# 6.6

# Propagating quantum microwaves

# 6.6 Propagating quantum microwaves

## Superconducting quantum circuits

→ Confined quantum states of light

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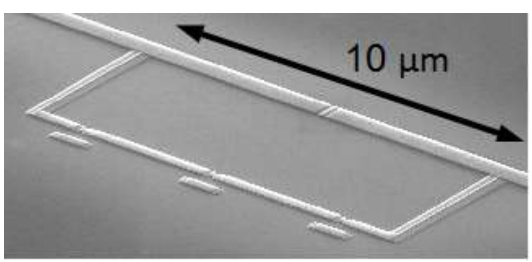
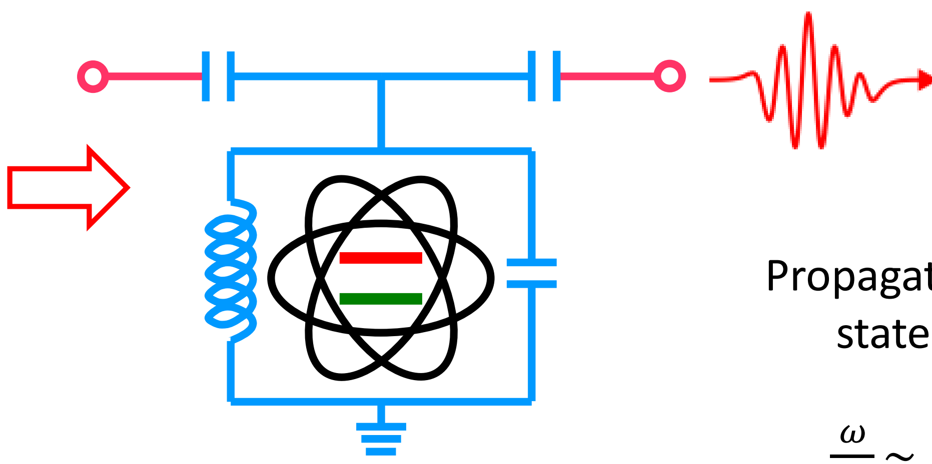



→ Propagating quantum states of light

input-output formalism - coupling to external multimode environment

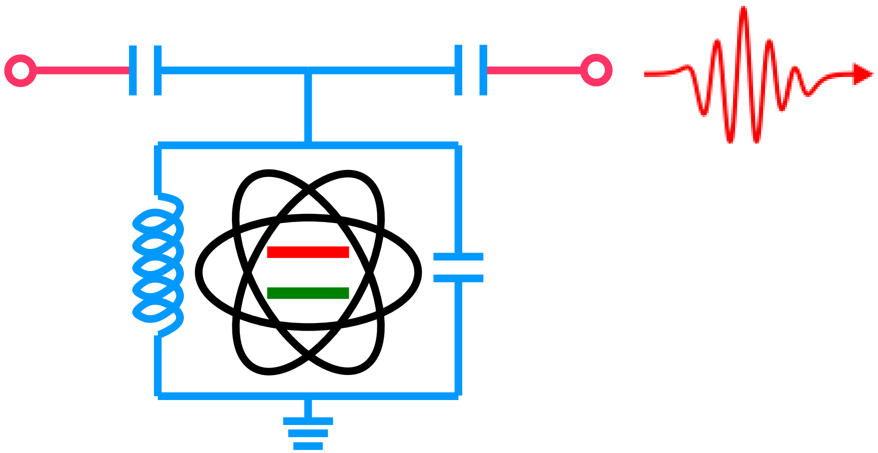


Propagating microwave states (photons)

$$\frac{\omega}{2\pi} \sim 1 - 10 \text{ GHz}$$

# 6.6 Propagating quantum microwaves





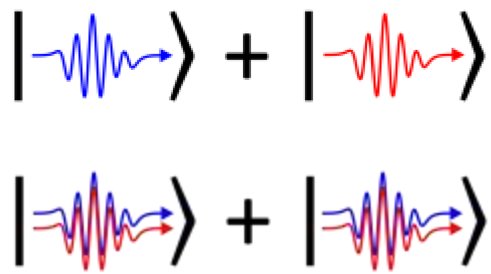
Propagating microwave states (photons)

$$\frac{\omega}{2\pi} \sim 1 - 10\ \text{GHz}$$

Fundamental and applied questions:

→ Does emitted microwave radiation exhibit quantum properties?

→ Such as, commutation relations, superposition, entanglement?




Quantum optics:

→ Yes, expected due to field quantization

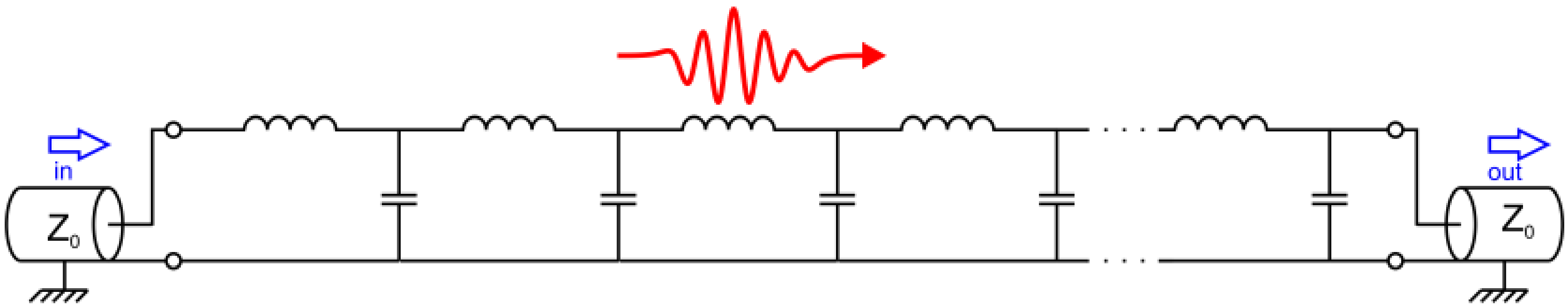
→ Confirmed by experiments

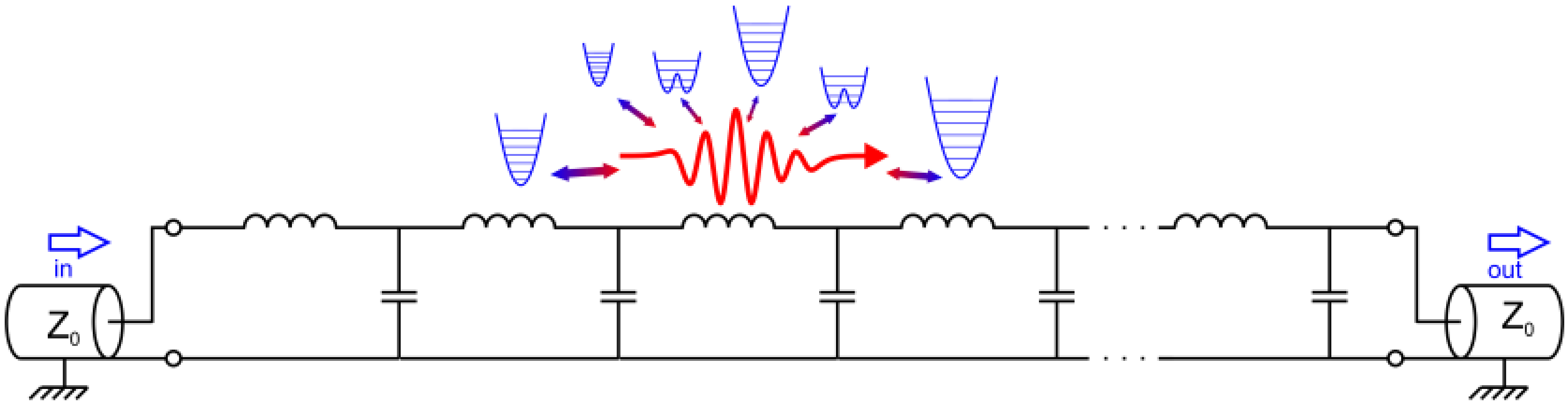
Quantum propagating microwaves:

→ Expected in analogy to optics

→ Experimental proof required!

## Fundamental technological considerations & obstacles




Microwave losses

→ May inhibit observation of quantum properties of propagating microwaves

→ May prohibit applications of quantum microwaves (in communication or sensing)

# 6.6 Propagating quantum microwaves

## Fundamental technological considerations & obstacles

Superconducting cables

→ Losses per km?

