Emanuele Polino, Mauro Valeri, Nicolò Spagnolo, and Fabio Sciarrino in
AVS Quantum Sci. 2, 024703 (2020). Published: 29 June 2020

Photonic Quantum Metrology

Presentation by Alexander Orlov,
Seminar on Advances in Solid State Physics, SS 2021,
18 May 2021

Metrology - a basic pillar of science

Metrology = Science of Measurement

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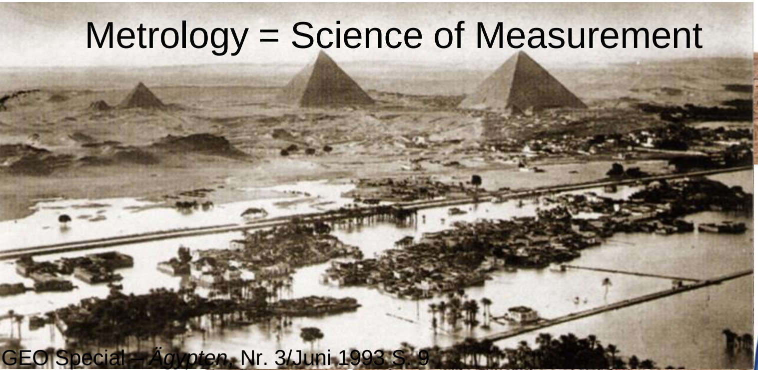
Meterology = branch of atmospheric
science, weather forecasting, …

Metrology - a basic pillar of science

Metrology = Science of Measurement



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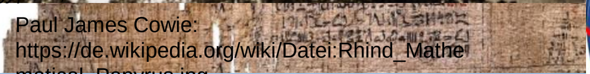

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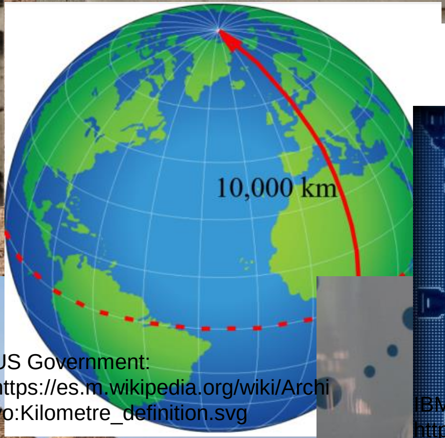

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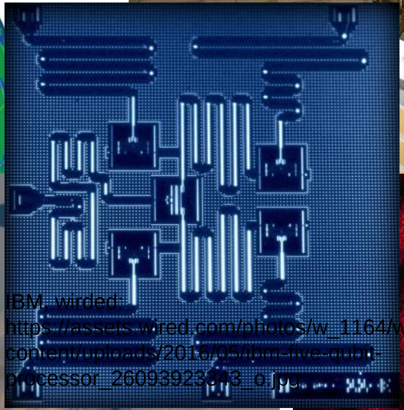

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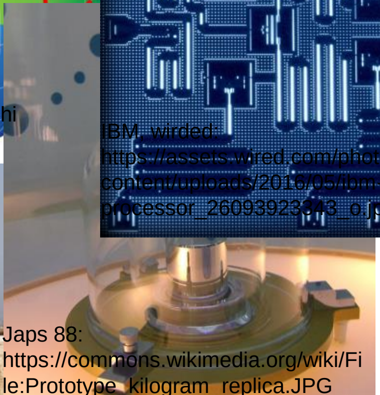

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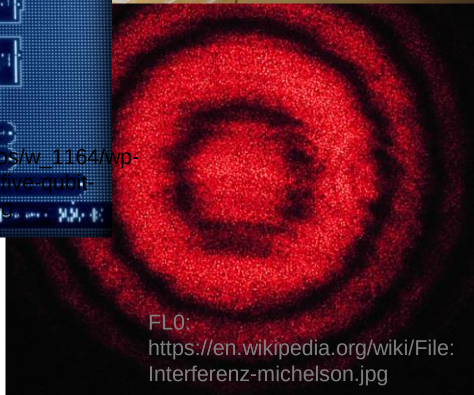

