Chapter 6

6 Flux Pinning and Critical Currents

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- power applications require high $T_c$, $B_{c2}$, and $J_c$
6.1 Power Applications of Superconductivity

6.1.1 Examples

- energy transport and storage

fault current limiter

SMES (2 MJ)
superconducting magnetic energy storage
6.1 Power Applications of Superconductivity

Comparison of the amount of space consumed by a superconducting cable (blue) with copper wires carrying the same amount of current.
6.1 Power Applications of Superconductivity

- superconducting magnets
  - high energy physics
  - magnetic resonance imaging
  - fusion
6.1 Power Applications of Superconductivity

Figure 1 – Cross section of the 45-T Hybrid Magnet. The magnet cryostat is about 8 feet in diameter and the large-diameter part is about 9 feet tall.
AMS-02 is the Alpha Magnetic Spectrometer, a superconducting particle physics experiment which will be launched on the Space Shuttle and installed on the International Space Station. The project is an international collaboration of 56 research institutes from 16 countries.
6.1 Power Applications of Superconductivity

6.1.2 Materials Requirements

- high $T_c$, $B_{c2}$, $J_c$
- manufacturability
- low cost
- availability (sustainability)
6.1.2 Materials Requirements

**material parameters:**

- **important low $T_c$ superconductors**
  - Material: NbTi, Nb$_3$Sn
  - 1:1 alloy, intermetallic compound
  - $T_c$: 9.6 K, 18 K
  - $B_{c2}(T=0)$: 10.5 - 15 T, 23 - 29 T

- **high $T_c$ superconductors**
  - Material: BSCCO, YBCO
  - Powder in Ag-tube, thin film on metal tape
  - $T_c$: 110 K, 91 K
  - $B_{c2}(T=0)$: $\sim$ 1000 T, 800 T

(application in commercial magnets)
6.1.2 Materials Requirements
### 6.1.2 Materials Requirements

**material parameters:**

<table>
<thead>
<tr>
<th>Material</th>
<th>Transition Temp (K)</th>
<th>Critical Field (T)</th>
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<tbody>
<tr>
<td>NbTi</td>
<td>10</td>
<td>15</td>
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<td>PbMoS</td>
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<td>Nb₃Al</td>
<td>18.7</td>
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<td>Nb₃(AlGe)</td>
<td>20.7</td>
<td>44</td>
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<tr>
<td>Nb₃Ge</td>
<td>23.2</td>
<td>38</td>
</tr>
</tbody>
</table>

From Blatt, Modern Physics
6.1.2 Materials Requirements

Advancing Critical Currents in Nb-Ti

- MFTF Conductor
- Best Production High Homogeneity, 1985.
- Best UW-Madison HT Multi-Filamentary Composite, '85.
- Best Small Scale HT Multi-Filamentary Composite '86
- Revised Equivalent SSC Strand Specification
- Nb-Ti: Best Industrial Scale Heat Treated Composites 1990
- Aligned ribbons, field parallel to ribbons.
- Furukawa APC '94, d_c=10.5nm
- Supercon APC/HT '95
- Nb-Ti: Nb-Ti/Nb (21/6) 390 nm multilayer '95 (5°) McCambridge et al. (Yale)
- Nb-Ti: 390 nm multilayer Nb-Ti/Nb (21/6), 50 µV/cm - McCambridge et al. (Yale) (0°)
- Nb-Ti: Nb-Ti/Ti (19/5) 370 nm multilayer '95 (0°), 50 µV/cm, N. Rizzo et al. LTSC'96 (Yale)
6.1.2 Materials Requirements

critical current density of superconductors:

