Master Thesis
Towards Tunable Coupling Between Two Superconducting Transmission Line Resonators

Xiaoling Lu

Supervisor: Prof. Dr. Rudolf Gross

Munich, August, 24th 2012
# Contents

1 Introduction .............................................. 1

2 Theory .................................................. 3
   2.1 Josephson Junction .................................. 3
      2.1.1 Josephson equations ............................ 3
      2.1.2 Josephson Junction coupling energy $E_J$ ....... 4
      2.1.3 RSCJ model of Josephson Junction ............... 5
      2.1.4 Magnetic field dependence of Josephson Junction .. 7
   2.2 Superconducting quantum interference devices ......... 9
      2.2.1 Josephson Junction based DC SQUID ............... 9
      2.2.2 Josephson Junction based RF SQUID ............... 10
   2.3 Transmission line resonator .......................... 11
   2.4 Coupling between two TLRs .......................... 13
      2.4.1 Quantum harmonic oscillators ..................... 14
      2.4.2 Tunable coupling between 2 TLRs .................. 14

3 Experiments .............................................. 17
   3.1 Experimental design consideration .................... 17
   3.2 Sample fabrication process ........................... 18
   3.3 Fabrication parameter optimization .................... 20
   3.4 Josephson Junction measurement ..................... 22
      3.4.1 Cryogenics ........................................ 22
      3.4.2 Measurement setup ................................ 23
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Results and discussion</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Josephson Junction</td>
<td>25</td>
</tr>
<tr>
<td>4.1.1</td>
<td>Current voltage characteristic</td>
<td>25</td>
</tr>
<tr>
<td>4.1.2</td>
<td>Magnetic field modulation</td>
<td>29</td>
</tr>
<tr>
<td>4.2</td>
<td>Josephson junction based SQUID</td>
<td>30</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Magnetic field modulation curve for DC SQUID</td>
<td>30</td>
</tr>
<tr>
<td>4.3</td>
<td>Coupling between 2 TLRs</td>
<td>32</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Eigenfrequency of two transmission parallel line resonators</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>Summary and outlook</td>
<td>35</td>
</tr>
<tr>
<td>5.1</td>
<td>Bottom electrode and TLRs fabrication</td>
<td>37</td>
</tr>
<tr>
<td>5.2</td>
<td>Trilayer deposition</td>
<td>37</td>
</tr>
<tr>
<td>5.3</td>
<td>SiOx deposition</td>
<td>38</td>
</tr>
<tr>
<td>5.4</td>
<td>Top electrode deposition deposition</td>
<td>38</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td></td>
<td>41</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Superconducting quantum circuits as newly developing field in recent 10 years is playing an important role for quantum computation and quantum simulation [1–3]. It has strong parallels to quantum optics but features large nonlinearities and therefore large coupling strengths. There are two paradigmatic circuits, LC quantum harmonic oscillator and Josephson junction based circuits, which exhibit linear and nonlinear property respectively. The Josephson junction based superconducting circuits with high nonlinearity is commonly built as quantum two-level system (typically, qubits [1]). Nevertheless, the linear elements like superconducting microwave resonators are important tools for quantum information processing (QIP) with superconducting circuits. They could act as quantum buses, simple quantum memories, or coupler between qubits. Certainly, they also allow for interesting quantum simulations. Resonator chains allow for the study of Bose-Hubbard-like Hamiltonians [4] and also the investigation of two dimensional lattice [3]. In the above applications, a tunable coupling between resonators is, while not always strictly necessary, in any case desirable.

With the finest prospect, the aim of this thesis is to intersect two transmission line resonators with the simple nonlinear circuit formed by a superconducting loop and a Josephson junction, namely, RF SQUID. It is expected to result in tunable coupling by means of a small amount of external magnetic field threading through the RF SQUID loop. The designed coupling strength ranges from zero (decoupling) to 200 MHz at the resonance frequency of 6 GHz [5]. By making use of optical lithography, thin film deposition and ion reactive etching these micro fabrication techniques, the Nb superconductor transmission line resonators and Nb/Al2O3/Nb Josephson junction(JJ) based RF SQUID are fabricated in accordance with the theoretical design considerations. The fabrication parameters is optimized synchronously with the sample characterization procedure, which is carried at low temperatures in a cryostat.

Taking an panoramic view of this thesis, the second chapter introduces the tunable coupling theory step by step, including Josephson junction, RF SQUID and coupling strength between two transmission line resonators. Guided by the design consideration, in third chapter, sample fabrication process and measurement are gradually carried out. The characteristics of Josephson junction, DC SQUID and coupling between two transmission line resonators, are analyzed and discussed in the last chapter.
to know if the performance of beam splitter comes up to the expectation.
Chapter 2

Theory

In this chapter, the critical element of superconducting circuits, the Josephson Junction, would be firstly introduced. Then the electrodynamics of direct current superconducting quantum interference device (DC SQUID) and radio frequency superconducting quantum interference device (RF SQUID) with Josephson Junctions intersected into a superconductor loop is discussed in detail. Lastly, the implementation method to realize the controllability of the tunable coupling between two superconducting transmission line resonators (TLRs), as the critical part of the thesis, is exhaustively studied from theoretical aspect.

2.1 Josephson Junction

At low temperature, two electrons in superconductor are tend to bond together to form an cooper pair due to the electron-phonon interaction, which is responsible for the superconductivity. A significant percent of cooper pairs can only exist when the ambient temperature $T$ is much lower than the critical temperature $T_c$ to avoid the thermal energy breaking the pairs. All cooper pairs in the superconductor can, in a phenomenological approach, be described by a single macroscopic wavefunction with a single phase $\theta$. They can tunnel coherently through a thin insulating barrier and meanwhile, experience a phase drop $\varphi$. In a Josephson junction structures of sequential superconductor-insulator-superconductor (SIS) (see Fig. 2.1), this process is exploited to build a nonlinear element, more precisely a nonlinear inductance. By making use of this property, Josephson junction evolves to be one of the key building blocks of superconducting quantum circuits.

2.1.1 Josephson equations

In order to understand the electrical performance of Josephson junctions, it is beneficial to comprehend the 1$^{\text{st}}$ and 2$^{\text{nd}}$ Josephson equation. Essentially, they state that the supercurrent density through a Josephson Junction varies sinusoidally with the phase difference $\varphi = \theta_2 - \theta_1$ across the junction in the absence of any scalar and
Figure 2.1: Sketch of a superconductor-insulator-superconductor (SIS) Josephson Junction

vector potentials. The voltage across the junction is proportional to the $\frac{\partial \varphi}{\partial t}$. These two relationships are given as following:

1\textsuperscript{st} Josephson junction equation (current phase relationship):

$$J_s(\varphi) = J_c \sin(\varphi) \quad (2.1)$$

Here, $J_c$ is the critical Josephson current density which is determined by the coupling strength between the two superconductors wave functions. The critical current $I_c = j_c A$ is the corresponding maximum supercurrent flows through the Josephson junction, in which, $A$ is the junction area.

2\textsuperscript{nd} Josephson junction equation (voltage phase relationship):

$$V = \frac{\Phi_0}{2\pi} \frac{\partial \varphi}{\partial t} \quad (2.2)$$

in which $\Phi_0 = \frac{h}{2e}$ is the magnetic flux quanta.