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Photonic Quantum Metrology

Estimation of unknown parameters

exploiting quantum resources

using photonic systems

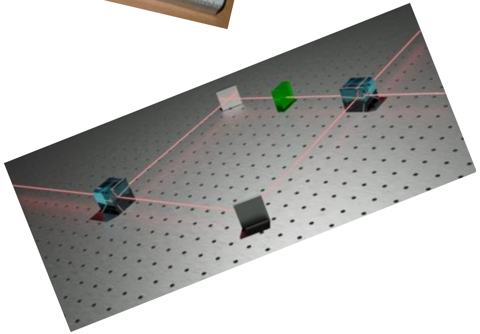
Agenda

1. **Metrology**
   a. The Measurement Process
   b. Limits on Accuracy
2. **Quantum Metrology** – Main Quantum Ressources
   a. Entanglement
   b. Squeezing
3. **Photonic Quantum Metrology**
   a. Photonic Degrees of Freedom
   b. Representation of Photonic States: Fock States, Quadratures and Wigner Functions
   c. Coherent States
   d. N00N States
   e. Squeezed States

Photonic Quantum Metrology

The Measurement Process

3 *(4)* Steps of a Measurement Process:

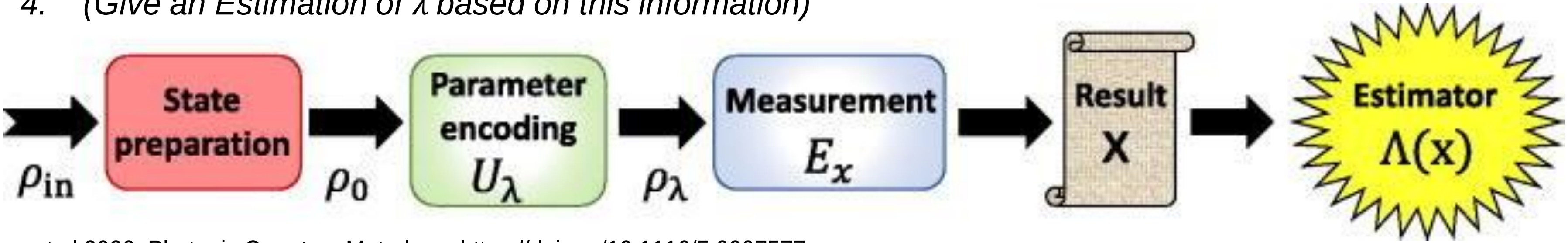


1. Prepare the probe state ρ_0 so that it is sensitive to variations of the unknown parameter λ

2. Let the probe interact with the system (through a unitary evolution U_λ)
→ The information about λ is now encoded on the probe state
$\rho_\lambda = U_\lambda \, \rho_0 \, U_\lambda^\dagger$

3. Extract the information about λ from the probe state ρ_λ

4. *(Give an Estimation of λ based on this information)*



Polino et al 2020: Photonic Quantum Metrology. https://doi.org/10.1116/5.0007577

Limits on Measurement Accuracy

Standard Quantum Limit

- Statistical (and systematic) errors

- Statistical errors can be accidental or fundamental (e.g. Shot Noise)

- Central Limit Theorem, Fisher Information: Lower Bound of the error scaling with $\frac{1}{\sqrt{N}}$

$$\Delta\lambda_{min} \geq \frac{1}{\sqrt{N}}$$

$N = 100 \Rightarrow \Delta\lambda_{min} = 0.1$

Heisenberg Limit

- Fundamental limit (originates from the Heisenberg Uncertainty Principle)