@ 4.2 K unless otherwise stated

- YBCO: Tape, || Tape-plane, SuperPower (Used in NHMFL tested Insert Coil 2007)
- YBCO: Tape, _ Tape Plane, SuperPower (Used in NHMFL tested Insert Coil 2007)
- Bi-2212: non-Ag $J_c$, 427 fil. round wire, Ag/SC=3 (Hasegawa ASC-2000/MT17-2001)
- Nb-Ti: Max @1.9 K for whole LHC NbTi strand production (CERN, Boutboul '07)
- Nb-Ti: Nb-47wt%Ti, 1.8 K, Lee, Naus and Larbaletier UW-ASC'96
- Nb$_3$Sn: Internal Sn OI-SL RRP 1.3 mm, ASC'02/ICMC'03
- Nb$_3$Sn: Bronze route int. stab. -VAC-HP, non-(Cu+Ta) $J_c$, Thoener et al., Erice '96.
- Nb$_3$Sn: 1.8 K Non-Cu $J_c$, Internal Sn OI-ST RRP 1.3 mm, ASC'02/ICMC'03
- Nb$_3$Al: RQHT+2 At.% Cu, 0.4m/s (Iijima et al. 2002)
- Bi 2223: Rolled 85 Fil. Tape (AmSC) B||, UW'6/96
- Bi 2223: Rolled 85 Fil. Tape (AmSC) B لبنان, UW'6/96
- MgB$_2$: 4.2 K "high oxygen" film 2, Eom et al. (UW) Nature 31 May '02
- MgB$_2$: Tape - Columbus (Grasso) MEM'06

- Nb$_3$Sn: 1.8 K Non-Cu $J_c$, Internal Sn OI
- Nb$_3$Sn: Bronze route int. stab. -VAC-HP
- Nb$_3$Al: RQHT+2 At.% Cu
- 2223 tape B||
- 2223 tape B||
- MgB$_2$: Rolled 85 Fil. Tape (AmSC) B||
- MgB$_2$: Rolled 85 Fil. Tape (AmSC) B||
- Nb$_3$Sn: Internal Sn OI
6.1.2 Materials Requirements

"engineering" critical current density of superconductors:

- YBCO B∥ Tape Plane
- YBCO B⊥ Tape Plane
- Nb-Ti
- RRP Nb₃Sn
- MgB₂
- Bronze Nb₃Sn

Graph showing applied field (T) vs. Jₑ (A/mm²) for various superconductors and filament compositions.
6.1.2 Materials Requirements

“engineering” critical current density of 100 m cable

- Nb-Ti: Example of Best Industrial Scale Heat Treated Composites ~1990 (compilation)
- Nb-Ti(Fe): 1.9 K, Full-scale multifilamentary billet for FNAL/LHC (OS-STG) ASC’98
- Nb-44wt.%Ti-15wt.%Ta: at 1.8 K, monofil. high field optimized, unpubl. Lee et al. (UW-ASC) ’96
- Nb-37Ti-22Ta: at 2.05 K, 210 fil. strand, 400 h total HT, Chernyi et al. (Kharkov), ASC2000
- Nb3Sn: Bronze route VAC 62000 filament, non-Cu 0.1µW·m 1.8 K Jc, VAC/NHMFL data courtesy M. Thoener.
- Nb3Sn: Non-Cu Jc Internal Sn OI-ST RRP #6555-A, 0.8m LTSW 2002
- Nb3Al: Nb stabilized 2-stage JR process (Hitachi,TML-NRIM,IMR-TU), Fukuda et al. ICMC/ICEC ’96
- Nb3Al: JAERI strand for ITER TF coil
- Bi-2212: non-Ag Jc, 427 fil. round wire, Ag/SC=3 (Hasegawa ASC2000+MT17-2001)
- Bi 2223: Rolled 85 Fil. Tape (AmSC) B||, UW'6/96
- Bi 2223: Rolled 85 Fil. Tape (AmSC) B\_\_, UW'6/96

At 4.2 K Unless Otherwise Stated

University of Wisconsin-Madison
Applied Superconductivity Center

December 2002 - Compiled by Peter J. Lee
6.1.3 Superconducting Wires and Tapes

fabrication of superconducting wires:
6.1.3 Superconducting Wires and Tapes

superconducting wires: NbTi, Nb$_3$Sn in Cu-matrix

Cable from high-$T_c$ superconductor

Multiple Traditional Copper Power Cables...

