{Comp}} S_{21}$ is nothing short of the EPR signal.

Figure 3.6a and b show $|\Delta_{\text{Comp}} S_{21}|$ as a function of $\mu_0 H_0$ and $\beta$ in false colour plot. The resonance shown in $|\Delta_{\text{Comp}} S_{21}|$ at positive magnetic field is shifted by around 180° in $\beta$ with respect to the negative magnetic field. In Fig. 3.6c - f, line cuts $|\Delta_{\text{Comp}} S_{21}|$ at $\beta = 10^\circ$, 100°, 190° and 280° are shown for each magnetic field direction. Compared with $\Delta_{\text{Mag}} |S_{21}|$ in the previous section, $|\Delta_{\text{Comp}} S_{21}|$ only exhibits a pure absorption-like signal. Note that absolute value of $|\Delta_{\text{Comp}} S_{21}|$ is calculated only after background subtraction, which results in the simple lineshape. At $\beta = 10^\circ$ and 190°, microwaves are circularly polarized, hence the EPR is excited for a magnetic field of a certain direction. At $\beta = 100^\circ$ the EPR absorption signal strength is nearly equal for both field directions. Judging from the result in Fig. 2.3, at $\beta = 10^\circ$ the microwaves rotate in clockwise direction. At $\beta = 10^\circ$, the spins in the DPPH sample precess in the counter-clockwise direction in negative magnetic field. The polarity of the microwave radiation is in good agreement with the assumption in Fig. 3.2.

The real part and imaginary part of $\Delta_{\text{Comp}} S_{21}$ consist of a dispersion-like and absorption-like functions. The mixing ratio of both functions is described by the comparative coefficient $k$ in Eq. (2.2). $k$ is the ratio of the amplitude of rf–magnetic fields in $x$–direction and $y$–direction, and thus allows the degree of circularity of the microwaves on top of the cross resonator to be evaluated. For example, $k = 1$ expresses that microwaves are purely circularly polarized at $\beta = \pm 90^\circ$ since the amplitude of microwaves for both $x$– and $y$–component are the same. If $k \approx 0$ at certain $\beta$, or much larger than 1, the microwaves are almost linearly polarized. To investigate $k$, expressing the magnitude and phase of the susceptibility is mandatory. Now we write $S_{21}$ as

$$S_{21} = A \exp(i\Phi) \chi_{11}$$

(3.6)

where $\Phi$ is the phase delay of EPR signal. Note that $\chi_{11}$ contains the dispersion-like and absorption-like functions as specified in Eq. (1.12). Figure 3.7 shows $A$ and $\Phi$ (black circles) derived from a simultaneous fit of the complex $S_{21}$ to real and imaginary part of the EPR signal. From Eq. (2.6), $A$ and $\Phi$ however can also be calculated as follows:

$$A \propto ||\mu_0 h_0 (ke^{i\beta} + i\eta)e^{i\theta}|| = |\mu_0 h_0| \sqrt{1 + k^2 + 2\eta k \sin(\beta)},$$

$$\Phi = \text{Arg}(\mu_0 h_0 (ke^{i\beta} + i\eta)e^{i\theta}) = \text{Arg}(\mu_0 h_0 (ke^{i\beta} + i\eta)) + \theta,$$

(3.7)

where $\mu_0 h_0$ is the complex value of the rf–magnetic fields on top of cross resonator and $\theta$ is the phase offset intrinsically generated due to the geometry of the cross resonator. As both, $A$ and $\Phi$ depend on $k$, we determine $k$ from a simultaneous fit. The dashed lines in Fig. 3.7a and b show the fitting result for each phase difference $\beta$, and we obtain the comparative factor $k = 1.30$ for positive $\mu_0 H_0$ and $k = 1.16$ for negative $\mu_0 H_0$ when the power and relative power are 0 dBm and 0 dBc, respectively. In this condition, $k$ is larger than 1, and this means that microwave applied at port 3 is bigger than at port 1.
Figure 3.6: (a) and (b) Absolute value of $\Delta_{\text{Comp}}S_{21}$ (Eq. 3.5) (with subtraction of complex background) with sweeping $\mu_0H_0$ and $\beta$. Panel (c) - (f) show $|\Delta_{\text{Comp}}S_{21}|$ as a function of $\mu_0H_0$ at the resonance frequency 9.5 GHz. Red and blue color indicate EPR signal recorded for positive and negative magnetic field, respectively. At $\beta = 10^\circ$ and $190^\circ$, the microwaves are circularly polarized and rotate in the clockwise and counter-clockwise direction, respectively.

Figure 3.7c shows the microwave polarity (now calculated using the $A$-values obtained from the full complex–valued fit) as a function of $\beta$. The results indicate that an 86% degree of polarization can be achieved for the given conditions. To achieve the purely circularly polarized microwaves, the adjustment of the relative power is mandatory. The asymmetry of the polarization may be due to phase delay of the equipment used in the experiment that depends on the magnetic field. The position of the sample may also provoke the asymmetry, which is difficult to evaluate precisely.

3.1.3 EPR as a function of the relative amplitude

In Fig. 3.1, $S_{21}$ is slightly smaller than $S_{23}$ on the dipole mode. This result indicates that the microwave amplitude from port 3 is larger than from port 1. In order to obtain the maximal circularly polarized microwaves at the sample position, the stimuli from port 1 and 3 on dipole mode need to be matched in amplitude. To achieve this, we modify the
relative stimulus power $P_{\text{relative}}$ of the two ports from $-7 \text{ dBc}$ to $10 \text{ dBc}$. Figure 3.8 shows $k$ as a function of relative power by dots. The relative voltage $V_{\text{relative}}$ of two microwave stimuli which determines the relative coefficient $k$ is given by $V_{\text{relative}} \propto P_{\text{relative}}$ in dB scale, so that $k$ is exponential to the relative power $P_{\text{relative}}$ and the relation is described as

$$k = k_0 \exp(a P_{\text{relative}}) + k_{\text{offset}}$$

(3.8)

where $k_0$ is the value of $k\big|_{P_{\text{relative}}=0}$, $a$ is the proportionality factor for $P_{\text{relative}}$ and $k_{\text{offset}}$ is the offset value. Dashed line in Fig. 3.8 shows fitting result with Eq. (3.8). Using the fit parameters $k_0 = 1.4$, $a = 0.104$ and $k_{\text{offset}} = -0.117$, we obtain $P_{\text{relative}} = -2.2 \text{ dBc}$ for which $k = 1$, i.e., where the circularly polarized microwaves are created on top of the cross resonator. On the other hand, when $P_{\text{relative}} = 10 \text{ dBc}$, the microwaves behave essentially as being linearly polarized.