- Precision enhancement by exploiting quantum ressources

- Quantum Fisher Information: Lower Bound of the error scaling with $\frac{1}{N}$

$$\Delta\lambda_{min} \geq \frac{1}{N}$$

$N = 100 \Rightarrow \Delta\lambda_{min} = 0.01$

Photonic Quantum Metrology

Quantum Resources in the Measurement Process

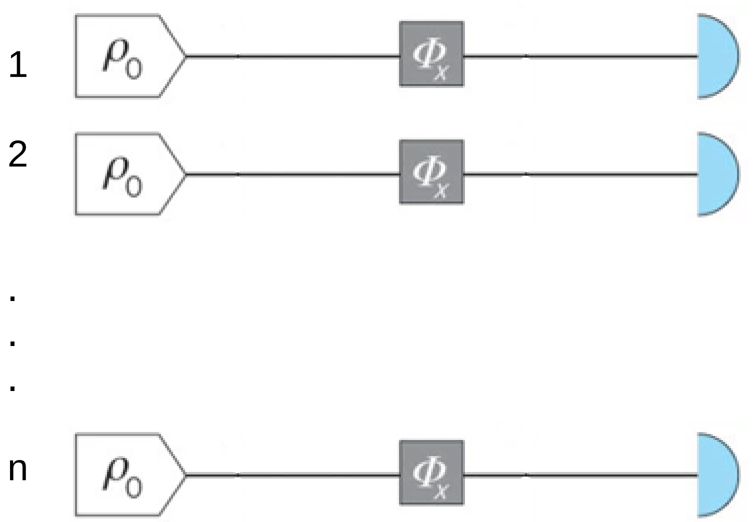
3 main steps of a Measurement Process:
(Estimate unknown parameter x)

1. Preparation of the probe state ρ_0

2. Interaction of the probe with the system

$$\rho_x = \Phi_x \, \rho_0 \, \Phi_x^\dagger$$

3. Extract the information about x from the probe state ρ_x



Giovanetti et al 2011: Advances in Quantum Metrology. https://doi.org/10.1038/nphoton.2011.35

Quantum Resources in the Measurement Process

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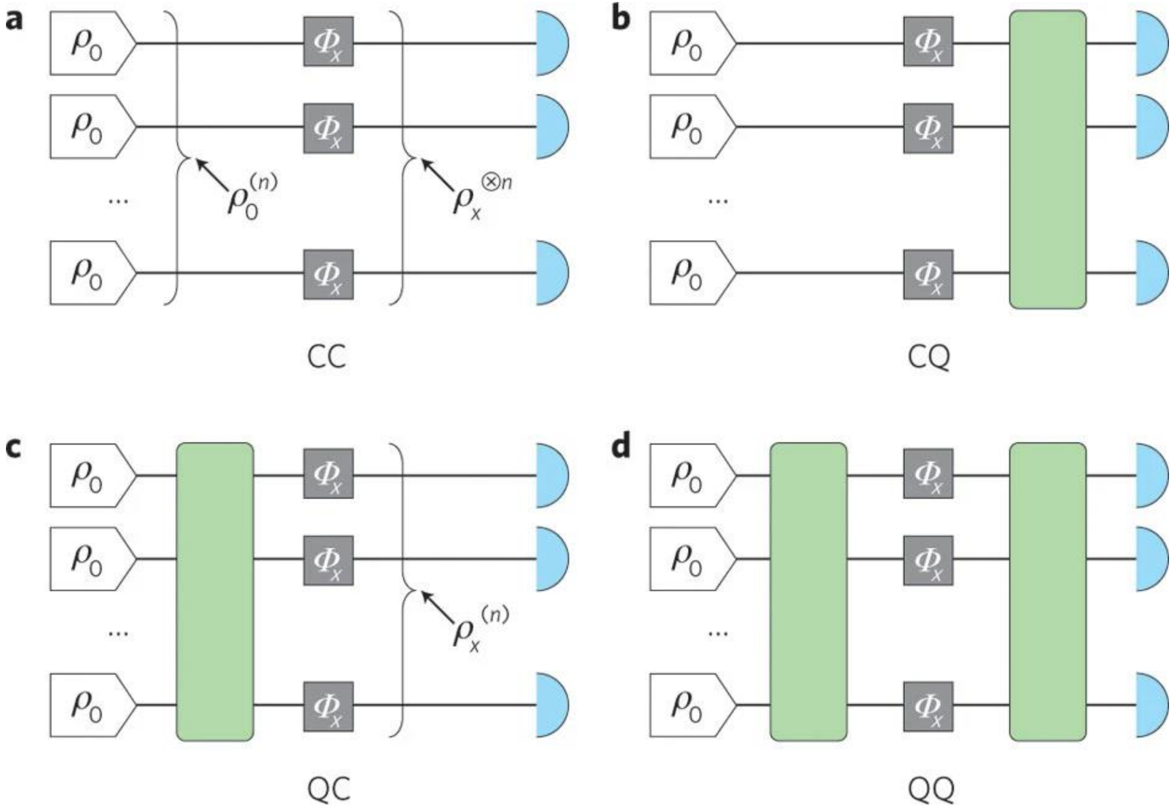