...replaced by one Power Equivalent HTS Cable
Figure 2 – Conductors for the three subcoils of the Superconducting Outsert Magnet (A, B, and C) were jacketed in special stainless-steel alloys at Gibson Tube. More than 6 km of conductor were used in these coils.
6.1.3 Superconducting Wires and Tapes

- Sumitomo
- AMSC's 344 Superconductors
- Stainless pipe
- Electrical insulation
- High temperature superconductor
- LN$_2$
6.1.3 Superconducting Wires and Tapes

*high-\( T_c \) wires:*

*can be made*

- Ag is too expensive and too soft
- HTS have higher critical fields

\[ \text{But it's 70\% silver!} \]
6.1.3 Superconducting Wires and Tapes

Preparation of multi-filamentary BiSrCaCu-oxid (2223) tapes in Ag/AgMg-sheath by the powder in tube method. Investigation of the structural and superconducting properties.

cross-section of 61 filamentary tape

600m tape on a coil for $J_c$-measurement
6.1.3 Superconducting Wires and Tapes

HTS cable
The "AmpaCity" project has been kicked off:
The RWE Group and its partners are just about to replace a 1-kilometre-long high-voltage cable connecting two transformer stations in the Ruhr city of Essen with a state-of-the-art superconductor solution. This will mark the **longest superconductor cable installation in the world**. As part of this project, the Karlsruhe Institute of Technology will analyse suitable superconducting and insulating materials.

The three-phase, concentric 10 kV cable will be produced by Nexans and is designed for a transmission capacity of 40 megawatts.
6.1.3 Superconducting Wires and Tapes

"Garching-technology":

- low cost process for large area deposition of HTS films
- high quality and reproducibility

http://www.theva.com/
6.1.3 Superconducting Wires and Tapes

coated conductor:

superconducting film on flexible steel tape
6.1.3 Superconducting Wires and Tapes

G. Hammerl et al., APL 81, 3209 (2002)
6.1.3 Superconducting Wires and Tapes

pulsed laser deposition
Substrate manipulators

Fiber for IR laser heating

Excimer laser optics

AFM/STM system

casing of RHEED
screen and camera

target manipulators

Pyrometer

Operator tool

Atomic oxygen source

Laser Molecular Beam Epitaxy, Garching 2006
L-MBE: Fully automated target holder

K.-W. Nielsen, S. Geprägs, Th. Brenninger
Fully automated control of beam profile

S. Geprägs,
M. Opel,
Th. Brenninger

computer controlled variation of beam profile and power density
6.1.4 Superconducting Bulk Material

bulk superconductors as “permanent magnets”
6.1.4 Superconducting Bulk Material
6.1.4 Superconducting Bulk Material
6.1.4 Superconducting Bulk Material

**Jap. Yamanashi MAGLEV-System**
(42.8 km long test track between Sakaigawa and Akiyama)

maximum velocity: 581 km/h (02. 12. 2003)
6.2 Critical Current of Superconductors

- increase of supercurrent density results in increase of velocity of superconducting electron

  critical current density: \textit{kinetic energy} = \textit{binding energy of Cooper pairs}

\textit{depairing critical current density}

- increase of supercurrent results in Lorentz force on flux lines in mixed state of type-II superconductors

  critical current density: \textit{Lorentz force} = \textit{pinning force}

\textit{depinning critical current density}
6.2.1 Depairing Critical Current Density

**Revision: Ginzburg-Landau Theory:**

- Minimization of free enthalpy of superconductor:
  - Integration of enthalpy density over whole volume of superconductor
  - Minimization by variation of $\Psi$ and $A$

**Ginzburg-Landau Equations:**

1. GL-equation

\[
\mathbf{J}_s = q_s n_s \left\{ \frac{\hbar}{m_s} \nabla \theta (\mathbf{r}) - \frac{q_s}{m_s} \mathbf{A} (\mathbf{r}) \right\}
\]

2. GL-equation

\[
\frac{\hbar^2}{2m_s \alpha} \left( \frac{1}{i} \nabla - \frac{q_s}{\hbar} \mathbf{A} \right)^2 \psi + \psi - |\psi|^2 \psi = 0
\]

2nd characteristic length scale

GL coherence length

\[
\xi_{GL} = \sqrt{\frac{\hbar^2}{2m_s \alpha}}
\]
6.2.1 Depairing Critical Current Density

• consider a thin wire with diameter \( d \ll \xi_{GL} \)

\[ \rightarrow \text{no amplitude variation of order parameter } \Psi \text{ across wire} \]

