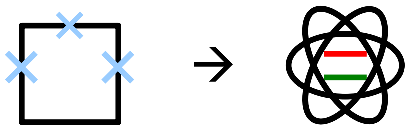
6.5

Circuit quantum electrodynamics

Analogy to quantum optics

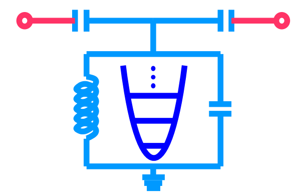
Superconducting quantum circuits (SQC)

Nonlinear circuits
Qubits, multilevel systems



Artificial atoms with
engineered level spectrum

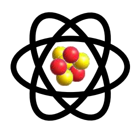
Linear circuits
Waveguides, resonators



Quantum light
(microwaves)

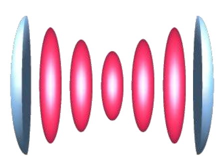
In **quantum optics** for 40+ years

→ Natural atoms/ions/molecules



→ Probed with laser light

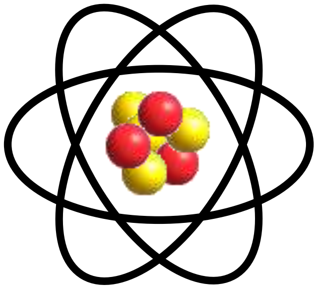
→ Quantum light in cavities

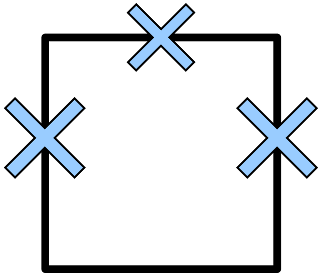


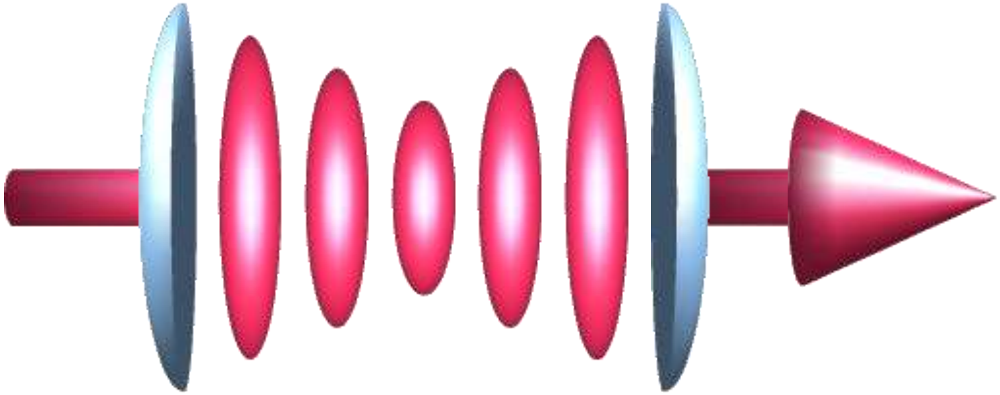
Cavity quantum electrodynamics (QED)

→ QIP, Qsim, Qcomm
→ Light-matter interaction
→ Test quantum mechanics

Analogy to quantum optics

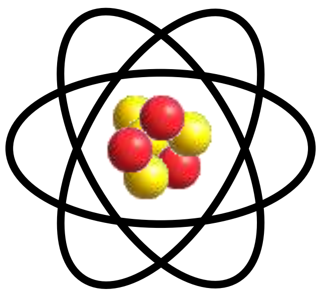


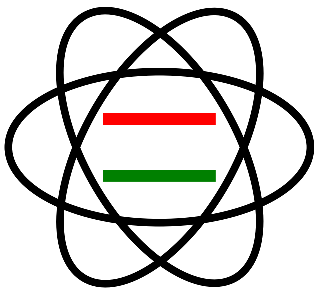




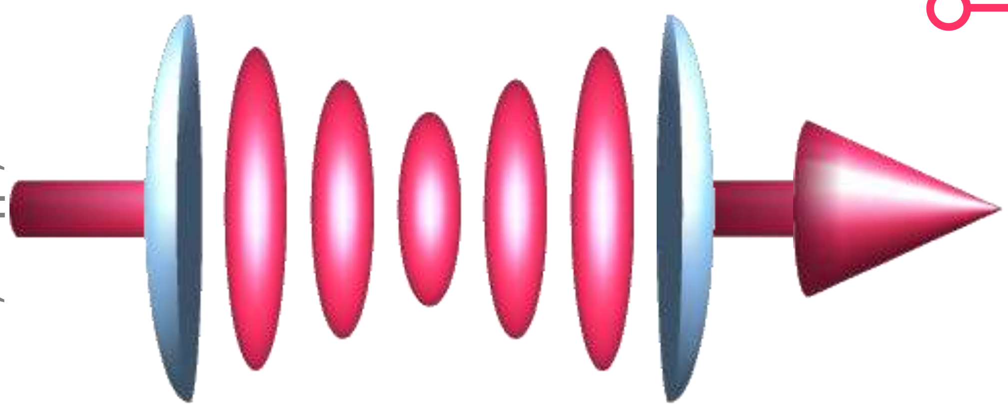
Analogy to quantum optics

(single)
natural
atom

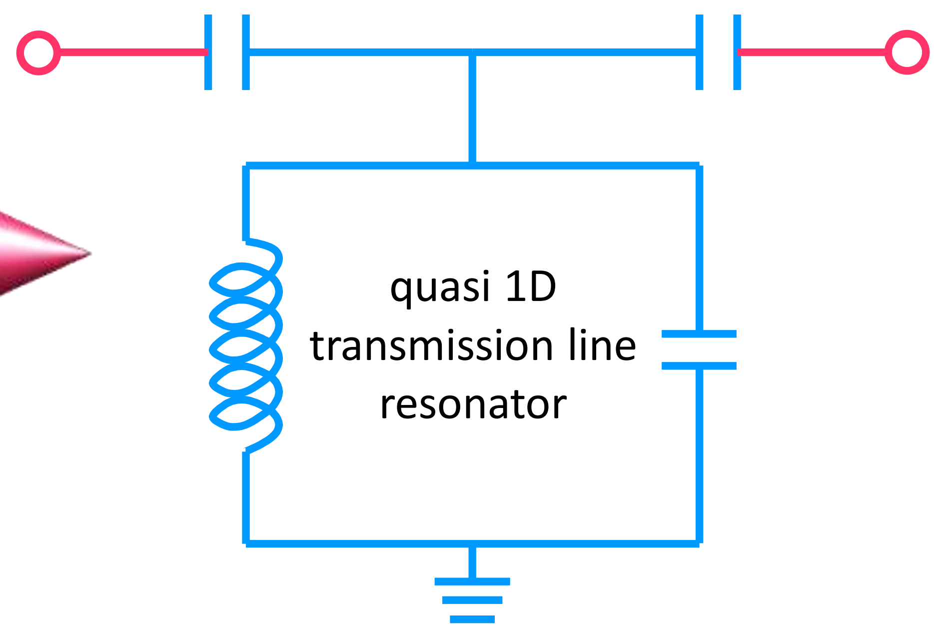




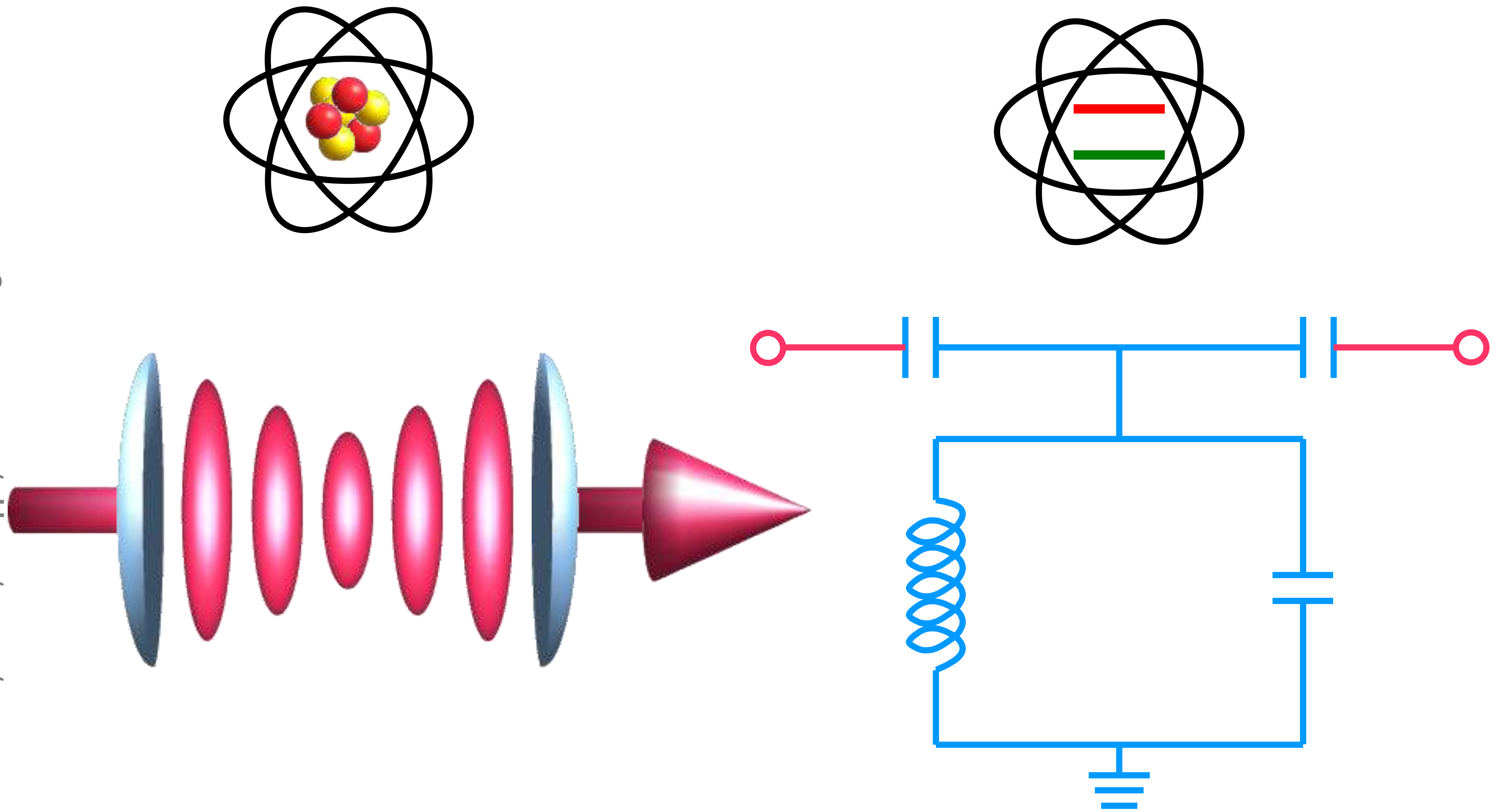
single
artificial
atom



3D optical cavity resonator

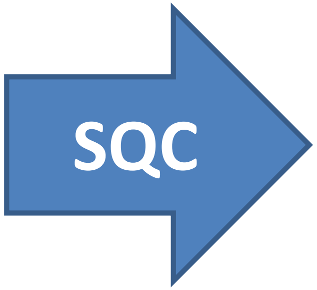


Analogy to quantum optics



Analogy to quantum optics

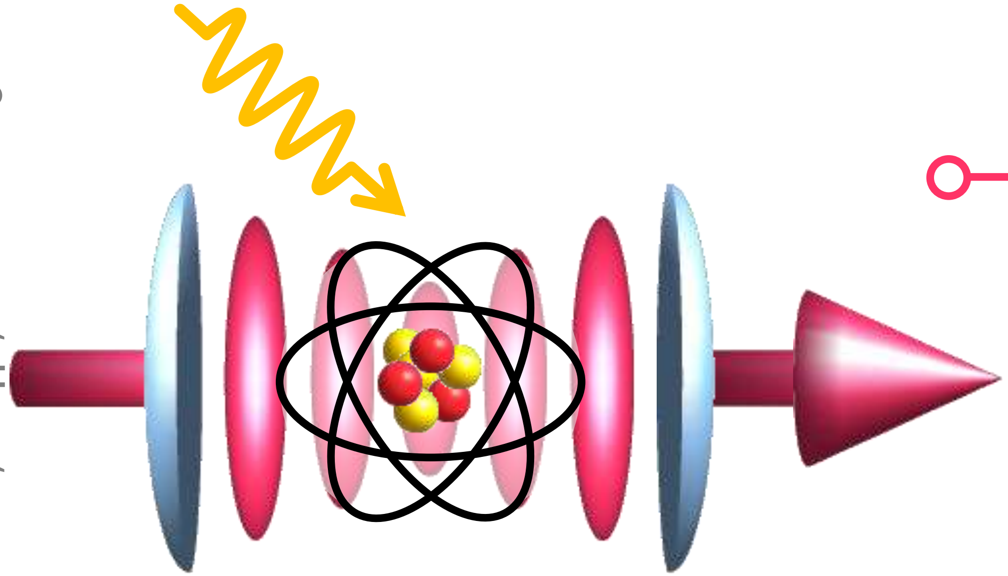
Cavity QED

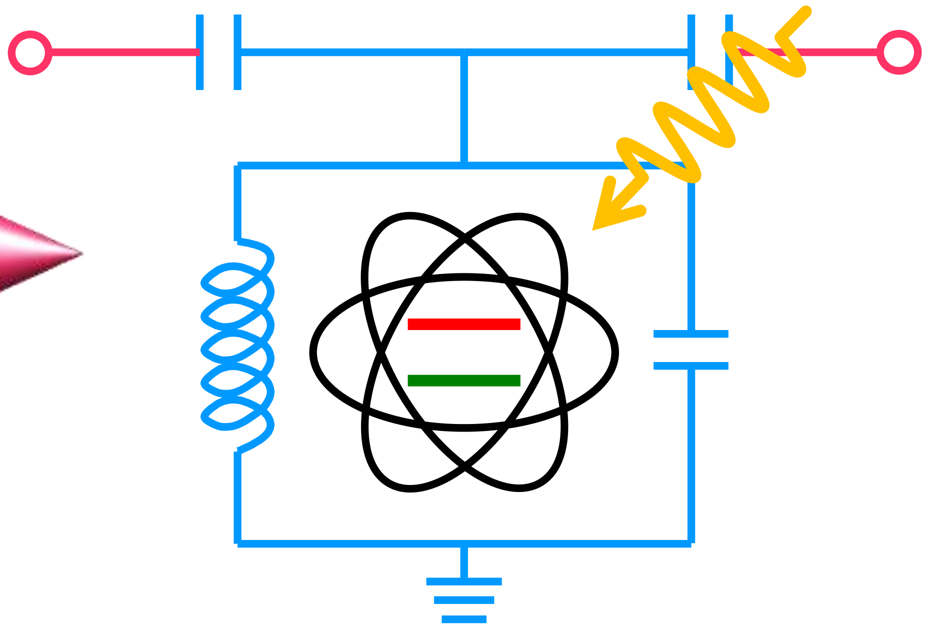


Circuit QED

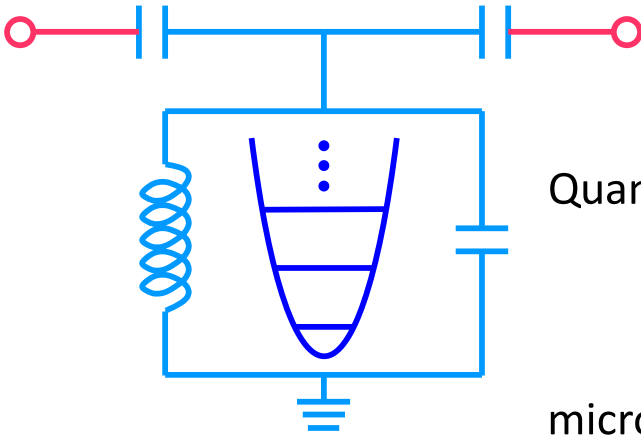
Quantum electrodynamics
with superconducting circuits

A. Blais et al., Phys. Rev. A 69, 062320 (2004)

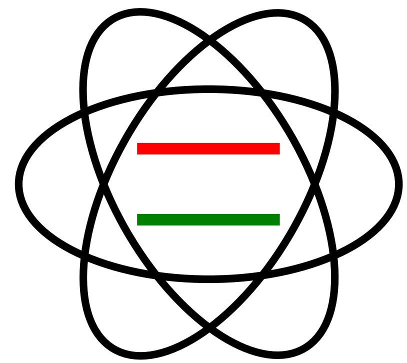




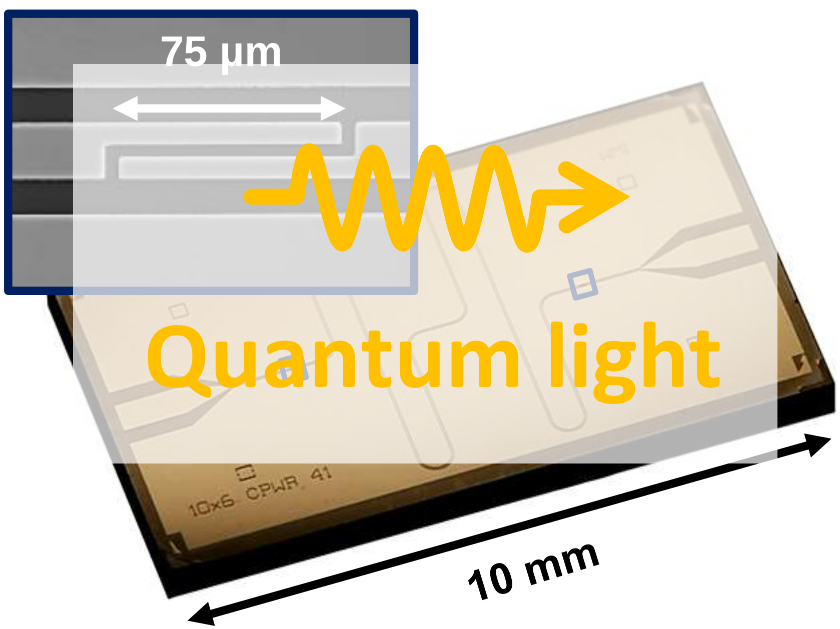
Quantum light and Artificial atoms



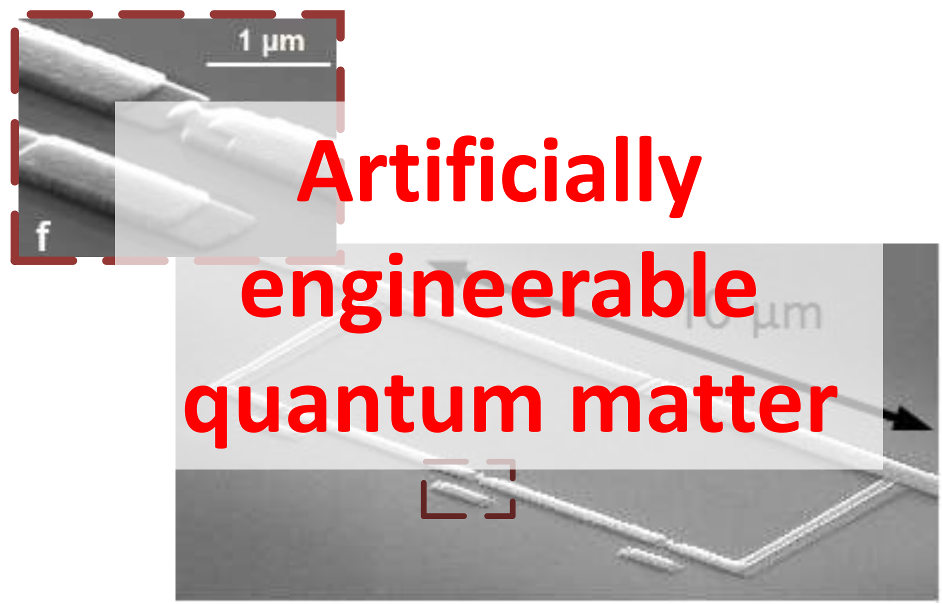
Linear circuits
↓
Quantum harmonic oscillator
↓
„Box“ for microwave photons



