
Applied Superconductivity:

Josephson Effect and Superconducting Electronics

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Chapter B

Imaging of the Maximum Josephson Current Density

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