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Preface

The present manuscripts is addressed to students in the 7. and 8. semester of the Technical University of Munich being interested in the course on *Superconductivity and Low Temperature Physics*. The material of the lecture on *Applied Superconductivity: Josephson Effect and Superconducting Electronics* completes that of the lecture on *Foundations of Superconductivity*, which focuses on the fundamental aspects of the phenomenon superconductivity.

The present manuscript is focused on weak superconductivity, in particular the foundations of Josephson effects and their applications in superconducting devices and circuits. After a brief introduction into macroscopic quantum systems it discusses the static and dynamic properties of small and large Josephson junctions. This provides the basis for the understanding of superconducting devices such as Superconducting Quantum Interference Devices (SQUIDs), microwave mixers and oscillators as well as of simple circuits in analog and digital electronics. The manuscript also briefly addresses the fabrication of Josephson junctions based on metallic superconductors and the novel high temperature superconductors. Finally, some actual topics such as superconducting quantum bits are discussed.

The manuscript is considered as a supplement to the lecture. It contains additional material to the topics discussed during the lecture. This material is provided to those students being interested to obtain a deeper insight into the field of applied superconductivity.

The manuscript has been started in the winter semester 2003/2004 and further completed during the subsequent semesters. Nevertheless, it still contains in typos and inaccuracies. The authors are grateful for hints concerning mistakes or conceptual errors (e-mail: Rudolf.Gross@wmi.badw.de).

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Rudolf Gross