Nonlinear circuits
↓
Quantum bit
↓
Artificial atom



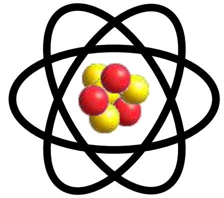
Quantum light

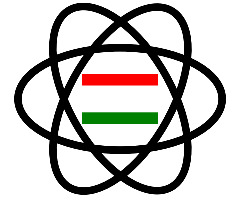


Artificially engineerable quantum matter

6.5 Circuit quantum electrodynamics

Natural vs. artificial atoms





	Natural atoms	Artificial solid-state atoms	
Size	Å	nm-cm	+
Transition frequencies	IR – UV	µ-wave	−
Design flexibility	None	Large	
Tunability	Small	Large	
Selection rules	Strict	Relaxed, controllable	+
Lasing or masing	Multiple atoms	Single atom	
Effective dipole moment	Small	Large	
Coherence times	Long (min, s)	Short (ns-ms)	−
Interaction strengths	Small (Hz-kHz)	**Large (MHz-GHz)**	+

6.5 Circuit quantum electrodynamics

Large interaction strengths in superconducting circuits – The qubit

	energy shift	natural atom	charge qubit	flux qubit
first order	$\Delta E_{\rm el}^{(1)}$	$\pm\mu_{\rm el}\cdot\delta E_{\rm ex}$ $\langle\mu_{\rm el}\rangle = 0$	$\pm\mu_{\rm el}\cdot\delta E_{\rm ex}$ $\mu_{\rm el} = ed$ $\sim 10^{-26}\text{-}10^{-25}\,{\rm Cm}$	—
	$\Delta E_{\rm mag}^{(1)}$	$\pm\mu_{\rm mag}\cdot\delta B_{\rm ex}$ $\mu_{\rm mag}\simeq\mu_{\rm B}$	—	$\pm\mu_{\rm mag}\cdot\delta B_{\rm ex}$ $\mu_{\rm mag} = I_{\rm p}A$ $\sim 10^{4}\text{-}10^{5}\,\mu_{\rm B}$
second order	$\Delta E_{\rm el}^{(2)}$	$-\frac{1}{2}\chi_{\rm el}\cdot\delta E_{\rm ex}^2$ $\chi_{\rm el}\simeq 4\pi\epsilon_0 a_{\rm B}^3$ $\sim 10^{-41}\,{\rm Cm^2/V}$	$-\frac{1}{2}\chi_{\rm el}\cdot\delta E_{\rm ex}^2$ $\chi_{\rm el}\simeq -2\frac{(ed)^2}{\Delta_{\rm ge}}$ $\sim 10^{-28}\text{-}10^{-26}\,{\rm Cm^2/V}$	—
	$\Delta E_{\rm mag}^{(2)}$	$-\frac{1}{2}\chi_{\rm mag}\cdot\delta B_{\rm ex}^2$ $\chi_{\rm mag}\simeq -\frac{e^2a_{\rm B}^2}{6m}$ $\sim 10^{-29}\,{\rm Am^2/T}$	—	$-\frac{1}{2}\chi_{\rm mag}\cdot\delta B_{\rm ex}^2$ $\chi_{\rm mag}\simeq -2\frac{(I_{\rm p}A)^2}{\Delta_{\rm ge}}$ $\sim 10^{-13}\text{-}10^{-11}\,{\rm Am^2/T}$

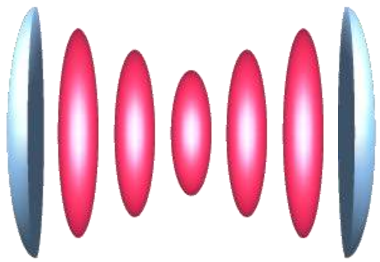
Large effective dipole moment

2^{nd}-order susceptibility still appreciable!

T. Niemczyk *et al.*, Supercond. Sci. Technol. **22**, 034009 (2009).

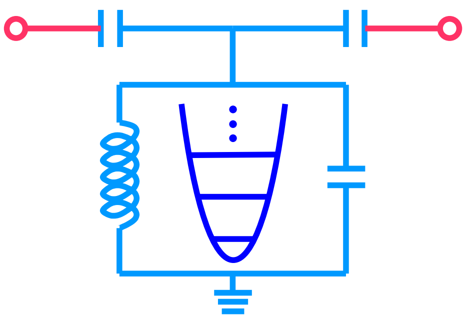
Large interaction strengths in superconducting circuits – The resonator

3D Optical cavities



Large mode volume (3D!)

Superconducting LC resonators



Small mode volume ($V_\mathrm{m} \approx 10^{-12}\ \mathrm{m}^3$)

Vacuum field amplitudes $E_0 = \sqrt{\frac{\hbar\omega_\mathrm{r}}{2\epsilon_0 V_\mathrm{m}}}$, $B_0 = \sqrt{\frac{\mu_0\hbar\omega_\mathrm{r}}{2V_\mathrm{m}}}$

E_0^2/ω_r, B_0^2/ω_r small

→ Small interaction strengths

E_0^2/ω_r, B_0^2/ω_r large

→ **Large interaction strength**

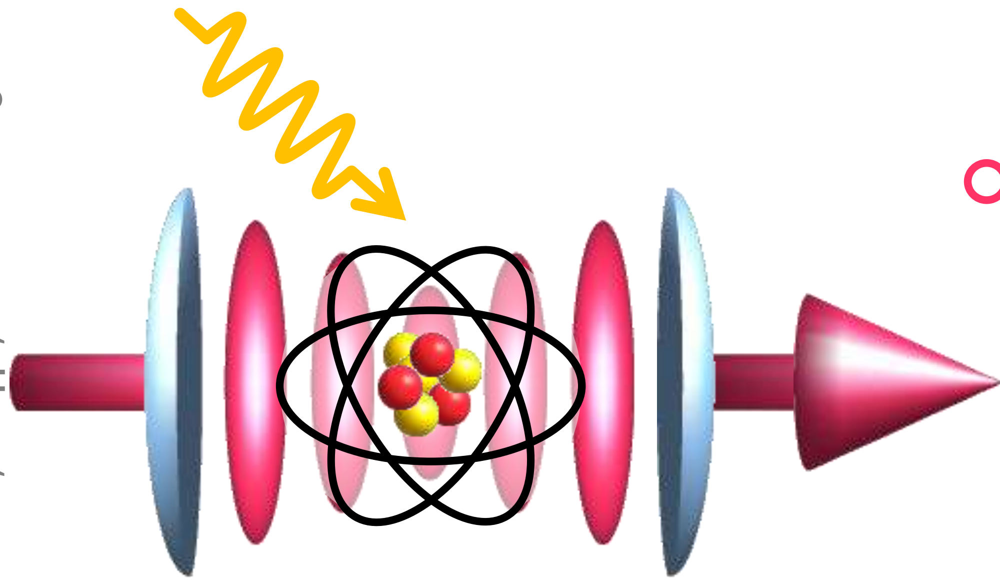
T. Niemczyk *et al.*, Supercond. Sci. Technol. **22**, 034009 (2009).

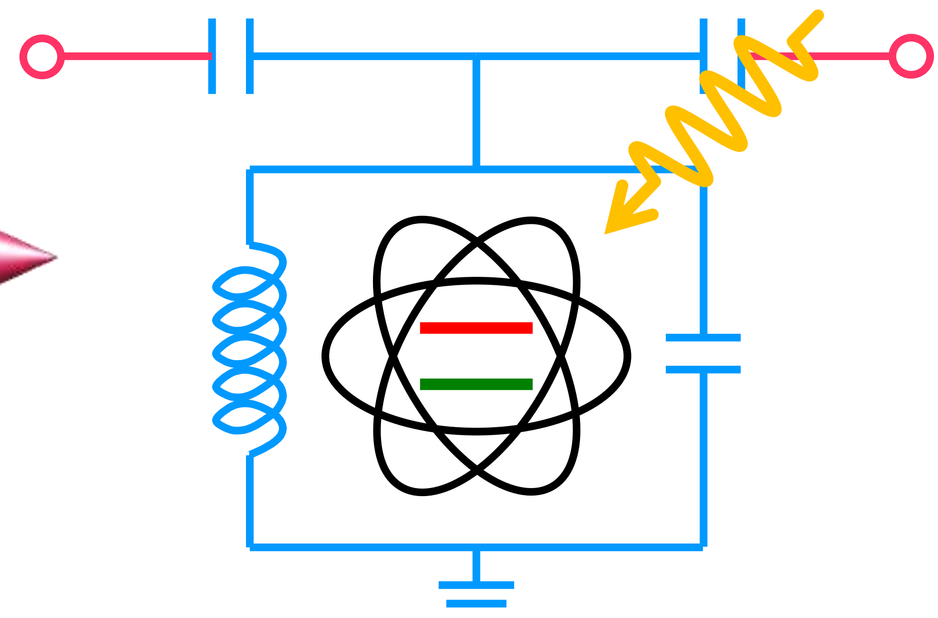
Circuit QED vs. cavity QED – Key advantages

→ Better prospect of scalability

→ Larger interaction strengths outweigh inferior quantum coherence

- → More quantum operations per coherence time
- → Fundamental quantum effects observable on single-photon level





6.5 Circuit quantum electrodynamics

The light-matter interaction Hamiltonian

→ Dipole interaction between atom and cavity field
→ Interaction energy $\hbar g$ → (Vacuum field amplitude) x (Atomic dipole moment)

$$\hat{H}_{\mathrm{qr}} = \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar\omega_{\mathrm{r}}\hat{a}^\dagger\hat{a} + \hbar g\left(\hat{a}^\dagger + \hat{a}\right)\underbrace{\left(\hat{\sigma}^- + \hat{\sigma}^+\right)}_{=\hat{\sigma}_x}$$

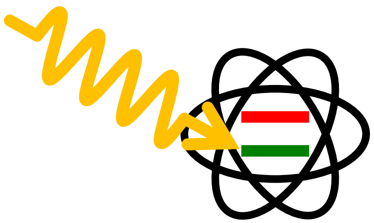
Known as the quantum Rabi model because of its relation to the Rabi Hamiltonian

6.5 Circuit quantum electrodynamics

Vaccum Rabi oscillations

→ Dipole interaction between atom and cavity field
→ Interaction energy $\hbar g$ → (Vacuum field amplitude) x (Atomic dipole moment)

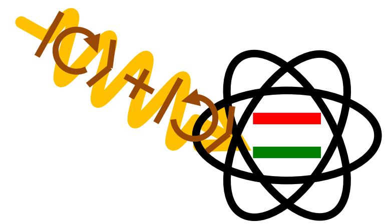
$$\hat{H}_{\mathrm{qr}} = \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar\omega_{\mathrm{r}}\hat{a}^\dagger\hat{a} + \hbar g\left(\hat{a}^\dagger + \hat{a}\right)(\hat{\sigma}^- + \hat{\sigma}^+)$$



Classical driving field
$\propto \cos\omega t \propto \left(e^{-i\omega t} + e^{+i\omega t}\right)$

→ RWA → Coherent population (Rabi) oscillations for $\omega = \omega_{\mathrm{q}}$

→ Driving field in the ground state ($|g_{\min}| = 0$)
→ No oscillations



Quantum driving field $\propto \left(\hat{a} + \hat{a}^\dagger\right)$

→ Full quantum version of Rabi Hamiltonian
→ RWA→ Quantum oscillations for $\omega_{\mathrm{r}} = \omega_{\mathrm{q}}$

→ Driving field ground state is the vacuum ($|g_{\min}| = |g_{\mathrm{vac}}| > 0$)
→ Vacuum Rabi oscillations

→ Vacuum Rabi oscillations describe the coherent exchange of an excitation between qubit and resoantor → Signature of a quantum system

The Jaynes-Cummings Hamiltonian for light-matter interaction

→ Dipole interaction between atom and cavity field
→ Interaction energy $\hbar g$ → (Vacuum field amplitude) x (Atomic dipole moment)

$$\widehat{H}_{\mathrm{qr}} = \hbar\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a} + \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar g\left(\hat{a}^{\dagger} + \hat{a}\right)(\hat{\sigma}^{-} + \hat{\sigma}^{+})$$

RWA $\quad g \ll \omega_{\mathrm{r}}, \omega_{\mathrm{q}}$

Jaynes-Cummings Hamiltonian

$$\widehat{H}_{\mathrm{JC}} = \hbar\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a} + \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar g\left(\hat{a}^{\dagger}\hat{\sigma}^{-} + \hat{a}\hat{\sigma}^{+}\right)$$

Interaction picture, $\widehat{U} = \mathrm{e}^{-it(\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a} + \frac{\omega_{\mathrm{q}}}{2}\hat{\sigma}_z)}$

$$\widehat{H}_{\mathrm{JC}}^{\mathrm{I}} = \hbar g\left(\hat{a}^{\dagger}\hat{\sigma}^{-}e^{i\delta t} + \hat{a}\hat{\sigma}^{+}e^{-i\delta t}\right),$$

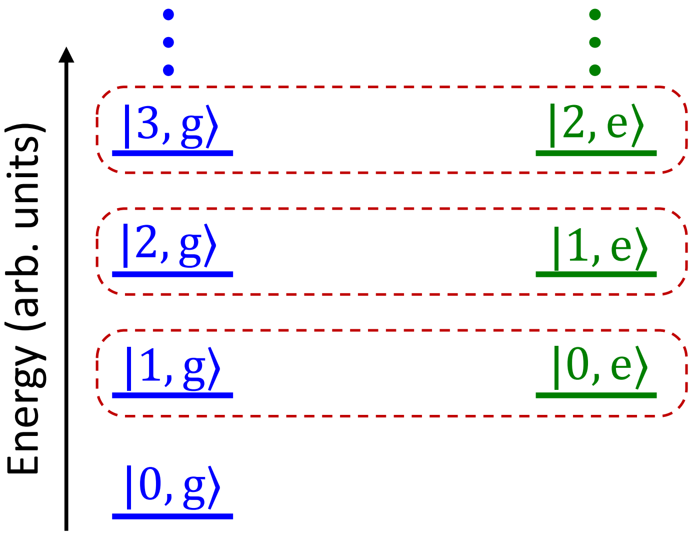
Detuning $\delta \equiv \omega_{\mathrm{q}} - \omega_{\mathrm{r}}$

Two regimes →
- $\delta = 0$ → Resonant regime
- $\delta \neq 0$ → Dispersive regime

Diagonalizing the Jaynes-Cummings Hamiltonian

$$\hat{H}_{\mathrm{JC}} = \hbar\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a} + \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar g\left(\hat{a}^{\dagger}\hat{\sigma}^{-} + \hat{a}\hat{\sigma}^{+}\right)$$

$$\begin{aligned} \hat{a}^{\dagger}|n\rangle &= \sqrt{n+1}\;|n+1\rangle \\ \hat{a}|n\rangle &= \sqrt{n}\;|n-1\rangle \end{aligned}$$



Intuition

→ Uncoupled Hamiltonian

$$\hat{H}_{\mathrm{JC}} = \hbar\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a} + \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z$$

→ Resonant regime ($\delta = 0$)

→ Level pairs $\{|n+1,\mathrm{g}\rangle, |n,\mathrm{e}\rangle\}$

Effect of the interaction term $\hat{H}_{\mathrm{JC,int}} = \hbar g\left(\hat{a}^{\dagger}\hat{\sigma}^{-} + \hat{a}\hat{\sigma}^{+}\right)$

→ Start in $|n+1,\mathrm{g}\rangle$ → $\left(\hat{a}^{\dagger}\hat{\sigma}^{-} + \hat{a}\hat{\sigma}^{+}\right)|n+1,\mathrm{g}\rangle = \sqrt{n+1}|n,\mathrm{e}\rangle$

→ Start in $|n,\mathrm{e}\rangle$ → $\left(\hat{a}^{\dagger}\hat{\sigma}^{-} + \hat{a}\hat{\sigma}^{+}\right)|n,\mathrm{e}\rangle = \sqrt{n+1}|n+1,\mathrm{g}\rangle$

→ $\hat{H}_{\mathrm{JC,int}}$ couples only states within the same level pair („JC doublet")

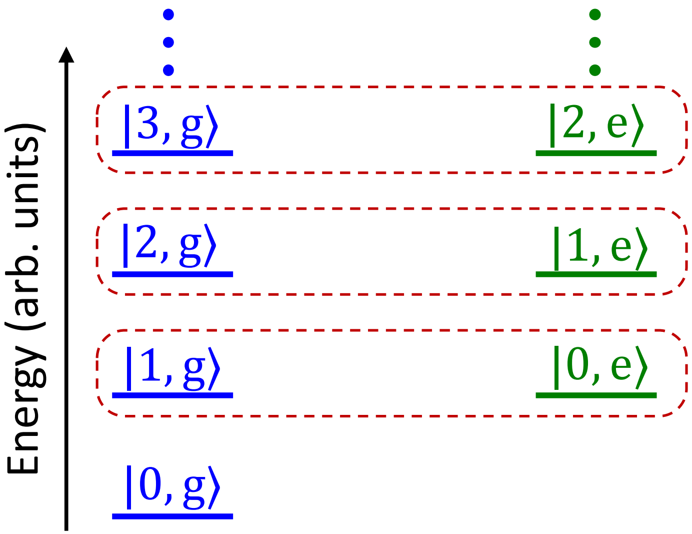
→ $\hat{H}_{\mathrm{JC}}$ decouples into infinite direct product of 2×2-matrices

$$\rightarrow \hat{H}_{\mathrm{JC},n}\begin{pmatrix} |n,\mathrm{e}\rangle \\ |n+1,\mathrm{g}\rangle \end{pmatrix} = \hbar\begin{pmatrix} n\omega_{\mathrm{r}} + \frac{\omega_{\mathrm{q}}}{2} & g\sqrt{n+1} \\ g\sqrt{n+1} & (n+1)\omega_{\mathrm{r}} - \frac{\omega_{\mathrm{q}}}{2} \end{pmatrix}\begin{pmatrix} |n,\mathrm{e}\rangle \\ |n+1,\mathrm{g}\rangle \end{pmatrix}$$

→ Result is general (only inspired by intuition!)