We now analyze the set of data taken with $P_{\text{relative}} = -2.0 \text{ dBc}$ shown in Fig. 3.9a and b. For the relative power, $k = 1.03$ close to 1, such that a high degree of circular polarization is to be expected. The local minimum value of $A$ is closer to 0 compared with the value of $A$ for $P_{\text{relative}} = 0.0 \text{ dBc}$. More specifically, more purely circularly polarized microwaves are created since the EPR is strongly excited in one direction of magnetic field whereas barely excited in other direction. Fitting the data described in the context of Fig. 3.7, we obtain $k = 0.96$ for positive magnetic field and $k = 0.93$ for negative magnetic field. These values are closer to 1 compared with $k$ when $P_{\text{relative}} = 0.0 \text{ dBc}$. Accordingly, we obtain the maximum polarization $P = 97\%$ (see Fig. 3.9c).
3.2 Retangular waveguide

Another way to create circularly polarized microwaves is using the TE$_{10}$ mode of a rectangular waveguide [3]. Depending on the position inside the rectangular waveguide, the polarity of the microwaves is different. For example, in the center of the waveguide, the microwaves are linearly polarized while off the axis center, they can have different degrees of ellipticity. One of the advantages of this experiment is that the frequency is not fixed at a resonator mode frequency and thus broadband FMR with the circularly polarized microwaves can be achieved. In this section, we show the FMR spectra of a YIG film at room temperature. In addition to that, we analyze the frequency dependency of polarization.
3.2.1 FMR measured in forward and backward transmission

The transmission $S_{21}$ and $S_{12}$ are measured with a VNA as described in Chapt. 2. We choose a frequency range from 6 GHz to 16 GHz and sweep the magnetic field $\mu_0 H_0$ with a step width of 1 mT. When viewed at the sample position along the direction of the magnetic field lines, the microwaves rotate in clockwise direction for a $S_{21}$ VNA measurement (see Fig. 2.6c). The YIG sample is set on the Teflon 2 mm lower than the center of $y$-axis (see Fig. 2.6b). The YIG sample dimensions are $2 \text{ mm} \times 2 \text{ mm}$ with thickness of $5 \mu \text{m}$ and we note that the finite sample size will limit the degree of polarization we can expect. The normal of the sample plane is parallel to the magnetic field direction. In order to remove the frequency dependent background, we consider the relative change of the transmission parameter

$$\Delta_{H_{\text{ref}}} S_{21} = S_{21}(f, \mu_0 H_0) - S_{21}(f, \mu_0 H_{\text{ref}}),$$

where $\mu_0 H_{\text{ref}}$ is a reference field at which no FMR signal is expected for the given frequency range.

Figure 3.10a and b show $\Delta_{H_{\text{ref}}} |S_{21}|$ as a function of the magnetic field and frequency with the reference field $\mu_0 H_{\text{ref}} = \pm 400 \text{ mT}$. We consider a field range from $400 \text{ mT}$ to $700 \text{ mT}$ and $-400 \text{ mT}$ and $-700 \text{ mT}$ where the FMR is expected to be seen. In both directions of $\mu_0 H_0$, the FMR absorption is clearly observed. Below $f = 6.6 \text{ GHz}$, no FMR absorption is observed since that frequency range is below the cut–off frequency of the wave guide where no microwave propagation is possible in the waveguide. To visualize the circularly polarization of the microwaves, the subtraction

$$\Delta_{\text{foldback}} S_{21} = S_{21}(f, \mu_0 H_0) - S_{21}(f, -\mu_0 H_0).$$

is useful (see Fig. 3.10c). For linearly polarized microwaves, no change of the resonant absorption is expected upon reversing the field. Therefore, no signal should be visible when calculating the difference of the absorption at positive and negative field. In the presented data there is, however, a clear signal which indicates elliptically polarized microwaves. The microwave propagation reversal reverses the polarization and thus the sign of the FMR signal is inverted (see Fig. 3.10f).

For a quantitative analysis, we follow the same approach as for the EPR on the cross resonator in Sect. 3.1. In detail, Eq. (3.5) is applied for the subtraction of background and Eq. (3.7) is used to obtain the amplitude $A$ and phase shift $\Phi$ of the FMR signal. When the simultaneous fit is conducted, initial value for the fitting should be already close to the fitting parameter. For the broadband FMR experiment, however, the relation between frequency and the amplitude $A$ of FMR signal is not easily estimated, so that not the complex valued $S_{21}$ but the real part of $S_{21}$ is used for fitting. Note that the background
Figure 3.10: (a) $\Delta H_{ref}|S_{21}|$ and (d) $\Delta H_{ref}|S_{12}|$ for negative field and (b) $\Delta H_{ref}|S_{21}|$ and (e) $\Delta H_{ref}|S_{12}|$ for positive field. (c) $\Delta_{\text{foldback}}|S_{21}|$ indicates that FMR for negative fields is larger than for positive field. On the other hand, (f) $\Delta_{\text{foldback}}|S_{12}|$ indicates that FMR for positive fields is larger than for negative field.

is field dependent as seen in Fig. 3.10, therefore a field dependent background,

$$S_{21}^{BG} = a_{BG}\mu_0 H_0 + b_{BG},$$

around the resonant field is taken into account. $a_{BG}$ and $b_{BG}$ is the slope and interception for characterizing the field dependent background. We restrict the analysis to the recommended frequency band for WR–90 waveguides from 8.2 GHz to 12.4 GHz to avoid the dispersion at the lowest cut–off frequency and coupling with the higher mode (see Sect. 2.2.1 for details).

The amplitude $A$ and the phase shift $\Phi$ thus obtained show a linear increase with frequency (see Fig. 3.11a and b). For the amplitude $A$, this can be explained by Eq. (2.5) since the detected induced voltage is proportional to the dynamic component of the magnetization $m(t)$, which, in turn, is proportional to frequency. To understand the frequency dependency of the phase shift, the phase delay $\tau_p$ of the propagating microwaves must be considered. The phase delay is the rate of change of the total phase shift with respect to frequency and described as

$$\tau_p = \frac{\Delta \Phi(f)}{\Delta f}.$$
In the rectangular waveguide with mechanical length $l_{\text{mech}}$ and the speed of microwaves $c_{\text{MW}}$, the phase is described as

$$\Phi = \frac{fl_{\text{mech}}}{c_{\text{MW}}} \times 360^\circ.$$  \hfill (3.13)

By inserting Eq. (3.13) into Eq. (3.12) the phase delay is given by $\tau_p = \frac{l_{\text{mech}}}{c_{\text{MW}}} \times 360^\circ$, and we obtain $\tau_p = 502^\circ$/GHz. Note that the frequency range for the fit is conducted up to 12 GHz in order to remove the contribution of phase jump at 12.2 GHz (see Fig. 3.11b). In this set-up, the length of the waveguide $l_{\text{mech}} = 24.5$ cm so that $\tau_p$ is estimated to be $294^\circ$/GHz which is in the same order of the experimental result. Figure 3.11c shows the polarization $P$ as a function of frequency. The FEM simulation in Chapt. 2 shows that the optimal position for circularly polarized microwaves does change with increasing frequency and hence, the polarity at a fixed position is frequency dependent. The value of the polarity (e.g. $P \approx 30\%$ at 9 GHz) in the experiment is in good agreement with in the simulation (cf. Fig. 2.9c). In addition to that, when the size of the sample is close to the wavelength of the microwave, the polarized microwaves applied to sample become inhomogeneous, and thus $P$ may decrease with increasing frequency as well. Because of the sample size (2 mm) compared to the wavelength of the microwaves (~ 30 mm) we expect that this is not the dominant effect for the reduction of the degree of circular polarization. When the propagation direction is positive, the FMR signal for negative field is bigger than positive field and vice versa. Taking into account the polarity of the magnetic field and the polarization of the microwaves, we can determine that the sign of g-factor of the YIG is positive, in agreement with the literature [29].

At 12.2 GHz, an extraordinary large amplitude $A$ and $360^\circ$ shift in $\Phi$ are observed. The wavelength of the microwaves at 12.2 GHz is $\lambda_{\text{res}} = \frac{c_{\text{MW}}}{12.2 \text{GHz}} = 24.6$ mm and the half wavelength of $\lambda_{\text{res}}$ roughly coincides with the short dimension direction, $b$, of the waveguide, and this means, resonant mode is excited in $b$-direction. Since the FMR signal is dominantly excited by standing mode in $b$-direction, which does not have the elliptical polarity, $P$ drops down to zero.