Standard Quantum Limit

$$\Delta\lambda_{min} \geq \frac{1}{\sqrt{N}}$$

Heisenberg Limit

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Quantum Resources in the Measurement Process

Entanglement

Bell States:

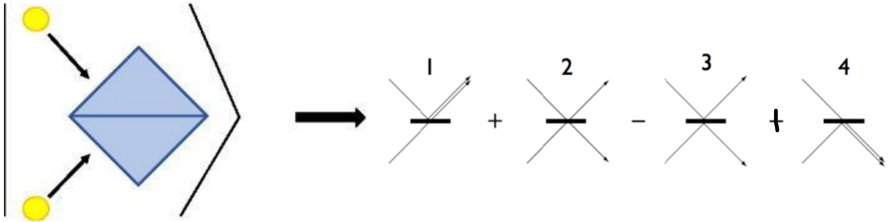
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)$$

Hong-Ou-Mandel Effect:



Polino et al 2020: Photonic Quantum Metrology. https://doi.org/10.1116/5.0007577

Pieter Kok: https://commons.wikimedia.org/wiki/File:Hong_Ou_Mandel_effect.png

Quantum Resources in the Measurement Process

Entanglement

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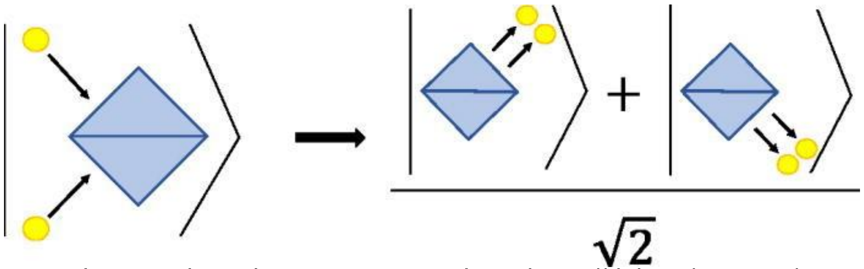
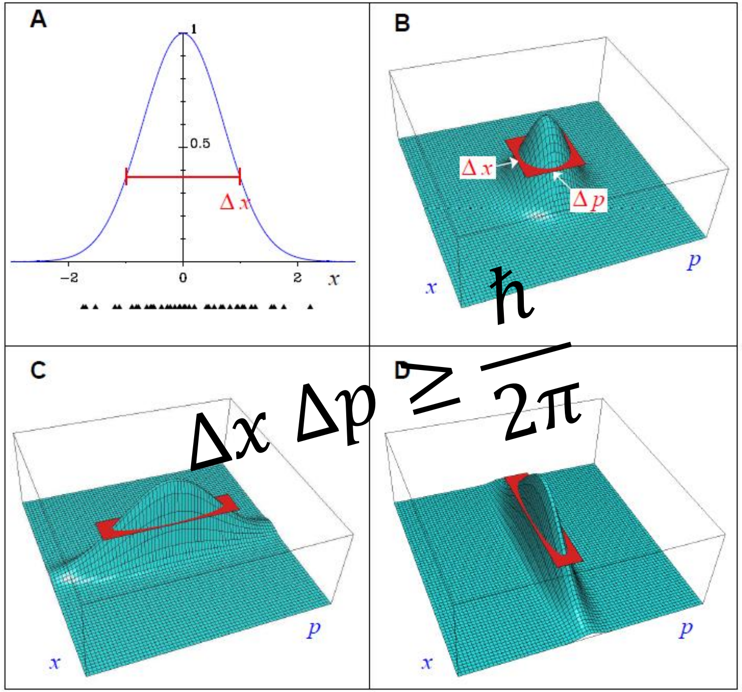


