6.6
Propagating quantum microwaves

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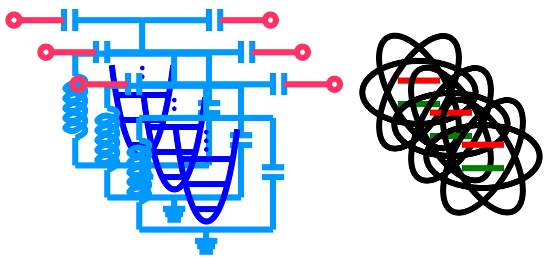
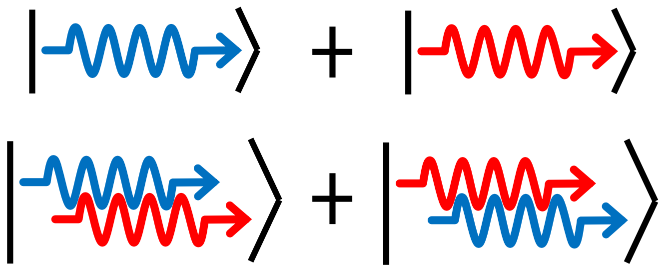


Quantum -  - 



Superconducting quantum circuits

→ Artificial quantum matter

→ Confined quantum states of light

Does the emitted microwave radiation exhibit quantum properties?

→ Commutation relations, superpositions, entanglement

→ Quantum optics

→ Yes, expected due to field quantization

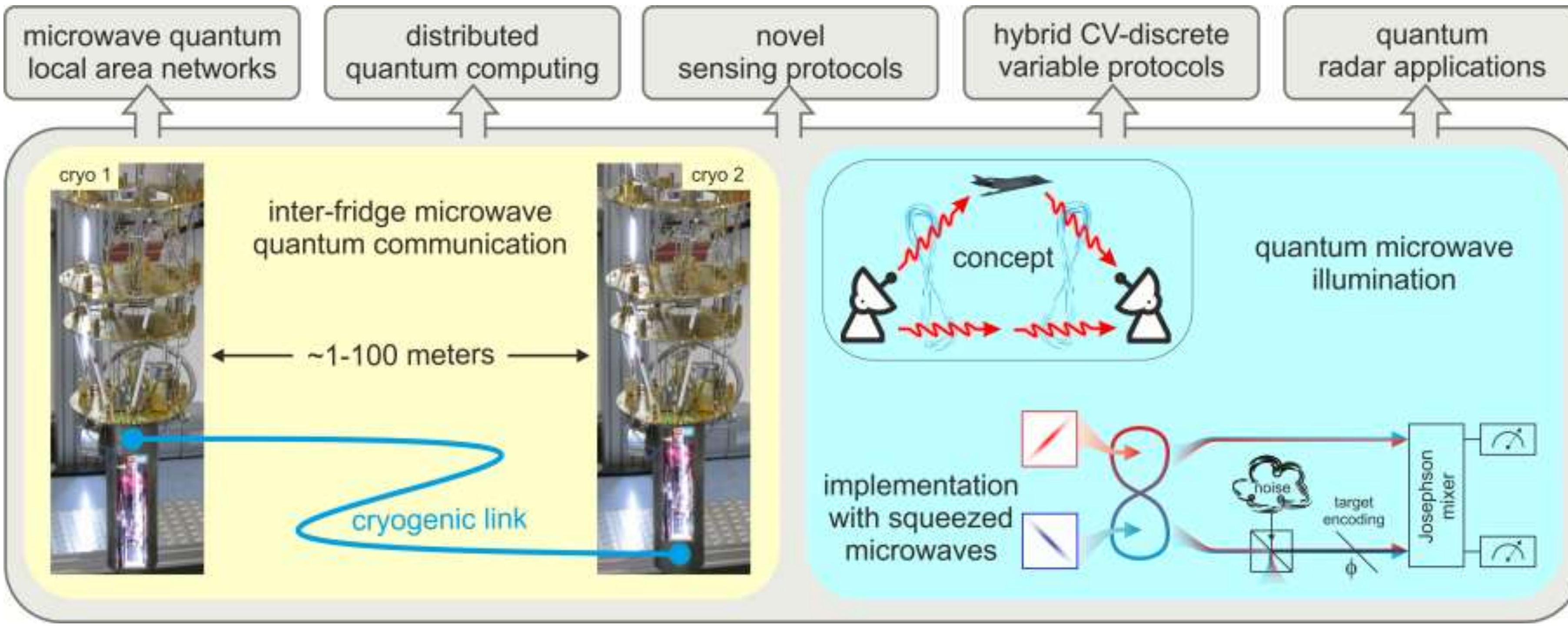
→ Confirmed by experiments

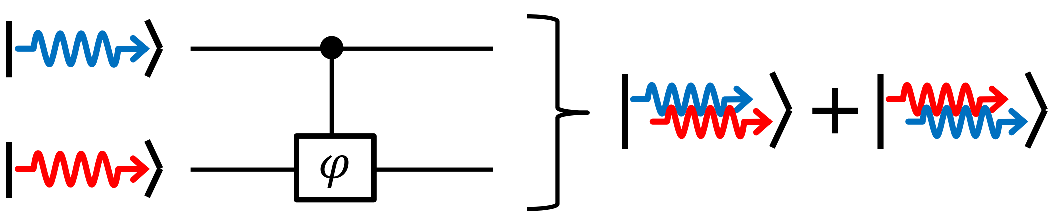
→ Microwaves → Expected in analogy to opics

→ Different technology → Experimental proof required!