In the experiment, we can also determine the frequency dependent full-width at half maximum (FWHM) $\mu_0\Delta H_{\text{res}}$ of the FMR absorption signal (see Fig. 3.12). This allows Gilbert damping constant $\alpha$ to be determined from a fit to Eq. (1.13). With the relation $\omega = 2\pi f$, Eq. (1.13) is written as

$$\mu_0\Delta H_{\text{res}} = \mu_0\Delta H_{\text{res},0} + \left| \frac{4\pi\alpha}{\gamma} \right| f,$$ \hfill (3.14)

where $\mu_0\Delta H_{\text{res},0}$ is the FWHM at $\mu_0H = 0$. $\alpha$ is determined by the slope of 2nd term in Eq. (3.14). By the fit up to 12 GHz, we obtain $\alpha = 0.00043 \pm 0.00012$ for positive field and $\alpha = 0.00138 \pm 0.00008$ for the negative field. Ideally the perfect single-crystal YIG bulks have the lowest damping of $3 \times 10^{-5}$ [30]. If the quality of crystalline structure
Figure 3.11: Panels (a) and (b) show the value of $A$ and $\Phi$ obtained from a fit of Eq. (3.6) (real part) to the real part of $S_{21}$. The red and blue symbols hereby indicate $A$ and $\Phi$ for positive and negative $\mu_0H_0$, respectively. (c) The degree of polarization, calculated using $A$–values by Eq. (3.4), as a function of $\beta$. Both $A$ and $\Phi$ increase with an increase of frequency while $P$ decreases. At 12.2 GHz where the extremely large peak for $A$ and the 360° phase shift in $\Phi$ are observed due to the waveguide resonance, the value of $P$ drops.

decreases, the microwave loss increases and the damping constant increases by a factor of 10 to 100 [31]. In our experiment, spin wave excitations overlapped to the uniform mode (FMR) might be also detected. Those mixed excitation signals result in the broader line width and higher damping constant. In order to distinguish the FMR signal from the spin wave excitation effect, magnetic field sweep with narrower step width ($\sim 0.01$ mT) must be conducted. The damping constant detected in the positive magnetic field is smaller than in positive field with the factor of 3. As seen in Fig. 3.12, below 9.5 GHz the line width of the FMR signal detected in the positive magnetic field is more noisy and does not follow the linear response to the microwave frequency as described in Eq. (3.14). In this frequency range, the polarization of the incident microwaves is relatively higher (see Fig. 3.11c), and thus the FMR is excited less strongly in positive magnetic field resulting in the less reliable fit to the weak FMR signal. In addition to that, since the FMR is strongly excited in the positive propagation, the signal–to–noise ratio in the negative field is higher than in positive field, and thus the damping constant calculated from FWHM in the negative field has a lower standard deviation than in the positive field.

3.2.2 FMR measured in reflection

The transmission and reflection parameters are detected simultaneously. Despite the fact that the reflection in the set–up is really low, the FMR signal is distinctly seen. Figure 3.13a, b and c show $A$, $\Phi$ and $P$ as a function of frequency, respectively. With Eq. (3.12) and Eq. (3.13), we obtain phase delay $\tau_p = 715°/\text{GHz}$. In transmission, the value of phase delay obtained by experiment and calculated with mechanical length are differed by the factor of 1.71. In consideration of this factor, the mechanical length
3.2 Rectangular waveguide

Figure 3.12: The FMR line width $\mu_0 \Delta H_{res}$ as a function of frequency (a) for negative (blue) and (b) for positive (red) magnetic field. The yellow dashed lines indicate the fit to Eq. (3.14). The FWHM of the FMR for positive field includes more noise than for negative field since the FMR for positive field is smaller.

in reflection is calculated to $l_{mech} = 34.8$ cm. The length between the sample and the connector is 19.4 cm, and thus the distance of the microwave traveling which is reflected at the sample position is calculated to 38.8 cm, and this distance is in good agreement with the value calculated from the experiment. In the reflection $S_{11}$, the impact of the circularly polarized drive is not visible, even though the spin system is excited by the circularly polarized incident microwaves. This result is based on the efficiency of the microwave propagation depending on the polarity. At the sample position, the FMR is excited strongly by polarized microwaves rotating clockwise for the negative magnetic field. The microwave radiation which is circularly polarized is consisted of one elliptically polarized microwaves rotating in the clockwise direction, and transmitted forward (detected in $S_{21}$), and the other rotating in counter-clockwise direction transmitted backward (detected in $S_{11}$). This microwave radiation emitted by the precessing magnetization in FMR can only excite the forward propagating mode efficiently, and FMR signal transmitted in backward propagation mode is detected weakly in reflection. When reversing the direction of the magnetic field, the FMR is weakly excited by the clockwise incident microwaves since the rotating direction of the magnetization precession excited by the FMR is also reversed. This weak microwave radiation, however, is composed of the elliptically polarized microwaves rotating in the counter-clockwise direction dominantly, and thus the microwaves can propagate backward more efficiently than forward. In sum, the detected signal is the same for both magnetic field direction and no polarization is detected in $S_{11}$ (reflection). Further details of the microwave propagation picture are outlined in Appendix B.

When looking at the amplitude and linewidth for transmission and reflection, it becomes obvious that the FMR data measured in reflection ($S_{11}$) is less noisy than the data taken in $S_{21}$ and the fit of the damping constant is much more reliable (see Fig. 3.14). In both cases, the FMR is dominantly excited by the forwards propagating microwaves. However,
Figure 3.13: (a) The amplitude $A$ and (b) the phase shift $\Phi$ obtained by fit with the real part in $S_{11}$ for positive (red) and negative (blue) magnetic field. (c) The polarity $P$ as a function of frequency. In $S_{11}$, no polarization can be detected even though the strength of the FMR is changed by the polarity of microwaves.

$S_{11}$ has less noise compared with the FMR signal amplitude than $S_{21}$ and $\mu_0 \Delta H_{\text{res}}$ has a linear response to the microwave frequency more consistently, and thus a more precise fit for investigating the FWHM of the FMR is achieved. We conclude that the low reflection of the rectangular waveguide is the good feature for acquiring $\alpha$ reliably even though the detected FMR signal is excited by identical microwaves in transmission and reflection.

Figure 3.14: The line width $\mu_0 \Delta H_{\text{res}}$ derived from the $S_{11}$ data as a function of frequency (a) for negative (blue) and (b) for positive (red) magnetic field. Compared with the data obtained from $S_{21}$, there is less noise and $\alpha = 0.00113$ for positive field and $\alpha = 0.00106$ for the negative field are consistent within the error.
Chapter 4

Discussion

4.1 Comparison of cross resonator and rectangular waveguide

Even though for both approaches presented in this work, circularly polarized microwaves have been successfully excited, the two approaches differ in detail. The fundamental difference is the physical principle behind the two approaches: In the cross resonator, two orthogonally linearly polarized TEM modes are separately excited with a fixed phase relationship. The superposition of the two modes in a confined spatial region then creates circularly polarized microwaves. In the rectangular waveguide, the traveling microwaves of a TE mode are utilized for the creation of intrinsic circularly polarized microwaves, and the polarity depends on the position of the sample relative to the waveguide axis.

