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Exercise to the Lecture

Superconductivity and Low Temperature Physics I
WS 2014/2015

7 Questions

Fundamental Properties

1. Which properties are characteristic for superconductors?
2. Discuss the difference between a perfect conductor and a superconductor.
3. How can a persistent current be generated and frozen in a superconducting ring?
4. How can we measure a vanishingly small resistivity?
5. How can the magnetization of a superconductor be measured quantitatively?
6. How was flux quantization measured?
7. Describe the magnetic field **H** and the magnetic flux density **B** inside and outside a superconductor. Discuss possible limiting cases.
8. Describe the **H**-field inside a superconducting sphere.
9. Which elementary superconductors do you know?
10. Do you know superconducting materials with $T_c > 30\,\mathrm{K}$?

Thermodynamics of Superconductors

11. How big is the difference of the free enthalpies in the superconducting and normal state?
12. Describe the change of the entropy S and the electronic heat capacity c_p at the transition temperature T_c.
13. Sketch the temperature dependence $c_p(T)$ for a conventional superconductor with an isotropic gap.
14. Discuss the implications of the exponential temperature dependence of c_p at low temperatures.

London Theory – Macroscopic Quantum

15. How can we derive London equations?
16. Discuss the meaning of the macroscopic wave function of a superconductor.
17. Write down the central results of the London theory (current-phase and energy phase relation) and discuss the conclusions. Which phenomena can be accounted for by the London theory?
18. How can the London penetration depth λ_L be derived? Which quantities determine λ_L?
19. How can we derive the London equations if we assume that a superconductor can be described by a macroscopic wave function?
20. Derive the London equations from the general expression of the supercurrent density derived from the macroscopic quantum model.
21. Derive the fluxoid quantization from the general expression of the supercurrent density derived from the macroscopic quantum model.
22. What is the difference between flux and fluxoid quantization? What are the experimental requirements for measuring flux quantization?
23. How can we derive the Josephson equation from the expression for the supercurrent density in a bulk superconductor?
24. Write down and discuss the Josephson equations.
25. What are potential applications of the Josephson effect?
26. Discuss how the critical current I_c and normal resistance R_n of a superonductor/insulator/superconductor Josephson junction depend on the thickness of the tunneling barrier. What is the relation of the product $I_c R_n$ to the energy gap?

Ginzburg-Landau Theory

27. Sketch the basic ideas of the Ginzburg-Landau (GL) theory.
28. How can we describe the phase transition into the superconducting state (homogeneous superconductor in zero applied magnetic field) within the Landau theory of phase transitions? How does the free enthalpy density look like?
29. How can we classify phase transitions?
30. Comment on the order of the phase transition into the superconducting state at various point in the phase diagram.
31. Which additional terms have to be taken into account when we discuss a spatially inhomogeneous superconductor in an external magnetic field? What is the meaning of these additional terms?
32. Discuss qualitatively how we can derive the GL equations from the expression for the free enthalpy density.

33. Compare the GL equations with the current-phase and energy-phase relation derived from the macroscopic quantum model. What are the analogies and differences?

34. Which characteristic length scales are associated with the GL equations? What is the order of magnitude of these length scales?

35. Explain the meaning of the GL coherence length ξ_{GL}.

36. How vary ξ_{GL} and λ_{GL} close to T_c? Can the temperature dependence be derived from the GL theory?

37. Discuss the GL parameter κ.

38. Discuss the range of validity of the GL theory.

39. Describe the differences between London and GL theory.

Type-I and Type-II Superconductors

40. What types of superconductors do we have to distinguish?

41. Sketch the magnetization curves of type-I and type-II superconductors having the same condensation energy?

42. Explain the origin of the fact that the one superconductor behaves as type-I and the other as type-II. Discuss the energy density of an superconductor/normal metal interface.

43. Which parameter can be used to distinguish between different type-I and type-II of superconductors?

44. How does the Ginzburg-Landau parameter change if we add impurities to a clean metallic superconductor?

45. Discuss the meaning of the lower and upper critical field.

46. How can we derive the lower and upper critical field? How are the fields related to the characteristic length scales ξ_{GL} and λ_{GL}?

47. Sketch the magnetization curve for a type-II superconductor with pinning.

48. Sketch the magnetization and the magnetic flux density inside a type-I and type-II superconductor as a function of the applied magnetic field.

49. How does the magnetic flux distribution look like in the mixed state of a type-II superconductor?

Microscopic (BCS) Theory

50. Describe the isotope effect and possible conclusions.

51. Sketch the Feynman diagram representing the interaction between two conduction electrons mediated by a phonon. Which other exchange bosons could do the job?

52. How can we qualitatively describe the attractive interaction between two conduction electrons via the exchange of virtual phonons? Which conditions have to be satisfied to make the mechanism work? Make an estimate of the interaction range. What would change if you go from a metal to a semiconductors?

53. Cooper pairs: Provide a qualitative argument why the attractive interaction between two conduction electrons is most efficient for electrons with opposite momentum. Which electrons can participate in the interaction process? How can we estimate the interaction energy? Which assumptions simplifications are usually made to keep the problem simple?

54. What is the order of magnitude of the attractive interaction energy? Which characteristic parameters determine the interaction energy?

55. Discuss the symmetry of the pair wave function. Which combinations of orbital and spin wave functions are possible?

56. Qualitatively discuss how we can construct a coherent many body state from bosons and fermions. What are the fundamental differences?

57. Which form has the BCS ground state?

58. What is the condensation energy density at $T = 0$? How it is related to the energy gap? Provide an intuitive argument for the relation.

59. What is the relation between the zero temperature energy gap and the critical temperature according to BCS theory?

60. Sketch the temperature dependence of the energy gap.

61. How can we measure the energy gap of a superconductor?

62. Discuss the energy dependence of the so-called coherence factors $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ close to the Fermi energy for $T = 0$. What is the meaning of the coherence factors?

63. Sketch the dispersion of the Bogoliubov quasiparticles. Elaborate on particle-hole mixing.

64. Discuss the density of states of the Bogoliubov quasiparticles in the superconducting state and compare it to the normal state density of states.

Flux Pinning and Critical Currents

65. What are the relevant materials parameters for power applications of superconductors?

66. Why can't we use type-I superconductors for superconducting wires and magnetic field coils?

67. How does the force acting on a flux line in the presence of a finite transport current density look like in a type-II superconductor?

68. How would the current-voltage characteristics of an ideal (no defects) type-II superconductor look like?

69. How can we pin magnetic flux lines in a type-II superconductor? What is the relation between the pinning force and the shape of the pinning potential? How should an optimum pinning center (maximum pinning force) look like?

High Temperature Superconductors