6.5 Circuit quantum electrodynamics

The Jaynes-Cummings Hamiltonian for light-matter interaction



$$\hat{H}_{\mathrm{JC},n}\begin{pmatrix}|n,\mathrm{e}\rangle\\|n+1,\mathrm{g}\rangle\end{pmatrix}=\hbar\begin{pmatrix}n\omega_\mathrm{r}+\frac{\omega_\mathrm{q}}{2} & g\sqrt{n+1}\\ g\sqrt{n+1} & (n+1)\omega_\mathrm{r}-\frac{\omega_\mathrm{q}}{2}\end{pmatrix}\begin{pmatrix}|n,\mathrm{e}\rangle\\|n+1,\mathrm{g}\rangle\end{pmatrix}$$

$$\delta\equiv\omega_\mathrm{q}-\omega_\mathrm{r}$$

$$\hat{H}_\mathrm{TLS}=\frac{\epsilon}{2}\hat{\sigma}_z+\frac{\Delta}{2}\hat{\sigma}_x\rightarrow E_\pm=\pm\frac{1}{2}\sqrt{\epsilon^2+\Delta^2}$$

$$\Theta_n\equiv\frac{1}{2}\tan^{-1}\frac{2g\sqrt{n+1}}{\delta}$$

$$\hat{H}_{\mathrm{JC},n}=\hbar\begin{pmatrix}n\omega_\mathrm{r}+\frac{\omega_\mathrm{q}}{2} & g\sqrt{n+1}\\ g\sqrt{n+1} & (n+1)\omega_\mathrm{r}-\frac{\omega_\mathrm{q}}{2}\end{pmatrix}=\hbar\omega_\mathrm{r}\left(n+\frac{1}{2}\right)\hat{1}_\mathrm{r}+\hbar\frac{\delta}{2}\hat{\sigma}_z+\hbar g\sqrt{n+1}\,\hat{\sigma}_x$$

Analytic diagonalization → $E_{n,\pm}=\hbar\omega_\mathrm{r}\left(n+\frac{1}{2}\right)\pm\frac{\hbar}{2}\sqrt{\delta^2+4g^2(n+1)}$

$E_{0,\mathrm{g}}=-\hbar\frac{\omega_\mathrm{q}}{2}$

Eigenstates →

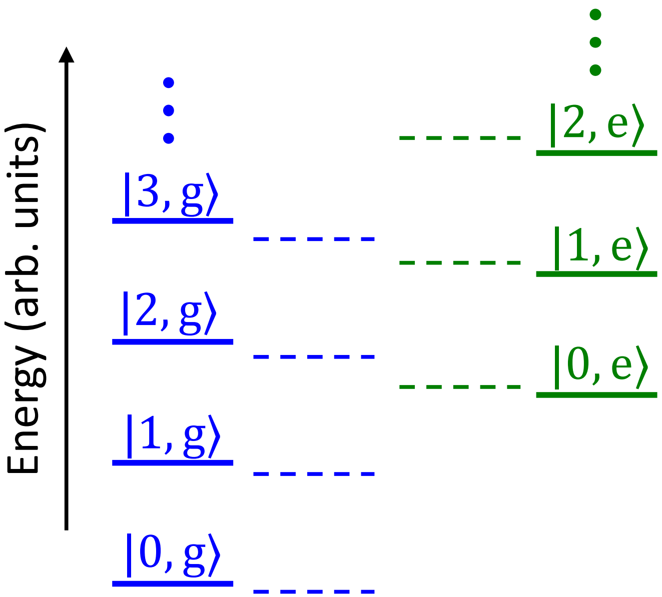
$$|n,+\rangle=\cos\Theta_n\,|n+1,\mathrm{g}\rangle-\sin\Theta_n\,|n,\mathrm{e}\rangle$$
$$|n,-\rangle=\sin\Theta_n\,|n+1,\mathrm{g}\rangle+\cos\Theta_n\,|n,\mathrm{e}\rangle$$

6.5 Circuit quantum electrodynamics

Dispersive regime of the JC Hamiltonian

$$\hat{H}_{\mathrm{JC}}^{\mathrm{int}} = \hbar g\left(\hat{\sigma}^{-}\hat{a}^{\dagger}e^{i\delta t} + \hat{\sigma}^{+}\hat{a}e^{-i\delta t}\right)$$

$$\delta \equiv \omega_{\mathrm{q}} - \omega_{\mathrm{r}}$$



Detuning $\delta \gg g$ → Qualitative discussion

→ No transitions (energy mismatch)

→ Heisenberg uncertainty allows for the creation of excitations from the vacuum for time $\delta t \simeq \frac{\hbar}{\delta E}$

→ Excitation needs to jump back

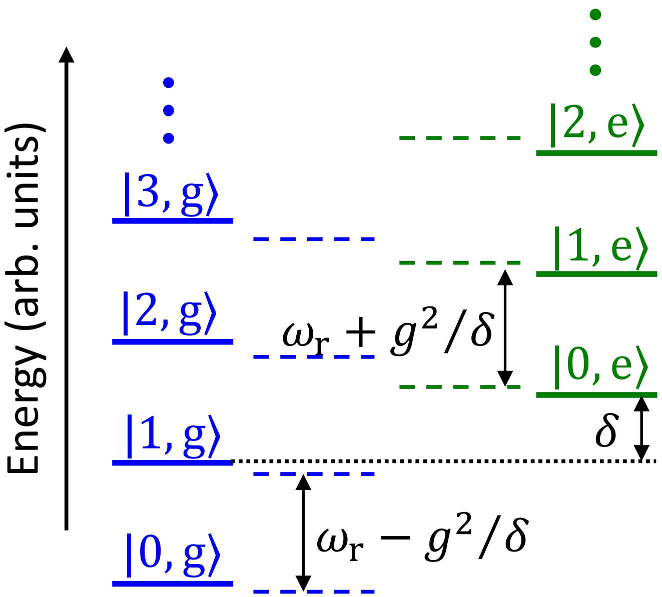
- → Level shifts
- → Second-order effect (enery scale g^2/δ)
- → Eigenstates still predominantly resonator-like or qubit-like with small admixture of other component

A. Blais *et al.*, Phys. Rev. A **69**, 062320 (2004)

Dispersive regime of the JC Hamiltonian

$$\widehat{H}_{\mathrm{JC}} = \hbar\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a} + \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar g\left(\hat{a}^{\dagger}\hat{\sigma}^{-} + \hat{a}\hat{\sigma}^{+}\right)$$

$$\widehat{H}_{\mathrm{JC}}^{\mathrm{int}} = \hbar g\left(\hat{\sigma}^{-}\hat{a}^{\dagger}e^{i\delta t} + \hat{\sigma}^{+}\hat{a}e^{-i\delta t}\right) \qquad \delta \equiv \omega_{\mathrm{q}} - \omega_{\mathrm{r}}$$



Detuning $\delta \gg g$ → Quantitative discussion

→ $\widehat{H}_{\mathrm{JC}}^{\mathrm{int}}$ has explicit time dependence

→ Transform $\widehat{H}_{\mathrm{JC}}$ via $\widehat{U} = e^{\frac{g}{\delta}(\hat{\sigma}^{+}\hat{a} - \hat{\sigma}^{-}\hat{a}^{\dagger})}$

→ $\hat{A} = \frac{g}{\delta}\left(\hat{\sigma}^{+}\hat{a} - \hat{\sigma}^{-}\hat{a}^{\dagger}\right)$

→ $\widehat{U}$ cancels first-order photon exchange terms $\hbar g\left(\hat{a}^{\dagger}\hat{\sigma}^{-} + \hat{a}\hat{\sigma}^{+}\right)$

→ Develop up to second order in $\frac{g}{\delta}$ → $\widehat{H}^{(2)} = \widehat{U}\widehat{H}_{\mathrm{JC}}\widehat{U}^{\dagger} = \sum_n \frac{\hat{A}^n}{n!}\widehat{H}_{\mathrm{JC}}\sum_m \frac{\left(\hat{A}^{\dagger}\right)^m}{m!} \equiv \sum_{n,m}\widehat{H}_{nm}$
$= \widehat{H}_{00} + \widehat{H}_{01} + \widehat{H}_{10} + \widehat{H}_{02} + \widehat{H}_{11} + \widehat{H}_{02}$

→ Neglect terms with $\left(\frac{g}{\delta}\right)^2$ → $\widehat{H}^{(2)} = \hbar\left(\omega_{\mathrm{r}} + \underbrace{\frac{g^2}{\delta}\hat{\sigma}_z}_{\text{Ac Stark / Zeeman shift}}\right)\hat{a}^{\dagger}\hat{a} + \frac{\hbar}{2}\left(\omega_{\mathrm{q}} + \underbrace{\frac{g^2}{\delta}}_{\text{Lamb shift}}\right)\hat{\sigma}_z$

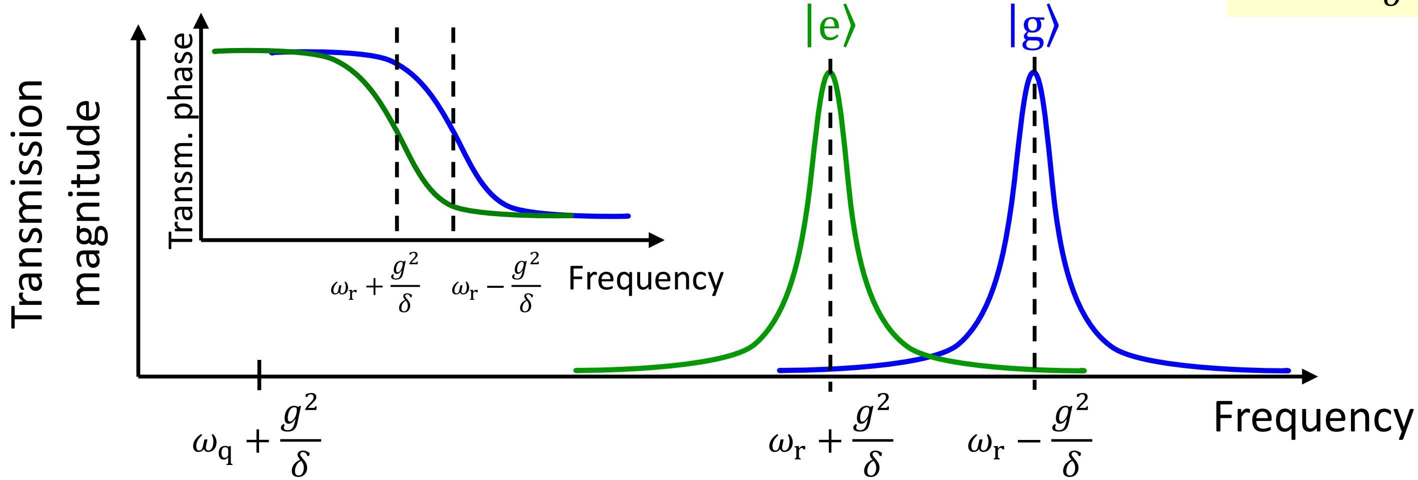
→ Qubit readout, dispersive gates, photon number calibration

A. Blais *et al.*, Phys. Rev. A **69**, 062320 (2004)

Dispersive readout

$$\hat{H}_{\mathrm{JC}}^{\mathrm{int}} = \hbar\frac{g^2}{\delta}\hat{\sigma}_z\hat{a}^\dagger\hat{a} + \frac{\hbar}{2}\frac{g^2}{\delta}\hat{\sigma}_z$$

$$\delta \equiv \omega_{\mathrm{q}} - \omega_{\mathrm{r}}$$



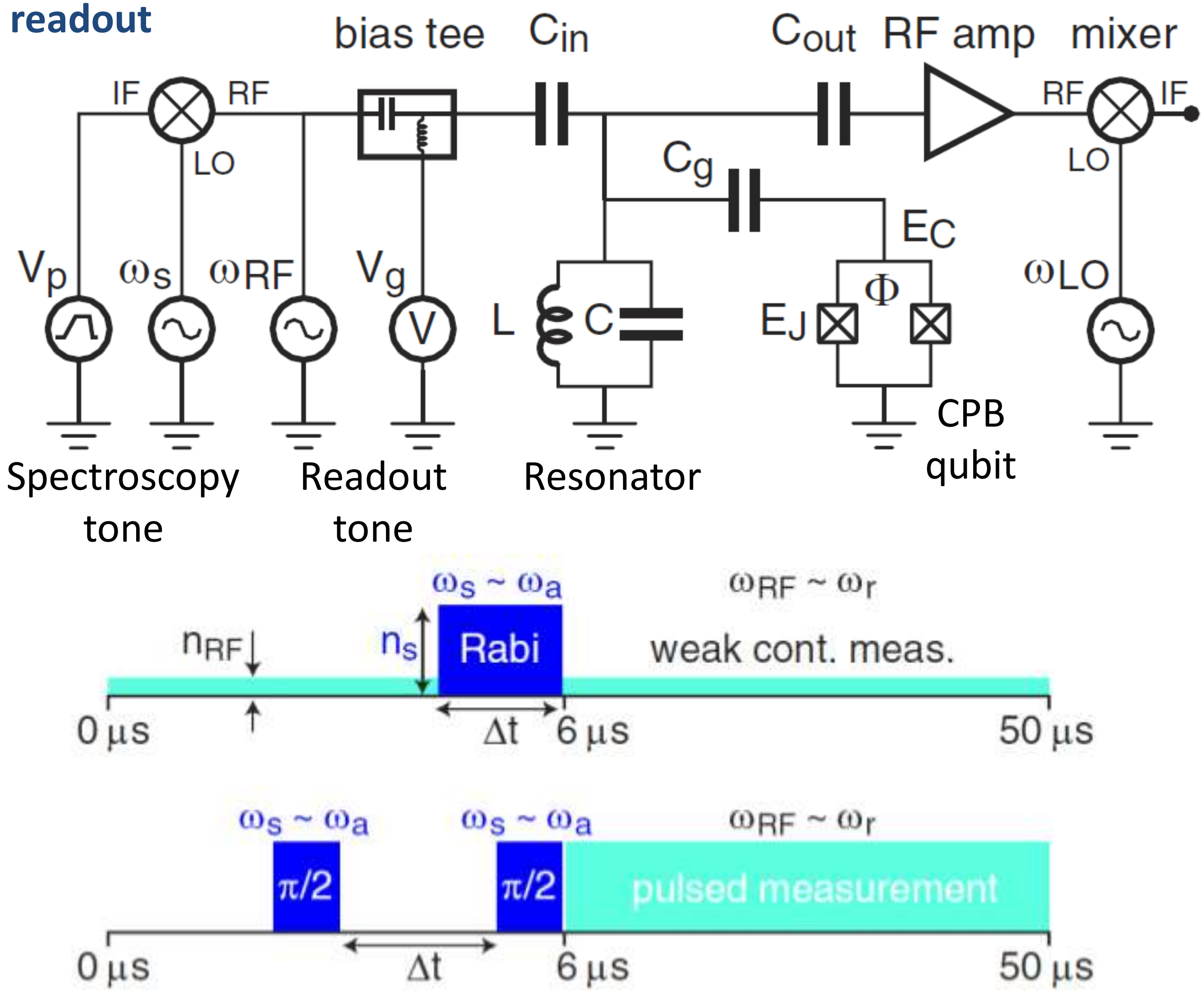
→ Two-tone spectroscopy

- → Send probe tone $\omega_{\mathrm{rf}} = \omega_{\mathrm{r}} + \frac{g^2}{\delta}$
- → Sweep spectroscopy tone ω_{s}
- → When the qubit is in $|\mathrm{e}\rangle$, the transmission magnitude will drop drastically from blue to green curve
- → Mixed state → Reduced shift, but still ok
- → Transmission phase can also be used!

→ Measurement operator $\hat{\sigma}_z$ commutes with qubit Hamiltonian

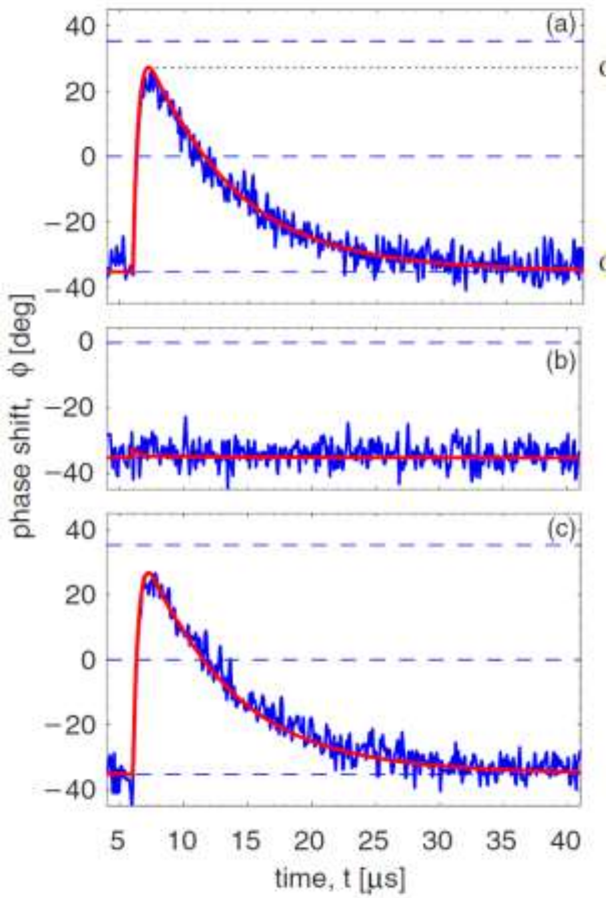
- → Quantum nondemolition measurement
- → Qubit wave function may be projected, but neither destroyed nor unknown