Although we investigated two quantitatively different samples (paramagnetic DPPH in the cross resonator, ferrimagnetic YIG in the waveguide), the property of the two approaches can be compared. Note that due to lack of time, it unfortunately was not possible to study one and the same sample in both set-ups. In our experiment, the DPPH sample were used for well-known standard sample, and the FMR experiment was demonstrated for the investigation of the YIG film sample of which the sign of the g-factor was unknown. The most apparent difference of the presented results lies in the maximal polarization possible. For the cross resonator, a circular polarization of 97% is achieved whereas the rectangular waveguide shows a typical polarization of only 31% although over a broadband frequency range (nearly 4 GHz). This difference has, however, to be regarded in more detail: For the cross resonator, all available parameters - that is the relative amplitude $P_{relative}$ and the phase difference $\beta$ of the power between two microwaves - are explored and the optimal operating point has been found. We estimate that perfect (> 99%) circularly polarized microwaves are possible in this configuration upon fine-tuning these parameters. The rectangular waveguide approach, however, has still a high potential for improvement. The major factor limiting the maximum polarization turns out to be the Teflon sample holder. With its high dielectric constant that creates an impedance jump in the waveguide and distorts the microwave magnetic field, the degree of circular
polarization is limited to the above mentioned 31%.

The sample holder and the position in the waveguide are, however, the only free parameters that exist for the waveguide approach which sets it apart from the cross resonator approach. The experimental set–up in an electromagnet and the needed optimization are therefore very simple. A two port Scalar Network Analyzer or a simple microwave source and detector device are all the equipment needed for this approach. The rectangular waveguide for the $\text{TE}_{10}$ mode excitation is already standardized and commercially available. Also, custom designs are fabricated with low technological requirements. As a result, the FMR experiment is simply conducted after placing the sample on the proper position. In the case of the cross resonator, on the other hand, more sophisticated electronics are necessary. For the sake of superimposing the two microwaves on the center of the cross resonator, two phase–locked microwave sources are needed. In our case a four port VNA with two phase locked internal sources is used. The circularly polarized microwaves are then achieved only after carefully optimizing the phase difference $\beta$ and the relative amplitude $P_{\text{relative}}$ of the stimuli. Phase sensitive detection is necessary for the precise analysis of the polarity. For the rectangular waveguide in contrast, only the propagation direction and the sample position higher or lower than the waveguide axis allow to unambiguously infer the polarization, such that a scalar measurement of microwave transmission is sufficient.

Magnetic resonance experiments on the cross resonator are conducted at a fixed frequency determined by the length and thus the resonance frequency of the coplanar resonators. The $\text{TE}_{10}$ mode, on the other hand, can propagate in the waveguide within the recommended range of the waveguide standard. Thus, FMR is available in a frequency band which typically spans 4 GHz to 8 GHz depending on the waveguide standard.

Comparing the sensitivity of both approaches is not possible with the presented data. For that, a quantitative study of the signal amplitude for various differently shaped samples needs to be conducted. Generally, however, it can be noted that the coplanar resonator has advantages over the rectangular waveguide for small samples: The microwave field is concentrated in a narrow region around the center conductor. The filling factor, the ratio of the microwave energy in the sample compared with the total microwave energy of the mode. For a CPW resonator, thin film sample can fulfill the one side of the resonator surface, and therefore the filling factor can be up to 1/2. For a WR–90 rectangular waveguide, the filling factor will rather be in the order of $10^{-7}$ (calculated by sample volume per volume of rectangular waveguide) for a typical thin film sample. Additionally the resonator quality factor of the coplanar cross resonator (although rather low with $Q = 34$) increases the microwave magnetic field strength and thus the signal–to–noise ratio as compared to the waveguide. Moreover, contrary to the macroscopic rectangular waveguide, the size of the cross resonator can be reduced further using photo–lithography, with which the CPW gaps can be reduced down to 20 $\mu$m. Integrated on–chip and also high frequency applications are therefore conceivable.
4.2 Possible improvements of the cross resonator

Relative power sweep

In Sect. 3.1, \( k \) is estimated for each \( P_{\text{relative}} \) and the optimal \( k \) for generating circularly polarized microwaves is experimentally determined. When just the optimal \( P_{\text{relative}} \) is required, sweeping the relative power is sufficient. To determine the optimal \( P_{\text{relative}} \), as in the previous chapter, a DPPH sample with \( g > 0 \) should be mounted on the cross resonator and a positive magnetic field \( (\eta = -1) \) applied. At \( \beta = 90^\circ \), the microwaves are circularly polarized in the counter-clockwise direction and in the ideal case the EPR is not excited. With Eq.(2.6), the EPR signal in this situation is described as

\[
S_{\text{EPR}} \propto \mu_0 h_0 \chi_{11}(ike^{i(90^\circ)} + 1) \\
\propto \mu_0 h_0 \chi_{11}(k - 1).
\] (4.1)

As expected, when \( k = 1 \), the microwaves are purely circularly polarized in the counter-clockwise direction, and therefore the EPR signal disappears. In Eq. (4.1), \( h_0 \) and \( \chi_{11} \) are constant while the absolute power \( P \) and the frequency is kept constant. \( k \), however, is a function of the relative power \( P_{\text{relative}} \) of the two channels. If the relation between the amplitude of EPR and \( P_{\text{relative}} \) is measured, the optimal value for purely circularly polarized microwaves can be extracted from a variation of \( P_{\text{relative}} \) alone.

We have tested this idea experimentally. For this analysis, first of all the resonant field has to be determined. This is done by performing a field sweep at fixed frequency \( f = 9.5 \text{ GHz} \) and fixed phase difference \( \beta = 190^\circ \) for our sample (see Fig. 4.1a). From this measurement, we extract the on–resonant field \( \mu_0 H_{\text{res}} = 340.3 \text{ mT} \) and choose \( \mu_0 H_{\text{offres}} = 337 \text{ mT} \) as off–resonant field.

The amplitude of the EPR is simply estimated as the relative value

\[
\Delta_{\text{EPR}}|S_{21}| = |S_{21}|(\mu_0 H = \mu_0 H_{\text{res}}) - |S_{21}|(\mu_0 H = \mu_0 H_{\text{offres}}).
\] (4.2)

Under the condition that amplitude of the EPR signal is much smaller than of the background, \( \Delta_{\text{EPR}}|S_{21}| \) for the analysis of the amplitude of the EPR signal is still correct (see Appendix A). We now conduct a sweep of the relative power at fixed frequency \( f = 9.5 \text{ GHz} \) and fixed phase difference \( \beta = 190^\circ \) for the on–resonant field and off-resonant field. In Fig. 4.1b \( \Delta_{\text{EPR}}|S_{21}| \) is shown for \( P_{\text{relative}} \) in the range from \(-5.0 \text{ dBC}\) to \(5.0 \text{ dBC} \). The exponential relation between the amplitude of the EPR and \( P_{\text{relative}} \) is observed. It can be derived with the help of Eq. (3.8) and Eq. (4.1) which gives for the EPR amplitude

\[
\Delta_{\text{EPR}}|S_{21}| \propto k_0 \exp(aP_{\text{relative}}) + k_{\text{offset}} - 1.
\] (4.3)

Note that \( P_{\text{relative}} \) is in dBC scale. In this case the optimal relative power where the signal vanished is \( P_{\text{relative}} = -2.1 \text{ dBC} \). This result is in good agreement with the result obtained.
Figure 4.1: (a) $|S_{21}|$ measured on with the DPPH sample on the cross resonator, as a function of magnetic field at $\beta = 190^\circ$ and $f = 9.5$ GHz with the subtraction of the background. The DPPH EPR peak is observed at $\mu_0 H_{res} = 340.3$ mT for each $P_{relative}$. The magnitude of the EPR signal gets smaller as $P_{relative}$ approaches its optimal value where $k = 1$. (b) $\Delta_{EPR} |S_{21}|$ as a function of $P_{relative}$. The amplitude of the EPR signal is exponentially proportional to $P_{relative}$, and from the fitting result, the optimal $P_{relative} = -2.1$ dBc for $k = 1$ is obtained.