Polino et al 2020: Photonic Quantum Metrology. https://doi.org/10.1116/5.0007577

Squeezing the Uncertainty



$$\Delta x\, \Delta p \geq \frac{\hbar}{2\pi}$$

Giovanetti et al 2004: Quantum Enhanced Measurements. https://doi.org/10.1126/science.1104149

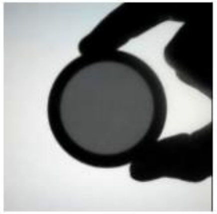
Photonic Quantum Metrology

Photonic Degrees of Freedom

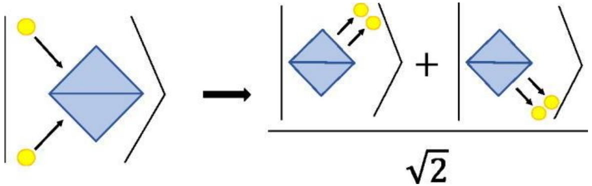
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)$$
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)$$

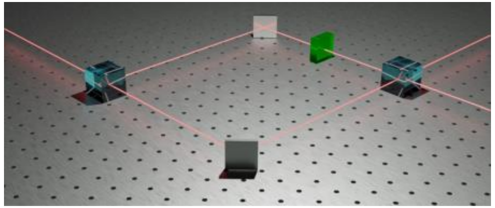
Which Photonic Degrees of Freedom can be used to encode quantum information?

1) Polarization („Spin Angular Momentum“)



2) Path Encoding





3) … Orbital Angular Momentum, time-bin, time-frequency, …

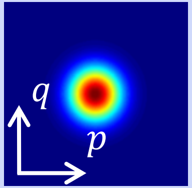
Polino et al 2020: Photonic Quantum Metrology. https://doi.org/10.1116/5.0007577

Basics & Representation of Photonic States

Hamiltonian of the quantized electromagnetic field:
(≙ quantum harmonic oscillator)

$$H_{em} = \sum_K \hbar\omega_{\vec{k}} \, (\underbrace{a_K^\dagger a_K}_{n_K} + \frac{1}{2})$$

Photonic States:

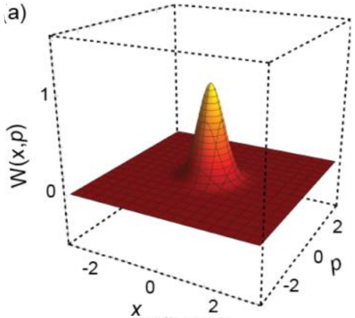
Fock States $\mid N_K \rangle$	**Continous Variable States**
- Eigenstates of the Hamiltonian $n_K \mid N_K \rangle = a_K^\dagger a_K \mid N_K \rangle = N_K \mid N_K \rangle$ - „Number states“	- Use Quadratures to define a quantum phase space: $E_{x_0}(t) = A\sin(\omega t + \phi_0) = P\sin(\omega t) + Q\cos(\omega t)$ → Quadratures $\hat{P}$, $\hat{Q}$: pair of conjugate variables analoguous to $\hat{x}, \hat{p}$ $[\hat{P}, \hat{Q}] = i \quad \Leftrightarrow \quad (\Delta P)(\Delta Q) \geq \frac{1}{4}$ - A state is described by a quasi-probability distribution in the phase space $W_\rho(P, Q)$  - Use x, p as quadratures: $x = \frac{a + a^\dagger}{\sqrt{2}} \qquad p = \frac{a - a^\dagger}{\sqrt{2}i} \Rightarrow \mathrm{H_{em}} = \sum_K \frac{\hbar\omega_{\vec{k}}}{4}(x_K^2 + p_K^2)$