• superconducting material is assumed homogeneous

• same current density along the wire

\[ \rightarrow \text{no amplitude variation of order parameter } \Psi \text{ along the wire} \]

\[ \Psi(r) = |\psi| e^{i\theta(r)} \]

• we use 1. and 2. GL equation:

\[
\begin{align*}
J_s &= q_s n_s \left\{ \frac{\hbar}{m_s} \nabla \theta(r) - \frac{q_s}{m_s} A(r) \right\} \\
\rightarrow J_s &= \frac{q_s}{m_s} |\psi|^2 \left( \frac{\hbar}{m_s} \nabla \theta - q_s A \right) = q_s |\psi|^2 v_s
\end{align*}
\]

\[
-\xi_{GL}^2 \left( \frac{1}{\hbar} \nabla - \frac{q_s}{\hbar} A \right)^2 \psi + \psi - |\psi|^2 \psi = 0
\]

\[
|\psi|^2 = \left| \frac{\psi}{\Psi_0} \right|^2
\]

\[
-\xi_{GL}^2 m_s^2 \frac{\hbar^2}{\hbar^2} \left( \frac{\hbar}{m_s} \nabla \theta - \frac{q_s}{m_s} A \right)^2 |\psi| + |\psi| - |\psi|^3 = 0
\]
6.2.1 Depairing Critical Current Density

- we obtain:

\[
|\psi|^2 = \left| \frac{\psi}{\psi_0} \right|^2 = \left( 1 - \frac{m_s^2 \xi_{GL}^2 v_s^2}{\hbar^2} \right) = \left( 1 - \frac{\frac{1}{2} m_s v_s^2}{|\alpha|} \right)
\]

condensation energy per Cooper pair

⇒ reduction of \(|\psi|^2\) is just proportional to ratio of kinetic and condensation energy

⇒ order parameter decreases due to additional kinetic energy of pairs

- expression for current density:

\[
J_s = q_s |\psi|^2 v_s = q_s |\psi_0|^2 \left( 1 - \frac{m_s^2 \xi_{GL}^2 v_s^2}{\hbar^2} \right) v_s
\]

- determine maximum of \(J_s\) by setting \(\partial J_s / \partial v_s = 0\):

\[
J_c = \frac{2}{3 \sqrt{3}} \frac{\hbar q_s}{m_s \xi_{GL}} q_s |\psi_0|^2 = \frac{\Phi_0}{3 \sqrt{3} \pi \mu_0 \lambda^2_L(T) \xi_{GL}(T)}
\]

\[
\Phi_0 = \hbar / q_s \quad \lambda^2_L(T) = m_s / \mu_0 |\psi_0|^2 q_s^2
\]
6.2.1 Depairing Critical Current Density

- $T$-dependence of $J_c$ determined by $T$-dependence of $\lambda_L$ and $\xi_{GL}$:

$$\lambda_L(T) = \frac{\lambda_{GL}(0)}{\sqrt{1 - \frac{T}{T_c}}}$$

$$\xi_{GL}(T) = \frac{\xi_{GL}(0)}{\sqrt{1 - \frac{T}{T_c}}}$$

close to $T_c$:

$$J_c \propto (1 - T/T_c)^{3/2}$$

- we can use

$$B_{cth}(T) = \frac{\Phi_0}{2\sqrt{2\pi} \xi_{GL}(T) \lambda_L(T)}$$

$$J_c = \frac{2\sqrt{2}}{3\sqrt{3}} \frac{B_{cth}}{\mu_0 \lambda_L} = 0.544 \frac{H_{cth}}{\lambda_L}$$

- note: according London theory we would expect $J_c = H_{cth}/\lambda_L$

(London theory does not take into account reduction of OP with increasing $J_s$)

**Pb:**

$B_{cth} \approx 80 \text{ mT, } \lambda_L \approx 40 \text{ nm}$

$\Rightarrow J_c \approx 8 \times 10^{11} \text{ A/m}^2$

**Nb:**

$B_{cth} \approx 200 \text{ mT, } \lambda_L \approx 40 \text{ nm}$

$\Rightarrow J_c \approx 2 \times 10^{12} \text{ A/m}^2$
6.2.1 Depairing Critical Current Density