The advantage of this method to estimate optimal $P_{relative}$ is that it is quick, requiring only a small number of measurements. The magnetic field sweep and the phase sweep must be conducted at least once to see the resonant field and to investigate the polarity as a function of phase at the center frequency. However, the power sweep is conducted for on-resonant field $\mu_0 H_{res}$ and off-resonant field $\mu_0 H_{offres}$ at the certain polarity afterwards which takes, for the data shown in Fig. 4.1b, only about 10 min, and then $P_{relative}$ corresponding to $k = 1$ is easily estimated.

Note that the complex background subtraction is not conducted. With Eq. (4.3) the relation between the EPR signal after complex background subtraction and $P_{relative}$ is written as

$$|\Delta_{EPR} S_{21}| \propto |k_0 \exp(a P_{relative}) + k_{offset} - 1|.$$ (4.4)

Around the optimal relative power, the EPR signal is hidden by background noise, and $|\Delta_{EPR} S_{21}|$ shows the amplitude of the noise and does not reach to zero, and thus the fit to Eq. 4.4 does not work successfully. On the other hand, for the simple magnitude background subtraction, absorption peak of EPR turns into absorption dip at the optimal relative power as seen in Fig. 4.1a, which can be observed as the sign change of $\Delta_{EPR} |S_{21}|$, so that the fit Eq. (4.3) can be conducted successfully in comparison with the fit to Eq. (4.4) with $|\Delta_{EPR} S_{21}|$.

**Cross coplanar waveguide**

Due to the coupling gaps which disrupt the signal line, the cross resonator allows microwaves to be transmitted only at certain frequencies. In the second harmonic mode,
a node of the standing wave develops at the center of the resonator where a sample is mounted. Therefore the EPR experiment with circularly polarized microwaves cannot be conducted, and thus the second lowest frequency which can be applied for the experiment is on third harmonic mode. In order to conduct the broadband EPR experiment, a cross coplanar waveguide which does not have coupling gaps could be a next step (see Fig. 4.2a). As for a conventional CPW, broadband experiments should then be possible with such a cross CPW. In this section, we discuss the features of the cross CPW in comparison with the cross resonator using CST simulations. For convenience, identical dimensions of the cross resonator, for instance, the length of signal line, the CPW gap, etc. are used.

The transmission parameter obtained from the simulation for a frequency range of 8 GHz to 12 GHz is shown in Fig. 4.2b. Unlike the cross resonator (dashed line), microwaves can be transmitted for the whole frequency range in the cross CPW (solid lines). However, the amplitude difference between $S_{21}$ and $S_{43}$ varies as a function of frequency. This shows that at the cross section, the ratio of the amplitude of rf–magnetic field between $x$– and $y$–direction varies as a function of frequency. To obtain the purely circularly polarized microwaves, the adjustment of the relative amplitude for each frequency is mandatory. Note that the cross talk ($S_{31}$) is also observed and this may interfere and reduce the amplitude of the microwaves on the center of the cross structure.

![Figure 4.2: (a) Schematic image of crossed CPW structure. Its shape is the same as the cross resonator except that there are no coupling gaps. (b) Scattering parameters as a function of frequency, as calculated with CST simulation. Compared with $S_{21}$ of the cross resonator (dashed line), the microwaves can be transmitted thorough the signal conductor of the cross CPW (solid lines) for whole frequency range. However, the amplitude of the transmission for $x$–direction ($S_{21}$) and $y$–direction ($S_{43}$) varies as a function of frequency.](image)

In Fig. 4.3a the amplitude of the rf–magnetic field of $y$–component as function of $x$–position at 9.67 GHz for the cross CPW (solid line) and cross resonator (dashed line) are shown. 9.67 GHz is the resonance frequency of the cross resonator on dipole mode. The microwaves are applied from port 1 with the power 27 dBm, and the center of the cross section is chosen as $x = 0$. The yellow cross hatching highlights the area where the
feed lines of the cross resonator are positioned. At \( x = 0 \) where the sample is mounted, the strength of the rf–magnetic field in \( y \)-direction is \( 10.7 \mu \text{T} \) for the cross CPW and \( 81.1 \mu \text{T} \) for the cross resonator. From the perspective of detecting the EPR signal, the cross resonator thus is superior to the cross CPW, as expected for the comparison of a resonator with quality factor \( Q > 1 \) and a waveguide.

The phase difference of the microwaves between \( x \)- and \( y \)-component also varies as a function of frequency at the center of the cross point. Figure 4.3b shows the difference of the phase \( \text{Arg} (S_{21} - S_{43}) \) (red line). \( \text{Arg} (S_{21} - S_{43}) \) does not have the linear response to frequency since the different signal line length of the two CPWs in our design result in different transmission characteristics. To obtain the purely circularly polarized microwaves with sweeping frequency, the phase difference between the microwaves from port 1 and port 3 must also be adjusted for each frequency.

On the CPW structure, only one TEM mode which does not have cut–off frequency is transmitted, so that the CPW allows broadband EPR over a wide frequency range. This sets it apart from the rectangular waveguide. An other solution to conduct broadband EPR experiments with circularly polarized microwaves is by using two isolated coplanar waveguide on different chips and placing them close to each other but orthogonally. The sample is mounted between the cross point of both signal lines. Even though the strength and phase difference between the two microwave stimuli must be adjusted, this approach will substantially reduce the expected cross talk as compared with the cross CPW (see Fig. 4.2b).

**Figure 4.3:** (a) The amplitude of rf–magnetic field in \( y \)-component as a function of \( x \)-positions for the cross CPW (solid line) and the cross resonator (dashed line). The amplitude of the microwave field of the cross CPW is smaller than of the cross resonator at the sample position \( x = 0 \). (b) Phase difference of the perpendicular propagating microwaves \( \text{Arg} (S_{21} - S_{43}) \) as a function of frequency.
4.3 Possible improvements of the rectangular waveguide

As mentioned in Sect. 2.2.3, the Teflon holder inside the rectangular waveguide destroys the circularity of the microwaves due to the sharp jump of the dielectric function at the sample position. Ideally, the measurement should be conducted with mounting the sample inside the rectangular waveguide without disturbing the microwave properties due to the sample mount. However, for the convenience to mount and remove the small sample in the waveguide, a sample holder is required. To lower its influence, we consider the Teflon holder whose thickness is reduced to 1 mm. It should be noted that the polarity of the microwaves is not influenced by the sample holder height (z-direction). With the previous sample holder, whose thickness is one half of the inner height of the waveguide, the reflection parameter has the maximal value of one at the cut-off frequency (dashed lines in Fig. 4.4). With higher frequency it then decreases to zero while the transmission parameter gradually increases to one. Considering the thinner Teflon holder (solid line), these changes happen much more quickly and hence, high transmission and low reflection are achieved in the recommended frequency range.

![Figure 4.4](image-url)

**Figure 4.4:** Transmission and reflection parameter for the Teflon sample holder 22 mm × 5 mm × 14 mm (dashed lines) and the thin Teflon holder 22 mm × 1 mm × 14 mm (solid lines). With the thinner Teflon holder, there is lower reflection and higher transmission, such that the waveguide behaves nearly like an empty rectangular waveguide.