2.1.2 Josephson Junction coupling energy $E_J$

Because of the finite overlap of the macroscopic wavefunction of the two weakly coupled superconductors, there supposed to be a corresponding finite energy stored in the Josephson Junction, when $I$ is ramping up from zero to $I_c$. This energy is so called Josephson junction coupling energy $E_j$. Its amplitude can be expressed as the integral of the external applied current $I$ and the subsequent generated voltage $V$ across the Josephson Junction in the time regime $[0, t_0]$:

$$E_j = \int_0^{t_0} I_s \cdot V dt \quad (2.3)$$

By substituting the 1\textsuperscript{st}(current-phase) and 2\textsuperscript{nd}(phase-voltage) Josephson equations, we obtain:

$$E_j = \frac{\Phi_0 I_c}{2\pi} (1 - \cos \varphi) \equiv E_{j0}(1 - \cos \varphi) \quad (2.4)$$
This Josephson coupling energy constitutes a characteristic energy scale of the junction. In order to operate the Josephson junction without influence of thermal noise associated to the ambient temperature $T$, we demand that:

$$E_J > k_B T$$ (2.5)

### 2.1.3 RSCJ model of Josephson Junction

![Figure 2.2](image)

Figure 2.2: Equivalent circuit for the Resistively and Capacitively Shunted Junction (RCSJ) model. The Josephson junction can be characterized by the inductance $L_s = L_c / \cos \varphi$ with $L_c = \hbar / 2e I_c$, and capacitance with $C$, the resistive channel approximately represented by a voltage and temperature independent conductance $1/R$. By applying external current $I$ on Josephson junction, voltage generated across the Josephson junction is measured.

The Resistively and Capacitively Shunted Junction Model (RCSJ) [6, 7] is universally used to describe the dynamics of the Josephson Junction in voltage state $V(t)$. Because the Josephson junction hinders the current passing by for $I > I_c$, it can be partially treated as a resistance. The junction configuration, superconductor-insulator-superconductor, proclaim itself as a parallel plate capacitor. Meanwhile, the Josephson effect can be represented by a inductor. Thus junction current flows through normal resistance $R_N$, capacitance $C$ and inductance $L_c$ three channels and it can be expressed as:

$$I = I_c \sin \varphi + \frac{1}{R} \frac{\Phi_0}{2\pi} \frac{d\varphi}{dt} + \frac{C}{2\pi} \frac{d^2\varphi}{dt^2}$$ (2.6)

Multiply by $\frac{\hbar}{2e}$ and using the Josephson coupling energy $E_{J0} = \frac{\hbar I_c}{2e}$, the Eq. 2.6 can be rewritten as:

$$\frac{d}{d\varphi} \{E_{J0}(1 - \cos \varphi - i\varphi)\} + \left(\frac{\hbar I_c}{2e}\right)^2 \frac{1}{R} \frac{\Phi_0}{2\pi} \frac{d\varphi}{dt} + \left(\frac{\hbar I_c}{2e}\right)^2 \frac{C}{2\pi} \frac{d^2\varphi}{dt^2} = 0$$ (2.7)

with $i = \frac{I}{I_c}$.
This equation is analogous to the equation of motions of particle along a washboard (see Fig. 2.3) with mass $M$ and damping $\eta$ in the potential $U$:

$$M \frac{d^2x}{dt^2} + \eta \frac{dx}{dt} + \nabla U = 0$$  \hspace{1cm} (2.8)

while the particle mass $M = \left(\frac{\hbar}{2\pi}\right)^2 C$, the damping $\eta = \left(\frac{\hbar}{2\pi}\right)^2 \frac{1}{R}$ and washboard potential $U = E_{j0}(1 - \cos \varphi - i\varphi)$.

![Figure 2.3: Analogy of the phase difference of Josephson junction which is biased by external signal and the damped motion of a particle with mass $M$ in the potential. The different color curves represent the changing trend of external current source comparing with the critical current of Josephson junction. When the external applied current exceeds the critical current of Josephson junction, the phase difference begins to change like the particle easily goes cross the potential barrier and runs down along the washboard. Then, the biased current begins to decrease, the phase difference varies slowly just as the small ball is hindering by the potential barrier. We notice that when $i = 0$, there is no phase difference and voltage generated across the Josephson junction because of no external current injection. This state is namely zero voltage state. For $i < 1$, the current is increasing but still does not exceed the critical current of Josephson junction, there is no motion of the phase difference neither because the particle is trapped within the potential barrier. Nevertheless, if the external applied current $I$ becomes larger than the critical current with $i > 1$, the potential barrier relaxes and the particle would roll down along the washboard. This situation indicates the phase difference in the Josephson junction is changing and...](image)
there is a voltage drop across the junction. Then, when the external applied current changes back below $I_c$ and down to zero, the potential to hinder the motion of particle would subsequently increase. But due to residual kinetic energy energy within the particle, the particle can keep running until the potential decreases to zero, depending a the Josephson junction type.

If we use normalized time $\tau = \frac{t}{2 e I_c R / h}$, the Eq. 2.6 could be rewritten as:

$$\beta_c \frac{d^2 \phi}{d\tau^2} + \frac{d\phi}{d\tau} + \sin \phi - i = 0$$ (2.9)

in which $\beta_c = \frac{2e}{R} I_c R^2 C$ is so called Stewart-McCumber parameter. For $\beta_c \gg 1$, the junction is in underdamped regime so that the particle will continue rolling until $i \approx 0$. In this case, the IVC shows hysteresis. But if $\beta_c \ll 1$, the junction is in overdamped regime which means the particle would stop running when the first potential barrier occurs. So there is no hysteresis current voltage curve (IVC) can be observed.

### 2.1.4 Magnetic field dependence of Josephson Junction

The IVC can be modulated by the external applied magnetic filed which is parallel to the Josephson junction (see Fig. 2.4(a)). The $I_s^m (B_y)$ dependence curve (see Fig. 2.4(b)) is equivalent to the diffraction pattern of a slit with width $L$ and constant transmission $I_c$. The corresponding critical current is given by:

$$I_s^m = I_c \left| \frac{\sin \frac{\pi \Phi}{\Phi_0}}{\pi \Phi / \Phi_0} \right|$$ (2.10)
Figure 2.4: (a) Sketch of a Josephson Junction with an external magnetic field parallel along the superconductor-insulator-superconductor trilayer. (b) Magnetic field dependence of the maximum Josephson current $I^m(B_y)$ of Josephson Junction.
2.2 Superconducting quantum interference devices

In superconducting loops, there are quantization conditions for the enclosed flux due to the fact that phase jumps are allowed only for Josephson junction. As a consequence, one can build quantum interference device. We denote the device consisting of a superconducting loop interrupted by Josephson junction as Superconducting Quantum Interference Devices (SQUIDs).

2.2.1 Josephson Junction based DC SQUID

![Equivalent circuit of a symmetric DC SQUID](image)

Figure 2.5: Equivalent circuit of a symmetric DC SQUID formed by two Josephson junction intersecting a superconducting loop. The currents passing through each junction are \( I_1 = I_c \sin \varphi_1 \) and \( I_2 = I_c \sin \varphi_2 \) respectively. The DC SQUID is biased by an external direct current \( I \) and the corresponding voltage drop \( V \) over the device is measured. In addition, an external applied magnetic field perpendicular to the DC SQUID loop can be applied.