6.6 Propagating quantum microwaves

(Envisioned) applications of propagating quantum microwaves





Quantum information processing

6.6 Propagating quantum microwaves

Fundamental technological considerations

Microwave losses may inhibit

→ Observation of quantum properties of propagating microwaves

→ Practical applications such as quantum microwave communication/illumination

Superconducting cables

→ Coherent propagation distance ℓ_{coh} sufficient?

  → In resonators, microwave signals travel back and forth many times before losing coherence ($T_1 \simeq 100\ \mu\mathrm{s} - 1\ \mathrm{ms}$)

  → $\ell_{\mathrm{coh}} \approx 3 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}} T_1 \simeq 10 - 100\ \mathrm{km}$ comparable to optics

→ Superconducting cables require cooling!

  → Short- or medium-distance applications certainly feasible

  → QIP platforms such as SQC also require cooling → Compatible

→ Technological compatibility to SQC

  → No frequency conversion losses

  → Natural candidate for chip-to-chip quantum communication between SQC

6.6 Propagating quantum microwaves

Fundamental technological considerations

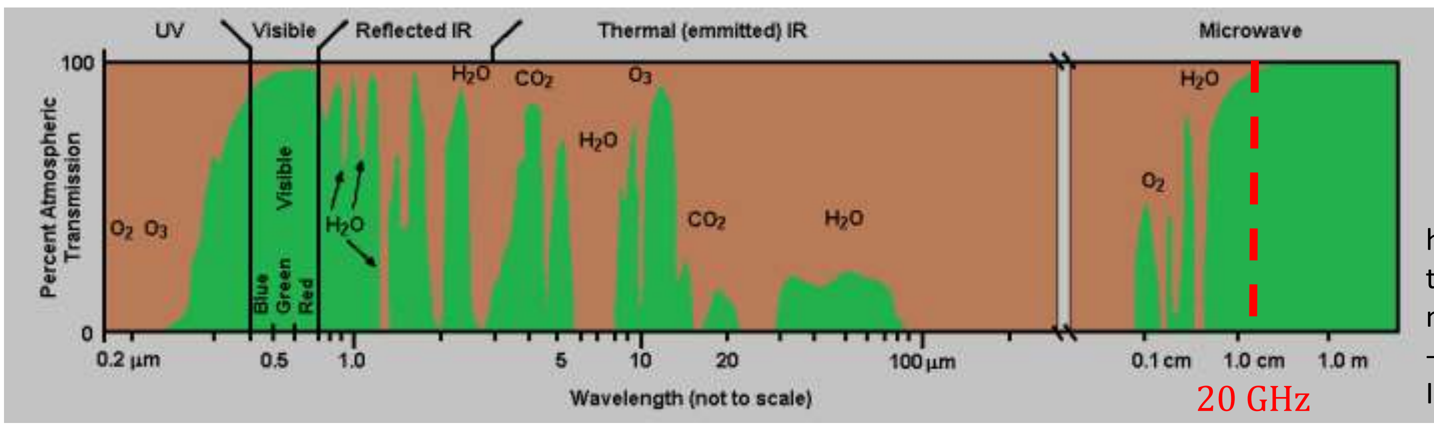
Microwave losses may inhibit

→ Observation of quantum properties of propagating microwaves

→ Practical applications such as quantum microwave communication/illumination

Free-space propagation

→ Atmospheric transparency windows



http://www.oneonta.edu/faculty/baumanpr/geosat2/RS-Introduction/RS-Introduction.html

→ Classical illumination with microwaves used for radar

→ Known to pass through clouds, fog, and rain

→ Typical frequencies $\simeq 20$ GHz ($\lambda = \frac{3\times10^8\frac{\mathrm{m}}{\mathrm{s}}}{f} \simeq 1.5$ cm)

→ Compatible with SQC (superconducting gap of aluminum still twice as large)

6.6 Propagating quantum microwaves

F. D. Walls, G. Milburn, *Quantum Optics* (Springer, Berlin, 2008).

Quantization of the electromagnetic field

→ Source-free (free field!) Maxwell equations

$$\boldsymbol{\nabla B} = 0, \quad \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad \boldsymbol{\nabla D} = 0, \quad \boldsymbol{\nabla} \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \qquad (\boldsymbol{B} = \mu_0 \boldsymbol{H}, \boldsymbol{D} = \epsilon_0 \boldsymbol{E}, \mu_0 \epsilon_0 = c^{-2})$$

→ Coulomb gauge ($\boldsymbol{\nabla A} = 0$) → $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$, $\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t}$