Figure 4.5a and b show the optimal position and the phase difference between the microwaves of x- and y-components at that position. The optimal position with thinner Teflon (orange dots) is very close to the case of the empty waveguide (green dots) while the optimal position for the previous holder (yellow dots) differs substantially. The simulations with the thin Teflon holder in the waveguide show, just like for the empty waveguides, a phase difference of around 90° over the whole frequency band. Hence, also the degree of circular polarization is over 90% in the whole allowed frequency band region at a fixed position of $y = -6$ mm (Fig. 4.5c). Taken together the microwave properties with the thinner Teflon sample holder are almost identical to those without Teflon, and a substantial improvement of the degree of circular polarization can be expected as compared to the
previous sample holder.

Figure 4.5: (a) The optimal position for the creation of circularly polarized microwaves and (b) phase difference between the microwaves of $x$– and $y$–component at the optimal position in the rectangular waveguide. With the thinner Teflon sample holder (orange dots), the microwave properties are very similar to those in the empty rectangular waveguide (green dots). The phase difference between $x$– and $y$–components is closer to the optimal value of 90° compared with the previous Teflon sample (yellow dots). (c) The polarity of the microwaves at $y = -6$ mm.
Chapter 5

Summary and outlook

The main focus of the thesis is to establish and characterize a method for the creation of circularly polarized microwaves. The goal is to create highly circularly polarized microwaves in a set-up compatible with low temperature experiments in order to enable a reliable determination of the sign and magnitude of the gyromagnetic ratio in ferrimagnetic sample as a function of temperature. We successfully assessed two different methods: Extrinsically superimposing two linearly polarized microwaves in a coplanar 2D structure and intrinsically generating circularly polarized modes in a 3D rectangular waveguide. For the 2D cross resonator, we employed an FEM simulation in order to predict the reflection and transmission characteristics (scattering parameters) of the structures. Based on the simulations, the structures were fabricated and optimized in–house on a low dielectric loss substrate using optical lithography. Experimentally, we were able to observe the resonant modes of the structure using vector network analysis. The scattering parameters recorded in experiment agreed very well with the FEM simulations. Given the good agreement, the FEM simulations also allowed us to predict the magnetic field distribution of the modes. We implemented a measurement routine to simultaneously and orthogonally excite linearly polarized modes in the crossed resonators. This led, at the crossing point, where the waves were superposed, to circularly polarized microwaves. We quantified the degree of polarization using EPR on a DPPH standard sample. By detecting the susceptibility as magnetization response to the microwaves, we found the optimal conditions to create the circularly polarized microwaves. By optimizing the relative power of both stimuli, we obtained microwave radiation with a degree of circular polarization of $P = 97\%$ at 9.5 GHz. Compared with the the polarization of 82% obtained by Henderson [6] and 80% by Mayer [28], the microwaves in our experiment were much better circularly polarized. In addition, our analysis allows to quantitatively assess the impact of the experimental parameters on the degree of circular polarization. Fine–tuning the experimental conditions as detailed in Chapt. 4 therefore will allow to generate circularly polarized microwaves with $P > 99\%$.

For the rectangular waveguide, a rectangular Teflon sample holder was inserted in the waveguide to place a ferrimagnetic YIG thin film sample. For this approach, FEM simulations were also performed in order to investigate how the polarization of the
microwaves changes with the sample position and microwave frequency. Within the recommended frequency range of the rectangular waveguide, the FMR signal was detected in transmission, and the microwave polarization was calculated as a function of frequency, where the typical polarization of 31% was observed. The decrease of the polarization as a function of the frequency expected from the simulation was also observed. We note that the linewidth of the FMR absorption was more precisely detected in reflection than in transmission and an accurate measure of the damping constant of the YIG sample was obtained. For the further improvement of the two methods, we proposed optimization routines and geometrical modifications of the cross resonator and a reduction of the Teflon holder height in Chapt. 4.

For both methods we conducted the experiments at room temperature. In order to conduct experiments at low temperature, the size of the device to generate the circularly polarized microwaves must be shrunk down to fit in the cryostat. For cross resonator, SMP connectors instead of end launches are preferable. If a variable frequency is desirable, the cross CPW instead of the cross resonator may be an alternative as discussed in Chapt. 4. For the rectangular waveguide, WR-42, with dimension of 10.668 mm × 4.318 mm which has smaller cross section but higher and wider recommended frequency range than WR-90, may fit in the cryostat.

For the temperature dependence experiment, an interesting group of materials are compensating ferrimagnets [32]: A ferrimagnet has (at least) two magnetic sublattices with neighboring magnetic moments pointing in opposite direction [33]. As in a ferromagnet, magnetic resonance is also excited in the ferrimagnet with the application of external magnetic field and microwaves. In a ferrimanget, however, there are multiple modes [34]: Ferromagnetic modes and exchange modes. In a low external magnetic field, the sublattices remain almost perfectly anti–parallel to each other, and the net magnetization precesses about the external (or more precisely the effective) field as in a ferromagnet. In this mode, the system behaves as a single lattice, so that the magnetization precesses in counter–clockwise direction. For typical laboratory fields of several hundreds of millitesla, the resonance frequency falls in the microwaves range as we observed in this thesis – YIG is a ferrimagnet. As in a ferromagnet, the amplitude of the magnetization depends on the temperature for each sublattice. For certain ferrimagnet, a so–called compensation temperature \( T_{\text{Comp}} \) exists where the total magnetization is zero [35]. At \( T_{\text{Comp}} \), the precessional direction of magnetization is reversed if anisotropies are neglected. In other words, the sign of the g–factor for a compensated ferrimagnet can change sign as a function of temperature. On the other hand, the exchange fields dominate the effective magnetic field for higher frequencies. The exchange field is contributed by neighboring magnetic moments in the other sublattice. For exchange fields of several hundred tesla, the resonance frequency falls in the teraherz range. Due to the direction of the net exchange field, the sublattice precesses in opposite direction compared to the low frequency mode. This effect can also be captured as a sign of the effective g–factor. In this mode, the resonant
frequency simply decreases as the temperature decreases due to the proportionality of the exchange field to the magnetization, and thus can be detected within the magnetic field range of a super conducting magnet. Clearly, experiments with circularly polarized microwave radiation in compensated ferrimagnets would be very interesting in the future.
Appendix A

Further Investigation of EPR signal with the background subtraction

A.1 Subtraction of magnitude background

In this chapter, the analysis in detail of $S_{21}$ of the cross resonator in Sect. 3.1 is discussed. For the sake of simplicity, $S_{21}$ is expressed as $S$. The detected signal $S$ is consisted of EPR signal $S_{EPR}$ and background signal $S_{BG}$, and is described as $S = S_{EPR} + S_{BG}$. At an off resonant field, $S_{EPR}$ is nearly 0 so that $S \simeq S_{BG}$. As in Sect. 3.1.1, $S_{BG}$ is composed of sinusoidal function with phase $\beta$, named $S_{BG}(\beta)$. With Eq. (3.2), the background signal is described as

$$S_{BG} = S_{BG,C}(1 + \delta \cos \beta + i\delta \sin \beta)$$

$$|S_{BG}| = |S_{BG,C}| \sqrt{1 + 2\delta \cos \beta + \delta^2} \simeq |S_{BG,C}|(1 + \delta \cos \beta),$$

where $S_{BG,C}$ is the constant value. The approximation $\sqrt{1+x} \simeq 1 + x/2$ under the condition $x \ll 1$ is performed. In the magnitude background, sinusoidal wave function appears as seen in Fig. A.1b. For whole the magnetic field, the sinusoidal background is continuously observed (see Fig. A.1a).