Continous Variable States

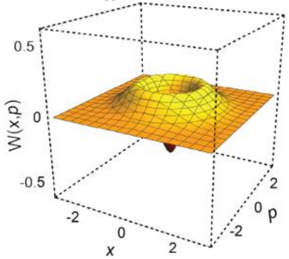
A state is described as a quasi-probability distribution in the phase space $W_\rho(P,Q)$ ("Wigner Function")

$$W(Q,P) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left\langle Q - \frac{\zeta}{2}\middle|\hat{\rho}\middle|Q + \frac{\zeta}{2}\right\rangle e^{ip\zeta} d\zeta$$

<u>Fock States</u>



Vacuum State $|0\rangle$

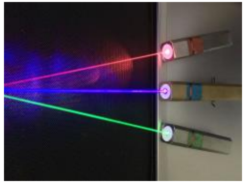


Fock State $|1\rangle$

<u>Coherent States</u>

- "Laser"
- Well defined frequency and phase
- mathematically: Eigenstates of the Annhilition Operator $a|\alpha\rangle = \alpha|\alpha\rangle$
- Subclass of the important class of Gaussian states (2 parameters) → *Minimal Uncertainty* $\Delta x\ \Delta p = \frac{\hbar}{2\pi}$
- "Displaced vacuum"
- Standard Quantum Limit

$$\Delta\lambda_{min} \geq \frac{1}{\sqrt{N}}$$



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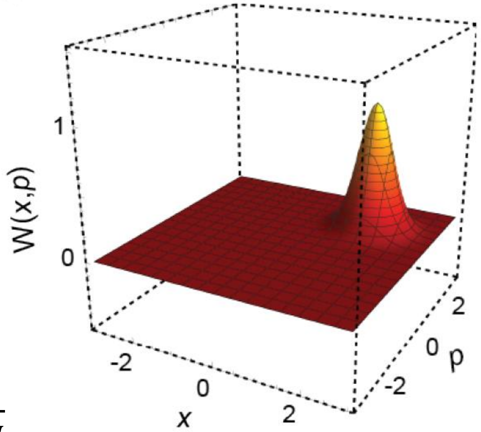


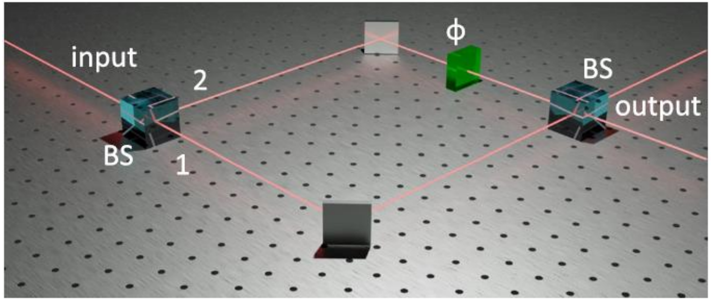
Goetz 2017: The Interplay of Superconducting Quantum Circuits and Propagating Microwave States

N00N States

<u>Definition</u>: "maximally entangled multipartite state distributed along two modes"

$$|\Psi\rangle_{N00N} = \frac{|N,0\rangle + e^{i\gamma}|0,N\rangle}{\sqrt{2}}$$

<u>Usage</u>:



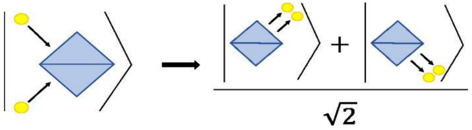
Evolution of an N00N-state in an ideal interferometer: $\frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}} \xrightarrow{U_\Phi} \frac{|N,0\rangle + e^{iN\Phi}|0,N\rangle}{\sqrt{2}}$

→ The phase shift is amplified by the number of photons N

Sensitivity: $\Delta\Phi_{N00N} \geq \frac{1}{N}$ → Heisenberg Limit

<u>Generation</u> of N00N states:

- N = 2: Hong-Ou-Mandel Effect → $|\Psi\rangle_{HOM} = \frac{i}{\sqrt{2}}(|2{,}0\rangle + |0{,}2\rangle)$