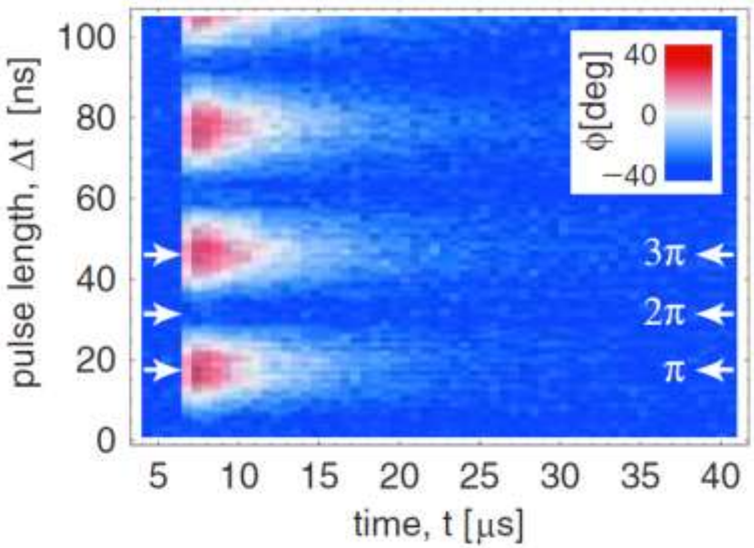
Dispersive readout

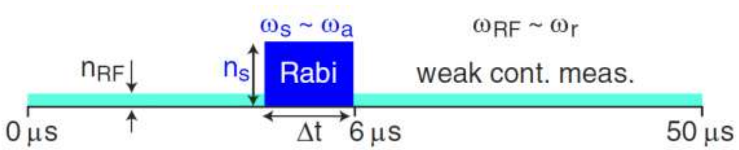


A. Wallraff *et al.*, Phys. Rev. Lett **95**, 060501 (2005)

Energy relaxation and driven Rabi oscillations



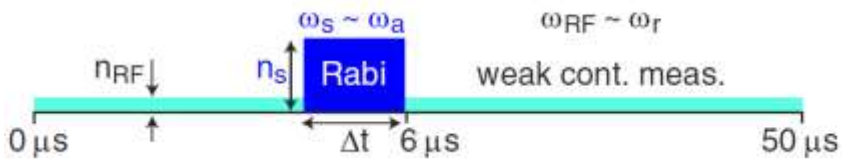


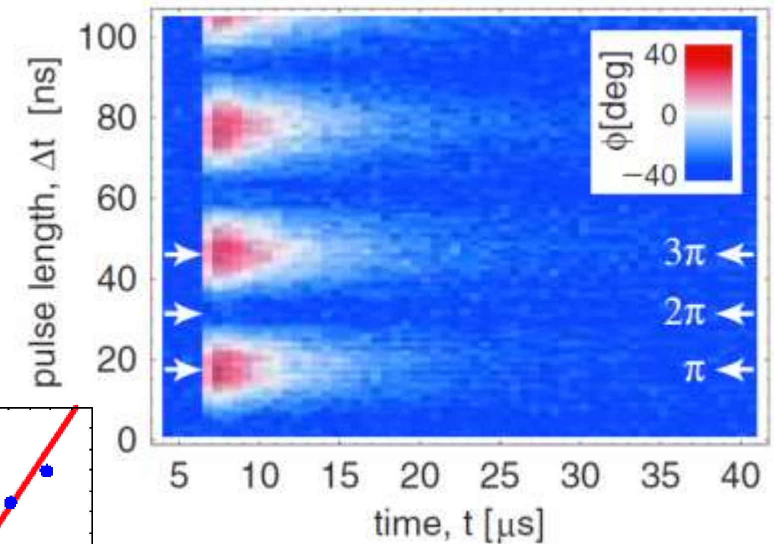


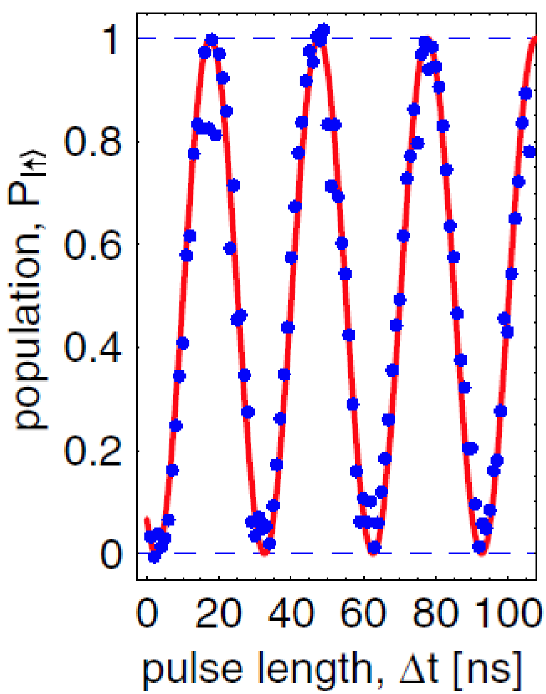
A. Wallraff *et al.*, Phys. Rev. Lett **95**, 060501 (2005)

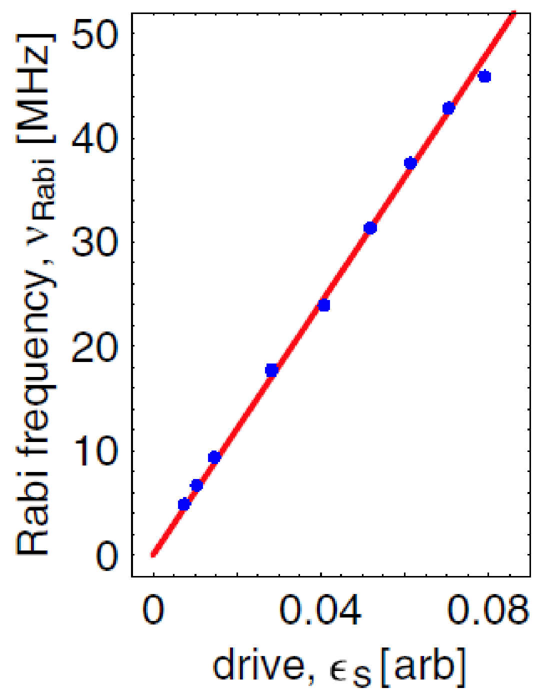
6.5 Circuit quantum electrodynamics

Energy relaxation and driven Rabi oscillations



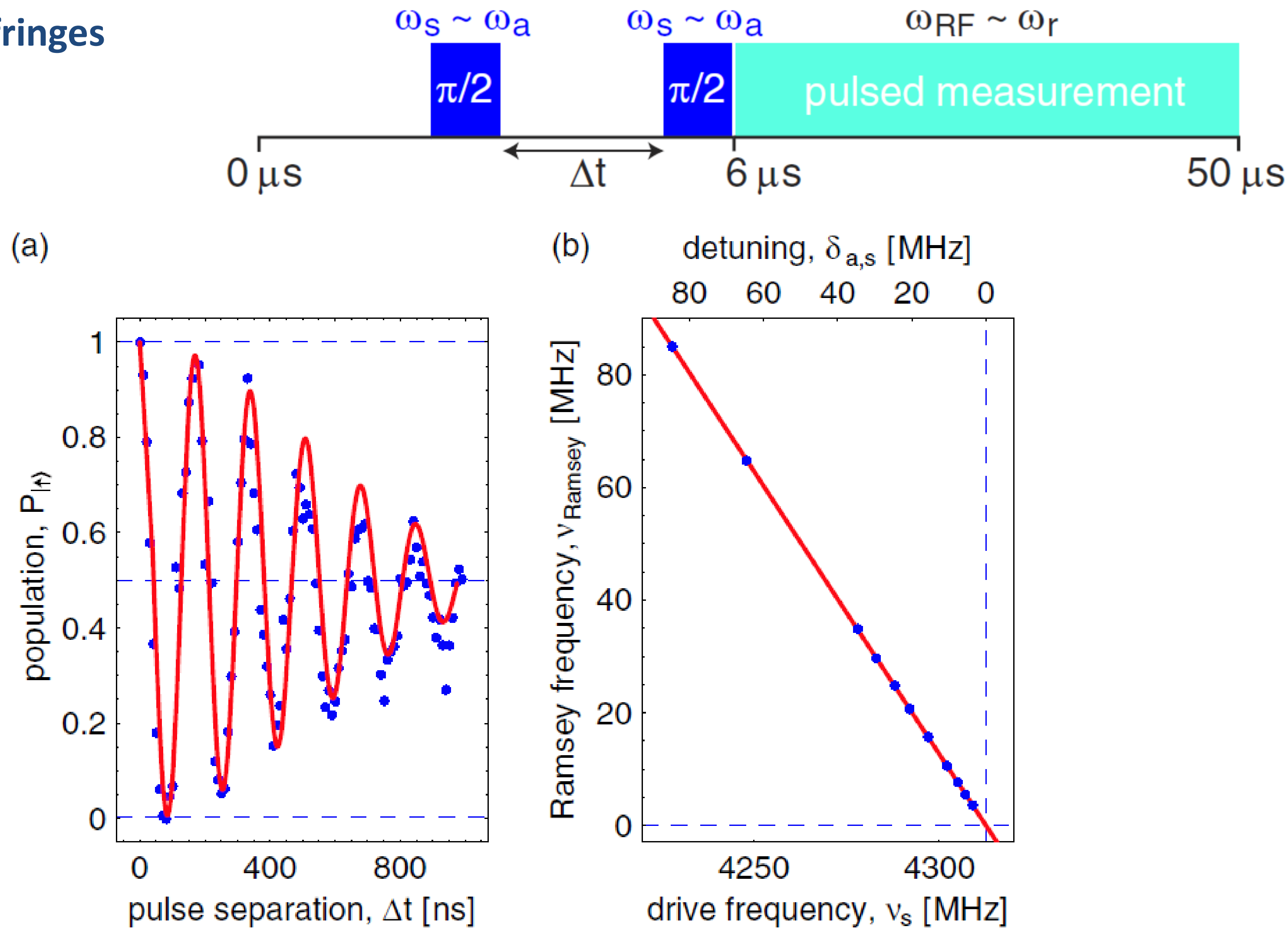






A. Wallraff *et al.*, Phys. Rev. Lett **95**, 060501 (2005)

Ramsey fringes

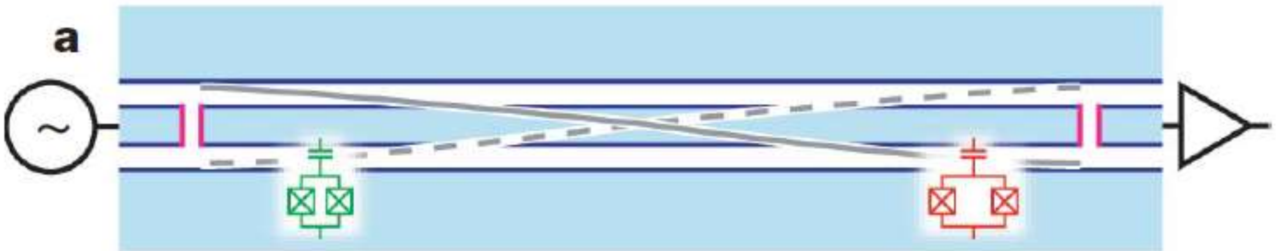


A. Wallraff *et al.*, Phys. Rev. Lett **95**, 060501 (2005)

Dispersive protocol for quantum state transfer

$\delta_{\mathrm{A,B}} \equiv \omega_{A,B} - \omega_{\mathrm{r}}$

$\hat{U} = e^{\frac{g}{\delta}(\hat{\sigma}_{\mathrm{A}}^{+}\hat{a} - \hat{\sigma}_{\mathrm{A}}^{-}\hat{a}^{\dagger} + \hat{\sigma}_{\mathrm{B}}^{+}\hat{a} - \hat{\sigma}_{\mathrm{B}}^{-}\hat{a}^{\dagger})}$



→ Couple two transmon qubits A and B to the same resonator

→ Use resonator as quantum bus for state transfer

→ $\hat{H} = \frac{\hbar\omega_{\mathrm{A}}}{2}\hat{\sigma}_z^{\mathrm{A}} + \frac{\hbar\omega_{\mathrm{B}}}{2}\hat{\sigma}_z^{\mathrm{B}} + \hbar\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a} + \hbar g\left(\hat{\sigma}_{\mathrm{A}}^{-}\hat{a}^{\dagger} + \hat{\sigma}_{\mathrm{A}}^{+}\hat{a}\right) - \hbar g\left(\hat{\sigma}_{\mathrm{B}}^{-}\hat{a}^{\dagger} + \hat{\sigma}_{\mathrm{B}}^{+}\hat{a}\right)$

→ Dispersive regime $g \ll \delta$

→ $\hat{H}^{(2)} = \frac{\hbar\omega_{\mathrm{A}}}{2}\hat{\sigma}_z^{\mathrm{A}} + \frac{\hbar\omega_{\mathrm{B}}}{2}\hat{\sigma}_z^{\mathrm{B}} + \hbar\left(\omega_{\mathrm{r}} + \frac{g^2}{\delta_{\mathrm{A}}}\hat{\sigma}_z^{\mathrm{A}} + \frac{g^2}{\delta_{\mathrm{B}}}\hat{\sigma}_z^{\mathrm{B}}\right)\hat{a}^{\dagger}\hat{a} + \underbrace{\hbar J\left(\hat{\sigma}_{\mathrm{A}}^{-}\hat{\sigma}_{\mathrm{B}}^{+} + \hat{\sigma}_{\mathrm{A}}^{+}\hat{\sigma}_{\mathrm{B}}^{-}\right)}$

Dispersive (2nd order) qubit-qubit coupling

$$J = \frac{g^2}{2}\left(\frac{1}{\delta_{\mathrm{A}}} + \frac{1}{\delta_{\mathrm{B}}}\right)$$

→ $\hat{\sigma}_{\mathrm{A}}^{-}\hat{\sigma}_{\mathrm{B}}^{+} + \hat{\sigma}_{\mathrm{A}}^{+}\hat{\sigma}_{\mathrm{B}}^{-}$ can be understood from $\hat{\sigma}_z = \hat{\sigma}^{+}\hat{\sigma}^{-} - \hat{\sigma}^{-}\hat{\sigma}^{+}$

→ Virtual from qubit A to the resonator and back to qubit B

→ Minus sign from different signs of the mode voltages at the qubit positions

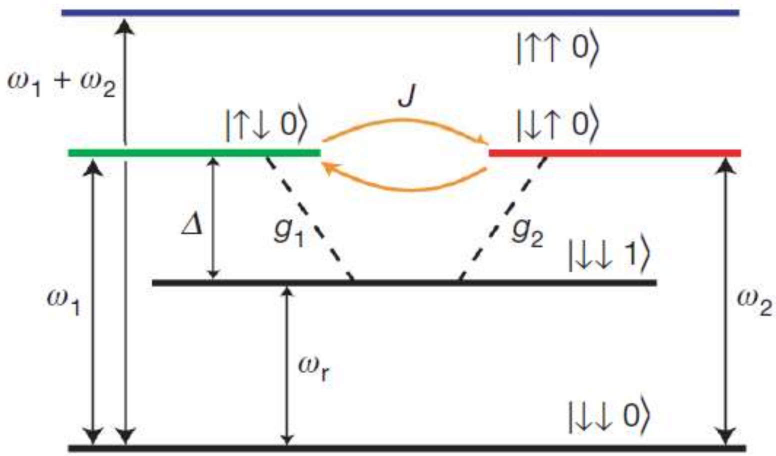
J. Majer *et al.*, Nature **449**, 443-447 (2007)

Dispersive protocol for quantum state transfer

$\delta_{\mathrm{A,B}} \equiv \omega_{A,B} - \omega_{\mathrm{r}}$

$\widehat{U} = e^{\frac{g}{\delta}(\hat{\sigma}_{\mathrm{A}}^{+}\hat{a} - \hat{\sigma}_{\mathrm{A}}^{-}\hat{a}^{\dagger} + \hat{\sigma}_{\mathrm{B}}^{+}\hat{a} - \hat{\sigma}_{\mathrm{B}}^{-}\hat{a}^{\dagger})}$

$$\widehat{H}^{(2)} = \frac{\hbar\omega_{\mathrm{A}}}{2}\hat{\sigma}_z^{\mathrm{A}} + \frac{\hbar\omega_{\mathrm{B}}}{2}\hat{\sigma}_z^{\mathrm{B}} + \hbar\left(\omega_{\mathrm{r}} + \frac{g^2}{\delta_{\mathrm{A}}}\hat{\sigma}_z^{\mathrm{A}} + \frac{g^2}{\delta_{\mathrm{B}}}\hat{\sigma}_z^{\mathrm{B}}\right)\hat{a}^{\dagger}\hat{a} + \hbar J(\hat{\sigma}_{\mathrm{A}}^{-}\hat{\sigma}_{\mathrm{B}}^{+} + \hat{\sigma}_{\mathrm{A}}^{+}\hat{\sigma}_{\mathrm{B}}^{-})$$



$$\frac{g}{2\pi} \simeq 105\ \mathrm{MHz}$$

$\downarrow$

$$\frac{J}{2\pi} \simeq \text{few } 10\text{'s of MHz}$$

→ Both qubits dispersively shift the resonator

→ Only virtual photons exchanged with the bus

→ Bus can also be used for readout without degrading qubit coherence

J. Majer *et al.*, Nature **449**, 443-447 (2007)

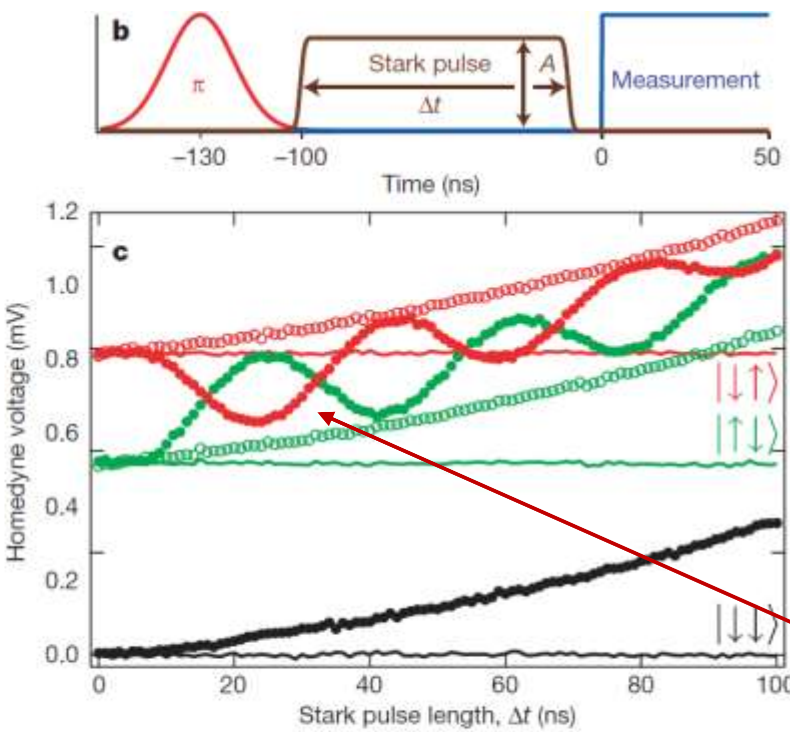
6.5 Circuit quantum electrodynamics

Dispersive protocol for quantum state transfer

$\delta_{\mathrm{A,B}} \equiv \omega_{A,B} - \omega_{\mathrm{r}}$

$\hat{U} = e^{\frac{g}{\delta}(\hat{\sigma}_{\mathrm{A}}^{+}\hat{a} - \hat{\sigma}_{\mathrm{A}}^{-}\hat{a}^{\dagger} + \hat{\sigma}_{\mathrm{B}}^{+}\hat{a} - \hat{\sigma}_{\mathrm{B}}^{-}\hat{a}^{\dagger})}$

$$\hat{H}^{(2)} = \frac{\hbar\omega_{\mathrm{A}}}{2}\hat{\sigma}_z^{\mathrm{A}} + \frac{\hbar\omega_{\mathrm{B}}}{2}\hat{\sigma}_z^{\mathrm{B}} + \hbar\left(\omega_{\mathrm{r}} + \frac{g^2}{\delta_{\mathrm{A}}}\hat{\sigma}_z^{\mathrm{A}} + \frac{g^2}{\delta_{\mathrm{B}}}\hat{\sigma}_z^{\mathrm{B}}\right)\hat{a}^{\dagger}\hat{a} + \hbar J(\hat{\sigma}_{\mathrm{A}}^{-}\hat{\sigma}_{\mathrm{B}}^{+} + \hat{\sigma}_{\mathrm{A}}^{+}\hat{\sigma}_{\mathrm{B}}^{-})$$



J. Majer *et al.*, Nature **449**, 443-447 (2007)

Protocol

→ Qubits detuned at different frequencies
  → Coupling off, $J \simeq 0$

→ Bring one qubit to $|\mathrm{e}\rangle$ using a π-pulse

→ Apply strong & detuned Stark shift pulse to bring qubits into resonance
  → $\frac{J}{2\pi} \simeq 23$ MHz for time Δt

→ After the pulse ($J \simeq 0$ again) send readout pulse to resonator

Results

→ Qubits coherently exchange population

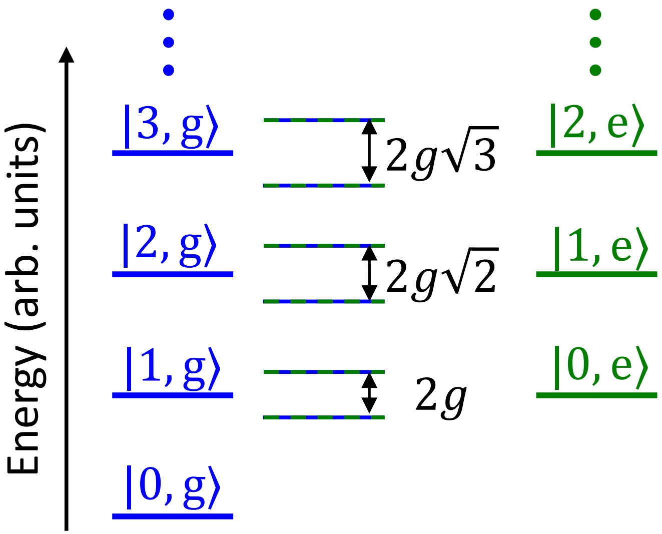
→ Curved slope due to residual Rabi drive by the off-resonant Stark tone

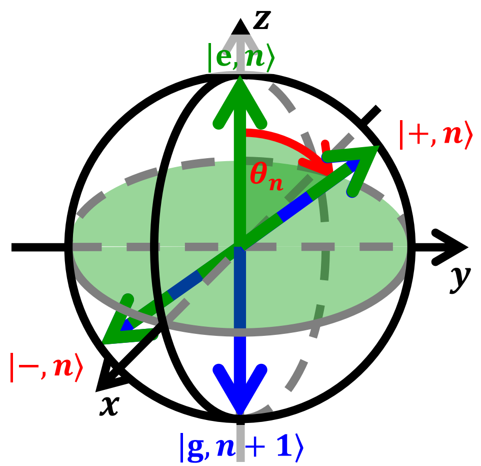
6.5 Circuit quantum electrodynamics

Resonant regime of the JC Hamiltonian

$\widehat{H}_{\mathrm{JC}}^{\mathrm{int}} = \hbar g\left(\hat{\sigma}^- \hat{a}^\dagger e^{i\delta t} + \hat{\sigma}^+ \hat{a} e^{-i\delta t}\right)$

$\delta \equiv \omega_{\mathrm{q}} - \omega_{\mathrm{r}}$





Detuning $\delta = 0$

→ $\widehat{H}_{\mathrm{JC}} = \hbar\omega_{\mathrm{r}} \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_{\mathrm{q}}}{2} \hat{\sigma}_z + \hbar g\left(\hat{a}^\dagger \hat{\sigma}^- + \hat{a} \hat{\sigma}^+\right)$

→ Degenerate levels $\{|n, \mathrm{e}\rangle, |n+1, \mathrm{g}\rangle\}$ split into JC doublets $\{|\pm, n\rangle\}$ due to coupling $g\sqrt{n+1}$

→ Ground state is the vacuum $|\mathrm{g}, 0\rangle$

Dynamics?