→ $\boldsymbol{A}(\boldsymbol{r}, t)$ satisfies wave equation $\boldsymbol{\nabla}^2 \boldsymbol{A} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{A}}{\partial t^2}$

→ Separate vector potential $\boldsymbol{A}(\boldsymbol{r}, t) = \boldsymbol{A}^{(+)}(\boldsymbol{r}, t) + \boldsymbol{A}^{(-)}(\boldsymbol{r}, t)$ into

- → Right-propagating components $\boldsymbol{A}^{(+)}(\boldsymbol{r}, t)$ varying with $e^{-i\omega t}$ for $\omega > 0$
- → Left-propagating components $\boldsymbol{A}^{(-)}(\boldsymbol{r}, t)$ varying with $e^{i\omega t}$ for $\omega > 0$

→ Restrict field to finite volume

- → $\boldsymbol{A}^{(+)}(\boldsymbol{r}, t) = \sum_k c_k \boldsymbol{u}_k(\boldsymbol{r}) e^{-i\omega_k t}$
- → Fourier coefficients c_k constant for free field
- → Vector mode functions $\boldsymbol{u}_k(\boldsymbol{r})$
  - → Satisfy wave equations $\left(\nabla^2 + \frac{\omega_k^2}{c^2}\right) \boldsymbol{u}_k(\boldsymbol{r}) = 0$
  - → Satisfy transversality condition $\boldsymbol{\nabla u}_k(\boldsymbol{r}) = 0$
  - → Form orthonormal set $\int_V dr\, \boldsymbol{u}_k^\star(\boldsymbol{r}) \boldsymbol{u}_{k'}(\boldsymbol{r}) = \delta_{kk'}$
  - → Depend onboundary conditions

F. D. Walls, G. Milburn, *Quantum Optics* (Springer, Berlin, 2008).

Quantization of the electromagnetic field

→ General example for boundary conditions
  → Periodic (travelling waves)
  → Reflecting walls (standing waves)

→ Here: Plane wave functions suitable for cubic volume with side lengths L
  → $\boldsymbol{u}_k(\boldsymbol{r}) = \frac{1}{L^{3/2}} \hat{\boldsymbol{e}}^{(\lambda)} e^{i\boldsymbol{k}\boldsymbol{r}}$
  → Typically no polarization in microwaves propagating in waveguides
    → Polarization vector $\hat{\boldsymbol{e}}^{(\lambda)} = \hat{\boldsymbol{e}}$
  → Wave vector $\boldsymbol{k} = \left(k_x, k_y, k_z\right)$ with $k_{x,y,z} = \frac{2\pi}{L} n_{x,y,z}$ and $n_{x,y,z} \in \mathbb{Z}$
  → $\hat{\boldsymbol{e}}$ perpendicular to $\boldsymbol{k}$

→ Quantization of classical Fourier amplitudes
  → $a_k, a_k^\star \rightarrow \hat{a}_k, \hat{a}_k^\dagger$ with commutation relations $\left[\hat{a}_k, \hat{a}_{k'}^\dagger\right] = \delta_{kk'}$
  → $\boldsymbol{A}(\boldsymbol{r}, t) = \sum_{\boldsymbol{k}} \sqrt{\frac{\hbar}{2\omega_k \epsilon_0}} \left[\hat{a}_k \boldsymbol{u}_k(\boldsymbol{r}) e^{-i\omega t} + \hat{a}_k^\dagger \boldsymbol{u}_k^\star(\boldsymbol{r}) e^{i\omega t}\right]$
  → $\boldsymbol{E}(\boldsymbol{r}, t) = i \sum_{\boldsymbol{k}} \sqrt{\frac{\hbar\omega_k}{2\epsilon_0}} \left[\hat{a}_k \boldsymbol{u}_k(\boldsymbol{r}) e^{-i\omega t} - \hat{a}_k^\dagger \boldsymbol{u}_k^\star(\boldsymbol{r}) e^{i\omega t}\right]$
  → Hamiltonian $\widehat{H} = \frac{1}{2} \int d\boldsymbol{r} \, (\epsilon_0 \boldsymbol{E}^2 + \mu_0 \boldsymbol{H}^2) = \sum_k \hbar\omega_k \left(\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2}\right)$
  → Quantum states $|\psi_k\rangle$ of each mode can now be discussed independently!

6.6 Propagating quantum microwaves

Continuos variables (CV) vs. discrete variables (DV)

Annihilation operator $\hat{a} \equiv \frac{\omega_\mathrm{r} C\hat{\Phi} + i\hat{q}}{\sqrt{2\omega_\mathrm{r} C\hbar}}$

Creation operator $\hat{a}^\dagger \equiv \frac{\omega_\mathrm{r} C\hat{\Phi} - i\hat{q}}{\sqrt{2\omega_\mathrm{r} C\hbar}}$

$$\hat{H}_\mathrm{HO} = E_\mathrm{kin} + E_\mathrm{pot} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega_\mathrm{r}^2 \hat{x}^2$$

$$\hat{P} \equiv \sqrt{\frac{1}{2\hbar m\omega_\mathrm{r}}}\hat{p}^2,\ \hat{Q} \equiv \sqrt{\frac{m\omega_\mathrm{r}}{2\hbar}}\hat{x}^2$$