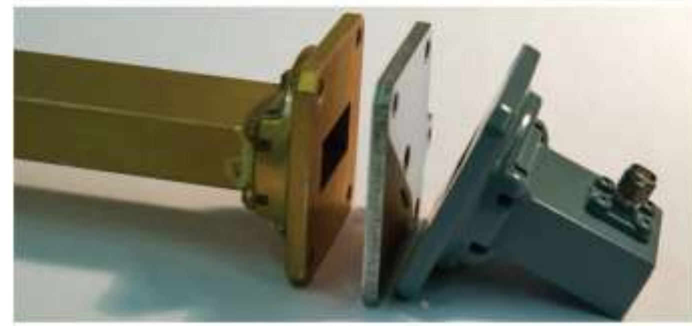
→ 0.002 dB/m for coaxial cables
→ 0.005 dB/m for 3D-waveguides
→ 0.001 dB/m for commercial optical fibres

} comparable

→ Coherent propagation distance $\ell_{\rm coh}$ sufficient?

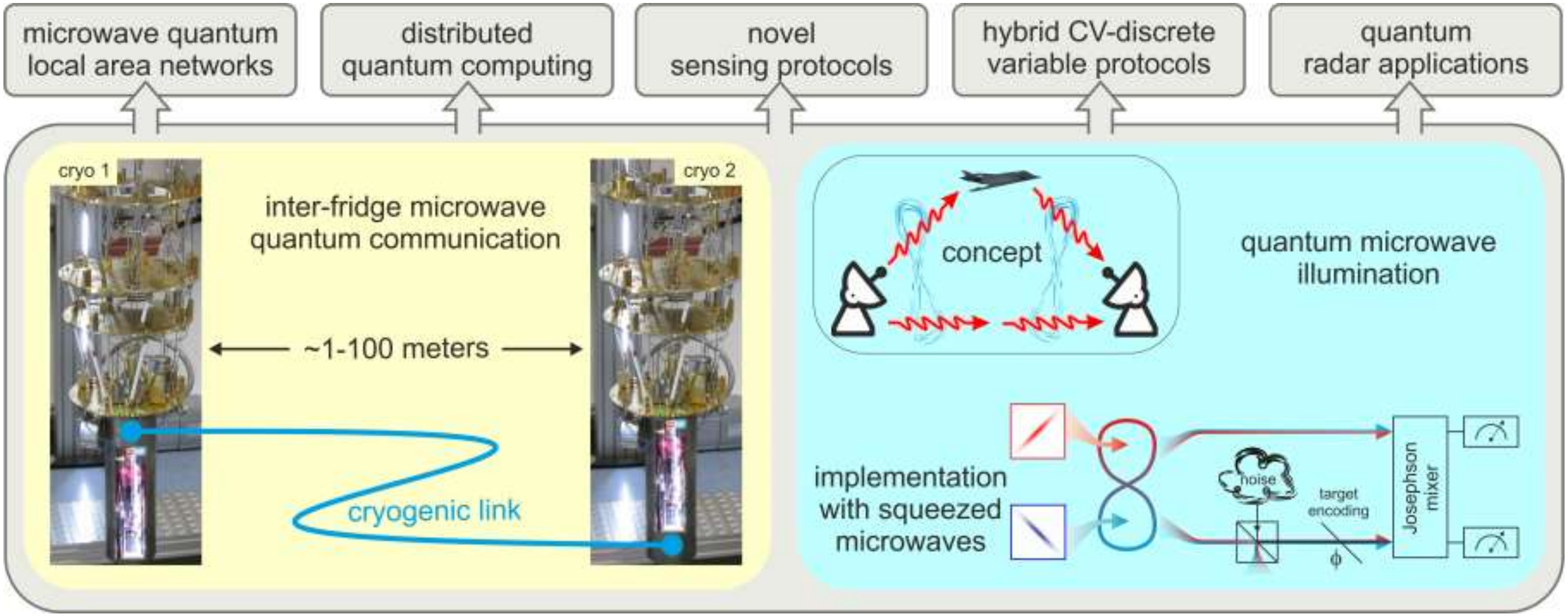
→ Furthermore, in resonators, microwave signals travel back and forth many times before losing coherence ($T_1 \simeq 100\ \mu\text{s} - 1\ \text{ms}$)

→ $\ell_{\rm coh} \approx 3 \times 10^8 \frac{\rm m}{\rm s} T_1 \simeq 10 - 100\ \text{km}$
comparable to optics



M. Pfeifer, Bachelor thesis, TUM (2020).
E. Xie, PhD thesis , TUM (2019).
P. Kurpiers et al.,EPJ Quantum Technol. 4, 8 (2017).

## Envisioned applications of propagating quantum microwaves



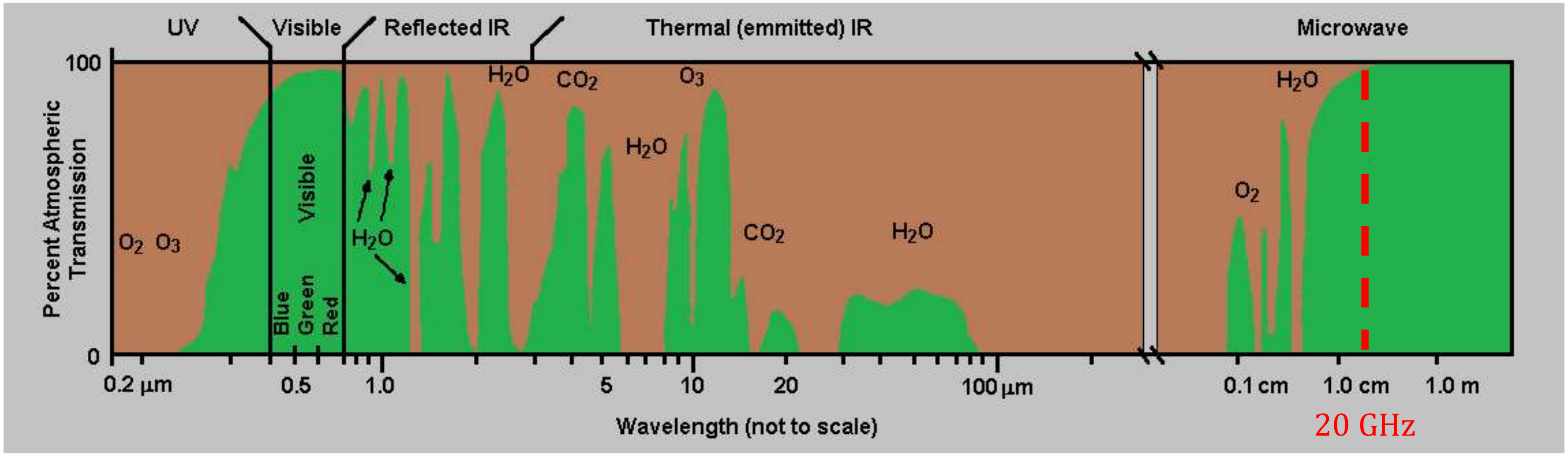

→ Superconducting cables require cooling!
  → Short- or medium-distance applications certainly feasible
  → QIP platforms such as SQC also require cooling → compatible!

→ Technological compatibility to SQC
  → No frequency conversion losses
  → Natural candidate for chip-to-chip quantum communication with SQC

## Fundamental technological considerations & obstacles

### Free-space propagation

→ Atmospheric transparency windows




→ Classical illumination with microwaves used for radar

→ Known to pass through clouds, fog, and rain

→ Typical frequencies $< 20$ GHz $\left(\lambda = \frac{3\times10^8\frac{\mathrm{m}}{\mathrm{s}}}{f} \simeq 1.5\ \mathrm{cm}\right)$

→ Compatible with SQC (superconducting gap of aluminum still twice as large)

# 6.6 Propagating quantum microwaves



## Quantization of the electromagnetic field

F. D. Walls, G. Milburn, *Quantum Optics* (Springer, Berlin, 2008)

→ Source-free (free field!) Maxwell equations

$$\boldsymbol{\nabla B} = 0, \quad \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad \boldsymbol{\nabla D} = 0, \quad \boldsymbol{\nabla} \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \qquad (\boldsymbol{B} = \mu_0 \boldsymbol{H}, \boldsymbol{D} = \epsilon_0 \boldsymbol{E}, \mu_0 \epsilon_0 = c^{-2})$$

→ Coulomb gauge ($\boldsymbol{\nabla A} = 0$) → $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$, $\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t}$