→ Create non-eigenstate, e.g., $|n, \mathrm{e}\rangle$ by nonadiabatically detuning the qubit, sending a π-pulse and tuning it back to resonance

→ Coherent population exchange between qubit and resonator $|n, \mathrm{e}\rangle \leftrightarrow |n+1, \mathrm{g}\rangle$

→ Vacuum Rabi oscillations with n-photon Rabi frequency $\Omega_{\mathrm{n}} \equiv 2g\sqrt{n+1}$

A. Blais *et al.*, Phys. Rev. A **69**, 062320 (2004)

6.5 Circuit quantum electrodynamics

Coherent dynamics

→ Dynamics governed by interaction Hamiltonian and initial state $|\Psi_0\rangle \equiv |\Psi_\mathrm{q}(t=0), \Psi_\mathrm{r}(t=0)\rangle$

→ On resonance ($\omega_\mathrm{q} = \omega_\mathrm{r}$) → $\hat{H}_\mathrm{JC}^\mathrm{int} = \hbar g\left(\hat{\sigma}^- \hat{a}^\dagger + \hat{\sigma}^+ \hat{a}\right)$ time-independent

→ $|\Psi_{|\Psi_0\rangle}(t)\rangle \equiv |\Psi_\mathrm{q}(t), \Psi_\mathrm{r}(t)\rangle = \mathrm{e}^{igt(\hat{\sigma}^- \hat{a}^\dagger + \hat{\sigma}^+ \hat{a})}|\Psi_0\rangle = \sum_n \frac{(igt)^n}{n!}\left(\hat{\sigma}^- \hat{a}^\dagger + \hat{\sigma}^+ \hat{a}\right)^n |\Psi_0\rangle$

→ Initial state is ground state, $|\Psi_0\rangle = |\mathrm{g}, 0\rangle$

→ $|\Psi_{|\mathrm{g},0\rangle}(t)\rangle = \sum_n \frac{(igt)^n}{n!}\left(\hat{\sigma}^- \hat{a}^\dagger + \hat{\sigma}^+ \hat{a}\right)^n |\mathrm{g}, 0\rangle$

→ $\left(\hat{\sigma}^- \hat{a}^\dagger + \hat{\sigma}^+ \hat{a}\right)^n |\mathrm{g}, 0\rangle = \begin{cases} |\mathrm{g}, 0\rangle & \text{for} \quad n = 0 \\ 0 & \text{for} \quad n > 0 \end{cases}$

→ $|\Psi_{|\mathrm{g},0\rangle}(t)\rangle = |\mathrm{g}, 0\rangle$

→ No time evolution when starting from an eigenstate!

6.5 Circuit quantum electrodynamics

Coherent dynamics

$$|\Psi_{|\Psi_0\rangle}(t)\rangle = \sum_n \frac{(igt)^n}{n!}\left(\hat{\sigma}^-\hat{a}^\dagger + \hat{\sigma}^+\hat{a}\right)^n |\Psi_0\rangle$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

→ Initial state is $|\mathrm{e},n\rangle$

→ Coherent dynamics in doublet $|\pm,n\rangle$

→ $|\Psi_{|\mathrm{e},n\rangle}(t)\rangle = \sum_n \frac{(igt)^n}{n!}\left(\hat{\sigma}^-\hat{a}^\dagger + \hat{\sigma}^+\hat{a}\right)^n|\mathrm{e},n\rangle$

→ $\left(\hat{\sigma}^-\hat{a}^\dagger + \hat{\sigma}^+\hat{a}\right)^m|\mathrm{e},n\rangle = \begin{cases} \sqrt{n+1}^{2n}\,|\mathrm{e},n\rangle & \text{for} \quad m = 2n \\ \sqrt{n+1}^{2n+1}|\mathrm{g},n+1\rangle & \text{for} \quad m = 2n+1 \end{cases}$

→ $|\Psi_{|\mathrm{e},n\rangle}(t)\rangle = \sum_n (-1)^n \frac{\left(g\sqrt{n+1}t\right)^{2n}}{2n!}|\mathrm{e},n\rangle + i\sum_n (-1)^n \frac{\left(g\sqrt{n+1}t\right)^{2n+1}}{(2n+1)!}|\mathrm{g},n+1\rangle$

→ $|\Psi_{|\mathrm{e},n\rangle}(t)\rangle = \cos\left(g\sqrt{n+1}t\right)|\mathrm{e},n\rangle + i\sin\left(g\sqrt{n+1}t\right)|\mathrm{g},n+1\rangle$

→ Qubit and resonator exchange an excitation at rate $g\sqrt{n+1}$

→ Vacuum Rabi oscillations

→ Initial state has arbitrary cavity component, $|\Psi_{|\mathrm{e},\Psi_\mathrm{r}\rangle}\rangle \equiv |\mathrm{e}\rangle \otimes \sum_n c_n|n\rangle = \sum_n c_n|\mathrm{e},n\rangle$

→ $|\Psi_{|\mathrm{e},n\rangle}(t)\rangle = \sum_n c_n\left[\cos\left(g\sqrt{n+1}t\right)|\mathrm{e},n\rangle + i\sin\left(g\sqrt{n+1}t\right)|\mathrm{g},n+1\rangle\right]$

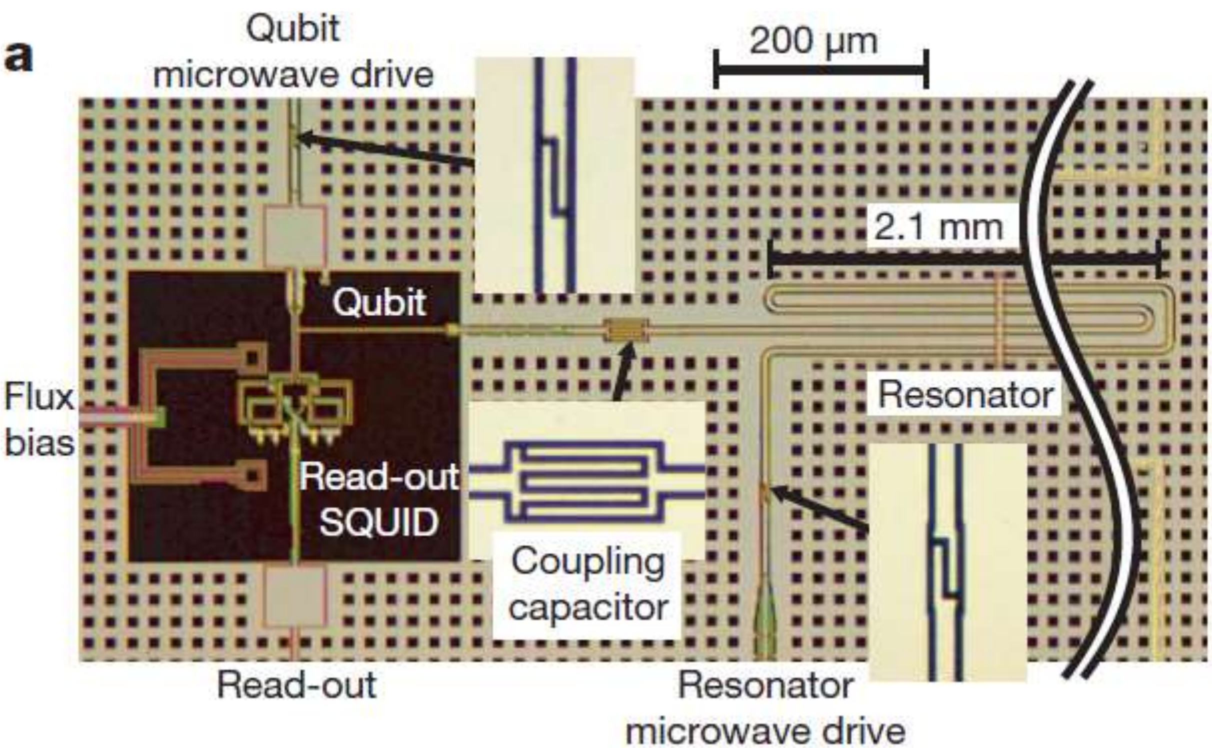
→ Superposition of noncommensurate oscillations

→ Beatings, collapse and revivals etc.

Experimental demonstration of vacuum Rabi oscillations

→ Phase qubit capacitively coupled to a coplanar waveguide resonator

→ Readout method: Switching method using a dedicated readout SQUID



→ Coupling strength

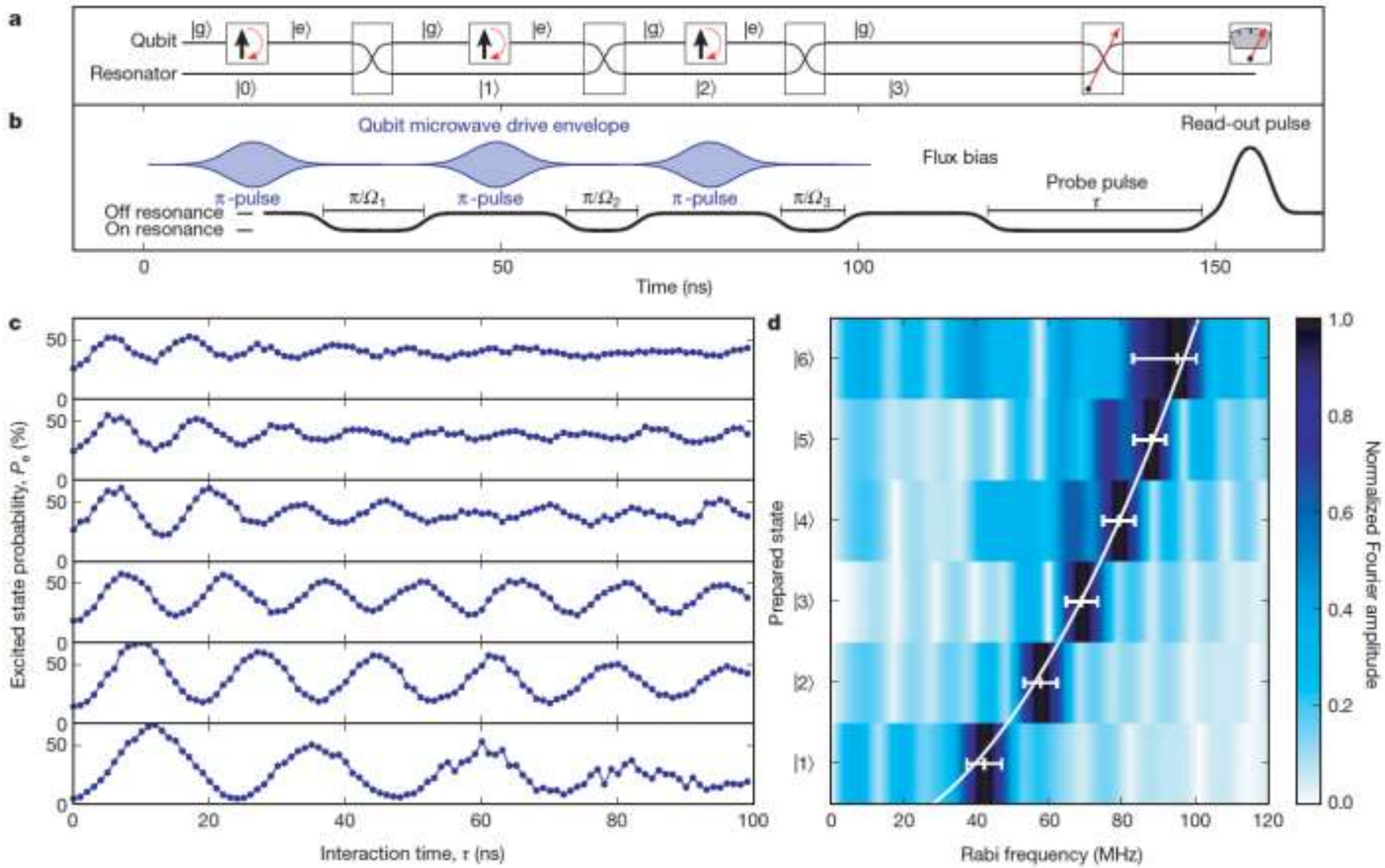
→ $\frac{g}{2\pi} = 36\ \mathrm{MHz}$

→ Coherence times

→ $T_1^{\mathrm{q}} \simeq 550\ \mathrm{ns}$
$T_2^{\mathrm{q}} \simeq 100\ \mathrm{ns}$
$T_1^{\mathrm{r}} \simeq 1\ \mu\mathrm{s}$
$T_2^{\mathrm{r}} \simeq 2\ \mu\mathrm{s}$

M. Hofheinz *et al.*, Nature **454**, 310-314 (2004)

Experimental demonstration of vacuum Rabi oscillations

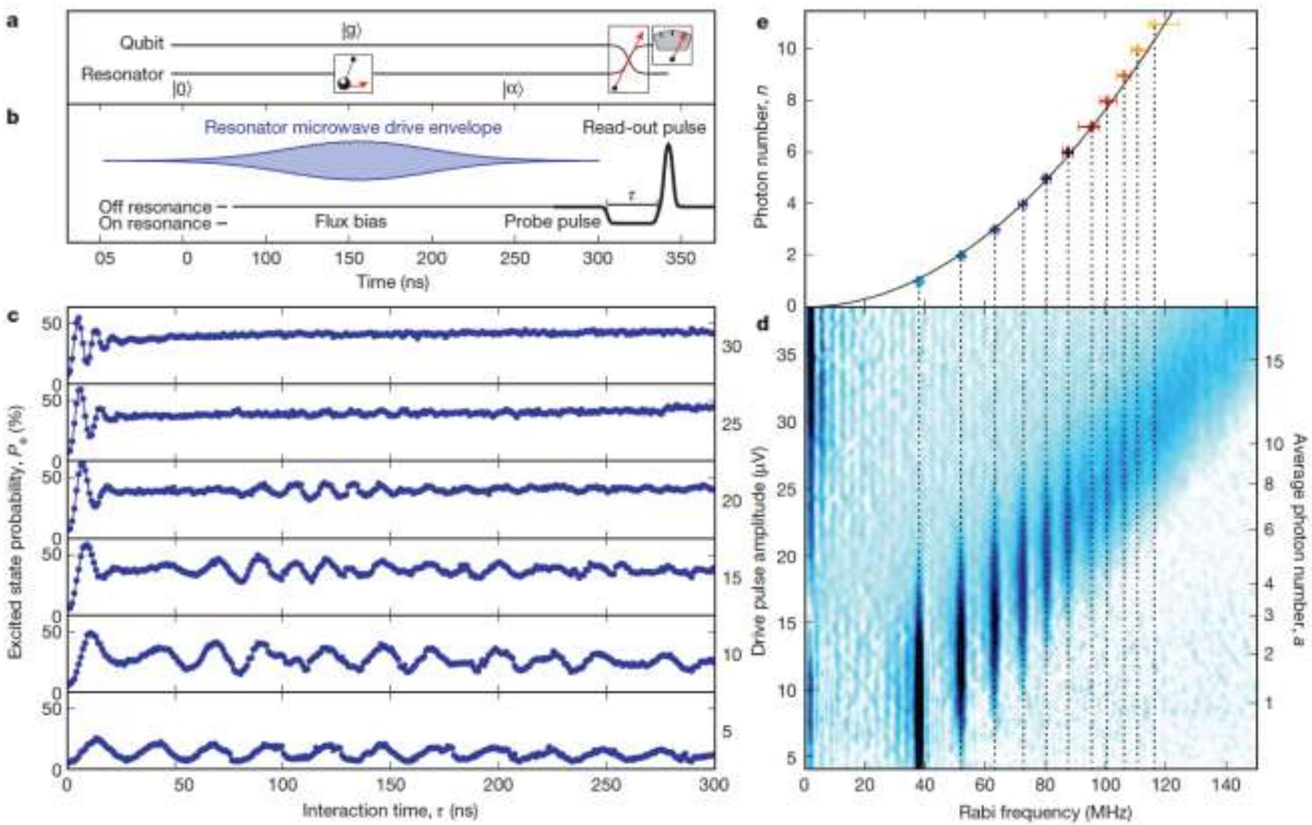


→ Sequential generation of Fock states putting one excitation after another

→ Confirms $\sqrt{n+1}$-dependence of the vacuum Rabi frequency

M. Hofheinz *et al.*, Nature **454**, 310-314 (2004)

Experimental demonstration of vacuum Rabi oscillations

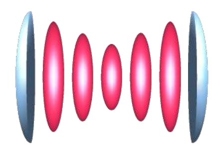


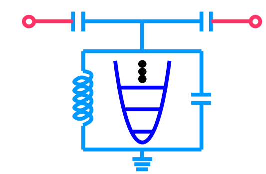
→ Initial state $|g, \alpha\rangle$ → Beatings and collapse & revival features observed

