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# Applied Superconductivity:

## Josephson Effect and Superconducting Electronics

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## **Chapter C**

# **Numerical Iteration Method for the Calculation of the Josephson Current Distribution**

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