→ $\boldsymbol{A}(\boldsymbol{r}, t)$ satisfies wave equation $\boldsymbol{\nabla}^2 \boldsymbol{A} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{A}}{\partial t^2}$

→ Separate vector potential $\boldsymbol{A}(\boldsymbol{r}, t) = \boldsymbol{A}^{(+)}(\boldsymbol{r}, t) + \boldsymbol{A}^{(-)}(\boldsymbol{r}, t)$ into

- → right-propagating components $\boldsymbol{A}^{(+)}(\boldsymbol{r}, t)$ varying with $e^{-i\omega t}$ for $\omega > 0$
- → left-propagating components $\boldsymbol{A}^{(-)}(\boldsymbol{r}, t)$ varying with $e^{i\omega t}$ for $\omega > 0$

→ Restrict field to finite volume

- → $\boldsymbol{A}^{(+)}(\boldsymbol{r}, t) = \sum_k c_k \boldsymbol{u}_k(\boldsymbol{r}) e^{-i\omega_k t}$
- → Fourier coefficients $c_k$ constant for free field
- → vector mode functions $\boldsymbol{u}_k(\boldsymbol{r})$
  - → satisfy wave equations $\left(\nabla^2 + \frac{\omega_k^2}{c^2}\right) \boldsymbol{u}_k(\boldsymbol{r}) = 0$
  - → satisfy transversality condition $\boldsymbol{\nabla u}_k(\boldsymbol{r}) = 0$
  - → form orthonormal set $\int_V dr\, \boldsymbol{u}_k^\star(\boldsymbol{r}) \boldsymbol{u}_{k'}(\boldsymbol{r}) = \delta_{kk'}$
  - → depend on boundary conditions

# 6.6 Propagating quantum microwaves



## Quantization of the electromagnetic field

F. D. Walls, G. Milburn, *Quantum Optics* (Springer, Berlin, 2008)

→ General example for boundary conditions
  → periodic (travelling waves)
  → reflecting walls (standing waves)

→ Here: plane wave functions suitable for cubic volume with side lengths $L$
  → $\boldsymbol{u}_k(\boldsymbol{r}) = \frac{1}{L^{3/2}} \hat{\boldsymbol{e}}^{(\lambda)} e^{i\boldsymbol{kr}}$
  → wave vector $\boldsymbol{k} = (k_x, k_y, k_z)$ with $k_{x,y,z} = \frac{2\pi}{L} n_{x,y,z}$ and $n_{x,y,z} \in \mathbb{Z}$
  → $\hat{\boldsymbol{e}}^{(\lambda)}$ perpendicular to $\boldsymbol{k}$
  → typically no polarization in microwaves propagating in waveguides
    → polarization vector $\hat{\boldsymbol{e}}^{(\lambda)} = \hat{\boldsymbol{e}}$

→ Quantization of classical Fourier amplitudes
  → $a_k, a_k^\star \rightarrow \hat{a}_k, \hat{a}_k^\dagger$ with commutation relations $[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta_{kk'}$
  → $\boldsymbol{A}(\boldsymbol{r}, t) = \sum_{\boldsymbol{k}} \sqrt{\frac{\hbar}{2\omega_k \epsilon_0}} [\hat{a}_k \boldsymbol{u}_k(\boldsymbol{r}) e^{-i\omega t} + \hat{a}_k^\dagger \boldsymbol{u}_k^\star(\boldsymbol{r}) e^{i\omega t}]$
  → $\boldsymbol{E}(\boldsymbol{r}, t) = i \sum_{\boldsymbol{k}} \sqrt{\frac{\hbar\omega_k}{2\epsilon_0}} [\hat{a}_k \boldsymbol{u}_k(\boldsymbol{r}) e^{-i\omega t} - \hat{a}_k^\dagger \boldsymbol{u}_k^\star(\boldsymbol{r}) e^{i\omega t}]$
  → Hamiltonian $\hat{H} = \frac{1}{2} \int d\boldsymbol{r}\, (\epsilon_0 \boldsymbol{E}^2 + \mu_0 \boldsymbol{H}^2) = \sum_k \hbar\omega_k \left(\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2}\right)$
  → quantum states $|\psi_k\rangle$ of each mode can now be discussed independently!

# 6.6 Propagating quantum microwaves

## Continuos variables (CV) vs. discrete variables (DV)

Annihilation operator $\hat{a} \equiv \frac{\omega_{\mathrm{r}} C \hat{\Phi} + i\hat{q}}{\sqrt{2\omega_{\mathrm{r}} C \hbar}}$

Creation operator $\hat{a}^{\dagger} \equiv \frac{\omega_{\mathrm{r}} C \hat{\Phi} - i\hat{q}}{\sqrt{2\omega_{\mathrm{r}} C \hbar}}$

$$\widehat{H}_{\mathrm{HO}} = E_{\mathrm{kin}} + E_{\mathrm{pot}} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega_{\mathrm{r}}^2 \hat{x}^2$$

$$\hat{P} \equiv \sqrt{\frac{1}{2\hbar m \omega_{\mathrm{r}}}} \hat{p}^2, \; \hat{Q} \equiv \sqrt{\frac{m\omega_{\mathrm{r}}}{2\hbar}} \hat{x}^2$$

Classical single-mode electromagnetic waves $A\cos(\omega t + \phi)$

→ equivalent description $P\cos\omega_k t + \mathrm{Q}\sin\omega_k t$
with field quadratures $Q = A\cos\phi$ and $P = A\sin\phi$

→ in engineering, *P* is often called *I*

→ field quadratures analogous to momentum/position in mechanics

→ field quantization → $[\hat{Q}, \hat{P}] = \frac{i}{2} \quad \Leftrightarrow \quad (\Delta P)(\Delta Q) \geq \frac{1}{4}$

Single-mode quantum field → $\widehat{H}_{\mathrm{HO}} = \hat{P}^2 + \hat{Q}^2 = \hbar\omega_k \left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)$

**Continuous-variable basis**

→ Set of eigenstates of either $\hat{P}$ or $\hat{Q}$ also forms basis

→ Natural states are Gaussian states (coherent, squeezed, thermal)

**Discrete-variable basis**

→ Fock basis $\{|n\rangle\}$

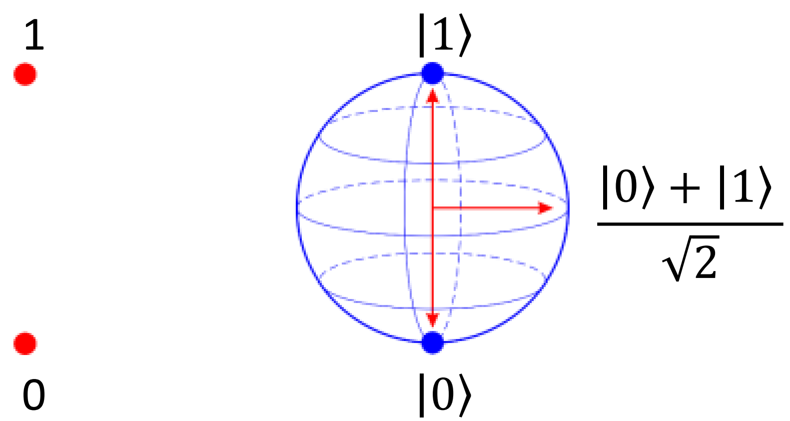
→ Single photons $|1\rangle$ are the natural quantity of interest

→ Any quantum state can be expressed either in CV or in DV

→ Any quantum task (QIP, QSim, Qcomm, QIllu) can be expressed in CV or DV

→ Nevertheless a particular basis may be more suitable for a particular problem

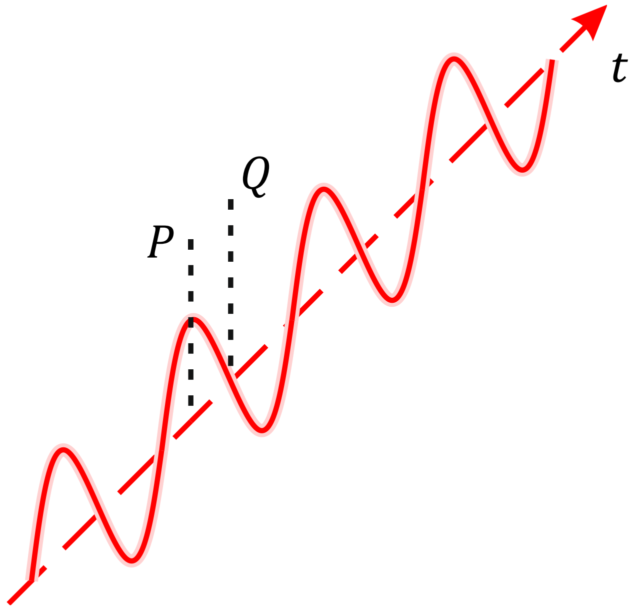
Discrete variables (**DV**)




Classical bit    Quantum bit (Qubit)

Continuous variables (**CV**)

$$A\cos(\omega t + \varphi) = P\cos(\omega t) + Q\sin(\omega t)$$




As an alternative to the discrete encoding, one can use the eigenstates $\{|x\rangle\}$ of a continuous-valued operator $\hat{x}$.