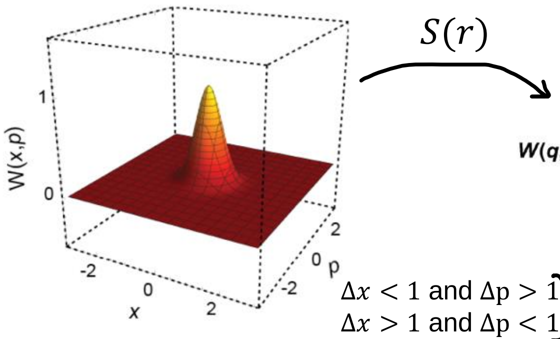


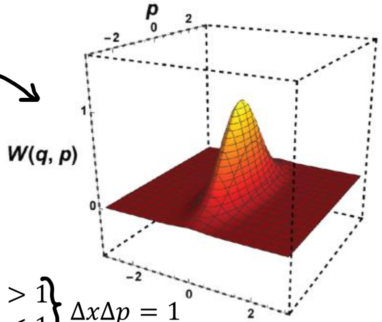
- N > 2… difficult

Polino et al 2020: Photonic Quantum Metrology. https://doi.org/10.1116/5.0007577

Squeezed States

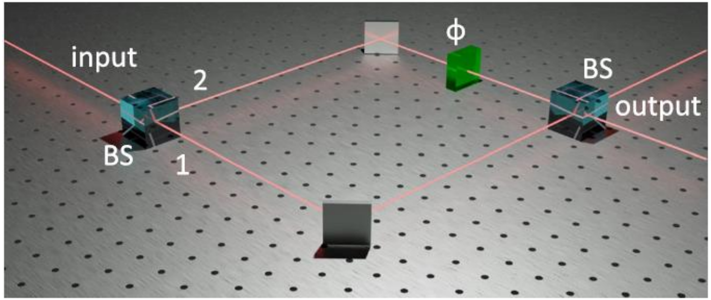
Idea: Squeeze the uncertainties of a Gaussian state





$\Delta x < 1$ and $\Delta p > 1$
$\Delta x > 1$ and $\Delta p < 1$
} $\Delta x \Delta p = 1$

Usage:



$$\Delta\Phi_{squeezed} \geq \frac{1}{2\sqrt{2}} \frac{1}{\sqrt{\langle n\rangle^2 + \langle n\rangle}}$$

Mathematically:
Squeezing Single Mode Operator: $S(r) = e^{\frac{1}{2} r a^{\dagger 2} - \frac{1}{2} r^* a^2}$ with $r = |r| e^{i\Theta}$

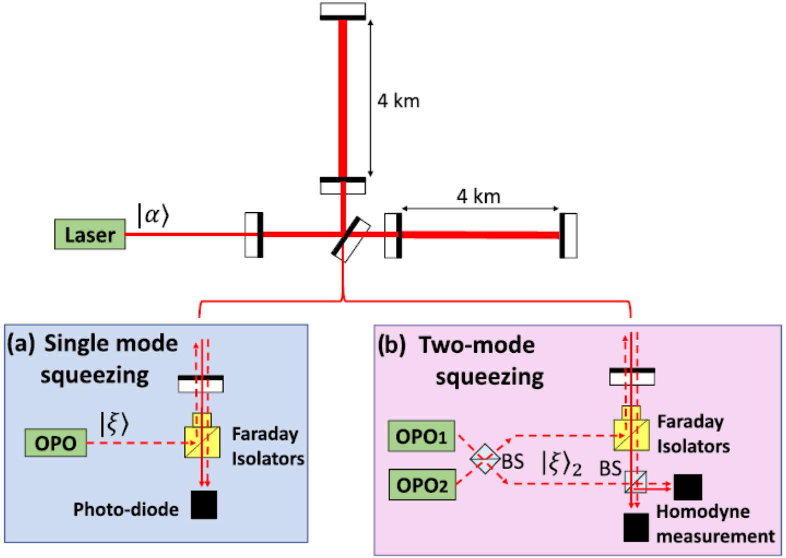
$$\Delta p^2 \xrightarrow{S(|r|,\Theta)} \frac{\Delta p^2}{e^{|r|}} \qquad \Delta x^2 \xrightarrow{S(|r|,\Theta)} e^{|r|} \Delta x^2$$