The DC SQUID consists of two identical Josephson Junctions which phase difference \( \varphi_1 \) and \( \varphi_2 \) connected in parallel by a superconducting loop. It is a very sensitive magnetometer [8] that can detect flux changes up to \( 5 \times 10^{-18} T \) [9, 10] and therefore be used to read out the superconducting flux qubits [11]. The DC SQUID is biased by DC current to detect the modulated output current as a function of external applied magnetic field. The equivalent circuit is shown in Fig. 2.5. By applying the Kirchhoff’s law on the condition of the phase change along the closed contour loop is \( 2\pi n \), the supercurrent can be written as Eq. 2.11. For the detail of calculation, one can refer to the manuscript book [12].

\[
I_m^s = 2I_c \cos \left( \pi \frac{\Phi_{ext}}{\Phi_0} \right) \sin \left( \varphi_1 + \pi \frac{\Phi_{ext}}{\Phi_0} \right) \tag{2.11}
\]

as a result, the maximum current in the superconductor loop can be written as below:

\[
I_m^s = 2I_c \left| \cos \left( \pi \frac{\Phi_{ext}}{\Phi_0} \right) \right| \tag{2.12}
\]
However, in the above expression a full modulation of the cosine to zero is only possible if the effect of self-induced flux are small and can be ignored. This condition can be formalized via:

\[
\beta_L \equiv \frac{2LI_c}{\Phi_0} \leq \frac{2}{\pi}
\]  \hspace{1cm} (2.13)

here \(L\) is the inductance of the loop. In other words, the flux generated by the circulating current is smaller than the flux quantum and can be neglected. The corresponding magnetic field dependence curve is show in Fig. 2.6.

![Figure 2.6: Theoretical Magnetic field dependence curve of DC SQUID.](image)

### 2.2.2 Josephson Junction based RF SQUID

![Figure 2.7: The RF SQUID formed by a single Josephson junction intersecting a superconducting loop.](image)
Another type of superconducting quantum interference device, the RF SQUID is formed by a superconducting loop containing only a single Josephson junction (see Fig. 2.7). The SQUID loop is inductively coupled to the coil of an LC resonant circuit which is excited by a RF current. The circulating current in the loop can be calculated as follows [8] Eq. 2.14 and the corresponding curve is given in Fig. 2.8.

\[ I_s = -I_c \sin \frac{2\pi \Phi_{\text{ext}}}{\Phi_0} \]  

(2.14)

The corresponding screening parameter \( \beta_{L,\text{rf}} \) of RF SQUID reads as follows and self inductance can be neglected as long as:

\[ \beta_{L,\text{rf}} = \frac{2\pi LI_c}{\Phi_0} \leq 1 \]  

(2.15)

### 2.3 Transmission line resonator

A superconducting transmission line resonator is made of a superconductor line interrupted by two capacitors at its ends (shown in Fig. 2.9). The large impedance mismatch makes each of these capacitors causes is analogous to a dielectric mirror, since it is lossless and reflects most incident radiation, but transmits a small amount. Through this "mirror", photons can be added to the cavity or allowed to leak out.

In the microwave domain, a transmission line resonator can be treated as a LC oscillator (shown in Fig. 2.10).
Figure 2.9: Sketch of a superconducting transmission line resonator made of a superconducting line interrupted by two capacitors at its ends.

Figure 2.10: Circuit model of a superconducting TLR, where $C_r$ and $L_r$ are the capacitance and inductance per unit length of the superconducting line respectively. Typically, an input signal is fed on one end and the output is detected on the other one.

Restricting ourselves to the lowest mode, the resonant frequency is:

$$\omega_0 = \frac{\pi}{\sqrt{LC}}$$

in which $L$ and $C$ are the total inductance and capacitance of TLR respectively.

In order to observe the quantum effect, the quantum energy of oscillator should be greater than the thermal fluctuations.

$$\hbar \omega_0 \gg k_B T$$

The reflection mirrors defining the resonator are capacitive loads with impedance $Z_L$. Their associated reflection coefficients are:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

in which

$$Z_0 = \sqrt{L_r / C_r}$$
is the characteristic impedance. In practice, this reflection coefficient defines the external quality factor $Q_{\text{ext}}$. All other losses are described by the internal quality factor $Q_{\text{int}}$. The total quality factor is:

\[
Q = \frac{\omega_i}{\Delta \omega_i}
\]  

(2.20)

where $1/Q = 1/Q_{\text{ext}} + 1/Q_{\text{int}}$. A high quality factor is prerequisite to guarantee the photon stay inside the TLR without damping out in comparable short time.

### 2.4 Coupling between two TLRs

![Figure 2.11: Diagram of beam splitter. The two superconducting TLRs are identical with respect to their length $2l$, width $W$ and two capacitors $C_k$ at the ends. The distance between the TLR and ground plane is $G$. There is a rectangular RF SQUID intersecting in between the two TLRs with length $\Delta x$ and width $\Delta y$ galvanically coupled to both TLRs.

When approaching one TLR parallel close to another, they would interact with each other due to the first order coupling. Generally, a ring or rectangular loop is inserted in between them as a transfer tool enhances the coupling strength due to second-order effects. However, the two above cases are less than perfect because of their couplings could not be flexibly controlled. For a pure ring, there is only positive inductance. It means the direction of the current circling in the ring depends on the way that how the external applied magnetic filed is applied. However, a tunable and even negative inductance can be obtained just by inserting a Josephson junction into the loop, which means the direction of the current circling in the loop can be forwards and reverse just by external magnetic field with a fixed direction. Based on this property, in this thesis, we establish the fabrication process for a Josephson junction based superconductor loop, so called RF SQUID which is located in between two TLRs. The goal is to realize a controlled coupling between the TLRs by precisely adjusting the applied external magnetic field threading the loop [5]. The geometric structure of such a tunable beam splitter is shown in Fig 2.11.

In the following section, we firstly introduce the quantum harmonic oscillator. After that, we describe the quantum behavior of the RF SQUID coupled to two supercon-
ducting TLRs. For achieving the flexible controllability between two TLRs, the main factors which decide coupling strength are intensively discussed.

### 2.4.1 Quantum harmonic oscillators

For analyzing the quantum properties of this Josephson junction based TLRs circuit, it benefits to solve the problem with harmonic quantum oscillator as a start.

The fundamental mode of a TLR can be written as:

\[ H = \hbar \omega_0 \left( a^\dagger a + \frac{1}{2} \right) \]  

(2.21)

The eigenenergies are:

\[ E = \hbar \omega_0 \left( n + \frac{1}{2} \right) \]  

(2.22)

Here the \( a^\dagger \) and \( a \) are bosonic creation and annihilation operators, and \( a^\dagger a = n \in \mathbb{N}_0 \) is the number of photons in the TLR.