- Gedanken experiment: what is the critical current of a Pb rod with large diameter $d$?

$$d = 1\text{cm}$$

Critical current $I_c = J_c \cdot A = J_c \cdot \pi (d/2)^2 \approx 6 \times 10^7 \text{ A}$

London theory:

$$B_z(x) = B_{\text{ext},z} \exp\left(-\frac{x}{\lambda_L}\right) \quad J_{s,y}(x) = J_{s,0} \exp\left(-\frac{x}{\lambda_L}\right)$$
6.2.1 Depairing Critical Current Density

- supercurrent flow only within surface layer of thickness $\lambda_L$

$\lambda_L \approx 40 \text{ nm}$

$d = 1 \text{ cm}$

critical current:

$$I_c = J_c \cdot A = J_c \cdot \pi d \lambda_L \approx 1 \times 10^3 \text{ A}$$

technical critical current density:

$$J_c = \frac{I_c}{\pi (d/2)^2} \approx 10 \text{ A/mm}^2$$

comparable to Cu-wire

- solution:
  - use multifilament wire with $d < \lambda_L$
    $\Rightarrow$ difficult to fabricate
  - use type-II superconductor
6.2.2 Depinning Critical Current Density

- **type-II superconductors:**
  - partial field penetration above $B_{c1}$
  - $B_i > 0$ for $B_{ext} > B_{c1}$
  - Shubnikov phase between $B_{c1} \leq B_{ext} \leq B_{c2}$
  - upper and lower critical fields $B_{c1}$ and $B_{c2}$

\[ \kappa < \frac{1}{\sqrt{2}} \quad \text{Typ – I Supraleiter} \]
\[ \kappa > \frac{1}{\sqrt{2}} \quad \text{Typ – II Supraleiter} \]

- in mixed state field penetrates the superconductor

- current flow is not restricted to thin surface layer

- high values of $B_{c2}$
6.2.2 Depinning Critical Current Density

- extreme type-II superconductors ($\kappa \gg 1$) have very high $B_{c2} \rightarrow$ high field operation

<table>
<thead>
<tr>
<th>Element</th>
<th>Al</th>
<th>In</th>
<th>Nb</th>
<th>Pb</th>
<th>Sn</th>
<th>Ta</th>
<th>Tl</th>
<th>V</th>
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<tbody>
<tr>
<td>$B_{cth}$ [mT]</td>
<td>10.49</td>
<td>28.15</td>
<td>206</td>
<td>80.34</td>
<td>30.55</td>
<td>82.9</td>
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<td>$\lambda_L(0)$ [nm]</td>
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<td>65</td>
<td>32-45</td>
<td>40</td>
<td>50</td>
<td>35</td>
<td>40</td>
<td></td>
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<tr>
<td>$\kappa_\infty$</td>
<td>0.03</td>
<td>0.06</td>
<td>$\sim$ 0.8</td>
<td>0.4</td>
<td>0.1</td>
<td>0.35</td>
<td>0.3</td>
<td>0.85</td>
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</table>

<table>
<thead>
<tr>
<th>Verbindung</th>
<th>NbTi</th>
<th>Nb$_3$Sn</th>
<th>NbN</th>
<th>PbIn (2-30%)</th>
<th>PbIn (2-50%)</th>
<th>Nb$_3$Ge</th>
<th>V$_3$Si</th>
<th>YBa$_2$Cu$_3$O$_7$ (ab-Ebene)</th>
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<tbody>
<tr>
<td>$T_c$ [K]</td>
<td>$\approx$ 10</td>
<td>$\approx$ 18</td>
<td>$\approx$ 16</td>
<td>$\approx$ 7</td>
<td>$\approx$ 8.3</td>
<td>23</td>
<td>16</td>
<td>92</td>
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<tr>
<td>$B_{c2}$ [T]</td>
<td>$\approx$ 10.5</td>
<td>$\approx$ 23-29</td>
<td>$\approx$ 15</td>
<td>$\approx$ 0.1-0.4</td>
<td>$\approx$ 0.1-0.2</td>
<td>38</td>
<td>20</td>
<td>160±25</td>
</tr>
<tr>
<td>$\lambda_L(0)$ [nm]</td>
<td>$\approx$ 300</td>
<td>$\approx$ 80</td>
<td>$\approx$ 200</td>
<td>$\approx$ 150</td>
<td>$\approx$ 200</td>
<td>90</td>
<td>60</td>
<td>$\approx$ 140 ± 10</td>
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<tr>
<td>$\kappa_\infty$</td>
<td>$\approx$ 75</td>
<td>$\approx$ 20-25</td>
<td>$\approx$ 40</td>
<td>$\approx$ 5-15</td>
<td>$\approx$ 8-16</td>
<td>30</td>
<td>20</td>
<td>$\approx$ 100 ± 20</td>
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6.2.2 Depinning Critical Current Density