Classical single-mode electromagnetic waves $A\cos(\omega t + \phi)$

→ Equivalent description $P\cos\omega_k t + \mathrm{Q}\sin\omega_k t$
with field quadratures $Q = A\cos\phi$ and $P = A\sin\phi$

→ In engineering, *P* is often called *I*

→ Field quadratures analogous to momentum/position in mechanics

→ Field quantization → $[\hat{Q}, \hat{P}] = \frac{i}{2} \quad \Leftrightarrow \quad (\Delta P)(\Delta Q) \geq \frac{1}{4}$

Single-mode quantum field → $\hat{H}_\mathrm{HO} = \hat{P}^2 + \hat{Q}^2 = \hbar\omega_k\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$

Continuous-variable basis

→ Set of eigenstates of either $\hat{P}$ or $\hat{Q}$ also forms basis

→ Natural states are Gaussian states (coherent, squeezed, thermal)

Discrete-variable basis

→ Fock basis $\{|n\rangle\}$

→ Single photons $|1\rangle$ are the natural quantity of interest

→ Any quantum state can be expressed either in CV or in DV

→ Any quantum task (QIP, QSim, Qcomm, QIllu) can be expressed in CV or DV

→ Nevertheless a particular basis may be more suitable for a particular problem

6.6 Propagating quantum microwaves

E. P. Menzel, PhD thesis (Tu München, 2013).

Expressing quantum microwave states

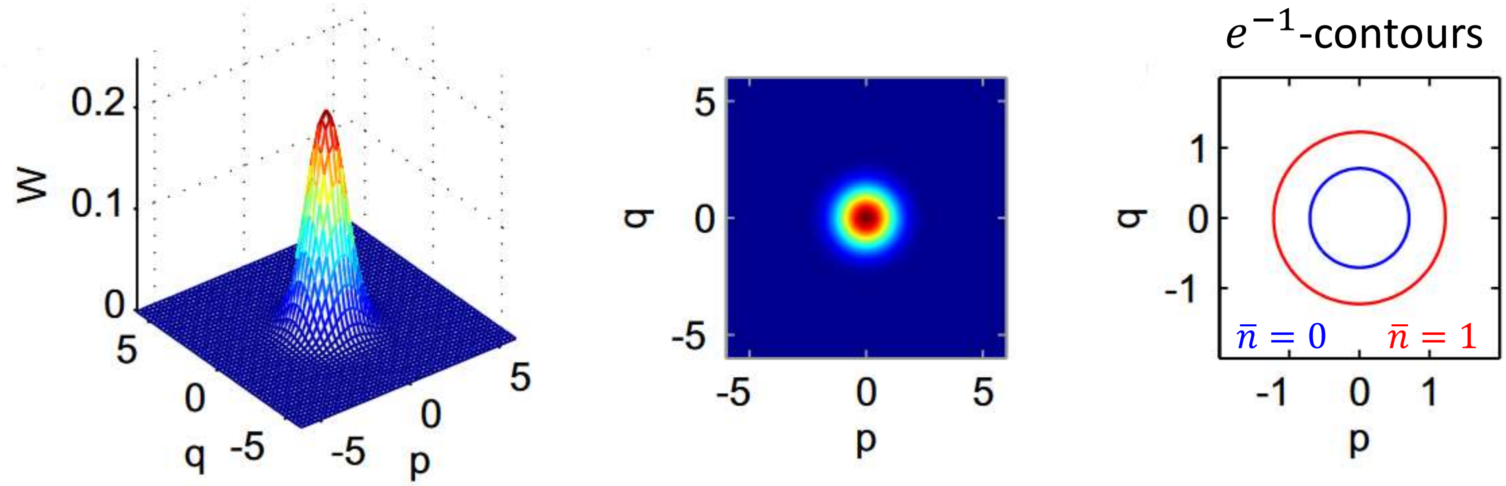
→ General quantum state described by the desity matrix $\hat{\rho} = \sum_{\Psi} P_{\Psi} |\Psi\rangle\langle\Psi|$
- → P_{Ψ} = Classical probability to be in state $|\Psi\rangle$ → $P_{\Psi} > 0$ and $\sum_{\Psi} P_{\Psi} = 1$
- → Expectation value of operator $\hat{O}$ → $\langle\hat{O}\rangle = \mathrm{Tr}\left[\hat{O}\hat{\rho}\right]$
- → Normalization → $\mathrm{Tr}[\hat{\rho}] = 1$
- → Complex matrix entries → Not easy to visualize