# 6.6 Propagating quantum microwaves

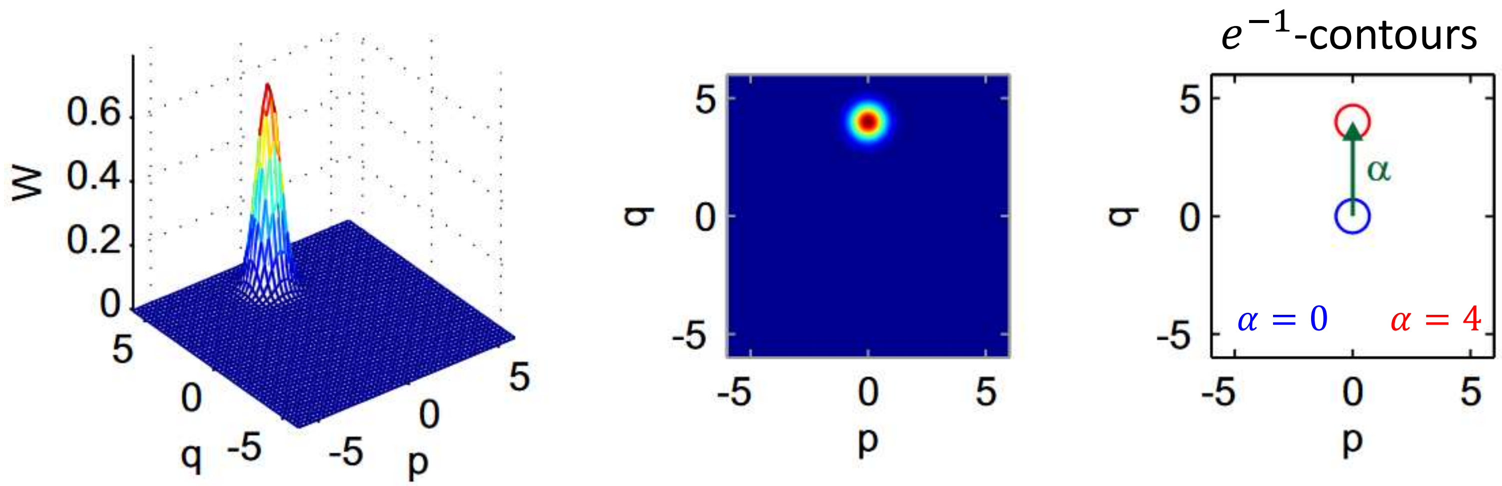


## Expressing quantum microwave states

→ General quantum state described by the desity matrix $\hat{\rho} = \sum_{\Psi} P_{\Psi} |\Psi\rangle\langle\Psi|$
- → $P_{\Psi}$ = Classical probability to be in state $|\Psi\rangle$ → $P_{\Psi} > 0$ and $\sum_{\Psi} P_{\Psi} = 1$
- → expectation value of operator $\hat{O}$ → $\langle \hat{O} \rangle = \mathrm{Tr}[\hat{O}\hat{\rho}]$
- → normalization → $\mathrm{Tr}[\hat{\rho}] = 1$
- → complex matrix entries → not easy to visualize

→ Phase space representation of a quantum state
- → ideal classical states → points in phase space
- → noisy classical states
  - → ordinary probability distribution $P(q,p)$ in phase space
  - → $q,p$ are the phase space variables associated with $Q,P$
  - → $P(q,p)dqdp$ is the probability to find the system in state $(q,p)$
- → quantum states
  - → Heisenberg uncertainty relation $(\Delta P)(\Delta Q) \geq \frac{1}{4}$ as "quantum noise"
  - → in general, requires also negative probability densities
  - → quasi-probability distribution $W(q,p)$ with $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(q,p)dqdp = 1$
  - → Wigner function $W(q,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle q - \frac{\zeta}{2}|\hat{\rho}|q + \frac{\zeta}{2}\rangle\, e^{ip\zeta} d\zeta$ and $\zeta \in \mathbb{R}$

## Wigner function examples



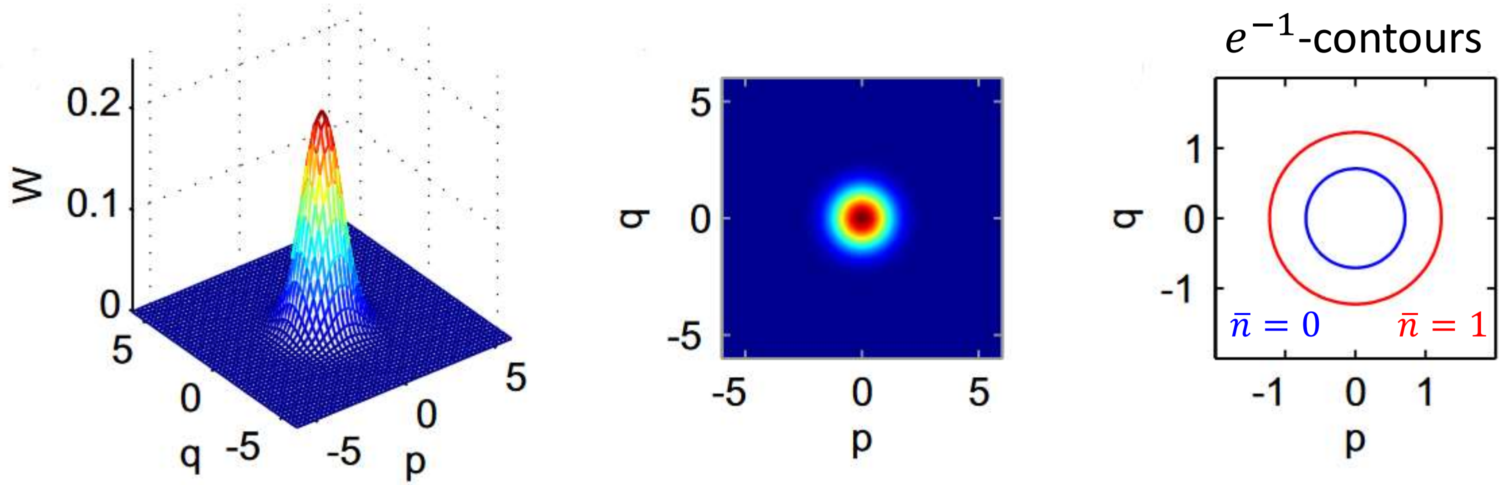

Coherent state $|\alpha\rangle \equiv |Q + iP\rangle = \widehat{D}(\alpha)|0\rangle$

→ Produced by displacement operator $\widehat{D}(\alpha) \equiv \mathrm{e}^{\alpha\hat{a}^\dagger - \alpha^\star \hat{a}}$ applied to vacuum

→ $W_{\mathrm{coh}}(q,p) = \frac{2}{\pi} e^{-2[(q-Q)^2+(p-P)^2]} > 0$

→ Coherent states are "most classical" quantum states - do not generate entanglement