- how does the transport current interact with the vortex lattice?
6.2.2 Depinning Critical Current Density

- interaction of a transport current with the vortex lattice:

- force on single pair:
  \[ \mathbf{F}_L = q_s \mathbf{v}_s \times \mathbf{B} \]

  \[ \mathbf{f}_L = \rho_s \mathbf{v}_s \times \mathbf{B} = \mathbf{J} \times \mathbf{B} \]

  - force density, charge density, total current density

  \[ \mathbf{f}_L \] results in force on charge carriers, which cannot leave conductor

\[ \textit{flux motion perpendicular to applied transport current} \]
6.2.2 Depinning Critical Current Density

- **origin of force acting on a flux line** (plausibility check)

- **Benoulli’s principle:**
  an increase in the speed of the fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy

- analogy to Magnus force

- without transport current: zero net force
- contributions of circulating current cancel

\[ \text{core of flux line} \]

\[ \text{small total superfluid velocity} \rightarrow \text{small total } F_L \]

\[ \text{large total superfluid velocity} \rightarrow \text{large total } F_L \]

\[ \text{net force} \]
6.2.2 Depinning Critical Current Density

- calculation of the total Lorentz force on single flux line:

\[ \mathbf{F}_L = \int f \, dV = L \int_A \mathbf{f} \, dA \]

\[ \mathbf{F}_L = L \int_A \mathbf{J}_{tr} \times \mathbf{B} \, dA \text{ homogeneous} \]

\[ \mathbf{F}_L = L \mathbf{J}_{tr} \times \oint_A \mathbf{B} \, dA \]

\[ \mathbf{F}_L = L \mathbf{J}_{tr} \times \Phi_0 \]

- force per unit length of flux line:

\[ \frac{\mathbf{F}_L}{L} = \mathbf{J}_{tr} \times \Phi_0 \]

- \( \mathbf{F}_L \) causes motion of the flux line
  - what is the velocity \( \nu_L \) of the flux line (depends on damping)
  - what is the work done by the Lorentz force
6.2.2 Depinning Critical Current Density

- **Faraday's law of induction:**
  The induced electromotive force (EMF) in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit.

- **Electromotive force** (EMF):

  
  \[
  \text{EMF} = \frac{1}{e} \oint_{\partial A} \mathbf{F} \cdot d\ell = -\frac{d}{dt} \int_{A} \mathbf{B} \cdot d\mathbf{A}
  \]

  
  
  \[
  = \frac{1}{e} \oint_{\partial A} (e\mathbf{E} + e\mathbf{v}_L \times \mathbf{B}) \cdot d\ell
  \]

  
  
  \[
  \text{EMF} = 0 \quad \text{(no flux change)}
  \]

  
  \[
  \mathbf{E} = -\mathbf{v}_L \times \mathbf{B} = \mathbf{B} \times \mathbf{v}_L
  \]

  
  - **Dissipation by moving flux line:**
    - motion with velocity \( \mathbf{v}_L \) induces electric field: \( \mathbf{E} = \mathbf{B} \times \mathbf{v}_L \)
    - \( \mathbf{E} \) is parallel to \( J_{\text{tr}} \) \( \rightarrow \) acts like resistive voltage
6.2.2 Depinning Critical Current Density

*other point of view: power balance*

- generated power for
  - single flux line: \( P_1 = F_L \cdot v_L \)
  - \( N \) flux lines: \( P_N = N \cdot F_L \cdot v_L \)

- power balance:

\[
P_N = N \cdot F_L \cdot v_L = UI_{tr}
\]