→ Phase space representation of a quantum state
- → Ideal classical states → Points in phase space
- → Noisy classical states
  - → Ordinary probability distribution $P(q,p)$ in phase space
  - → q,p are the phase space variables associated with Q,P
  - → $P(q,p)dqdp$ is the probability to find the system in state (q,p)
- → Quantum states
  - → Heisenberg uncertainty relation $(\Delta P)(\Delta Q) \geq \frac{1}{4}$ as “quantum noise”
  - → In general requires also negative probability densities
  - → Quasi-probability distribution $W(q,p)$ with $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(q,p)dqdp = 1$
  - → Wigner function $W(q,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle q - \frac{\zeta}{2}|\hat{\rho}|q + \frac{\zeta}{2}\rangle\, e^{ip\zeta} d\zeta$ and $\zeta \in \mathbb{R}$

6.6 Propagating quantum microwaves

E. P. Menzel, PhD thesis (TU München, 2013).

Wigner function examples



Thermal state $\hat{\rho}_{\mathrm{th}}$ with $\bar{n} \equiv Tr\left[\hat{a}^{\dagger}\hat{a}\hat{\rho}_{\mathrm{th}}\right] = 1$

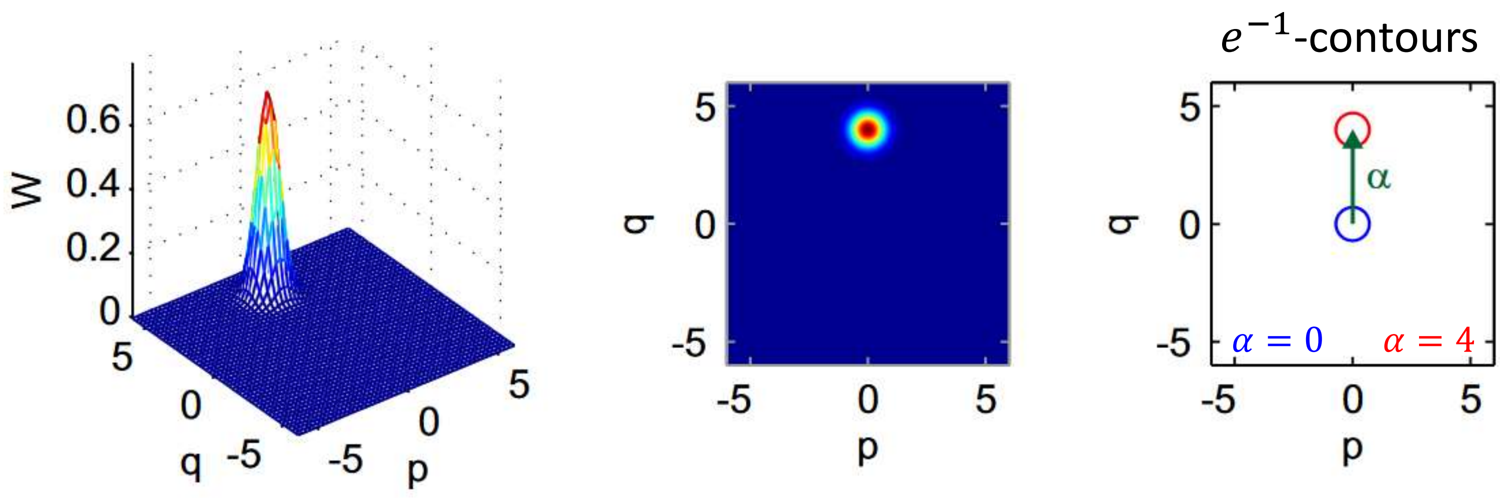
→ $W_{\mathrm{th}}(q,p) = \frac{1}{\pi\left(\bar{n}+\frac{1}{2}\right)} e^{-\frac{q^2+p^2}{\bar{n}+\frac{1}{2}}} > 0$

→ In the high-temperature limit $k_{\mathrm{B}}T \gg \hbar\omega_k$, thermal states can usually be considered to be classical

→ Thermal states are aslo classical in the sense that they do not generate entanglement when applied to a beam splitter with vacuum in the other input

E. P. Menzel, PhD thesis (TU München, 2013).

Wigner function examples



Coherent state $|\alpha\rangle \equiv |Q + iP\rangle = \widehat{D}(\alpha)|0\rangle$

→ Produced by the displacement operator $\widehat{D}(\alpha) \equiv \mathrm{e}^{\alpha\hat{a}^\dagger - \alpha^\star \hat{a}}$ applied to the vacuum