M. Hofheinz *et al.*, Nature **454**, 310-314 (2004)

Strong coupling regime

→ Definition motivated goal of observing quantum coherent dynamics
→ Vacuum Rabi dynamics happens at rate g
→ Qubit decays at rate $\gamma \approx \Gamma_1 + \Gamma_\varphi$, resonator at rate κ
→ $g > \kappa + \gamma$ required to see at leat one Vacuum Rabi flop
  → Strong coupling regime
  → „Order-of-magnitude“ citerion





Parameter	Symbol	3D optical	3D microwave	1D circuit
Resonance or transition frequency	$\omega_r/2\pi,\ \Omega/2\pi$	350 THz	51 GHz	10 GHz
Vacuum Rabi frequency	$g/\pi,\ g/\omega_r$	220 MHz, 3×10^{-7}	47 kHz, 1×10^{-7}	100 MHz, 5×10^{-3}
Transition dipole	d/ea_0	~1	1×10^3	2×10^4
Cavity lifetime	$1/\kappa, Q$	10 ns, 3×10^7	1 ms, 3×10^8	160 ns, 10^4
Atom lifetime	$1/\gamma$	61 ns	30 ms	2 μs
Atom transit time	t_{transit}	$\geq 50\ \mu$s	100 μs	∞
Critical atom number	$N_0 = 2\gamma\kappa/g^2$	6×10^{-3}	3×10^{-6}	$\leq 6\times10^{-5}$
Critical photon number	$m_0 = \gamma^2/2g^2$	3×10^{-4}	3×10^{-8}	$\leq 1\times10^{-6}$
Number of vacuum Rabi flops	$n_{\mathrm{Rabi}} = 2g/(\kappa+\gamma)$	~10	~5	$\sim10^2$

Large coupling strength outweighs restricted coherence of superconducting circuits

A. Blais *et al.*, Phys. Rev. A **69**, 062320 (2004)

Selection rules & Symmetry breaking

$$\widehat{H}_{\mathrm{TLS}} = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$$

→ Selection rules ↔ Concept of parity

- → Interaction can be dipolar, quadrupolar,…
- → Dipolar interaction
  - → Only odd transitions (between levels of different parity) are allowed

→ Natural atoms

- → Symmetry of atomic potential → Strict selection rules
- → Dipolar → EM field naturally couples to dipole moment
- → Quadrupolar → Requires strong field gradient (crystal fields)

→ Quantum circuits

- → Normally dipolar, quadrupolar for gradiometric designs
- → Well-defined parity only for symmetric qubit potential
- → Selection rules not always present

→ Experimental access to selection rules

- → Multi-photon excitations

Y-X Liu *et al.*, Phys. Rev. Lett. **95**, 087001 (2005).
F. Deppe *et al.*, Nature Phys. **4** , 686-691 (2008).
T. Niemczyk *et al.*, ArXiv:1107.0810 (2011)

6.5 Circuit quantum electrodynamics

Selection rules & Symmetry breaking

$\hat{H}_{\mathrm{TLS}} = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$

$\omega_{\mathrm{q}} = \sqrt{\epsilon^2 + \Delta^2}$

$\sin\theta = \frac{\Delta}{\hbar\omega_{\mathrm{q}}}, \cos\theta = \frac{\varepsilon}{\hbar\omega_{\mathrm{q}}}$

→ Parity operator $\hat{\Pi}$

- → Eigenvalues $+1$ (even parity) and -1 (odd parity)
- → Qubit parity operator is $\hat{\Pi} = -\hat{\sigma}_z$
  - → $\hat{\Pi}|\mathrm{g}\rangle = |\mathrm{g}\rangle$ and $\hat{\Pi}|\mathrm{e}\rangle = -|\mathrm{e}\rangle$
- → Parity related to symmetry of the potential
  - → Resonator potential symmetric (parabola!)
    - → Parity well defined, $\hat{\Pi}_{\mathrm{r}} = \mathrm{e}^{i\pi\hat{a}^\dagger\hat{a}}$
  - → Flux qubit potential only symmetric at degeneracy point $\varepsilon = 0$
    - → For $\varepsilon \neq 0$, parity is no longer well defined

→ Parity gives rise to selection rules

- → Interaction operators $\hat{B}_\pm$ exhibit symmetries!
- → Even operator $\hat{B}_+$ $\leftrightarrow$ $\hat{\Pi}\hat{B}_+\hat{\Pi} = +\hat{B}_+$ $\leftrightarrow$ Commutator $[\hat{\Pi}, \hat{B}_+] = 0$

  Odd operator $\hat{B}_-$ $\leftrightarrow$ $\hat{\Pi}\hat{B}_-\hat{\Pi} = -\hat{B}_-$ $\leftrightarrow$ Anticommutator $\{\hat{\Pi}, \hat{B}_+\} = 0$
- → $\langle\Psi_{\mathrm{odd}}|\hat{B}_+|\Psi_{\mathrm{even}}\rangle = \langle\Psi_{\mathrm{odd}}|\hat{\Pi}\hat{B}_+\hat{\Pi}|\Psi_{\mathrm{even}}\rangle = -\langle\Psi_{\mathrm{odd}}|\hat{B}_+|\Psi_{\mathrm{even}}\rangle = 0$

  $\langle\Psi_{\mathrm{even}}|\hat{B}_+|\Psi_{\mathrm{odd}}\rangle = \langle\Psi_{\mathrm{even}}|\hat{\Pi}\hat{B}_+\hat{\Pi}|\Psi_{\mathrm{odd}}\rangle = -\langle\Psi_{\mathrm{even}}|\hat{B}_+|\Psi_{\mathrm{odd}}\rangle = 0$
- → Transitions between levels of different parity forbidden for even operators
- → Investigate qubit-driving-field coupling operators

6.5 Circuit quantum electrodynamics

Selection rules & Symmetry breaking

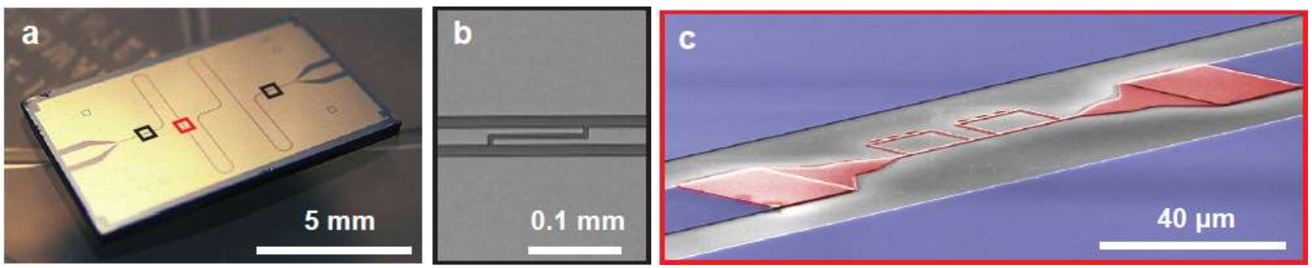
$$\widehat{H}_{\mathrm{TLS}} = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$$

$$\omega_{\mathrm{q}} = \sqrt{\epsilon^2 + \Delta^2}$$

$$\sin\theta = \frac{\Delta}{\hbar\omega_{\mathrm{q}}}, \cos\theta = \frac{\varepsilon}{\hbar\omega_{\mathrm{q}}}$$

→ Example: Flux qubit inductively coupled to resonator

$$\rightarrow \widehat{H}^{(1)} = \underbrace{\frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x}_{\text{Qubit}} + \underbrace{\hbar\omega_r \hat{a}^\dagger \hat{a}}_{\text{Resonator}} + \underbrace{\hbar g\, \hat{\sigma}_z(\hat{a}^\dagger + \hat{a})}_{\text{JC coupling}} + \underbrace{\frac{\Omega}{2}\hat{\sigma}_z \cos\omega t}_{\text{Qubit drive}}$$



First-order Hamiltonian using $\omega_{\mathrm{q}} = \omega$

$$\rightarrow \widehat{H}^{(1)} = \hbar\frac{\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar\omega_{\mathrm{r}}\hat{a}^\dagger\hat{a} - \hbar g \sin\theta\left(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+\right) - \frac{\Omega}{4}\underbrace{\sin\theta\left(\hat{\sigma}^- e^{i\omega t} + \hat{\sigma}^+ e^{-i\omega t}\right)}$$

One-photon Rabi drive can excite qubit for any ϵ, including $\epsilon = 0$

Second-order Hamiltonian using $\omega_{\mathrm{q}} = 2\omega$

$$\rightarrow \widehat{H}^{(2)} = \hbar\frac{\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar\omega_{\mathrm{r}}\hat{a}^\dagger\hat{a} - \hbar g \sin\theta\left(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+\right)$$

$$\underbrace{-\frac{\Omega^2}{4\Delta}\sin^2\theta\cos\theta\left(\hat{\sigma}^- e^{i2\omega t} + \hat{\sigma}^+ e^{-i2\omega t}\right) - \frac{\Omega^2}{8\Delta}\sin^3\theta\,\hat{\sigma}_z}$$

Two-photon Rabi drive can excite qubit only for $\epsilon \neq 0$

Selection rules & Symmetry breaking

$\widehat{H}_{\mathrm{TLS}} = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$

$\omega_{\mathrm{q}} = \sqrt{\epsilon^2 + \Delta^2}$

$\sin\theta = \frac{\Delta}{\hbar\omega_{\mathrm{q}}}, \cos\theta = \frac{\varepsilon}{\hbar\omega_{\mathrm{q}}}$

→ Qubit degeneracy point ($\varepsilon = 0$) interaction picture

→ One-photon drive operator $\hat{\sigma}_x$ has odd parity, $\left\{-\hat{\sigma}_{\mathrm{z}}, \frac{\Omega}{4}\hat{\sigma}_{\mathrm{x}}\right\} = 0$

→ One-photon transitions allowed

→ Two-photon drive operator has even parity, $\left[-\hat{\sigma}_z, \frac{\Omega^2}{8\Delta}\hat{\sigma}_{\mathrm{z}}\right] = 0$

→ Two-photon transitions forbidden

→ Dipolar selection rules

→ Artificial atom behaves like natural atom

→ Away from qubit degeneracy point ($\varepsilon \neq 0$) interaction picture

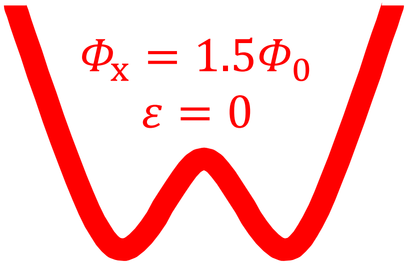
→ $\hat{\sigma}_z \rightarrow \cos\theta\,\hat{\sigma}_z - \sin\theta\,\hat{\sigma}_x$ → Drive operator does not have a well-defined parity

→ Both one- and two-photon transitions allowed

→ Artificial atom different from natural atom

→ Physics in circuit QED goes beyond the physics of cavity QED

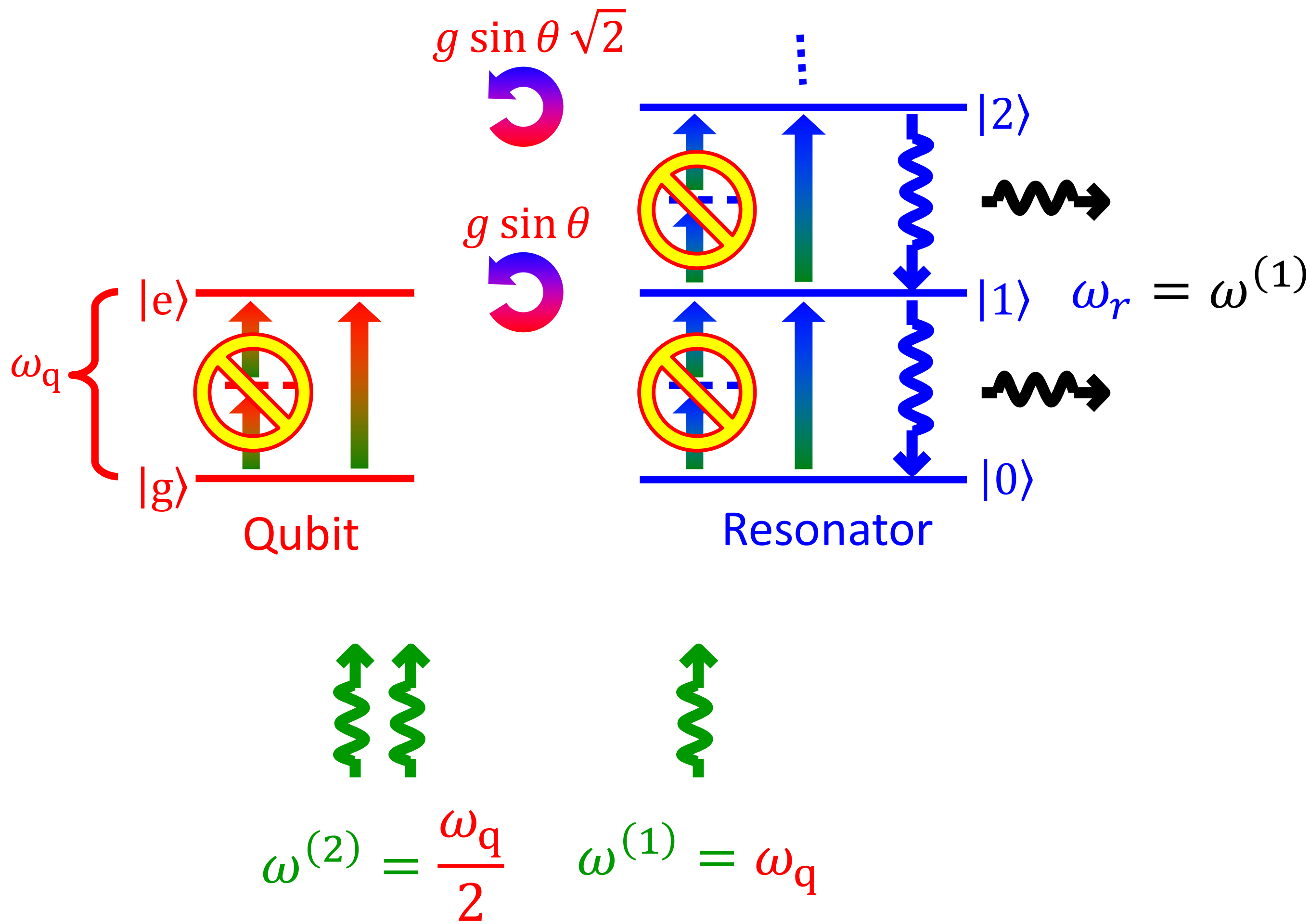
Selection rules & Symmetry breaking



qubit potential symmetric

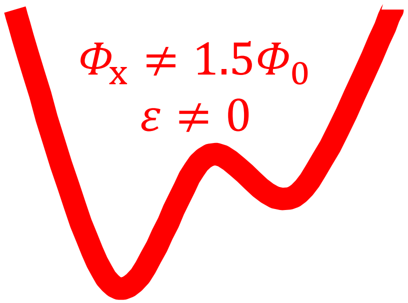
dipolar selection rules
↓
no transitions under two-photon driving

Example: $\omega_q = \omega_r$



Selection rules & Symmetry breaking

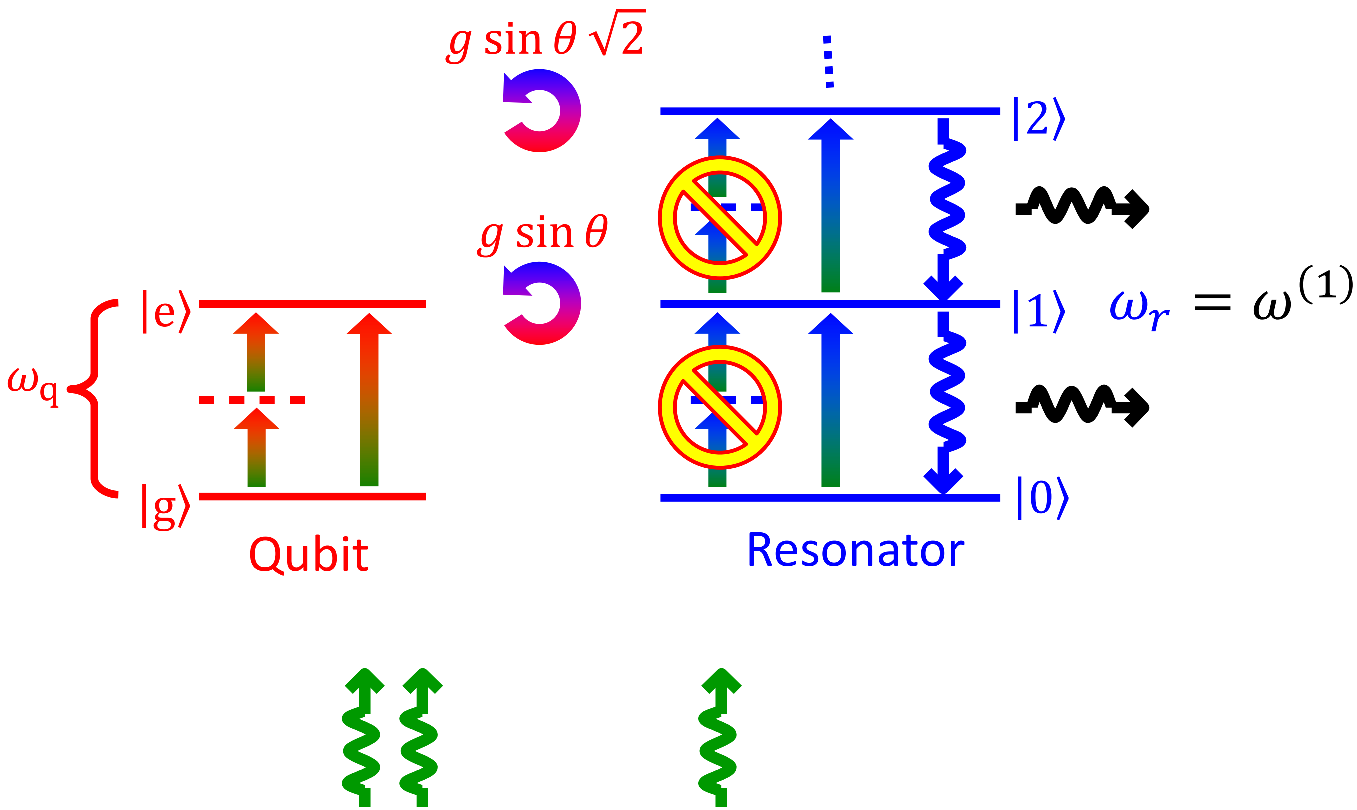
Example: $\omega_\mathrm{q} = \omega_\mathrm{r}$