When the two quantum harmonic oscillators get close, the degenerate eigenmodes in the two parallel TLR at same frequency splits due to the coupling \( \hbar g \). This is similar to the situation that the electron orbit of same energy level split into two orbits when two atoms get close to each other. The split modes \( \omega_1 \) and \( \omega_2 \) can be are expressed as:

\[ \omega_1 = \omega + g, \quad \omega_2 = \omega - g \]  

(2.23)

### 2.4.2 Tunable coupling between 2 TLRs

For the setup shown in Fig. 2.11 with two TLRs intersected by a RF SQUID, except the Hamiltonian of each TLR, the coupling Hamiltonian \( (H_{\text{int}}) \) should also be taken into account, thus the total Hamiltonian reads:

\[ H_{\text{eff}} = H_{\text{TLR1}} + H_{\text{TLR2}} + H_{\text{int}} = \hbar \omega (a^\dagger a + \frac{1}{2}) + \hbar \omega (b^\dagger b + \frac{1}{2}) + 2\hbar g \left( a^\dagger b + ab^\dagger \right) \]  

(2.24)

Here the \( a^\dagger \) and \( a \) are bosonic creation and annihilation operators in 1\textsuperscript{st} transmission line resonator 1, the \( b^\dagger \) and \( b \) are bosonic creation and annihilation operators in 2\textsuperscript{nd} transmission line resonator.

In the equation 2.24 the first term is the Hamiltonian of 1\textsuperscript{st} TLR, the second term is the Hamiltonian of 2\textsuperscript{nd} TLR, and the third one is the coupling energy between the two TLRs with the photon hoping frequency \( g/2\pi \). The frequency \( g \) represents the
coupling strength between the two TLRs. A large g indicates the photon hops very frequently from one TLR to another and the coupling strength is correspondingly large. After further derivation, we obtain the coupling strength g [5]:

$$g = \frac{2\pi^2}{\Phi_0} Z \left( \frac{\Delta x}{2l} \right)^2 I_c \cos \frac{2\pi \Phi}{\Phi_0}$$

(2.25)

To enhance the coupling strength g, one way is to increase the impedance of transmission line $Z = \sqrt{\frac{L_r}{C_r}}$. Another way is to manipulate the geometry $\Delta x$ of RF SQUID. However, a trade-off has to be found between the screening parameter $\beta_{L,rf} \equiv \frac{2LI_c}{\Phi_0}$ and large $\Delta x$. This is because the increase of impedance or geometry parameter $\Delta x$ of TLR implies the increase of the SQUID loop inductance $L$, and the critical current $I_c$ of JJ should be decreased to guarantee the $\beta_{L,rf} \ll 1$. But the design of Josephson junction with low critical current is challenging with respect to the fabrication process. A significant part of the experimental work in this thesis is devoted to this issue.
Chapter 3

Experiments

In this chapter, the development, optimization and characterization ofNb/Al-Al₂O₃/Nb Josephson junction and its related devices would be discussed. Of the first importance, experimental design consideration would be described in the first section. Then, the concrete fabrication steps would be stated in the second section. Combining with the problems confronted in the process of experiments, the parameter optimization of beam splitter would be commented and summarized in the third section. For the sample characterization, the measurement setup and cryostat technology are presented in the last section.

3.1 Experimental design consideration

The goal of our work is to develop a beam splitter with a coupling strength $g$ ranging from zero to approximate 200 MHz. Simulations are carried out to obtain optimized device parameters as a guideline for the fabrication process. In what follows, we give an explicit description of experimental design consideration.

For the detection of weak microwave signals, cryogenic low-noise components such as circulators and amplifiers are commercially available in the frequency range of $4 \sim 8$ GHz. There is consistent with the more fundamental consideration that the resonator frequencies should be smaller than the superconducting gap $2\Delta$. For pure Niobium, this value varies from 2.81 to 3.14 meV(approximately 68 GHz). For Al used in superconducting Qubits, its the energy gap is about 6 GHz. As a consequence, in this work we choose resonator frequency around 6 GHz. Thus, the length of the TLRs is $\lambda/2 \approx 9000 \mu m$.

To get rid of thermal smearing effect $hf \geq k_B T$, the experimental temperature should be controlled below 290 mK. Thus the coupling energy of Josephson junction must be larger than $h \nu = 4 \times 10^{-24}$ J. Since we are going to characterize the sample at $T=4.2K$, the recommended critical current of a Josephson junction for our circuit is $1.5 \mu A$ in accordance with the Eq. 2.5 $E_j > k_B T$ and Eq. 2.4 $E_j = h I_c / 2e$. Due to the smallest area of Josephson junction fabricated by optical lithography is $4 \mu m^2$, the critical current density of Josephson junction should be larger than $30 A/cm^2$. However, in order to
have negligible screening effect (refer to Eq. 2.13 $\beta_L \equiv 2LI_c/\Phi_0 \leq 1$), the Josephson critical current $I_c$ is not allowed to be arbitrarily large otherwise the SQUID loop would not fully tunable. The inductance of SQUID loop is determined by the its geometry shape parameter $\Delta x$ and $\Delta y$. Up to now, the mutual connection between the superconductor loop geometry and the critical current is the heart of the matter to realize the target coupling strength.

Furthermore, in order to be able to turn off the coupling between two TLRs completely, the absolute value of the geometric coupling $g_g$ should be smaller than that of beam splitter coupling $g_{BS}$. After combining the above conditions with the coupling strength Eq. 2.25 from section 2.5.2 to implement simulations, a reasonable critical current of approximately $1.5\mu A$ and the SQUID geometry parameters $\Delta x = \Delta y = 100 \mu m$ are chosen to be our fabrication goal and design parameter of optical mask template respectively.

### 3.2 Sample fabrication process

A schematic overview of the sample fabrication process is shown in Fig. 3.1. At first, the two parallel superconducting TLRs and bottom electrode made of Nb are created by DC magnetron sputtering, optical lithography, and reactive ion etching techniques with layer thickness of 100 nm on the Si substrate. This fabricated Nb structure is shown in Fig. 3.1(a). On one arm of the open bridge, the Nb/Al-Al$_2$O$_3$/Nb trilayer is designed and deposited by optical lithography and DC magnetron sputtering system with the 50nm lower Nb, 4nm Al and 50nm top Nb layers, see Fig. 3.1(b). Before the top Nb layer deposition, the insulator layer Al$_2$O$_3$ is prepared by introducing oxygen in the sputtering chamber to oxidize the surface layer of Al at room temperature. The thickness of generated Al$_2$O$_3$ is around 1nm. Afterward, another insulator layer SiO$_x$ surrounding the Josephson junction is prepared by optical lithography, reactive ion etching and magnetron sputtering to get rid of current leakage, see Fig. 3.1(c). The last step, 100nm top electrode Nb layer is created by optical lithography and magnetron sputtering, see Fig. 3.1(d). The optical lithography steps in the fabrication process are all using positive photoresist. Additionally, in between the two successive thin film deposition process, ion gun is used to get rid of the oxide layer to generate ideal layer to layer contact. Residual photo resist is removed by lift off process in step (b), (c) and (d). A detailed list of the processing parameter can be found in the Appendix A.

The whole fabrication process is carried out on the Si substrate with the size $6 mm \times 10 mm$. There are two additional test structures on the diagonal corner of the chip. One is single Josephson junction with the size $2.5 \times 2.5 \mu m^2$, another is DC SQUID with two identical Josephson junction (both are $2.5 \times 2.5 \mu m^2$) interrupted square loop with the size $100 \times 100 \mu m^2$. In the center is two TLRs intersected by a RF SQUID which is the same as test structure DC SQUID except with only one single junction. The final chip appearance is shown in Fig. 3.2.