## Wigner function examples



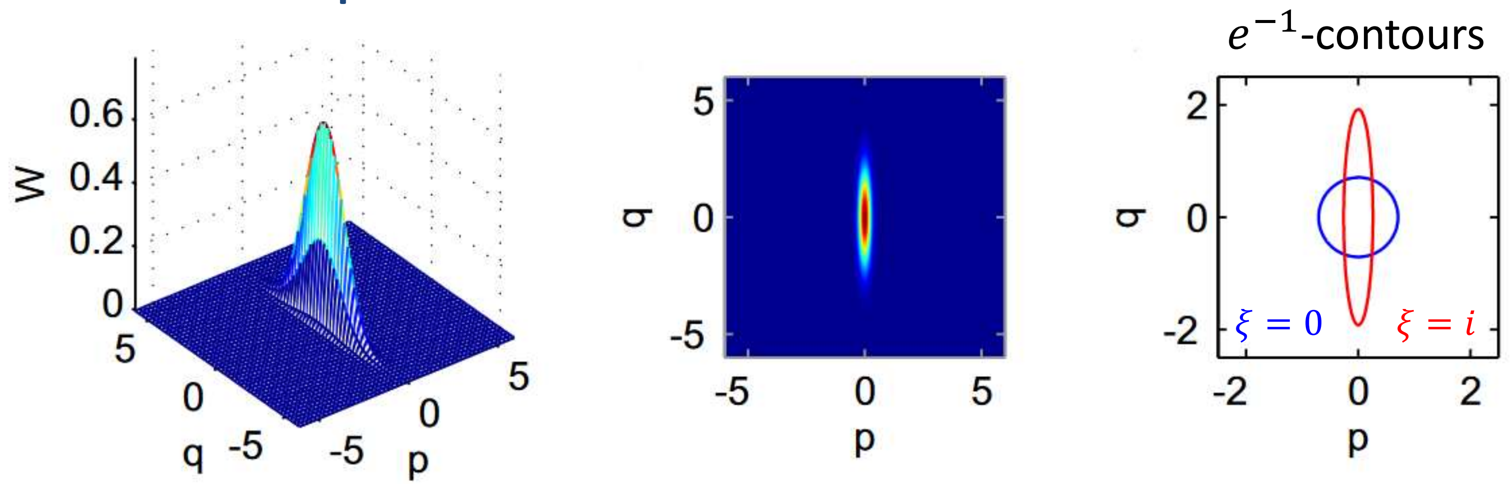

Thermal state $\hat{\rho}_{\mathrm{th}}$ with $\bar{n} \equiv Tr\left[\hat{a}^{\dagger}\hat{a}\hat{\rho}_{\mathrm{th}}\right] = 1$

→ $W_{\mathrm{th}}(q,p) = \frac{1}{\pi\left(\bar{n}+\frac{1}{2}\right)} e^{-\frac{\mathrm{q}^2+\mathrm{p}^2}{\bar{\mathrm{n}}+\frac{1}{2}}} > 0$

→ In the high-temperature limit $k_{\mathrm{B}}T \gg \hbar\omega_k$, thermal states can be considered to be classical

→ Thermal states are also classical - do not generate entanglement

## Wigner function examples




Squeezed vacuum determined by complex squeezing parameter $\xi \equiv re^{i\varphi}$

→ Produced by squeezing operator $\hat{S}(\xi) \equiv \mathrm{e}^{\frac{1}{2}\left[\xi^{\star}\hat{a}^{2} - \xi\left(\hat{a}^{\dagger}\right)^{2}\right]}$ applied to vacuum

→ $r$ determines the amount of squeezing and $\varphi$ the squeezing direction in phase space

→ $W_{\mathrm{sq}}(q,p) = \frac{2}{\pi} e^{-(e^{2r}+e^{-2r})|q+ip|^{2} - \frac{1}{2}(e^{2r}-e^{-2r})(e^{-i\varphi}|q+ip|^{2} + e^{i\varphi}|q-ip|^{2})} > 0$

→ Since $(\Delta P)^2 < \frac{1}{4}$, one must have $(\Delta Q)^2 \geq \frac{1}{4(\Delta P)^2}$ to satisfy the Heisenberg relation

→ Squeezed states are nonclassical - produce entanglement when applied to a beamsplitter with vacuum at the other input port

# 6.6 Quantum entanglement



Not entangled, separable (product) states: $|\Psi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle$

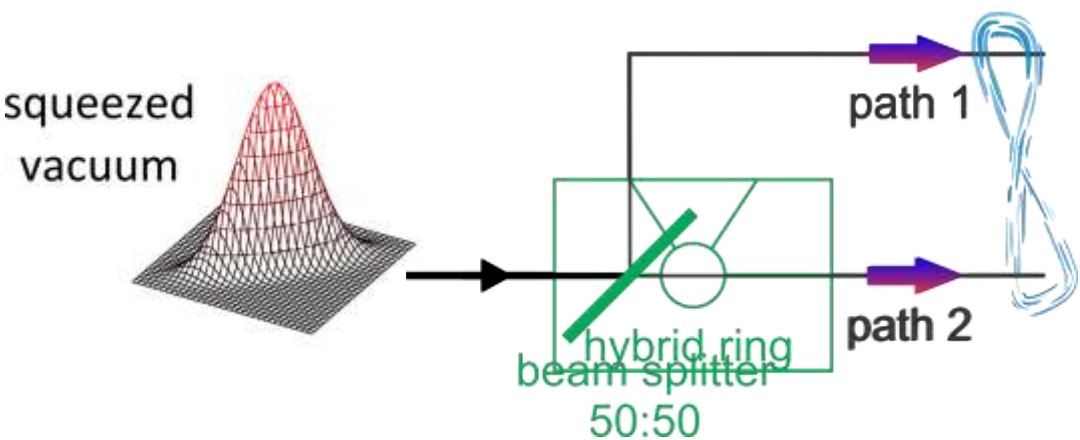
QE is a unique property of a **composite system**, where one cannot describe its subparts independently of each other.

DV - Bell state:

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

CV - two-mode squeezed state:

$$(\hat{q}_1 + \hat{q}_2)|\Psi\rangle_{TMS} = \delta(q_1 + q_2)$$
$$(\hat{p}_1 - \hat{p}_2)|\Psi\rangle_{TMS} = \delta(p_1 - p_2)$$





practical applications of entanglement:

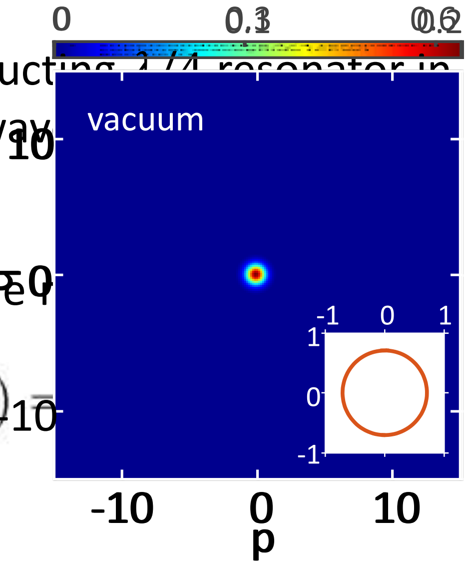
→ quantum communication

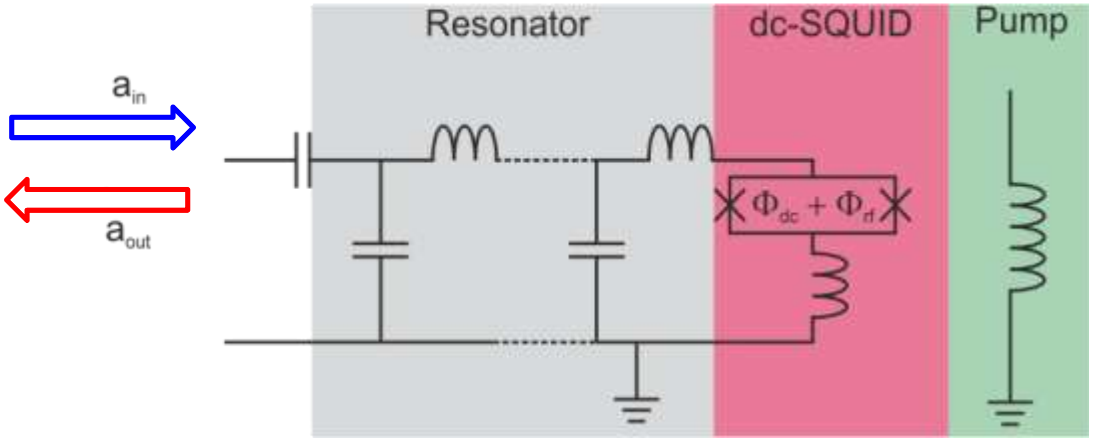
→ quantum sensing

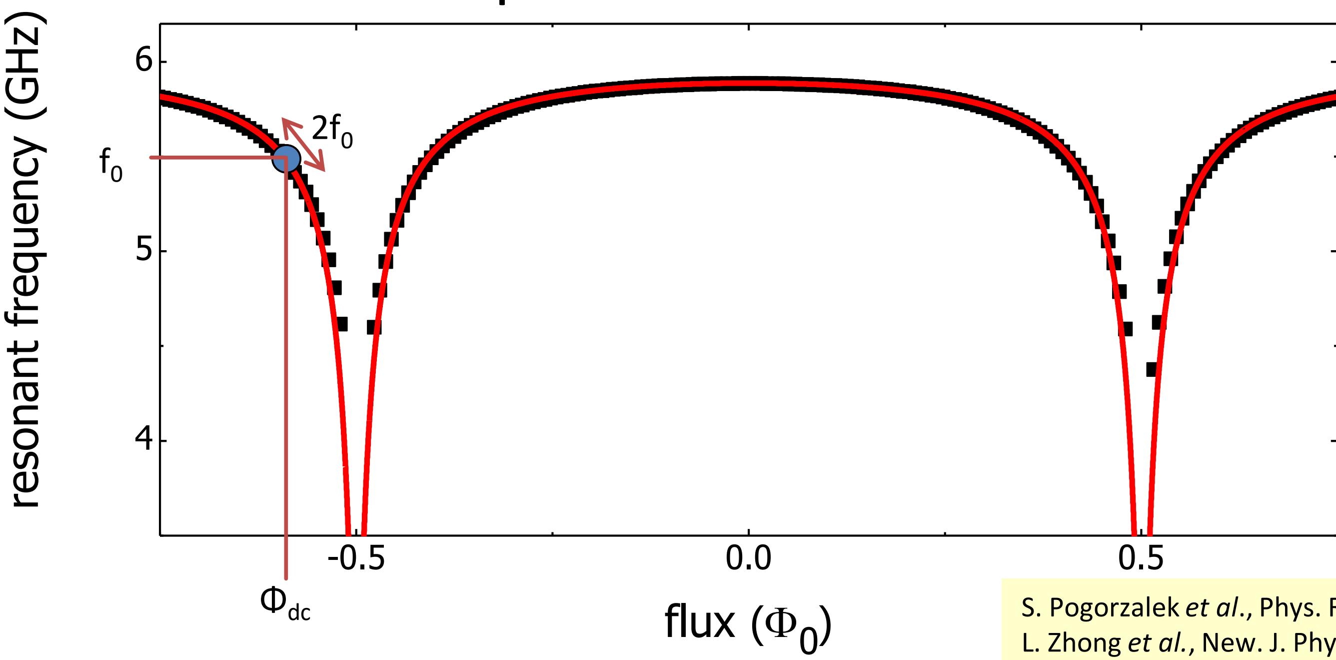
# 6.6 Generation of squeezed microwaves

- superconducting λ/4 resonator in coplanar wa geometry
- flux-tunable r

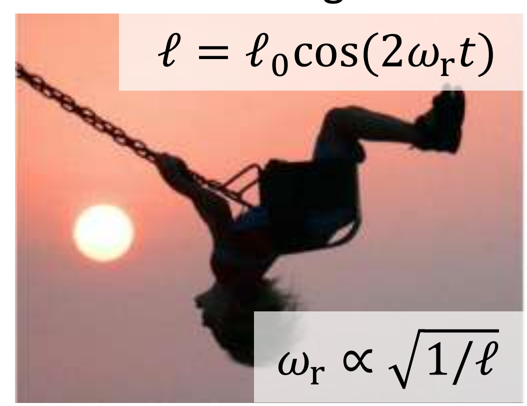
$L_{\mathrm{SQUID}}(\Phi) =$



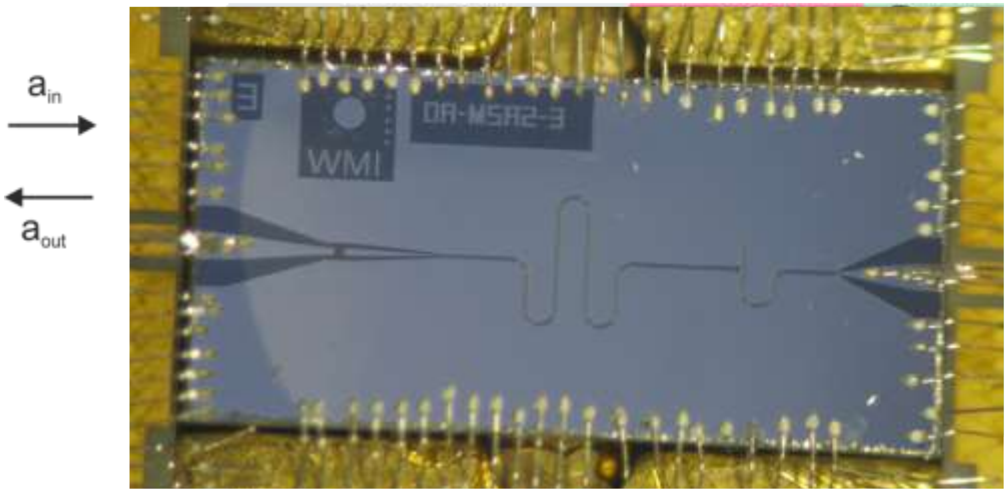




Classical analogue: child on a swing

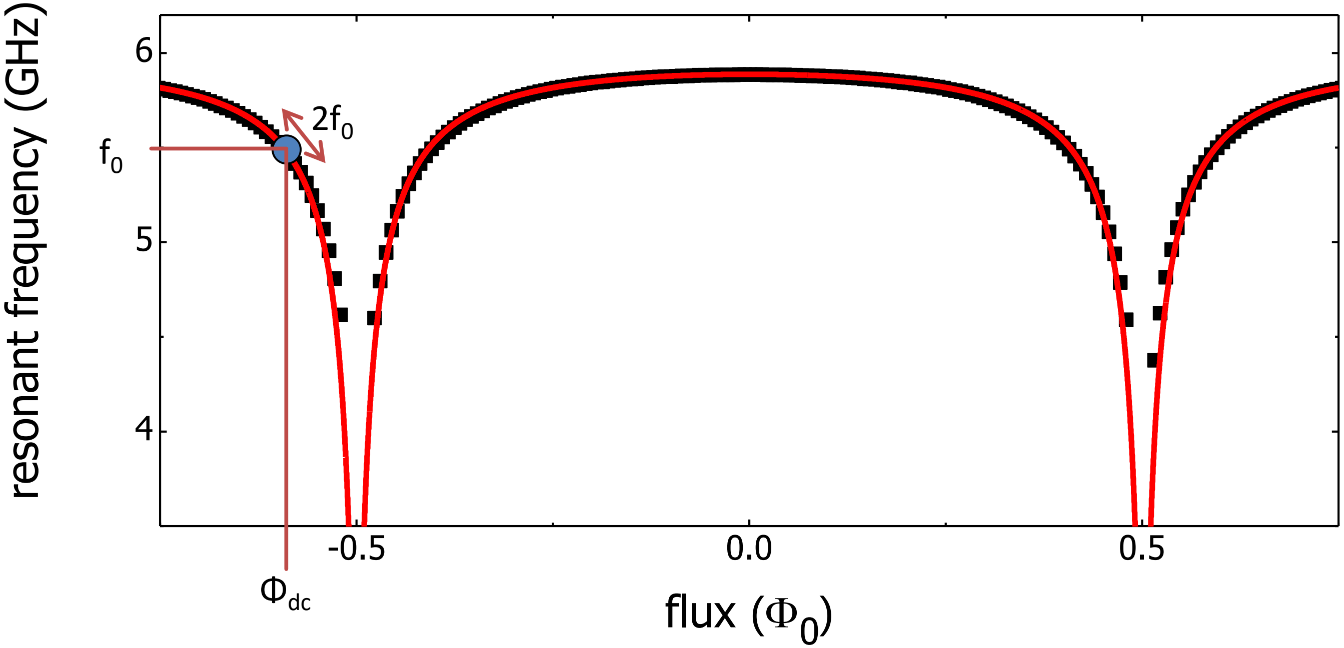



S. Pogorzalek *et al.*, Phys. Rev. Applied **8**, 024012 (2017)
L. Zhong *et al.*, New. J. Phys. 15, 125013 (2013)

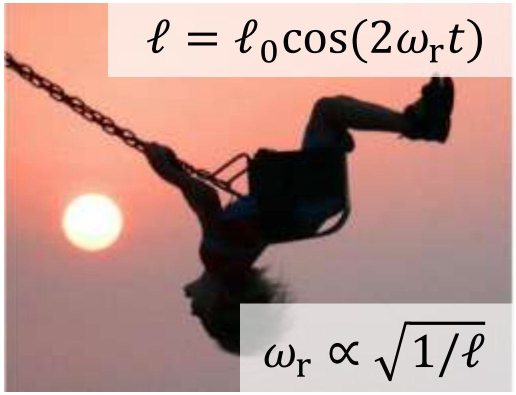



- $\lambda/4$ resonator in coplanar waveguide (CPW) geometry
- flux-tunable resonant frequency

$$L_{\mathrm{SQUID}}(\Phi) = \frac{\Phi_0}{4\pi I_c \left|\cos\left(\pi \frac{\Phi}{\Phi_0}\right)\right|}$$