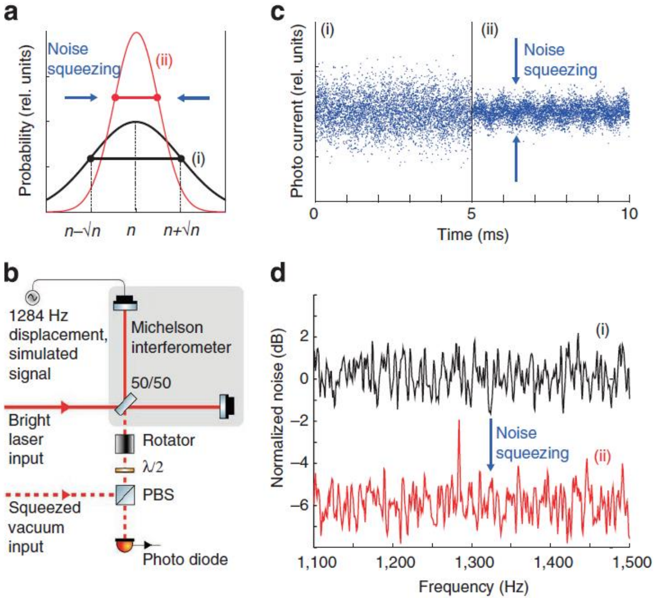
Two Mode Squeezing Operator: $S_2(r) = e^{r^* a_{K_1} a_{K_2} - r a_{K_1}^{\dagger} a_{K_2}^{\dagger}}$

Goetz 2017: The Interplay of Superconducting Quantum Circuits and Propagating Microwave States

Polino et al 2020: Photonic Quantum Metrology. https://doi.org/10.1116/5.0007577

Squeezed States for the detection of gravitational waves





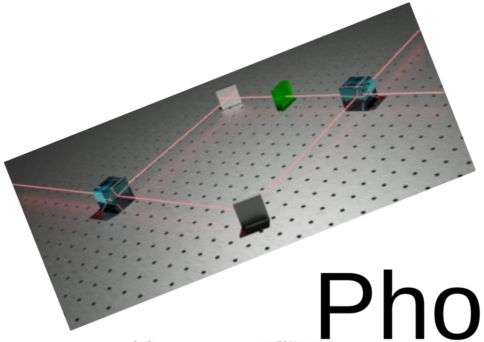
Polino et al 2020: Photonic Quantum Metrology. https://doi.org/10.1116/5.0007577

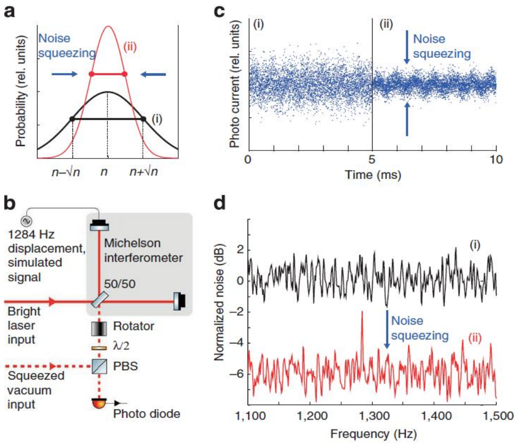
Schnabel et al 2010: Quantum metrology for gravitational wave astronomy. https://doi.org/ 10.1038/ncomms1122

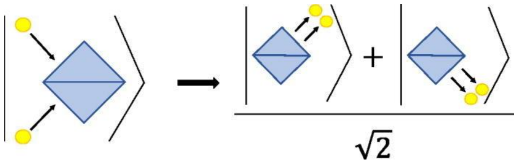
Photonic Quantum Metrology

Standard Quantum Limit: $\boldsymbol{\Delta\lambda_{min}} \geq \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{N}}}$

Heisenberg Limit: $\boldsymbol{\Delta\lambda_{min}} \geq \frac{\boldsymbol{1}}{\boldsymbol{N}}$







Hong-Ou-Mandel Effect

Estimation of unknown parameters exploiting quantum ressources using photonic systems