During the whole fabrication process, one must have noticed that we did not explicitly state the thickness of insulator layer Al$_2$O$_3$, SiO$_x$ and the area of Josephson junction while other structure parameters are constant. That’s because these three parameters
Figure 3.1: (a) Nb bottom electrode and TLR deposition: (i) deposit Nb layer on the Si substrate. (ii) define the Nb bottom electrode and TLR structure by photoresist pattern which is created by optical lithography. (iii) etching the Nb layer to create bottom electrode and TLR according to the defined photoresist pattern. (iv) cleaning the residual photoresist. (b) Nb/Al$_2$O$_3$/Nb Josephson junction deposition: (i) define the Josephson junction structure (position, size and shape) by optical lithography. (ii) deposit Nb/Al$_2$O$_3$/Nb trilayer by ion gun, oxidation technique and thin film deposition technologies. (iii) lift off the residual photoresist. (c) SiO$_x$ insulator layer deposition: (i) define the SiO$_x$ insulator layer structure (position, size and shape) by optical lithography. (ii) etching the openings area uncovered by photoresist pattern to create SiO$_x$ region to realize complete insulation. (iii) deposit SiO$_x$ by thin film deposition technology. (iv) lift off the residual photoresist. (d) Nb top electrode deposition: (i) define the top electrode structure (position, size, shape) by optical lithography. (ii) deposit Nb top electrode by ion gun and thin film deposition technology. (iii) lift off the residual photoresist.
Figure 3.2: Microscopic image of chip appearance, the fabrication process is carried out on a Si chip with the size $6 \text{mm} \times 10 \text{mm}$. On the upper right corner, it is the microscope image of a single JJ with the junction area $2.5 \times 2.5 \mu\text{m}^2$. On the lower left corner and in the center are the DC SQUID and two TLRs interrupted by a RF SQUID respectively with the same junction size $2.5 \times 2.5 \mu\text{m}^2$ and loop size $100 \times 100 \mu\text{m}^2$.

are critical and of great importance to the performance of Josephson junction. The more details about how to manipulate them to achieve the our project target would be discussed intensively in next section.

### 3.3 Fabrication parameter optimization

Because the critical current of Josephson junction is one of the important factors in determining the coupling strength between two TLRs, it is necessary to discuss how the fabrication parameters mentioned in the previous section affect the critical current.

The critical current of a junction depends exponentially on the oxide layer thickness. This can be understood from the Ambegaokar-Baratoff relation [13]. $\text{Al}_2\text{O}_3$ as the only resistive layer in Josephson junction, its resistivity ($R_N$) would certainly influence the critical current density of JJ and their relationship follows the equation:
In the fabrication process, the \( O_2 \) partial pressure and oxidation time decide the adsorption and absorption amount of \( O_2 \) molecule to the surface of Al, subsequently, influence the thickness of oxidation layer and its resistivity. As a result, it is obvious that the critical current of Josephson junction is inversely related to the \( O_2 \) partial pressure and oxidation time. But it does not depend independently on one of them but on their product. The relationship is as follow:

\[
I_c \propto j_c \propto (P \cdot t)^{(-\kappa)}
\]  

(3.2)

Because of different the fabrication procedures, the value of \( \kappa \) had been reported ranges from 0.4 to 1.5 \([14]\). In our case, the estimated \( \kappa \) value is approximately 0.4, this relationship is given in accordance with the equation 3.2 \([14]\). Furthermore, \( I_c \) is proportional to the junction area which is determined by pattern size of optical mask in the \( SiO_x \) optical lithography step. For our mask aligner, diffraction limits the minimum feature size to approximately 0.5 \( \mu m \). In the real fabrication, however, also other effects such as, for example, the resist properties or inhomogeneous exposure due to different materials limit the minimum feature size to approximately 2.5 \( \mu m \).

\( SiO_x \) layer is deposited to get rid of the short cut between the top and bottom electrode. But the use of this layer does not indicate the thicker, the better, because the thick layer would introduce unimaginable sharp spikes shown in Fig. 3.3 which would cut off the contact between the top electrode and the Josephson junction to induce invalid device. By considering the above contradiction, in \( SiO_x \) thin film deposition process, the deposition thickness should be chosen cautiously.
3.4 Josephson Junction measurement

Referring to Fig. 3.4, the sample characterize works in the following manner: the sample should be firstly glued on the PCB sampler holder, see Fig. 3.4(a) and fixed onto the bonding machine, see Fig. 3.4(b). The electrodes of interested structures on the chip are then connected with the Cu metal paddles of sample holder by Al bonding wires. Mounting this sample holder into the metal shield of cryostat insert stick (lower part of Fig. 3.4(c)), the signal input/output path between sample and measurement setup is basically created and ready for measurement. The whole measurement is carried out below the superconductor critical temperature $T_c$ by immersing the cryo insert stick inside the cryostat. A liquid $^4$He cryostat is normally used to create ambient temperature of approximately 4.2 K, see Fig. 3.4(d). Lower $T$ measurement down to 500 mK, it requires assistance of $^3$He evaporation cryostat, see Fig. 3.4(e).

3.4.1 Cryogenics

Figure 3.4: Main components for sample characterization. (a)PCB sample holder (b)Al wire bonding machine (c)sample insert with two connection ports, the right one is for connecting the measurement setup, the left one is for Temperature sensor connection (d) $^4$He cryostat (e)500 mK cryostat.

Associating with simplified diagram Fig. 3.5(a), operation principle of 500 mK cryostat is briefly stated. All the refrigerating performance is carried out under vacuum, the pre-chamber (7) need to be pumped to certain vacuum level before inserting the sample insert (6) through the plate valve (8). With sample staying in the cryostat,
4\(^4\)He gas is introduced in the thermal isolation chamber (red) through port II (2) and pumped out, when the chamber temperature reaches to 4.2 K as the outer 4\(^4\)He reservoir (dark blue). Afterwards, by connecting the pump to port II (3), the 4\(^4\)He is absorbed through the capillary (10) from outer 4\(^4\)He chamber and expands into the middle chamber (purple). This process, so called Joule-Thompson effect, decreases the chamber temperature down around 1.5 K. To achieve even lower temperature, the 3\(^3\)He installation (see Fig. 3.5(b)) is compulsory. The 3\(^3\)He stored by the gas handling system is injected into the inner chamber (light blue) where it will be condensed. Evaporating this 3\(^3\)He liquid by pumping the inner volume produce a typical 3\(^3\)He liquid temperature of around 500 mK for the sample measurement.

![Diagram of 500 mK cryostat](image)

**Figure 3.5**: (a)Diagram of 500 mK cryostat. (1)Inlet of 4\(^4\)He reservoir (2)Vacuum pump port I (3)Vacuum pump port II (4)Inlet of 3\(^3\)He (5)Outlet of 4\(^4\)He reservoir (6)Sample insert (7)Pre-chamber (8)Plate valve (9)Outlet of capillary (10)Inlet of capillary (11)Metal shield. (b) 3\(^3\)He gas handling system.