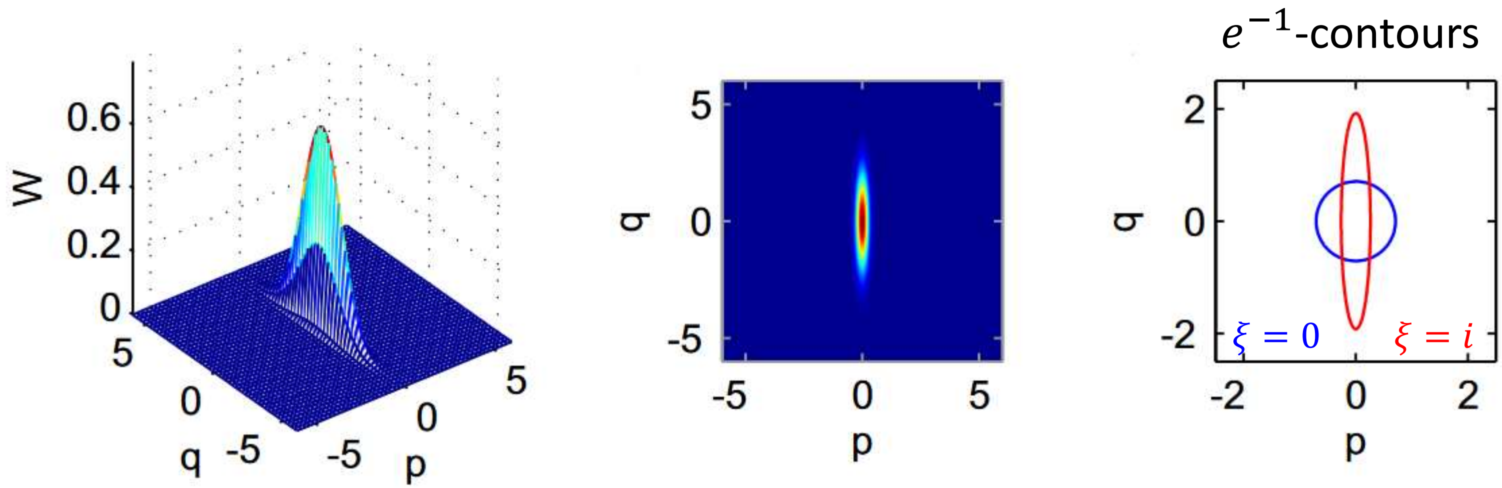
→ $W_{\mathrm{coh}}(q,p) = \frac{2}{\pi} e^{-2[(q-Q)^2+(p-P)^2]} > 0$

→ Coherent states are classical in the sense that the expectation values of the field operators obey the equations of motions for the actual field operators in the Heisenberg picture

6.6 Propagating quantum microwaves

E. P. Menzel, PhD thesis (Tu München, 2013).

Wigner function examples



Squeezed vacuum determined by complex squeezing parameter $\xi \equiv re^{i\varphi}$

→ Produced by the squeezing operator $\hat{S}(\xi) \equiv \mathrm{e}^{\frac{1}{2}\left[\xi^\star \hat{a}^2 - \xi\left(\hat{a}^\dagger\right)^2\right]}$ applied to the vacuum

→ r determines the amount of squeezing and φ the squeezing direction in phase space

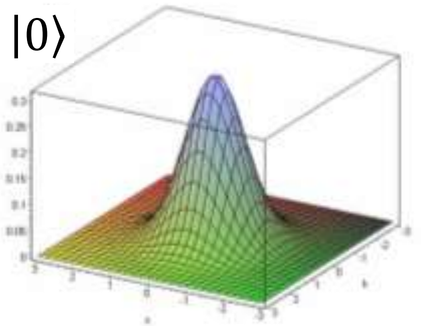
→ $W_{\mathrm{sq}}(q,p) = \frac{2}{\pi} e^{-\left(e^{2r}+e^{-2r}\right)|q+ip|^2 - \frac{1}{2}\left(e^{2r}-e^{-2r}\right)\left(e^{-i\varphi}|q+ip|^2 + e^{i\varphi}|q-ip|^2\right)} > 0$

→ Because $(\Delta P)^2 < \frac{1}{4}$, one must have $(\Delta Q)^2 \geq \frac{1}{4(\Delta P)^2}$ to satsify the Heisenberg relation

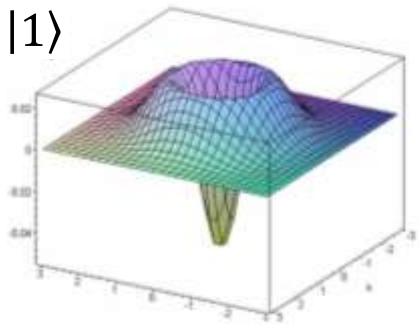
→ Squeezed states are nonclassical in the sense that they produce entanglement when applied to a beamsplitter with vacuum at the other input port

Wigner function examples

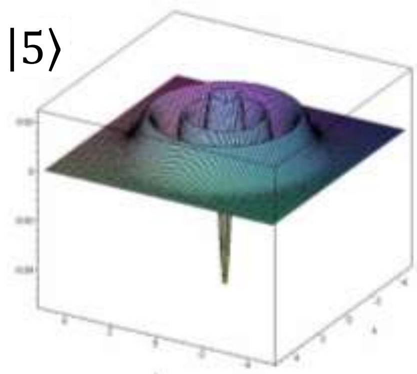
$|0\rangle$



$|1\rangle$



$|5\rangle$



Number states

→ Have a radially symmetric Wigner function

→ The vacuum $|0\rangle$ is a Gaussian state with the vacuum variace of $0.5\hbar\omega\_k$

→ Finite number states $|n \geq 1\rangle$ are nonclassical

→ Because their Wigner function can become negative

→ Because they when applied to a beamsplitter with vacuum at the other input port

6.6 Propagating quantum microwaves

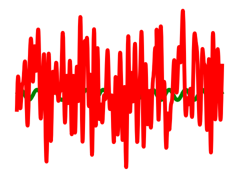
State reconstruction of propagating quantum microwaves

Difficult task!