The magnitude of detected signal $S$ is not simply described as $|S| = |S_{EPR}| + |S_{BG}|$, but as

$$|S| = |S_{EPR} + S_{BG}|$$

$$= |(\text{Re}(S_{EPR}) + \text{Re}(S_{BG})) + i(\text{Im}(S_{EPR}) + \text{Im}(S_{BG}))|$$

$$= \sqrt{|\text{Re}(S_{EPR}) + \text{Re}(S_{BG})|^2 + |\text{Im}(S_{EPR}) + \text{Im}(S_{BG})|^2}$$

$$= \sqrt{|S_{EPR}|^2 + |S_{BG}|^2 + 2(\text{Re}(S_{EPR}) \text{Re}(S_{BG}) + \text{Im}(S_{EPR}) \text{Im}(S_{BG}))}$$

(A.2)
To simplify the equation, the assumption,
\[
\sqrt{x + y} - \sqrt{y} = \frac{\sqrt{x + y} - \sqrt{y}}{\sqrt{x + y} + \sqrt{y}} \approx \frac{x}{2\sqrt{y}} \quad (x \ll y),
\]
is considered. When \( x = |S_{\text{EPR}}|^2 + 2[\Re(S_{\text{EPR}}) \Re(S_{\text{BG}}) + \Im(S_{\text{EPR}}) \Im(S_{\text{BG}})] \) and \( y = |S_{\text{BG}}|^2 \) are inserted into Eq. (A.3) under the condition \( |S_{\text{EPR}}| \ll |S_{\text{BG}}| \), the magnitude background subtracted signal is calculated as
\[
\Delta_{\text{Mag}}|S| = |S| - |S_{\text{BG}}| \approx \frac{|S_{\text{EPR}}|^2 + 2[\Re(S_{\text{EPR}}) \Re(S_{\text{BG}}) + \Im(S_{\text{EPR}}) \Im(S_{\text{BG}})]}{2\sqrt{|S_{\text{BG}}|^2}} \approx \frac{\Re(S_{\text{EPR}}) \Re(S_{\text{BG}}) + \Im(S_{\text{EPR}}) \Im(S_{\text{BG}})}{|S_{\text{BG}}|}.
\]
Note that \( |S_{\text{EPR}}|^2 \) is neglected. As discussed later, \( \Re(S_{\text{EPR}}) \) and \( \Im(S_{\text{EPR}}) \) consist of the dispersion–like and absorption–like functions. Both real part and imaginary part, the coefficient contributed by the sinusoidal background, and therefore in \( \Delta_{\text{Mag}}|S| \) the amplitude of the absorption–like and dispersion–like functions varies as a function of \( \beta \). As mentioned in Sect. 3.1.2 \( \Delta_{\text{Mag}}|S| \) is not suitable for the analysis of data.

A.2 Subtraction of complex background

After the complex background subtraction, \( \Delta_{\text{comp}}S = S_{\text{EPR}} \). With Eq.(1.12) and Eq.(3.6), the EPR signal of DPPH sample \((g > 0)\) is written as
\[
S_{\text{EPR}} \propto \mu_0 h_0 \chi_{11}(-i\eta k e^{i\beta} + 1) = |\mu_0 h_0| e^{i\theta}(D(\mu_0 H_0) - iA(\mu_0 H_0))(-i\eta k e^{i\beta} + 1),
\]
where $\chi_{11}$ is the diagonal component of susceptibility, $\mu_0 h_0$ is the strength of microwaves, $k$ is the the ratio of amplitude of microwaves in the $x$– and $y$–directions, $\theta$ is the phase property of $h_0$. Note that the case, $g > 0$, is dealt with so that $\eta = \eta_H$ corresponding to the sign of $\mu_0 H_0$ is processed. $D(\mu_0 H_0)$ and $A(\mu_0 H_0)$ are the dispersion-like and absorption-like functions included in $\chi_{11}$. Figure A.2 shows the real part and imaginary part of the complex scattering parameter $S_{21}$ after subtracting the complex background. The EPR signal in the positive field seems shifted by $180^\circ$ to in the negative field.

![Figure A.2](image_url)

**Figure A.2:** The real part and imaginary part in $\Delta_{\text{Comp}} S$ for positive and negative magnetic field as a function of phase difference $\beta$ of two stimuli.

The real part and imaginary part of $S_{\text{EPR}}$ are calculated to

\[
\begin{align*}
\text{Re}(S_{\text{EPR}})/|\mu_0 h_0| & = D(\mu_0 H_0)(\eta k \sin (\beta + \theta) + \cos \theta) + A(\mu_0 H_0)(-\eta k \cos (\beta + \theta) + \sin \theta), \\
\text{Im}(S_{\text{EPR}})/|\mu_0 h_0| & = D(\mu_0 H_0)(-\eta k \cos (\beta + \theta) + \sin \theta) + A(\mu_0 H_0)(-\eta k \sin (\beta + \theta) - \cos \theta).
\end{align*}
\]

(A.6)

In the single stimulus case where microwaves are applied from port 1 only ($k = 0$), the real part and imaginary part does not depend on $\beta$. To investigate precisely, the fit of this
EPR signal to the function $d_0 D(\mu_0 H_0) + a_0 A(\mu_0 H_0)$ is performed for the real part and the imaginary part separately. Figure A.3a and b show the amplitude of dispersion–like function $d_0$ and absorption–like function $a_0$ as a function of $\beta$ for both direction of the magnetic field. Upon reversal of the magnetic field, $d_0$ and $a_0$ are shifted by 180°. This behavior is explained by the sign parameter $\eta$ as a pre-factor to the sinusiodal function in Eq. (A.6). The offset of the sinusoidal function does not change when reversing the magnetic field direction. In addition to that, the sign of the offset of $d_0$ in imaginary part is same as of $a_0$ in real part while the sign the offset of $a_0$ in imaginary part is inverted to of $d_0$ in real part, which is also seen in Fig. A.3. Here, we showed that the theory outlined in Sect. 3.1 explains the observed complex signal in its entirety without the need for any further free parameters.

Figure A.3: (a) The amplitude of dispersion–like function $d_0$ and (b) absorption–like function $a_0$ as a function of $\beta$ for positive and negative magnetic field.
Appendix B

Directivity of the polarized microwaves in rectangular waveguide

B.1 propagation efficiency of incident microwaves and directivity of microwave radiation

The rectangular waveguide allows the microwaves to be transmitted in TE_{10} mode in the recommended frequency range. At the position off the center axis of the waveguide, only the elliptically polarized microwaves can be transmitted. Now we consider the propagation of the polarized microwaves $\mu h(t) = \mu_0 h_0(\cos \omega t, -k_0 \sin \omega t)$ ($k_0$: proportionality constant) at the sample position. In order to consider the microwaves which rotate in clockwise direction, $k_0 > 0$. This polarized microwaves consist of two circularly polarized microwaves, one rotates in the counter clockwise direction and the other in the clockwise direction, described as

$$\begin{pmatrix} \cos \omega t \\ -k_0 \sin \omega t \end{pmatrix} = \frac{-k_0 + 1}{2} \begin{pmatrix} \cos \omega t \\ \sin \omega t \end{pmatrix} + \frac{k_0 + 1}{2} \begin{pmatrix} \cos \omega t \\ -\sin \omega t \end{pmatrix}. \quad (B.1)$$