symmetry of qubit potential broken

one- and two-photon excitations coexist

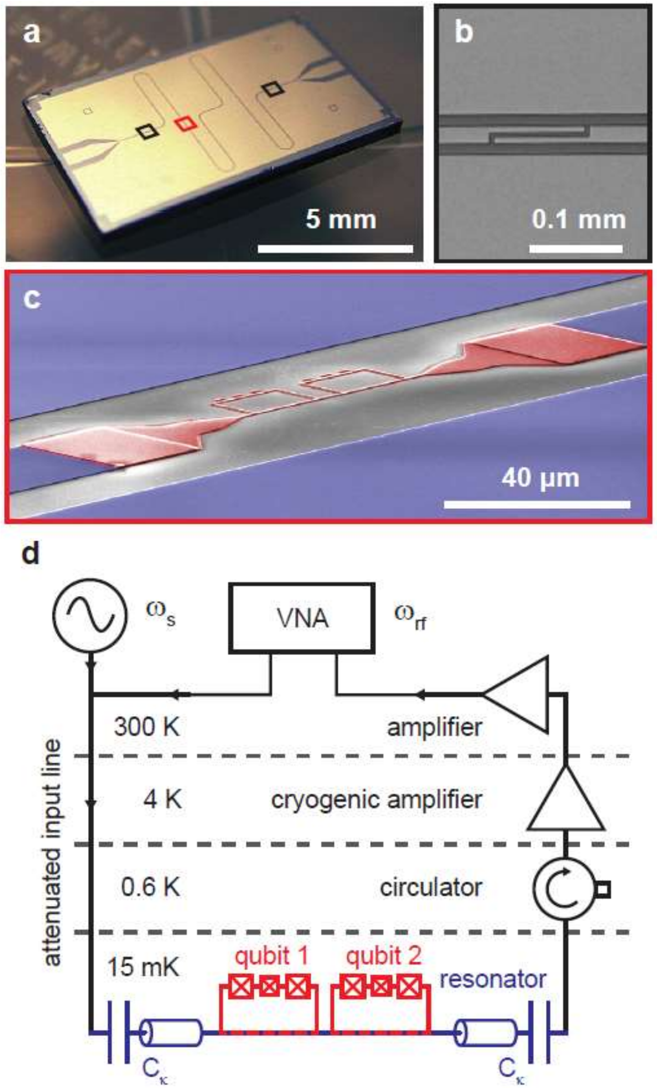
up-conversion dynamics

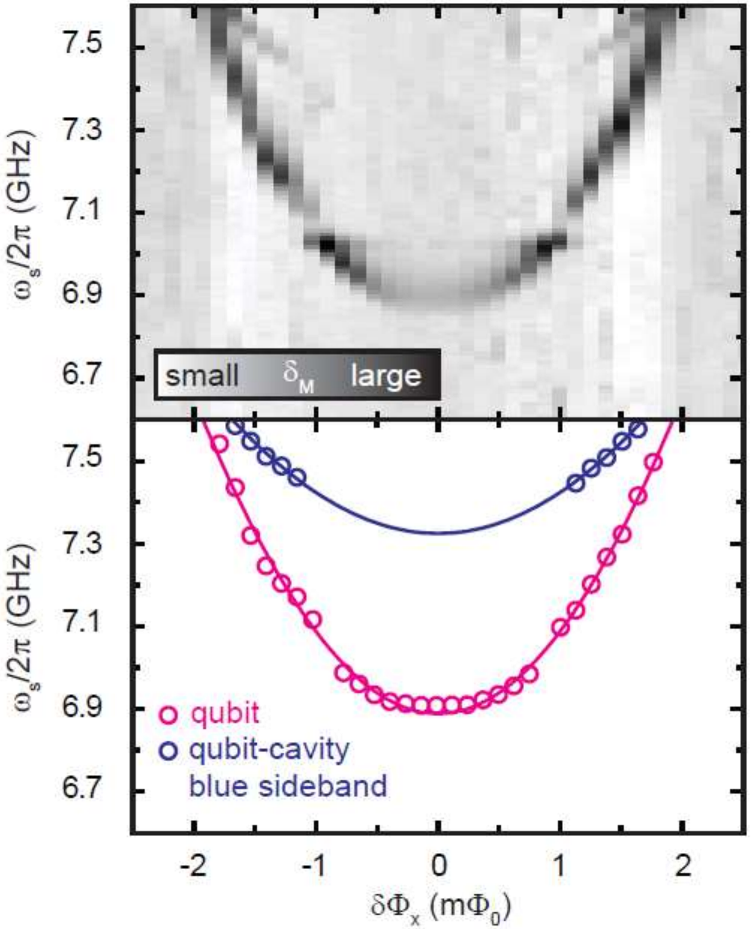


$$\omega^{(2)} = \frac{\omega_\mathrm{q}}{2} \qquad \omega^{(1)} = \omega_\mathrm{q}$$

Selection rules & Symmetry breaking

$$\hat{H}_{\mathrm{TLS}} = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$$

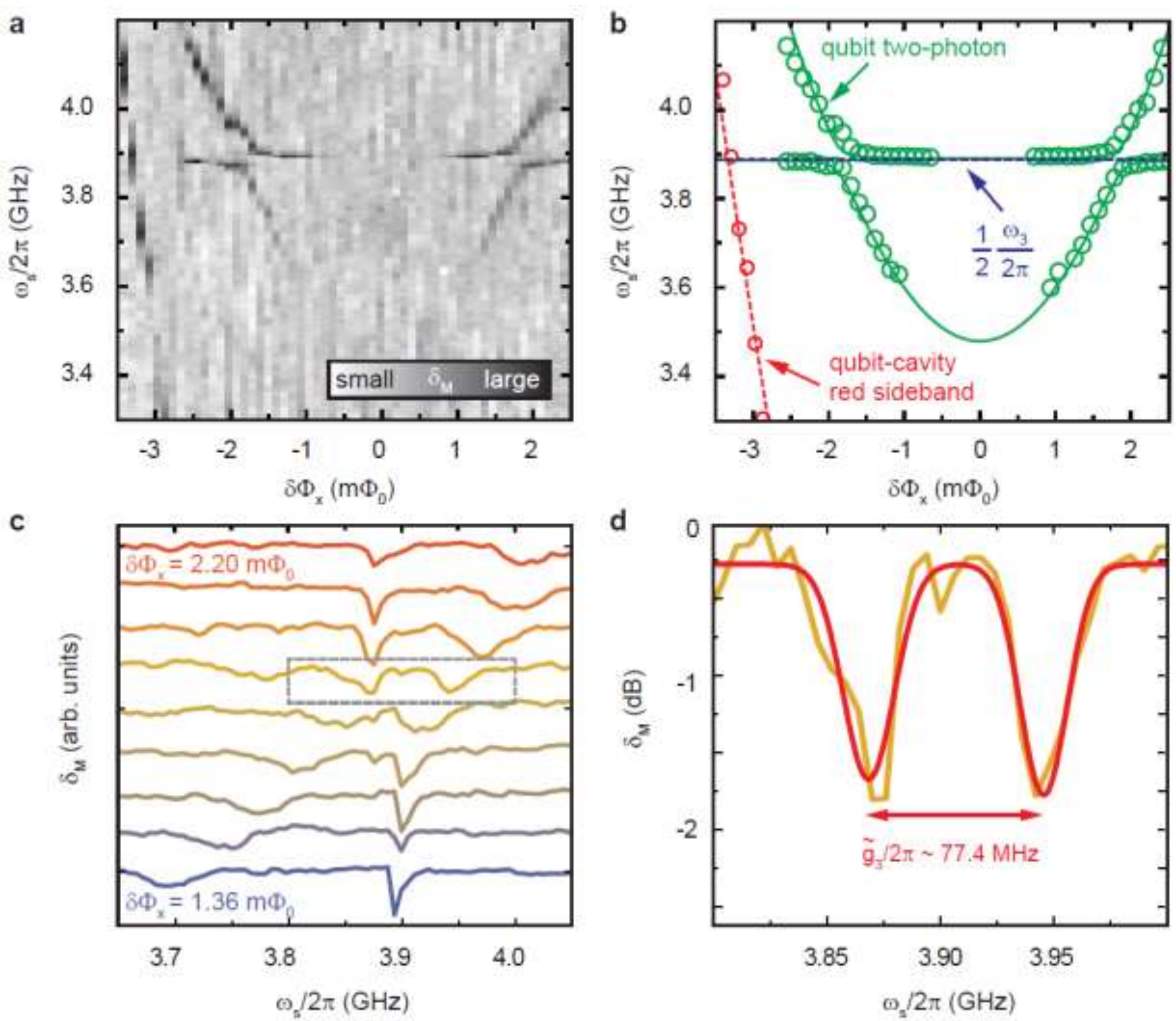




In two-tone spectroscopy, the one-photon dip is clearly visible at the qubit degeneracy point

Selection rules & Symmetry breaking

$$\hat{H}_{\mathrm{TLS}} = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$$



Two-photon dip not observed near the qubit degeneracy point

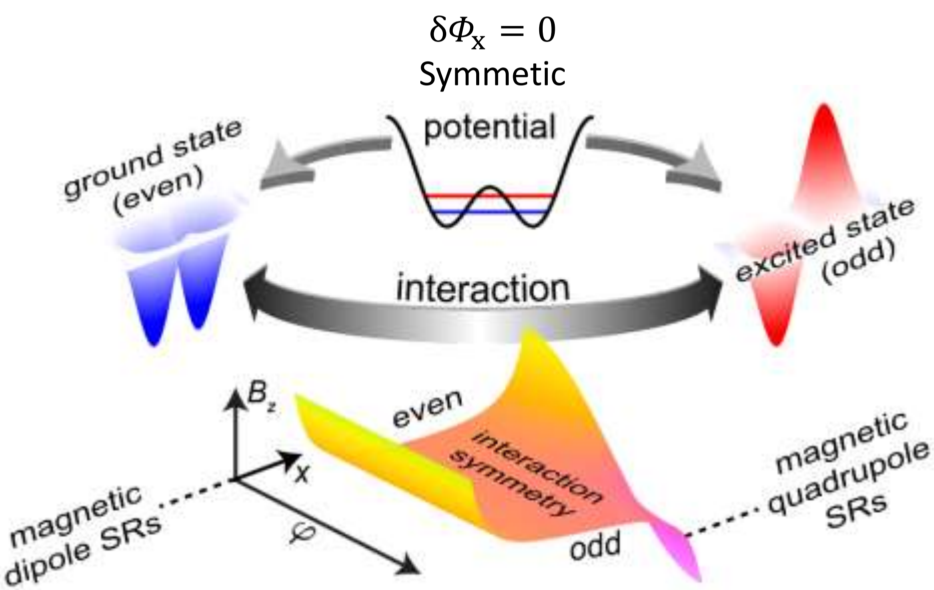
Bare qubit resonator anticrossing observable (no drive photons put into resonator)

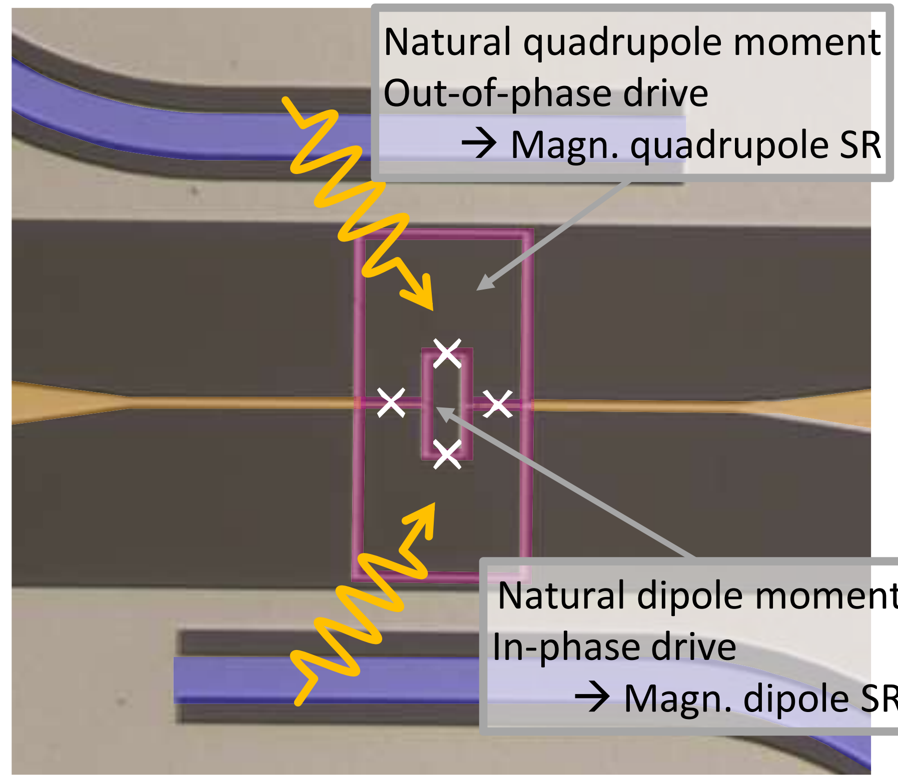
6.5 Quadrupole moment and selective driving

Gradiometric gap-tunable flux qubit

J. Goetz *et al.*, Phys. Rev. Lett. **121**, 060503 (2018)

$$\hat{H}_{\mathrm{TLS}} = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$$





→ Antenna pair allows for in-situ control of interaction parity via phase difference φ

→ Probe magnetic dipole & quadrupole SRs

→ Gradiometric $\varepsilon(\Phi_{\mathrm{x}})$ with tunable $\Delta(\Phi_{\mathrm{x}})$

→ Potential symmetry controlled via gradiometer loop

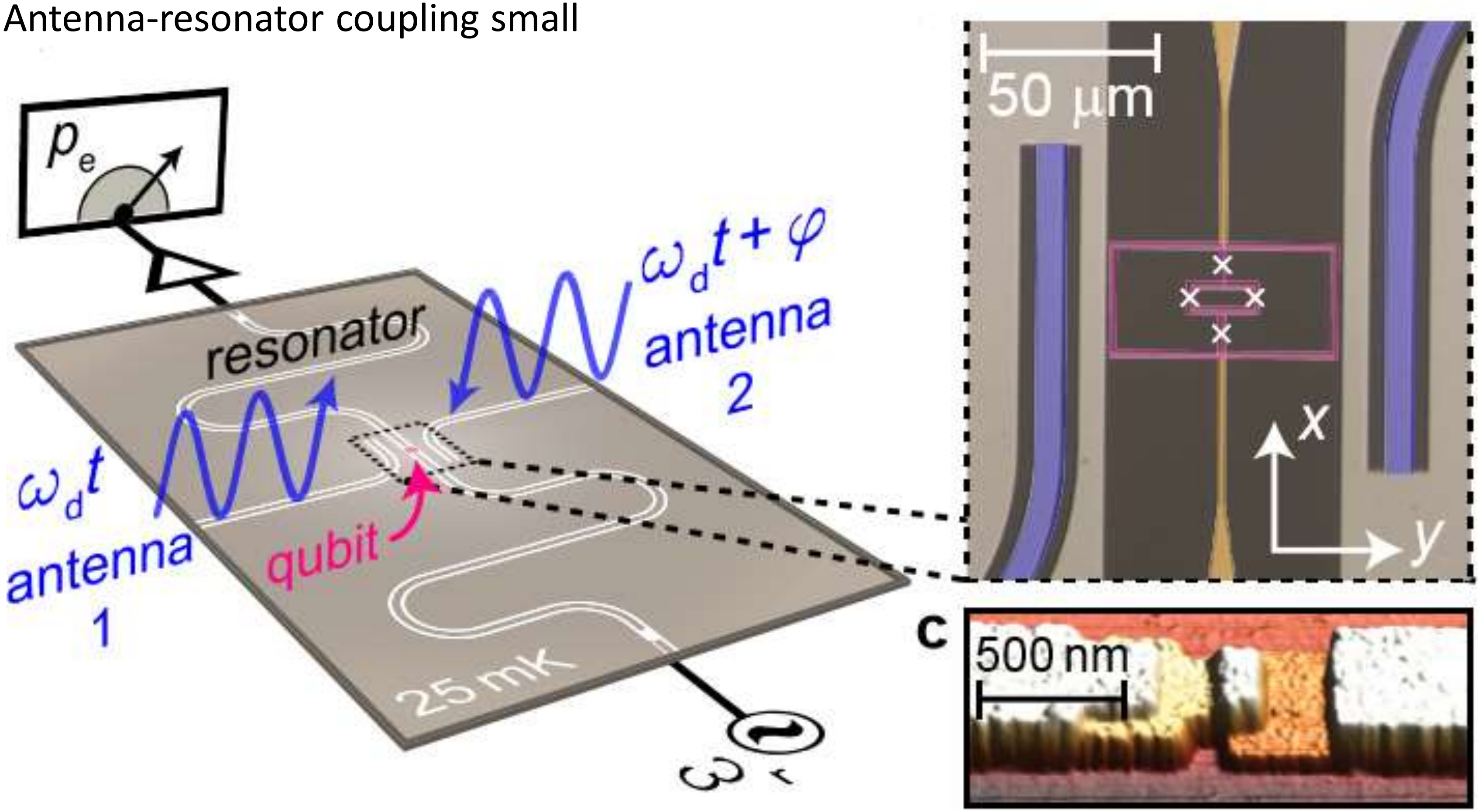
Readout → CPW Resonator

J. Goetz *et al.*, Phys. Rev. Lett. **121**, 060503 (2018)

$$\hat{H}_{\mathrm{TLS}} = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$$

→ Qubit at current antinode of fundamental mode

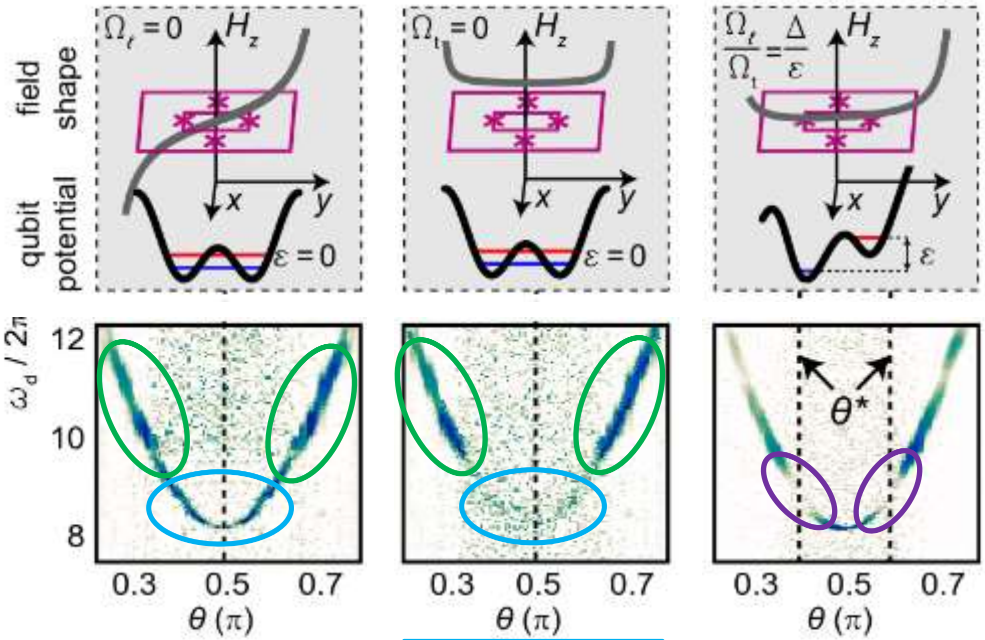
→ Antenna-resonator coupling small



Readout → CPW Resonator

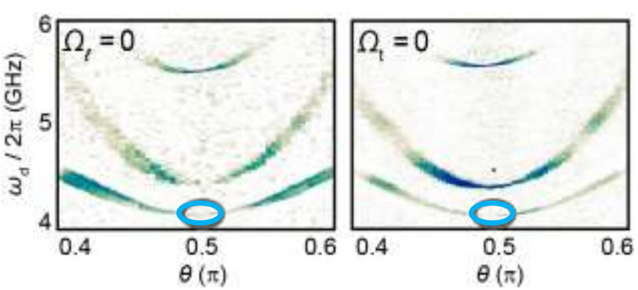
J. Goetz *et al.*, Phys. Rev. Lett. **121**, 060503 (2018)

$$\hat{H}_{\mathrm{TLS}} = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$$