- No efficient photon detectors

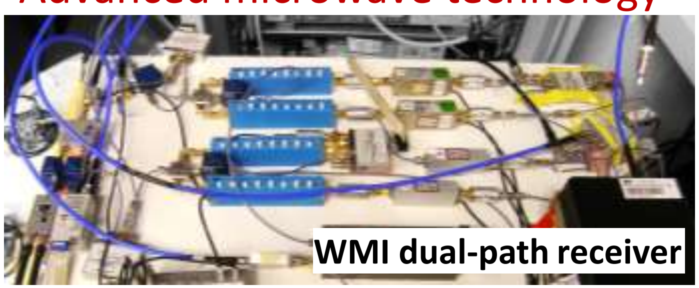
- Off-the-shelf linear amplifiers

- Add $\bar{n} \approx 10$ of noise to signal



Two approaches:
1. Parametric amplifiers
2. Signal recovery methods (measure signal moments)

Advanced microwave technology

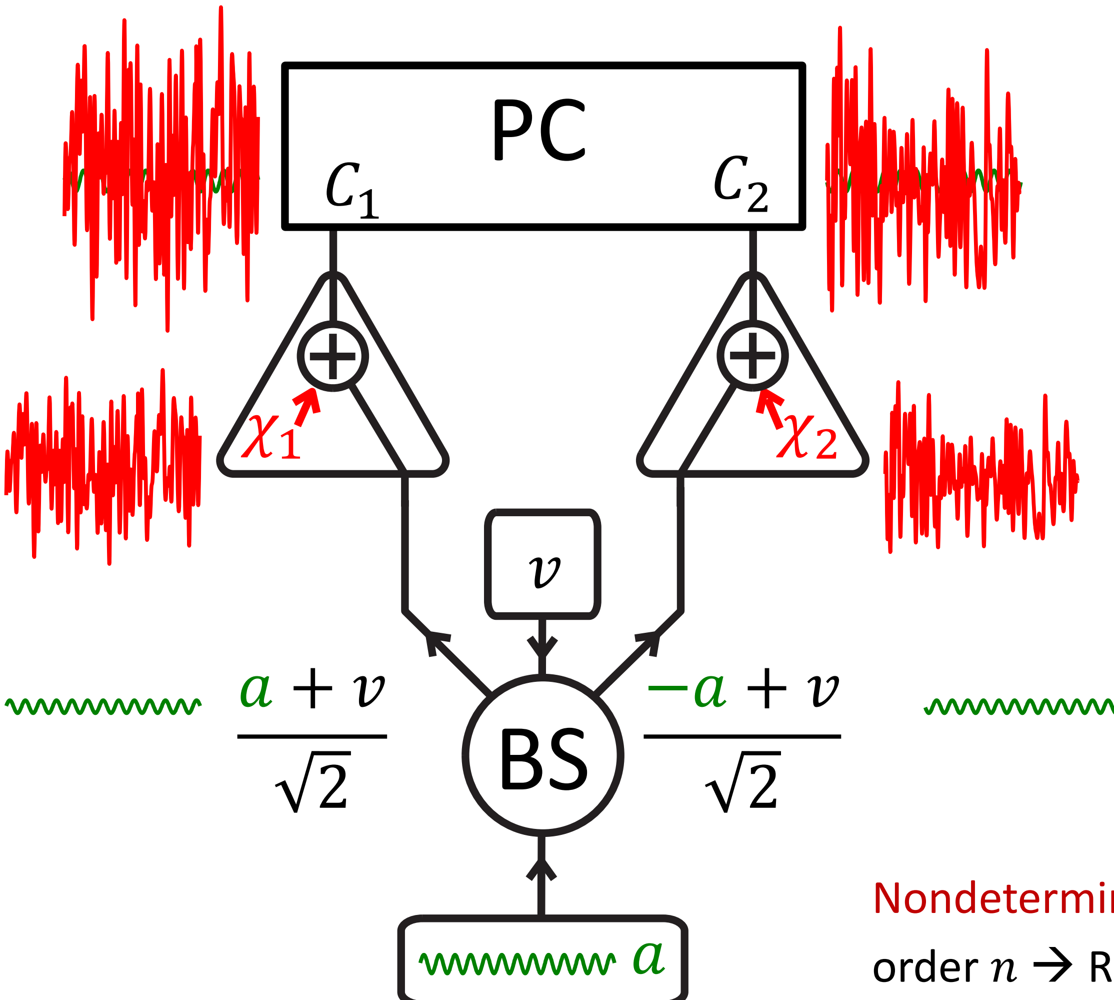


Realtime data processing

extremedesignsonline.com

Dual-path state reconstruction of propagating quantum microwaves

Kowledge of all moments is equivalent to knowlege of the Wigner function or density matrix.