For the simplicity, $\mu_0 h_0 = 1$ is conducted. The coefficient $(-k_0 + 1)/2$ and $(k_0 + 1)/2$ determine the strength of each circularly polarized microwave. Due to the rotating direction of elliptically polarized microwaves, component of the circularly polarized microwave rotating in the clockwise direction (2nd term) is dominant. When the external magnetic field is applied in the positive direction, the polarized microwaves which rotate in the counter-clockwise direction (1st term) excite the FMR (see Fig. B.1a). At the resonance frequency, the circularly polarized microwaves are radiated by the magnetization precession. In TE_{10} mode at the sample position, only the elliptically polarized component of the microwaves as $\mu_0(h_x, h_y) = \mu_0 h_0(\cos \omega t, -k_0 \sin \omega t)$ can be transmitted in the waveguide. This circularly polarized microwaves are also separated into the two elliptically
polarized microwaves, expressed as
\[
\frac{-k_0 + 1}{2} \left( \cos \omega t \right) = \frac{-k_0 + 1}{2} \left[ \frac{k_0 + 1}{2k_0} \left( \cos \omega t \right) + \frac{k_0 - 1}{2k_0} \left( -\cos \omega t \right) \right] = 1 - \frac{k_0^2}{4k_0} \left( \cos \omega t \right) + \frac{(k_0 - 1)^2}{4k_0} \left( \cos \omega t \right),
\]  
(B.2)

Note that the amplitude of circularly polarized microwaves described in Eq. (B.1) is taken into account. In TE_{10} mode, elliptically polarized microwaves rotating in the counterclockwise direction represented in 1st term can be propagated in the reflection direction (to port 1) and microwaves rotating in the clockwise direction represented in 2nd term can be transmitted forward (to port 2).

When reversing the magnetic field, 2nd term of the incident microwaves in Eq. (B.1) contribute to the FMR excitation (See Fig. B.1b). The microwave radiation which rotates in clockwise direction can also be separated into the elliptically polarized microwaves, described as
\[
\frac{k_0 + 1}{2} \left( -\sin \omega t \right) = \frac{k_0 + 1}{2} \left[ \frac{k_0 - 1}{2k_0} \left( \cos \omega t \right) + \frac{k_0 + 1}{2k_0} \left( -\cos \omega t \right) \right] = \frac{k_0^2 - 1}{4k_0} \left( \cos \omega t \right) + \frac{(k_0 + 1)^2}{4k_0} \left( \cos \omega t \right),
\]  
(B.3)

In considering for the strength of the transmitted microwave at port 1 and port 2, the polarity of the microwaves detected in \( S_{21} \) (transmission) and \( S_{11} \) (reflection) can be calculated. With Eq. (3.4), polarization \( P \) for transmission and reflection is described as,
\[
P(S_{21}) = \left| \frac{(k_0 - 1)^2/4k_0 - (k_0 + 1)^2/4k_0}{(k_0 - 1)^2/4k_0 + (k_0 + 1)^2/4k_0} \right| = \frac{|2k_0|}{k_0^2 + 1},
\]  
(B.4)
\[
P(S_{11}) = \left| \frac{(1 - k_0^2)/4k_0 - (1 - k_0^2)/4k_0}{(1 - k_0^2)/4k_0 + (1 - k_0^2)/4k_0} \right| = 0.
\]  
(B.5)

In transmission, polarization depends on how circularly polarized the incident microwaves are, and thus for \( k_0 = 1 \), the purely circularly polarized case, polarization turns into 100%. On the other hand, the polarization detected in reflection keeps zero even though the strength of the excited FMR signal is different for the both magnetic field direction. This is explained by the efficiency of the microwave propagation. For the negative magnetic field, the incident elliptically polarized microwaves are dominantly composed of the circularly polarized microwaves inducing FMR strongly. Then, the microwave radiation which mainly consists of the elliptically polarized microwave is propagated in the transmission direction efficiently, and in reflection direction the lower amplitude of FMR signal is detected. On the other hand, the incident microwaves are mostly composed of the microwaves which
excite FMR weakly in the positive magnetic field, and thus the microwave radiation
detected in transmission is low. However, the microwave radiation primarily consists of
the elliptically polarized microwaves which can be propagated backward efficiently, and
still strong FMR signal is detected at port 1. The polarity detected in reflection, therefore,
is zero. The case of the incident microwaves rotating in counter–clockwise direction can
be considered with the condition $k_0 < 0$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure_B1.png}
\caption{Schematic image of the incident polarized microwave propagation and the propagation of the
microwave radiation in (a) positive and (b) negative magnetic field. Due to the directiveity
of the microwaves, the impact of the circularly polarized drive is not visible, even though the
FMR strength is different for both magnetic field directions.}
\end{figure}

\section*{B.2 calculation of microwave polarization in rectangular
waveguide}

In general case, the phase between $x$ and $y$–components of the microwaves is not fixed by
90°. As introduced in Chapt. 2, with phase difference $\beta$, polarized microwaves are defined
as

$$\mu_0 \begin{pmatrix} h_x \\ h_y \end{pmatrix} = \begin{pmatrix} \cos \omega t \\ k \cos(\omega t + \beta) \end{pmatrix}$$

(B.6)

where $k$ is the proportionality factor, and thus $k$ and $\beta$ determine the polarity of the
microwaves. In order to calculated the polarization described in Eq. (3.4) with $k$ and $\beta$,
we consider the relation between $k_0$ used in Sect. B.1 and $k$ and $\beta$. Generally, flattening
of ellipse determined by $k_0$ does not change by rotating the ellipse by $\phi$. In the Cartesian
coordinate system, the orbit of this ellipse for $x$– and $y$– components defined by

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \cos \omega t \\ k_0 \sin \omega t \end{pmatrix},$$

(B.7)
is calculated using the relation $\cos^2 \omega t + \sin^2 \omega t = 1$ to
\[
\frac{\sin^2 \phi + k_0^2 \cos^2 \phi}{k_0^2} X^2 + \frac{2(k_0^2 - 1) \cos \phi \sin \phi}{k_0^2} XY + \frac{\cos^2 \phi + k_0^2 \sin \phi}{k_0^2} Y^2 = 1. \tag{B.8}
\]
Note that summation of coefficient of $X^2$ and $Y^2$ is
\[
\frac{\sin^2 \phi + k_0^2 \cos^2 \phi}{k_0^2} + \frac{\cos^2 \phi + k_0^2 \sin \phi}{k_0^2} = 1 + \frac{k_0^2}{k_0^2}. \tag{B.9}
\]
On the other hand, orbit of the projection of the polarized microwaves described in Eq. (B.6) to a Cartesian coordinate system is represented in
\[
\frac{k^2}{k^2 \sin^2 \beta} X^2 - \frac{2k \cos \beta}{k^2 \sin^2 \beta} XY + \frac{1}{k^2 \sin^2 \beta} Y^2 = 1. \tag{B.10}
\]
The summation of coefficient of $X^2$ and $Y^2$ is calculated to $(1 + k^2)/k^2 \sin^2 \beta$. When the orbit of this ellipse coincides with the orbit of polarized microwaves, the condition where the summation of coefficient of $X^2$ and $Y^2$ for Eq. (B.6) and Eq. (B.7) is same should be satisfied. With this necessary condition, $k_0$ is described as
\[
k_0 = \frac{k^2 \sin \beta}{\sqrt{1 + k^2 \cos^2 \beta}}. \tag{B.11}
\]
Inserting Eq. (B.11) to Eq. (B.4), the polarity detected by $S_{21}$ is calculated to
\[
P(S_{21}) = \frac{2k \sin \beta \sqrt{1 + k^2 \cos^2 \beta}}{1 + k^2}. \tag{B.12}
\]
Bibliography


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\(^1\)https://bar.wikipedia.org/wiki/Boarisch