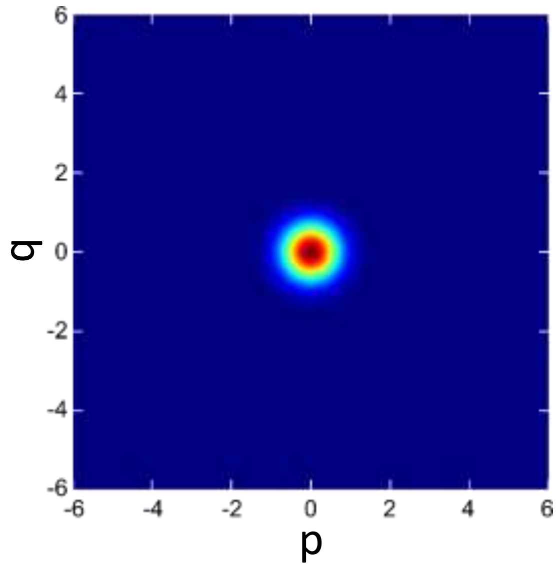

Classical analogue: child on a swing



**degenerate mode:**

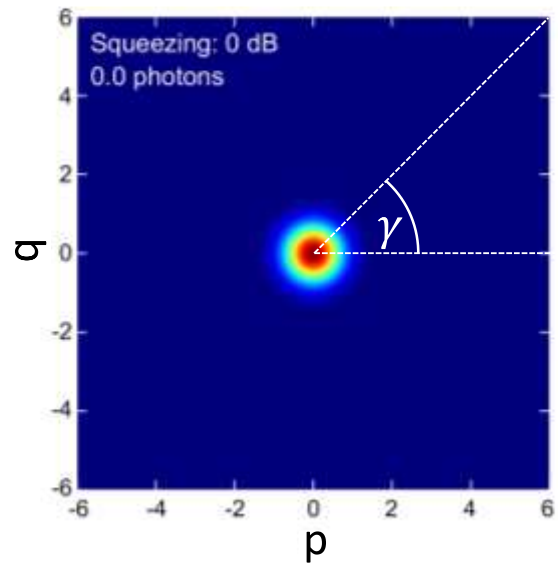
$$f_{\text{signal}} = f_{\text{pump}}/2$$

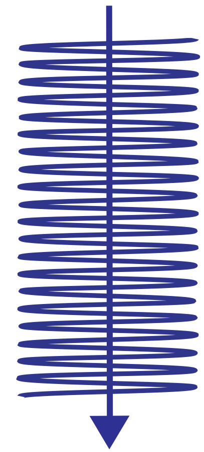
Vacuum state



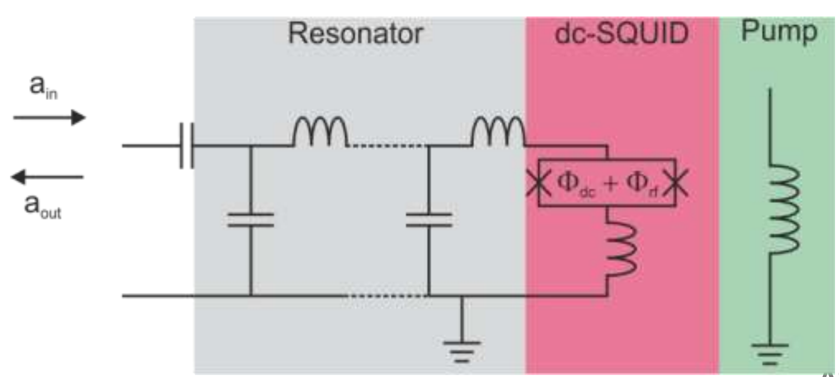

$\hat{S}(\xi)$ →

Squeezed vacuum state – simulation



JPA




Squeezing level

$$\mathcal{S} = -10\log_{10}\left[(\Delta X_{\text{sq}})^2/0.25\right]$$

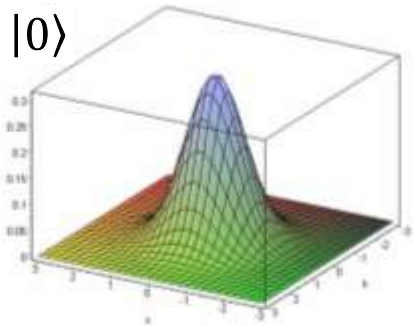
L. Zhong *et al.*, NJP **109**, 250502 (2012).
K. G. Fedorov *et al.*, PRL **117**, 020502 (2016).
S. Pogorzalek *et al.*, Phys. Rev. Appl. **8**, 024012 (2017)

# 6.6 Non-Gaussian microwave states

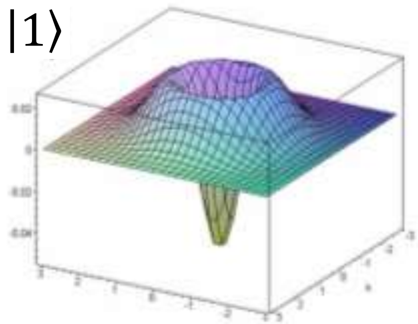


## Wigner function examples

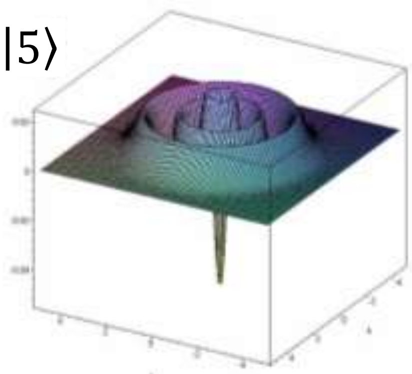
$|0\rangle$



$|1\rangle$



$|5\rangle$



## Fock (number) states

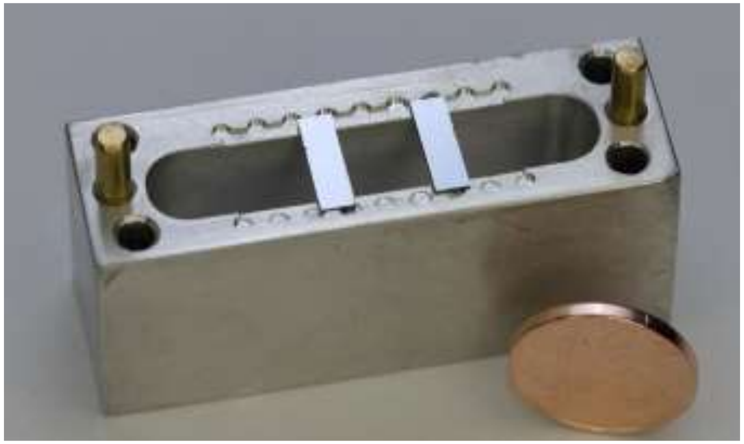
→ Have a radially symmetric Wigner function

→ The vacuum $|0\rangle$ is a Gaussian state with the vacuum variace of $0.5\hbar\omega$

→ Finite number states $|n \geq 1\rangle$ are nonclassical..

- → ..because their Wigner function can become negative
- → ..because they produce path-entanglement when applied to a beamsplitter with vacuum at the other input port

→ Typically generated employing qubits



# 6.6 Tomography of propagating microwaves



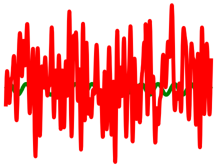
## State reconstruction of propagating quantum microwaves

Difficult task!