### 3.4.2 Measurement setup

The Fig. 3.6 is the connection schematic to characterize the IVC of single Josephson junction. We set sweeping voltage range on the computer(a), the voltage source(b) would deliver this order to current source(b). Then, the current source(c) applied corresponding current onto the Josephson junction. The generated voltage across the junction is not strong enough to be measured by voltmeter(e), therefore, need to amplified by voltage amplifier(d). Besides, a resistance thermometer(f) is connected with the sample to monitor the simultaneous experimental temperature. Since the voltage source, voltmeter and resistance thermometer are all connected with computer, the IVC can be read out immediately on the screen with the record of the experimental
conditions.

To investigate the performance of DC SQUID, a coil is needed to be located above the sample to generate magnetic field threading through the superconductor loop. Thus, an additional current source is necessary to insert into the connection circuits. Except this, all are the same as that of IVC measurement setup. We note that for measurements on single junctions, the magnetic field vector needs to be in the plane defined by the substrate surface.

**Figure 3.6:** The components diagram of measurement setup: (a) operation computer (b) voltage source (c) current source (d) voltage amplifier (e) voltmeter (f) resistance thermometer.
Chapter 4

Results and discussion

This chapter begins the presentation of experimental results. The first section is about the basic electromagnetic properties of Josephson junction. Then performance of SQUID is characterized. The performance of the coupling between two TLRs is investigated in the last section.

4.1 Josephson Junction

High quality Josephson junction are generally desirable for all applications, it is certainly crucial for this project. This part explains how Nb/Al-Al₂O₃/Nb Josephson junction is characterized and how the typical quality parameters are defined. Meanwhile, by revealing the physics behind the characteristic measurement, the direction of modulate the fabrication process to gain high quality parameters are more specified.

For a Josephson junction, the amount of information about its quality extractable from its current-voltage characteristic strongly depends on the value of the Stewart-McCumber parameter $\beta_c$. For overdamped junction with $\beta_c \ll 1$, not much information can be obtained. For underdamped with $\beta_c \gg 1$ in contrast, lots of information can be extracted from the IV curve.

The $I_cR_N$ product represents the tunnel strength of cooper pair, the ratio of subgap to normal resistance $R_{sg}/R_N > 10$, the characteristic voltage $V_m = I_cR_{sg} > 30$ mV, are commonly used parameters to represent high quality junctions.

4.1.1 Current voltage characteristic

The layer thickness of Nb/Al-Al₂O₃/Nb Josephson junction, which is shown in the following, is 50 nm/3 nm/1 nm/50 nm and the junction area is 20 µm × 20 µm. The microscopic image of fabricated single Josephson junction is shown in Fig. 4.1

The 20 × 20 µm² single Josephson junction is measured at 4.2 K. By applying the external current through the JJ, the output voltage is measured. The obtained IVC
Figure 4.1: Microscopic image of 20 µm × 20 µm bridge Josephson junction

shows a strong hysteresis (see Fig. 4.2) and it is can be explained by the washboard theory mentioned in section 2.2.3. Due to the fact that the kinetic energy stored in phase particle during the running state is not damped out after the current is lowered past $I_c$, so that its momentum can support the particle to get over the rising potential barrier and the movement only stops for a flat potential at $I_r \approx 0$. This is commonly known as retrapping to the zero voltage state and $I_r$ is called retrapping current. The occurrence of this hysteresis allows the observation that the quasiparticle resistance $R$ defined in RCSJ model does actually depends on the voltage. There are two resistances in different regimes are defined: for the voltage below the superconducting energy gap $V < 2\Delta/e$, quasiparticle conduction is only possible by assistance of thermal excitation, multi-photon process or pinholes in the tunneling oxide, so that the rather high subgap resistance $R_{sg}$ is observed. For the voltage $V > 2\Delta/e$, the energy supplied by the current source is sufficient to break up cooper pairs and support the quasiparticles tunnel from one side of the insulator to the unoccupied states on another side. This is so called ohmic regime with the normal resistance $R_N$. The large voltage jump occurs at the critical current $I_c$ at the gap voltage $V_{gap} = 2\Delta/e$. In our case, the critical current $I_c$ is 286.98 µA and corresponding gap voltage $V_{gap}$ is 2.7 mV. Because of the structure vanishing in optical lithography step, the actual junction area $A$ calculation related to the junction length $L$, follows $(L - 0.5)^2$. The current density is $j_c = I_c/A$ of JJ thus equals to 75.47 A/cm². When the external applied current is large enough to break up the cooper pairs, the Josephson junction present ohmic behavior with normal resistance $R_N = 4.74 \Omega$. When the external applied current is below the critical current, the Josephson junction has subgap resistance $R_{sg} = 80.46 \Omega$. Since the critical current $I_c$ of a Josephson junction scales with the junction area $A$ which is inversely proportional to the normal resistance $R_N$, the product $I_cR_N$ should have invariant value. This product is an important parameter to determine the cooper pair tunneling strength in the Josephson junction. The higher is this product the finer is the insulator barrier. However, if the straight line, whose slope yields the normal resistance in the first quadrant of the IVC intersects the current axis with a positive value, the quasiparticle conduction contribute the critical current so that
Figure 4.2: I-V curve for single Josephson junction with $A = 20 \times 20 \mu m^2$, at $T = 4.2$ K. The sample is highly underdamped with Stewart-McCumber parameter $\beta_c \gg 1$. The quality parameters: critical current $I_c$, gap voltage $V_{gap}$, subgap resistance $R_{sg}$ and normal resistance $R_N$ are all revealed in the curve.

$I_c R_N$ is no longer a measure for the strength of cooper pair tunneling anymore. For Josephson junctions, Ambegaokar and Baratoff drives an exact $I_c R_N$ expression from BCS theory [15]:

$$I_c R_N = \frac{\pi 2 \Delta(T)}{4e} \tanh \left[ \frac{2\Delta(T)}{4\kappa_B T} \right]$$  \hspace{1cm} (4.1)

For $T < 0.5T_c$, the superconducting gap in the weak coupling regime [16] is:

$$\frac{\Delta(T)}{\Delta(0)} \approx 1 - 3.33 \left( \frac{T}{T_c} \right)^{1/2} \exp \left( -1.76 \cdot \frac{T_c}{T} \right)$$  \hspace{1cm} (4.2)

Since Nb critical temperature $T_c = 9.25K$, the theoretical gap voltage $\Delta(T)$ at characterization temperature $T = 4.2$ K is deduced to be 2.96 mV with the Nb superconducting gap $\Delta (0) \approx 3.1$ meV [17]. As a result the theoretical product $I_c R_N$ is approximate 2.24 mV. However, the experimental gap voltage $V_{gap}$ and $I_c R_N$ are all smaller than the theoretical values. This is because the barrier potential created by insulator layer is not rectangular rather than sloped and also varies in three dimensions. To achieve the ideal tunneling, the AlO$_x$ should be homogeneous in space and contain as less
impurities as possible during the fabrication process to form more finer barrier. Besides, due to the affinity of Al to Nb, the Al layer planarizes the rather rough Nb surface \[18, 19\]. To get rid of this situation, Al thickness could be increased up to 7 nm \[20\].

Meanwhile, the ratio of subgap to normal resistance \(R_{sg}/R_N\) and voltage \(V_m = I_c R_{sg}\) are two additional quality parameters for Josephson junction. Being independent of junction geometry, they allow for the comparison among all the junctions made by same fabrication process. Technically, good quality is admitted with \(R_{sg}/R_N > 10\) and \(V_m = I_c R_{sg} > 30\) mV.