Expectation values of all signal moment up to order n

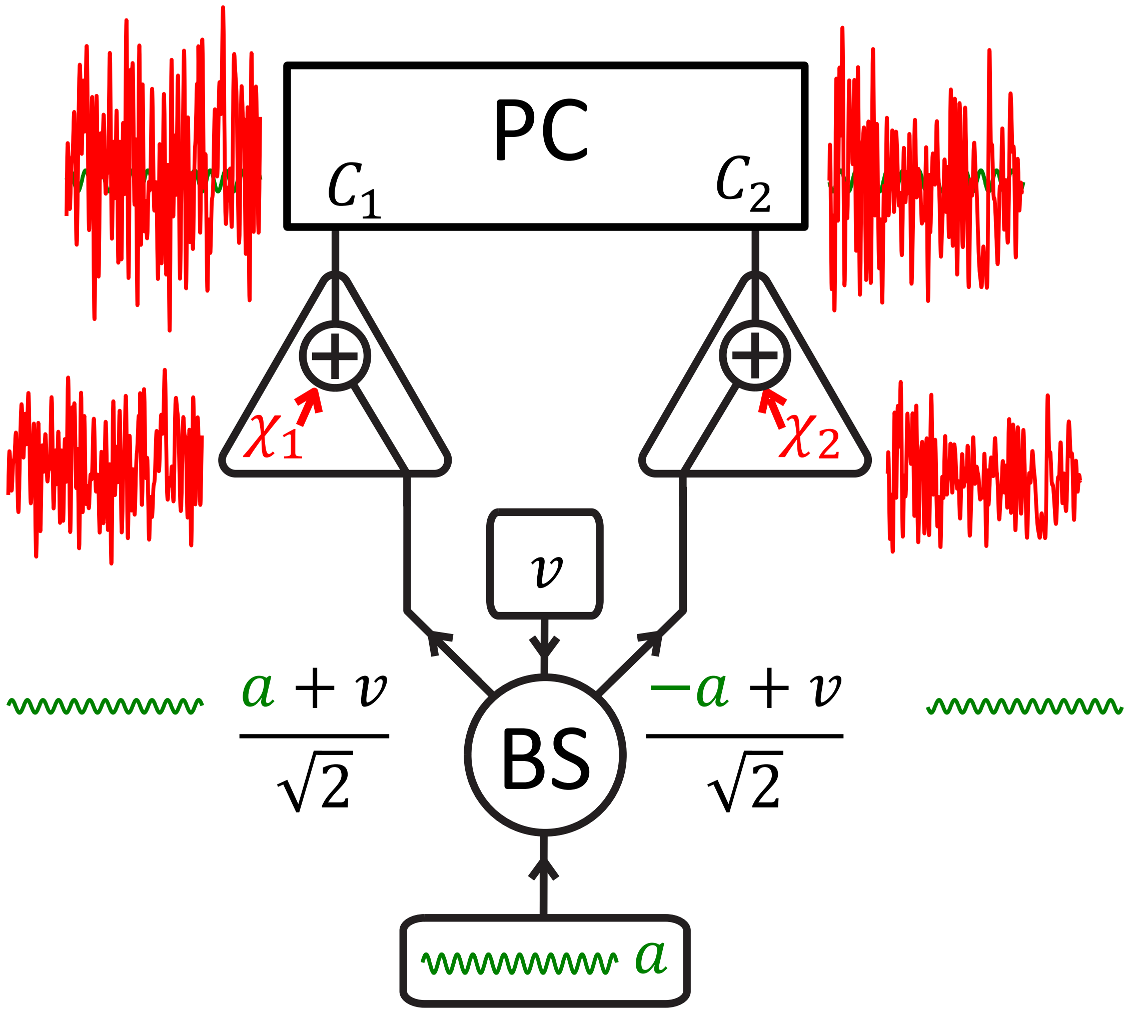
$$\langle C_1^{n-1} C_2 \rangle$$
$$\downarrow$$
$$\langle a^n \rangle, \langle \chi_1^n \rangle, \langle \chi_2^n \rangle$$

Iteratively obtain all signal & detector noise moments

Nondeterministic & quantum signals up to order n → Record all moments $\langle Q_1^j Q_2^k P_1^\ell P_2^m \rangle$ with $\mathrm{j} + \mathrm{k} + \ell + m \leq n$ and $\mathrm{j}, \mathrm{k}, \ell, m \in \mathbb{N}_0$

E.P. Menzel *et al.*, PRL **105**, 100401 (2010).

Dual-path state reconstruction of propagating quantum microwaves



Intuition for order $n = 2$

→ Use statistical independence of amplifier noise signals in different paths $\langle \chi_1 \chi_2 \rangle = \langle \chi_1 \rangle \langle \chi_2 \rangle = 0$

→ $\langle C_{1,2} \rangle = \langle a + \chi_{1,2} \rangle = \langle a \rangle + \langle \chi_{1,2} \rangle = \langle a \rangle$

→ $\langle C_{1,2}^2 \rangle = \langle (a + \chi_{1,2})^2 \rangle = \langle a^2 + a\chi_{1,2} + \chi_{1,2} a +$

E.P. Menzel *et al.*, PRL **105**, 100401 (2010).