\( F_L = L \cdot J_{tr} \times \Phi_0 \)

\[
E = \frac{U}{\ell} = \frac{N \cdot F_L \cdot v_L}{I_{tr} \cdot \ell} \cdot J_{tr} = \frac{N \cdot L \cdot (J_{tr} \times \Phi_0) \cdot v_L}{J_{tr} \cdot bL \cdot \ell} \cdot J_{tr} = \frac{N \cdot (J_{tr} \times \Phi_0) \cdot v_L}{b \cdot \ell} \cdot J_{tr} = [((J_{tr} \times B) \cdot v_L) \cdot J_{tr}
\]

\( \ell \) sample length
\( b \) sample width
\( bL \) cross-sectional area

\( \Rightarrow \) superconductor shows resistance: *flux-flow resistance*
6.2.2 Depinning Critical Current Density

- phase change due to flux motion:

- assumption: single flux line in sample

- phase difference between sample ends:

\[ \delta \theta_1 = \int_{\text{path}1} \nabla \theta \, dr \]

- integration path 2:

\[ \delta \theta_2 = \int_{\text{path}2} \nabla \theta \, dr = \int_{\text{path}2} - \int_{\text{path}1} + \int_{\text{path}1} = \oint \nabla \theta \, dr + \delta \theta_1 = 2\pi + \delta \theta_1 \]

- note: only the phase factor \( e^{i\theta} \) must be unambiguous
### 6.2.2 Depinning Critical Current Density

**moving flux line:**

- crossing of single flux line changes phase difference by \( \varphi = \theta_2 - \theta_1 = 2\pi \)

- required time for crossing: \( \delta t = \frac{b}{v_L} \)

\[
\frac{\partial \varphi}{\partial t} = N \frac{2\pi}{\delta t} = N \frac{2\pi}{b/v_L} = \frac{\Phi}{\Phi_0} \frac{2\pi}{b} v_L = \frac{B}{\Phi_0} \frac{b\ell}{b} \frac{2\pi}{b} v_L = B v_L \ell \frac{2\pi}{\Phi_0}
\]

*temporal change of phase difference*
6.2.2 Depinning Critical Current Density

• relation between change of phase difference and electric field:

\[ \frac{\partial \varphi}{\partial t} = B \, v_L \, l \, \frac{2\pi}{\Phi_0} = B \, v_L \, l \, \frac{2e}{\hbar} \]

- we make use of \( E = B \times v_L \) \( \rightarrow \) \( E = B v_L \)

\[ \hbar \frac{\partial \varphi}{\partial t} = 2e \, B \, v_L \, l = 2e \, E \, l = 2e \, V \]

"2. Josephson Equation"
6.2.2 Depinning Critical Current Density

- **power dissipation during vortex motion:**

(a) **pair breaking and recombination:**

- in front of flux-line: $|\Psi|$ decreases
  
  $\Rightarrow$ pairs have to break up

- pair breaking due to absorption of phonons:

- behind the flux-line: $|\Psi|$ increases
  
  $\Rightarrow$ recombination of pairs by phonon emission:

- finite phonon lifetime delays thermal equilibrium:

  *irreversible process $\Rightarrow$ friction, viscous flow*

- electric energy is transferred to heat
6.2.2 Depinning Critical Current Density

- power dissipation during vortex motion:

(b) eddy current losses:

\[-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} = \frac{\partial B}{\partial x} v_x\]

core of the flux line is considered as normal conductor

⇒ eddy current with ohmic losses

- both mechanisms: \( E \propto v_L \) ⇒ friction force
- balance between Lorentz and friction force: \( v_L \) becomes stationary

\[ v_L \propto J_{tr} \quad v_L \propto E \]

- we expect: \( J_{tr} \propto E \) Ohm’s law ⇒ zero critical current density
6.3 Flux Line Pinning

- experimental result:
  - $I_c$ depends on defect density
  - inhomogeneities pin flux-lines: \textit{flux pinning}
  - $\nu_L = 0$ is caused by flux pinning
    \[ \Rightarrow \text{no work, no dissipation} \]
    \[ \Rightarrow \text{no voltage drop} \]
6.3 Flux Line Pinning