The experimental measurement results with different junction areas, \(20 \times 20\) \(\mu\)m\(^2\), \(10 \times 10\) \(\mu\)m\(^2\) and \(5 \times 5\) \(\mu\)m\(^2\) are shown in Table 4.1. These three different sized junctions are fabricated by same fabrication process but local at different positions on one chip. We notice that the quality parameters, product \(I_c R_N\), \(R_{sg}/R_N\) and \(V_m\) decrease with the junction area reduction, which indicate that smaller junctions are more sensitive to barrier defects.

<table>
<thead>
<tr>
<th>(P_{ox}) (mbar)</th>
<th>(t_{ox}) (h)</th>
<th>(A) ((\mu)m(^2))</th>
<th>(I_c) ((\mu)A)</th>
<th>(V_g) (mV)</th>
<th>(R_N) ((\Omega))</th>
<th>(I_c R_N) (mV)</th>
<th>(I_c R_{sg}) (mV)</th>
<th>(R_{sg}/R_N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>3.0</td>
<td>380</td>
<td>287</td>
<td>2.70</td>
<td>4.75</td>
<td>1.59</td>
<td>24.6</td>
<td>15.5</td>
</tr>
<tr>
<td>1.0</td>
<td>3.0</td>
<td>90</td>
<td>59.8</td>
<td>2.72</td>
<td>19.3</td>
<td>1.16</td>
<td>10.2</td>
<td>8.83</td>
</tr>
<tr>
<td>1.0</td>
<td>3.0</td>
<td>20</td>
<td>8.33</td>
<td>1.83</td>
<td>53.6</td>
<td>0.45</td>
<td>1.62</td>
<td>3.64</td>
</tr>
</tbody>
</table>

Table 4.1: table: Electrical parameters of Josephson junctions with different junction area measured at 4.2 K. The oxidation pressure and time are all 1 mbar and 3 h

The 5 \(\mu\)m \(\times\) 5 \(\mu\)m bridge junction has critical current \(I_c\) around 8 \(\mu\)A, so the smaller junction area of approximately 2.5 \(\mu\)m \(\times\) 2.5 \(\mu\)m is needed to obtain the target critical current 1.5 \(\mu\)A. Except the fact the quality parameters of junction 5 \(\mu\)m \(\times\) 5 \(\mu\)m are highly below the mentioned standard, the fabrication yield decreases with decreasing junction size.(see Fig. 4.3). We can notice the proportion of vanishing structure is comparable huge for the small junction at the edge of the chip. Especially, the Josephson junction shown in Fig.4.3(c) completely lost its inherent rectangular appearance. So the target to fabricate even smaller junction 2.5 \(\mu\)m \(\times\) 2.5 \(\mu\)m confronts a huge quality and structure challenge. Nevertheless, as long as one observes obvious Josephson junction characteristic, it is meaningful to prepare DC SQUID and RF SQUID intersected in between 2 TLRs with 2.5 \(\mu\)m \(\times\) 2.5 \(\mu\)m Josephson junction.

![Figure 4.3: Single bridge Josephson junction with junction area (a) 20 \(\times\) 20 \(\mu\)m\(^2\) at the edge of the chip. (b)5 \(\times\) 5 \(\mu\)m\(^2\) in the center of the chip. (c)5 \(\times\) 5 \(\mu\)m\(^2\) at the edge of the chip.](image)
4.1.2 Magnetic field modulation

Since the Josephson junction area is small enough, the supercurrent flowing through Josephson junction is considered as homogeneous along the its cross-section yz plane. To investigate the magnetic field dependence of the maximum critical current of single Josephson junction, the sample holder is monted in a long superconducting coil, which generates the magnetic field with its direction parallel to the junction yz plane (see Fig. 4.4) and then the maximum supercurrent through the Josephson junction is modulated periodically by this magnetic field. The characteristic curve is shown in Fig. 4.5. For the zero magnetic field $\Phi_0 = 0$, the phase difference is constant and hence the center critical current is corresponding to the value derivated from the I-V curve in the previous section 2.2.4.

![Figure 4.4: The external applied magnetic field with its direction parallel to the yz plane of Josephson junction](image1)

![Figure 4.5: Fraunhofer pattern of Josephson junction obtained at 4.2 K with external applied magnetic filed parallel to the junction layer plane](image2)
4.2 Josephson junction based SQUID

4.2.1 Magnetic field modulation curve for DC SQUID

The DC SQUID made of two parallel Josephson junctions with size 2.5 \( \mu \text{m} \times 2.5 \mu \text{m} \) interrupting a Nb superconducting loop is shown in Fig. 4.6. Different from the case of single Josephson junction, the magnetic modulation of DC SQUID is carried out with the magnetic field perpendicular to its loop plane. By applying direct current through the DC SQUID, the measured magnetic dependence curve at different temperatures is shown in Fig. 4.7

We notice the \( I_c(B) \) curves are not modulated completely down to zero by the external applied magnetic field, which means the screening coefficient \( \beta_L \) is slightly too high and there is additional magnetic field generated by the superconductor loop, which interferes with the applied field. But the current modulation depth is 3.5 \( \mu \text{A} \) at 500 mK, we can infer that the sensitivity of DC SQUID reacts to the external magnetic field is acceptable and would not negatively affect the tunability of the beam splitter. Considering the influence of the temperature, the \( I_c(B) \) curve becomes smoother with the ambient temperature decreases. This is due to thermal fluctuation and ambient noise are contained at lower level while lowering the experimental \( T \). Concerning about the amplitude of supercurrent, it is mainly influenced by two factors, thermal excitation of \( E_j \) and superconductor energy gap, whose domination highly relies on the ambient temperature. Compared to the case in 500 mK and 1.5 K, the amplitude of supercurrent at 4.2 K is dramatically reduced. For this case due to the small \( I_c \) the thermal excitation of \( E_j \) plays the most significant role here. The phase of the junctions is exited, which leads to a voltage drop about the SQUID, even at a low driving current. Furthermore one has influences from other sources, because in the Gorter-Casimir two fluid model \([21, 22]\), cooper pairs density \( n_s \) depends on the experimental temperature:
Figure 4.7: $I_c(B)$ curves of DC SQUID at ambient temperature 500 mK, 1.5 K, 4.2 K

$$n_s = n_s(0) \left[ 1 - \left( \frac{T}{T_c} \right)^4 \right]$$ (4.3)

It indicates more than 4 percent of cooper pairs that in Nb at $T = 4.2$ K are broken by the thermal energy into quasiparticles, which could not contribute to the supercurrent, while only 0.1 percent cooper pair is destroyed at $T = 1.5$ K and even less at $T = 500$ mK. Referring to the Equ. 4.2, we notice superconductor energy gap decreases with reduction of ambient temperature $T$ and more possibility for the cooper pair to flow through the potential barrier. However, the thermal excitation still weights much more on influence of the supercurrent amplitude at $T = 4.2$ K than that of $T = 1.5$ K and $T = 500$ mK. For the later two temperatures, almost no cooper pairs are broken as a result the thermal excitation could be neglected and the reduction of the superconductor energy gap dominates. What is attractive is the period of the modulation at 4.2 K get expanded with its amplitude squeezing. The reason is might be the effective inductance $L$ of the loop is changing with the temperature. Except the reduction of surrpercurrent at higher temperature, the $I_c(B)$ is not symmetric for $T = 500$ mK and $T = 1.5$ K, meanwhile, the trough and crest of $I_c(B)$ curves also shift.
4.3 Coupling between 2 TLRs

4.3.1 Eigenfrequency of two transmission parallel line resonators

The frequency dependence of TLR was measured using a vector network analyzer. The sample was mounted on in a closed copper sample box and cooled in liquid \(^4\)He below the critical temperature of superconductors. The coupling is inductive coupling. Because of two TLRs are close to each other, the eigen frequency splits into two modes. From the measurement curve (see Fig. 4.8), the eigen frequencies of TLR are \(f_1 = 6.43\) GHz and \(f_2 = 6.50\) GHz. The quality factor is determined by the full width at half max of the spectrum and are found to be \(Q_1 = 627, Q_2 = 673\).