Magn. quadrupole SR

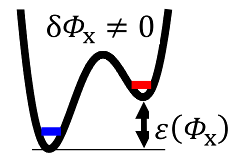
Magn. dipole SR



Two-photon signature

$\theta \equiv \mathrm{atan}\frac{\Delta}{\varepsilon} \neq \frac{\pi}{2}$

→ No selection rules (parity not well defined)

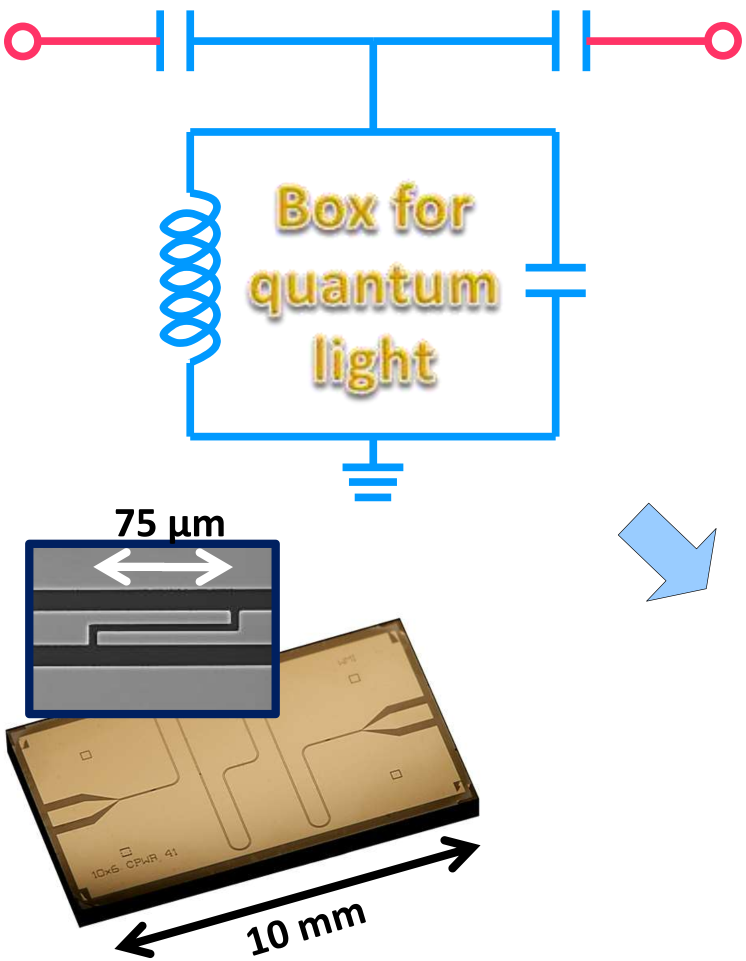


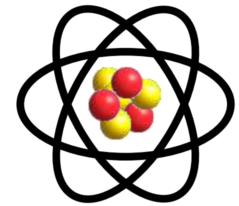
→ Quadrupolar SR restored at $\theta^\star \equiv \mathrm{atan}\frac{\Omega_\ell}{\Omega_t} \neq \frac{\pi}{2}$

- → At $\theta^\star$, the interaction operator in the qubit eigenframe is pure $\hat{\sigma}_z$
- → Interaction operator even
- → $|g\rangle \rightarrow |e\rangle$ transition forbidden
- → Restoration of SRs

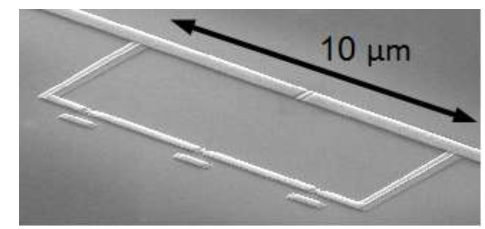
6.5 Circuit quantum electrodynamics

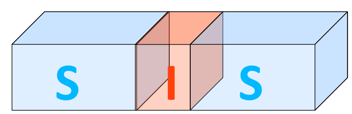
Ultrastrong light-matter coupling



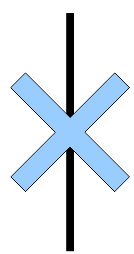


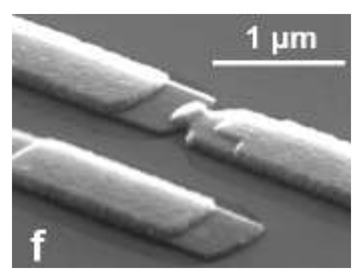
Engineered quantum matter





$L_J(\Phi)$





Unusual coupling

Ultrastrong light-matter coupling

→ Qubit-resonator coupling naturally determined by quantum Rabi Hamiltonian
  → $\widehat{H}_{\mathrm{qr}} = \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar\omega_{\mathrm{r}}\hat{a}^\dagger\hat{a} + \hbar g\left(\hat{\sigma}^+\hat{a} + \hat{\sigma}^-\hat{a}^\dagger + \hat{\sigma}^+\hat{a}^\dagger + \hat{\sigma}^-\hat{a}\right)$
→ Maximum coupling strengths
  → Linear coupling
    → Mutual inductance (capacitance) limited by self inductance (capacitance)
    → $g \leq \sqrt{\omega_{\mathrm{q}}\omega_{\mathrm{r}}}$
    → Simplified discussion → $\omega_{\mathrm{q}} = \omega_{\mathrm{r}} \equiv \omega$
      → Dimensionless coupling strength $g/\omega \leq 1$
  → Superconducting circuits
    → Nonlinear coupling via kinetic inductance or Josephson inductance
    → $g/\omega > 1$ expected to be possible

Ultrastrong light-matter coupling

→ Regimes of light-matter coupling
- → Jaynes-Cummings regime ($g/\omega \ll 1$)
  - → Coupling is small perturbation to self energy terms
  - → Excitation exchange dynamics (vacuum Rabi oscillations)
  - → Counterrotating terms can be neglected by RWA
  - → Number of excitations is a conserved quantity
  - → Ground state $|0, \mathrm{g}\rangle$
- → Ultrastrong coupling (USC) regime
  - → Deviations from JC Hamiltonian observable in experiment
  - → Ground state has contributions with $n > 0$
  - → Perturbative treatment still possible (modified population oscillation physics)
  - → In practice typically for $g/\omega \gtrsim 0.1$
- → Deep strong coupling regime $g/\omega > 1.5$
  - → Parity chain physics (coherent dynamics happens within two subgoups of states characterized by odd/even parity)
- → The „dark regime of USC“ ($g/\omega \simeq 1$)
  - → No intuition/analytics, only numerics

6.5 Circuit quantum electrodynamics

Ultrastrong light-matter coupling

$$\hat{H}_{\mathrm{JC}} = \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar\omega_{\mathrm{r}}\hat{a}^\dagger\hat{a} + \hbar g\left(\hat{\sigma}^+\hat{a} + \hat{\sigma}^-\hat{a}^\dagger\right)$$

↓ RWA breaks down

$$\hat{H}_{\mathrm{qr}} = \frac{\hbar\omega_{\mathrm{q}}}{2}\hat{\sigma}_z + \hbar\omega_{\mathrm{r}}\hat{a}^\dagger\hat{a} + \hbar g\left(\hat{\sigma}^+\hat{a} + \hat{\sigma}^-\hat{a}^\dagger + \hat{\sigma}^+\hat{a}^\dagger + \hat{\sigma}^-\hat{a}\right)$$

Ultrastrong coupling regime

→ Counterrotating term effects are observable in experiment

→ In practice typically for $\frac{g}{\omega} \gtrsim 10\%$

Experimental strategies

→ Dynamics (no more vacuum Rabi oscillations) → Complicated & demanding

→ Ground state photons → Demanding, few photons

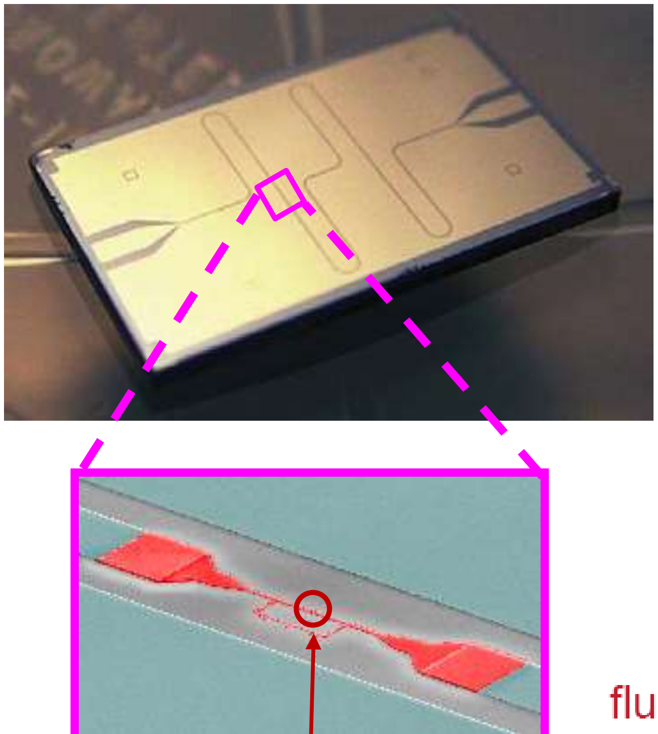
→ Spectroscopy (anticrossing) → Only quantitative change

→ Nonconservation of excitation number → Multimode setup, natural!

6.5 Circuit quantum electrodynamics

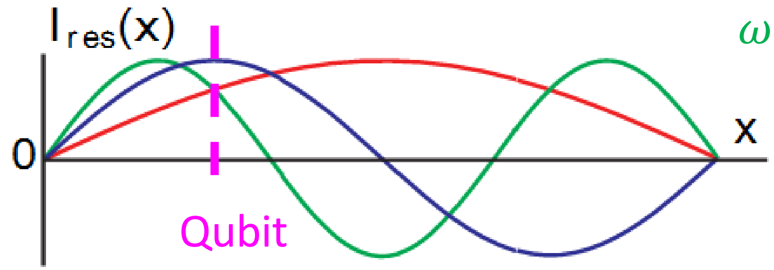
Ultrastrong light-matter coupling

Niobium CPW resonator



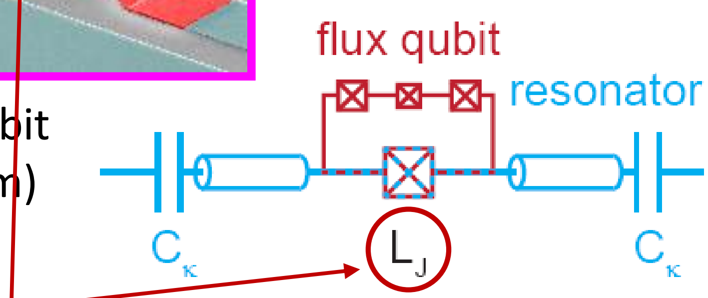
3JJ flux qubit
(aluminum)

$\omega_1/2\pi = 2.70\ \mathrm{GHz}$
$\omega_2/2\pi = 5.35\ \mathrm{GHz}$
$\omega_3/2\pi = 7.78\ \mathrm{GHz}$



4th harmonic has node
5th harmonic quite detuned

$$\frac{\hat{H}}{\hbar} = \frac{\omega_q}{2}\hat{\sigma}_z + \sum_{k=1}^{3}\omega_k \hat{a}_k^\dagger \hat{a}_k + \sum_{k=1}^{3} g_k \hat{\sigma}_x\left(\hat{a}_k^\dagger + \hat{a}_k\right)$$

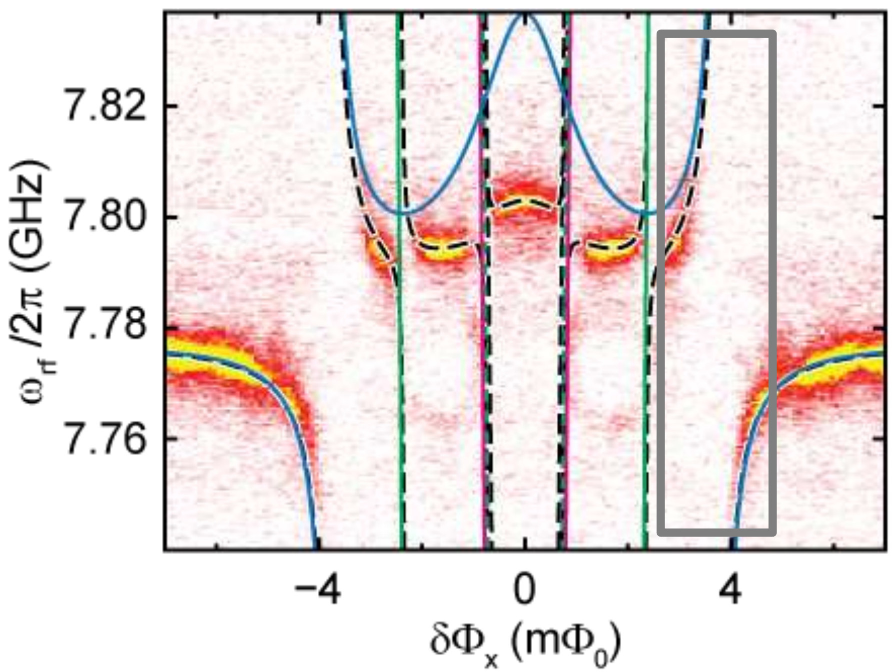


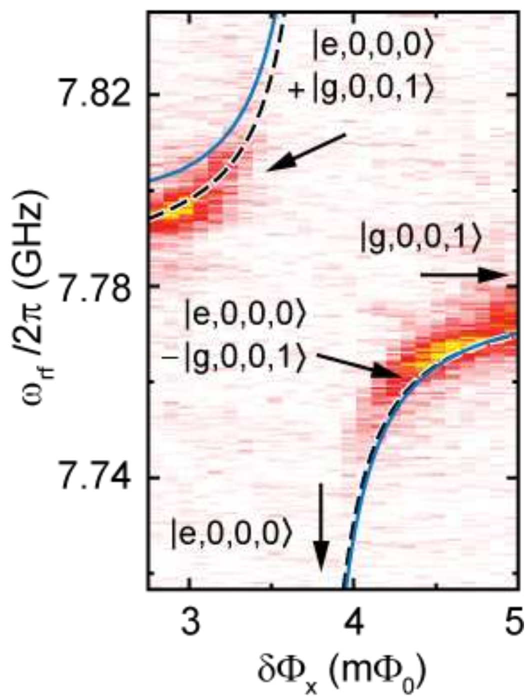
Measure frequency-dependent transmission through resonator!

Use nonlinear L_J as mutual inductance between qubit an resonator!

Ultrastrong light-matter coupling

ω_{rf} = probe frequency
Color code → Transmission magnitude





Large anticrossings (expected for large g)

→ Almost unaffected, no qualitative change, could well be JC

→ Reason → Only single excitation involved!

→ This experiment → g/ω up to 0.12
(recently g/ω up to 1.4 has been demonstrated with flux qubits)

T. Niemczyk *et al.*, Nature Phys. **6**, 772 (2010).

Ultrastrong light-matter coupling

ω_{rf} = probe frequency
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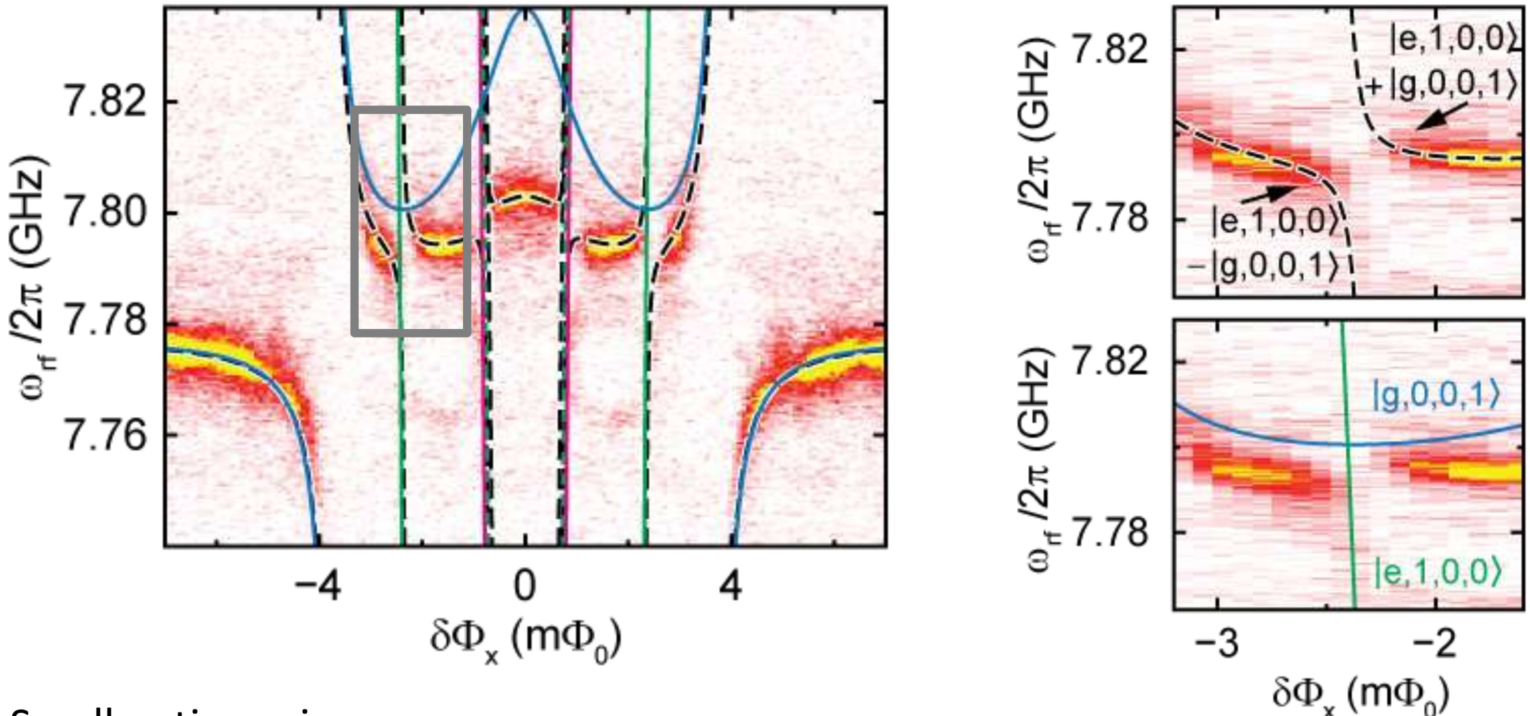

Small anticrossings

→ Qualitative difference to JC observable (anticrossing vs. crossing)

→ Reason: coupling of states with different number of excitations

→ Experimental proof for existence of counterrotating wave terms

T. Niemczyk *et al.*, Nature Phys. **6**, 772 (2010).