- No efficient photon detectors

- Off-the-shelf linear amplifiers

- Add $\bar{n} \approx 10$ of noise to signal

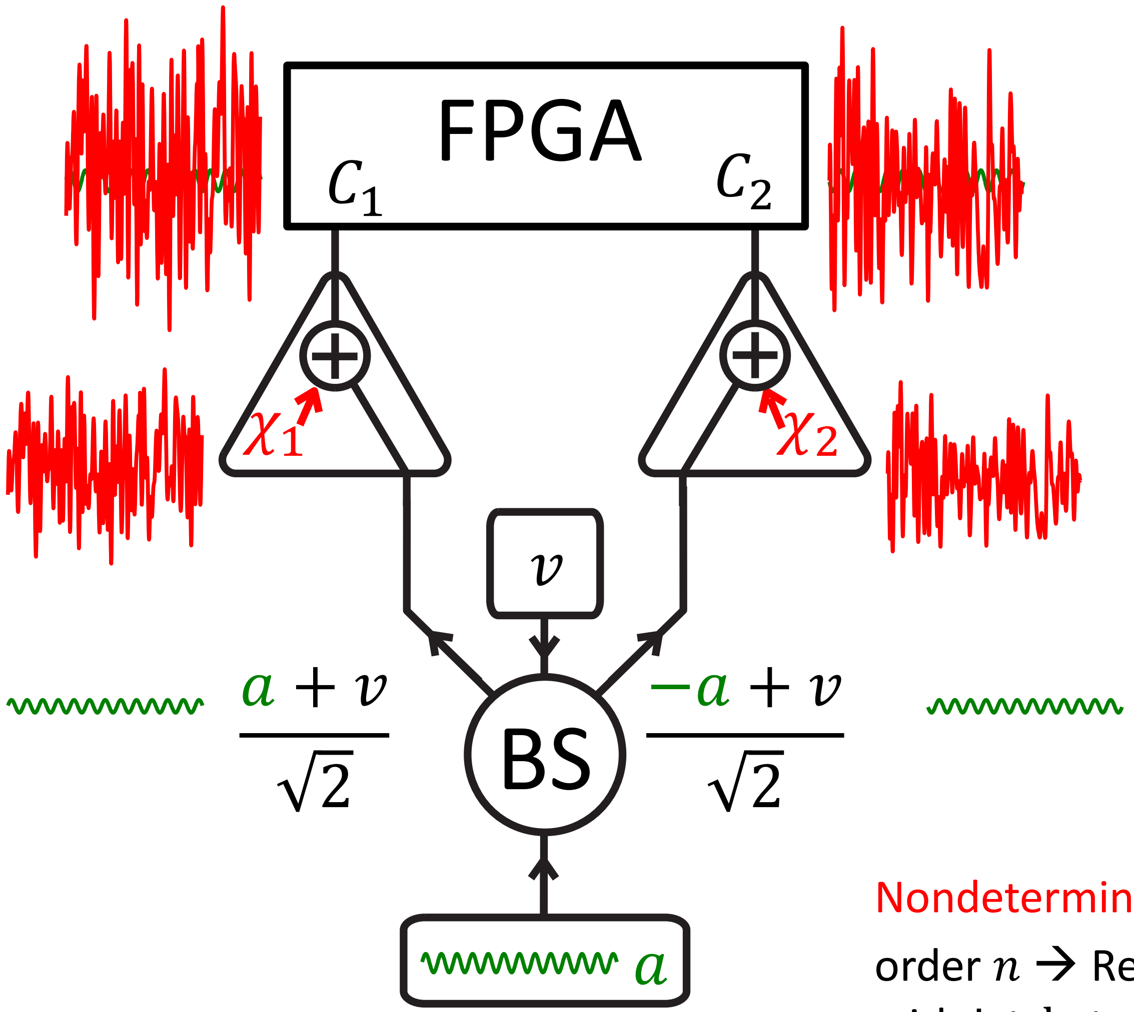


Two common approaches:
1. Parametric amplifiers
2. Signal recovery methods (measure signal moments)

# 6.6 Tomography of propagating microwaves

## Dual-path state reconstruction of propagating quantum microwaves

Kowledge of all moments is equivalent to knowlege of the Wigner function or density matrix.




Expectation values of all signal moment up to order $n$

$$\langle C_1^{n-1} C_2 \rangle$$

$$\downarrow$$

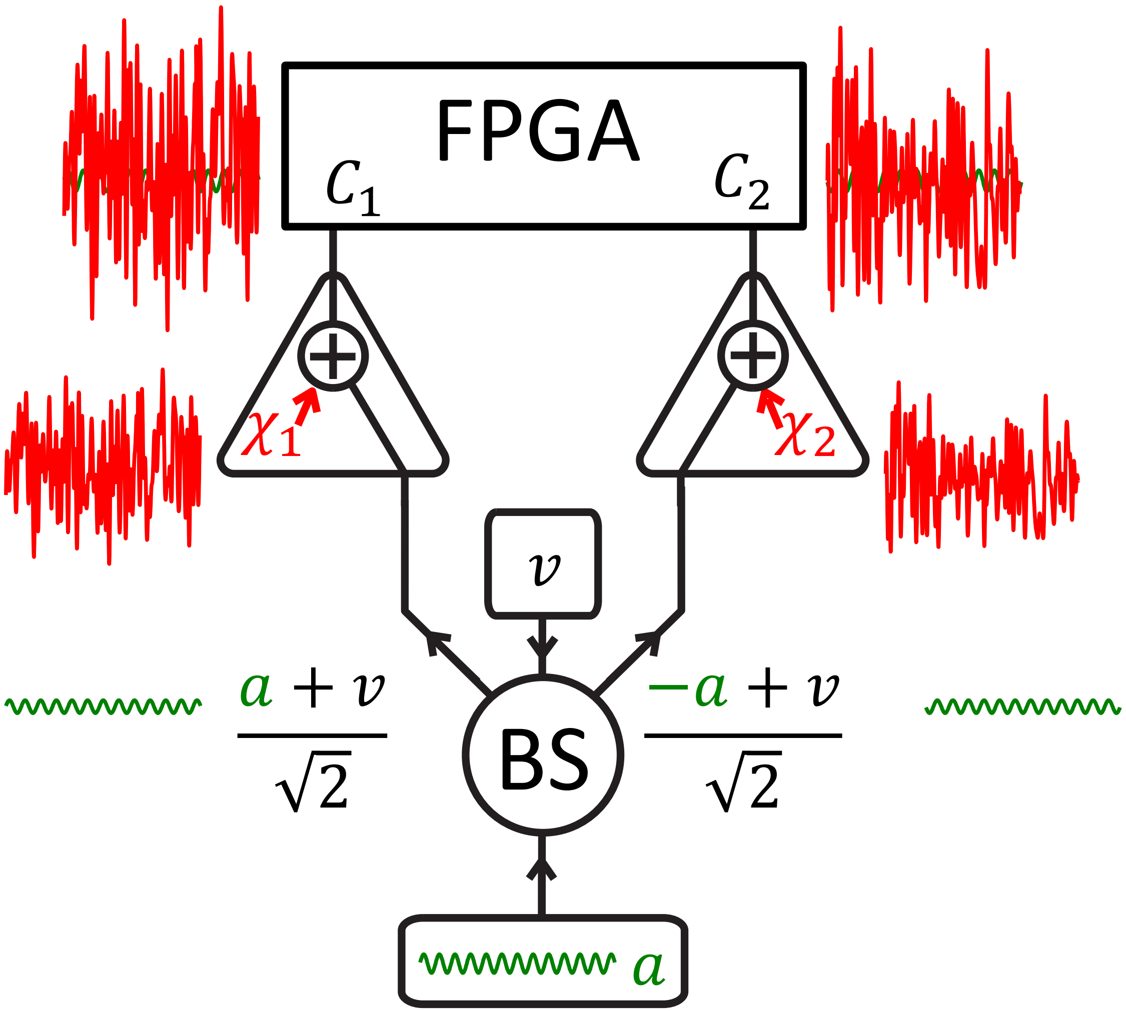
$$\langle a^n \rangle, \langle \chi_1^n \rangle, \langle \chi_2^n \rangle$$

Iteratively obtain all signal & detector noise moments

Nondeterministic & quantum signals up to order $n$ → Record all moments $\langle Q_1^j Q_2^k P_1^\ell P_2^m \rangle$ with $\mathrm{j}+\mathrm{k}+\ell+m \leq n$ and $\mathrm{j},\mathrm{k},\ell,m \in \mathbb{N}_0$

E.P. Menzel *et al.*, PRL **105**, 100401 (2010).

## Dual-path state reconstruction of propagating quantum microwaves




Intuition for order $n = 2$

→ Use statistical independence of amplifier noise signals in different paths $\langle \chi_1 \chi_2 \rangle = \langle \chi_1 \rangle \langle \chi_2 \rangle = 0$

→ $\langle C_{1,2} \rangle = \langle a + \chi_{1,2} \rangle = \langle a \rangle + \langle \chi_{1,2} \rangle = \langle a \rangle$ 

→ $\langle C_{1,2}^2 \rangle = \langle (a + \chi_{1,2})^2 \rangle = \langle a^2 + a\chi_{1,2} + \chi_{1,2} a +$

E.P. Menzel *et al.*, PRL **105**, 100401 (2010).