![Figure 4.8: Frequency mode split at \(f = 6.47\) GHz for two identical parallel TLRs. The two split mode frequency are \(f_1 = 6.43\) GHz and \(f_2 = 6.50\) GHz respectively with quality factor \(Q_1 = 627\) and \(Q_2 = 673\). The coupling strength is \(g = (f_1 - f_2)/2 = 35\) GHz.](image)

As what had been discussed in the theoretical section 2.5.2, the geometry coupling strength varies with the structure which is used to enhance the coupling strength between two superconducting TLRs. In order to know the variation tendency, a comparison was made among the two separated TLRs with no structure in between, Nb bridge and Nb ring intersected TLRs while the length for what the resonators go in parallel and the distance of two TLRs there are all \(500 \mu\)m and \(100 \mu\)m respectively. And the length of the ring and bridge \(\Delta x\) is \(100 \mu\)m. Measuring with the vector network analyzer, the frequency spectrum of three different structures normalized at their center frequencies were showed in Fig. 4.9. The geometry coupling strength
$2g = \omega_1 - \omega_2$ in these three cases are 35MHz, 46MHz, 62MHz and it increase with the intersection ratio rises.

![Diagram showing mode split for different structures between two parallel TLRs.](image)

**Figure 4.9:** Frequency mode split for different structures in between two parallel TLRs. A comparison of coupling strength is made by intersecting the TLRs with three structures, nothing, Nb bridge and Nb ring. The coupling strength are $g_1 = 35$ GHz, $g_2 = 46$ GHz, $g_3 = 62$ GHz respectively. The three small pictures show the coupling region of the resonators.

The coupling strength of beam splitter should be larger than that of geometry coupling to realize the decoupling. If not, the coupling could not be turned off completely by external applied magnetic field, which indicated the controllability is unqualified. This is expected to be well feasible from simulations [23]. The actual fabrication and characterization of such a device would have exceeded the temporal scope of this thesis and is left as the next step.
Chapter 5

Summary and outlook

The geometry of two tunable coupled TLRs acting as a microwave beam splitter and the main requirements for achieving coupling strengths between 0 and 200 MHz is discussed intensively from theoretical point of view. For the chosen design of a 6 GHz resonator, the ideal practical $I_c$ of JJ deduced from the simulation should be around 1.5 $\mu$A. This value is roughly obtained by optimizing the fabrication parameters, junction size and insulator layer thickness. The experimental result for 2.5 $\mu m \times 2.5 \mu m$ bridge Josephson junction in DC SQUID has critical current around 1.5$\mu$A at 4.2k. At the same measurement temperature, the correspond DC SQUID has 2.5$\mu$A modulation depth responding to the external applied magnetic field, which is acceptable for beam splitting. Finally, the feasibility of beam splitting has been verified by checking that the geometric coupling strength in the actual device geometry is smaller than the simulated tunable coupling strength. The logical next step is to combine the JJ technology and the coupled-resonator layout developed in this work into an actual device and demonstrate that the coupling can be tuned via an external magnetic field.

In conclusion, the thesis work paves the way for the beam splitter as an effective tool to comprehend the photon-photon interaction and also contributes the qubits applications in the field of quantum simulations and quantum information processing.
Appendix A

In the following is the details of fabrication process of tunable beam splitter.

5.1 Bottom electrode and TLRs fabrication

100nm Nb deposition on Si substrate.
AZ 5214 optical resist deposition with 8000 rpm, baking at 110 °C for 70s.
Sidewall removal (30s UV exposure + 90s AZ MIF 726 development).
Structuration (UV exposure dose = 36 mJ/cm² + 70s AZ MIF 726 development).
Reactive ion etching 70s
Cleaning process (1. Acetone, 70 for 30mins + Ultrasonic, 2mins, repeat this step one more time. 2. Acetone TE, Ultrasonic 2mins + Acetone PA, Ultrasonic 2mins + Isopropanol, Ultrasonic 2mins.)

5.2 Trilayer deposition

AZ 5214 optical resist deposition with 4000 rpm, baking at 110 °C for 70s.
Float exposure (UV exposure dose = 4 mJ/cm² baking 130 °C for 120s).
Structuration (UV exposure dose = 45 mJ/cm² + 70s AZ MIF 726 development.)
Ion gun to get rid of NbOₓ.
Sputtering 50nm Nb.
Sputtering 4nm Al.
Al oxidation to generate 1nm $\text{Al}_2\text{O}_3$.
Sputtering 50nm Nb.
Lift off (1. Acetone, 70 for 30mins + Ultrasonic, 2mins, repeat this step one more time. 2. Acetone TE, Ultrasonic 2mins + Acetone PA 2mins + Isopropanol, Ultrasonic 2mins.).

5.3 $\text{SiO}_x$ deposition

$\text{AZ6612}$ optical resist deposition with 4000 rpm, baking at 110 °C for 70s.
Sidewall removal (30s UV exposure + 90s AZ MIF 726 development).
Structuration (UV exposure dose = 45 mJ/cm$^2$ + 30s AZ MIF 726 development.
Reactive ion etching 70s
Lift off as previous one.

5.4 Top electrode deposition deposition

$\text{AZ6612}$ optical resist deposition with 4000 rpm, baking at 110 °C for 70s.
Structuration (UV exposure dose = 60 mJ/cm$^2$ + 30s AZ MIF 726 development.
Ion gun to get rid of $\text{NbO}_x$.
Sputtering 500nm Nb.
Lift off as previous one.
Bibliography


Acknowledgments

I would like to thank Professor Rudolf Gross. He absorbed me in this Qubits research group so that I have the chance to get deeper comprehension about quantum physics which actually confused me before. His profound knowledge and sharp review about the physics problems impresses me and demonstrates the scholar’s elegance.

Dr. Frank, Deppe, as my tutor taught me how to grab the essence of the problems and how to be a qualified seeker for the unknown. Hereto, I want to thank him sincerely. I believe his serious and cautious scientific attitude is an excellent example for me forever. I am deeply appreciated that he guided me to truly touch the beauty of quantum physics.

Many thanks to PhD. Karl Friedrich Wulschner because he is the one who guides and advises me personally and patiently on how to manipulate the experimental machines, analyzes the measurement data and extracts answers of various problems based on the basic physical principle. Even more, he helped me get used to the life in Munich since it is my first time to be here.

Additionally, I am very appreciated for the my family who support me and deliver me warmth selflessly.