- **pinning mechanism:**

  precipitate with small $\Delta$ or normal conducting

  precipitate:
  - vortex core causes no additional loss in condensation energy
  - motion of vortex core costs energy (condensation energy)
  - effective binding forced at precipitate $\Rightarrow$ "pinning force"
  - most effective, if defect size $\approx \xi$
6.3 Flux Line Pinning

- **Pinning force:** \[ F_p = -\frac{\partial V_p}{\partial r} \approx -\frac{E_{\text{con}}}{r_p} \rightarrow \text{reduce } r_p \text{ to increase } F_p \]
  - optimum: \( r_p \approx \xi \) (vortex size \( \approx \) pin size)

- For high-temperature superconductors:
  - \( \xi \approx 1 \text{ nm}, E_{\text{con}} \text{ large} \)
  - very small pinning sites required
  - large pinning force

- **Problem** for HTS: thermally activated escape of flux line (thermally activated flux flow: TAFF)
  - \[ E_{\text{con}}\xi^3 \approx k_B T \] (\( \xi \) small and \( T \) large)
6.3 Flux Line Pinning

• **so far:** pinning of a single flux line
• **now:** pinning of a flux line lattice

→ complicated problem:
- flux line lattice is elastic object
  
  *(stiffness of the lattice, flux lines can bend)*
- pinning potential is disordered

• **example:** net pinning force of completely stiff flux line lattice by statistical pinning potential vanishes

→ flux line lattice has to deform to adopt to pinning potential
→ even if a single flux line does not sit in a potential well, it is pinned by the interaction with the other flux lines (rigidity of the lattice)

→ **collective pinning theory**

see e.g.  
*Vortices in high-temperature superconductors*
G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur
Rev. Mod. Phys. 66, 1125 (1994)
6.3 Flux Line Pinning

- *surface pinning:*

- important for superconducting thin films
  - relative length difference of flux line at different positions may be large
6.3 Flux Line Pinning

• $J_c$ as a function of temperature and applied magnetic field:

- decrease of $J_c$ with increasing $B$: several flux lines per pinning site
- decrease of $J_c$ with increasing $T$: thermally activated flux motion, reduced condensation energy
6.4 Magnetization of Hard Superconductors

- **Flux line pinning in an external field:**

  - $B_{c1}$ and $B_{c2}$ stay the same
  - Pinning prevents flux motion, i.e. penetration and exit of flux lines
  - $B_i$ is inhomogeneous within the sample

**Diagram:**
- Magnetization curve
- Hysteresis loop
- Remanent flux
- Virgin curve
- Ideal SC
6.4 Magnetization of Hard Superconductors

- **field distribution in sample:**

![Graph showing field distribution and magnetization curves](image)

- sample surface: jump ↔ ideal magnetization curve
- within superconductor: field gradient ↔ gradient of flux density
- flux lines repel each other: motion if repulsion > pinning force
- gradient decreases with increasing magnetic field
6.4 Magnetization of Hard Superconductors

- *field distribution in sample (demagnetization):*

- gradient changes sign
- $B_a < -B_{c1}$: flux lines with opposite direction penetrate

*recombination with frozen-in flux lines inside the superconductor*
6.4 Magnetization of Hard Superconductors

- **Bean model:**
  - flux gradient ↔ shielding current
  - macroscopic average:
    \[ \nabla \times B_i = \mu_0 J_{\text{scr}} \]
    \[ \text{Ampère's law} \]
  - here:
    \[ \frac{\partial B_{i,z}}{\partial x} = \mu_0 J_{\text{scr,y}} \]
  - for small \( \partial B_{i,z}/\partial x \):
    \[ J < J_c \Rightarrow \text{flux lines are pinned} \]
  - for large \( \partial B_{i,z}/\partial x \):
    \[ J > J_c \Rightarrow \text{flux lines move} \]
  - motion until
    \[ J = J_c \]
    "critical state"
  - note:
    measurement of \( \partial B_{i,z}(x)/\partial x \): \( \Rightarrow J_c \)
    \( J_c \) decreases with increasing \( B_i \) \( \Rightarrow \) smaller slope
6.4 Magnetization of Hard Superconductors

- magneto-optical imaging of flux distribution