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Josephson Effect and Superconducting Electronics

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Contents

List of Figures

List of Tables

Chapter 6

The Josephson Voltage Standard

The realization of Josephson voltage standard is one of the successful applications of the Josephson effect. It is based on the voltage-frequency relation $V = \frac{h}{2e} f = \Phi_0 f$. Therefore, defining the unit 1 V on the basis of the Josephson effect reduces the calibration of a voltage to the determination of a frequency and the knowledge of the ratio $h/2e$ of the Plank's constant and twice the elementary charge. Since the frequency can be determined with extreme accuracy by the standard of an atomic caesium clock, only a convention for the value of $\Phi_0 = h/2e$ is needed to obtain a very good reproducibility of the voltage calibration. Based on this idea the development of a Josephson voltage standard was started already in the 1970s. After overcoming some technical difficulties the Josephson effect based voltage standards are used as primary standards in the national calibration laboratories since 1990. Meanwhile even industrial companies are using these standards. Since the Josephson voltage standards have no competitors in semiconductor electronics, they represent a successful application of the Josephson effect.

6.1 Voltage Standards

6.1.1 Standard Cells and Electrical Standards

For a long period electrochemical cells were used for establishing reference voltages. Initially, these cells were mainly used to provide reliable sources of electrical current. For example, following the ideas of **Galvani**,[1] **Volta** built the first practical electrochemical cell in 1794. These cells had been the only reliable sources for electrical current for more than 50 years.[2]

Only after the introduction of the ***Zn-Cu Daniell cell*** in 1836,[3,4] electrochemical cells were used as stable voltage sources for maintaining and disseminating the unit volt. There was a large effort in the late 19^{th} and the early 20^{th} century to establish a standard for electromotive force (emf) based on electrochemical reactions within chemical cells. For example, the first legal unit of voltage for the United States was based on the ***Zn-Hg Clark cell***, developed by **Latimer Clark** in 1872, with its output assigned a value of 1.434 international volts by the 1893 International Electric Congress. Public Law 105, passed by the U.S. Congress in 1894, made this the legal standard of voltage in the U.S. During the years between 1893 and 1905, the standard cell devised by **Edward Weston**[5] (Cd(Hg)/$CdSO_4$(aq),Hg_2S_4/Hg battery, see Fig. 6.1) was found to have many advantages over the Clark cell.[6] The ***Cd-Hg Weston cell*** consists of a cadmium amalgam anode and a mercury-mercurous sulfate cathode with a saturated cadmium sulfate solution as the electrolyte. It has a much better long-term stability, a smaller temperature coefficient and less hysteresis. Furthermore, its output voltage of 1.0186 V was a better approximation to 1 V. In 1908, at the London International Conference on Electrical Units and Standards, the Weston cell was officially adopted for maintaining the volt. Due to the improved electrochemical cells in 1910 the Rayleigh Committee recommended a more precise value for the maintenance of the volt. Similar improvements had been adopted for the ampère and the ohm at the London Conference two years earlier. The newly recommended units were called ***International Units***.

The Weston standard cell is sensitive to external parameters such as motion during transport, change in temperature or a small electrical current. When at times it was necessary to eliminate cells – due to changes in emf of a cell relative to the mean of the group – new cells could be added. In 1965 the National Reference Group of standard cells[7] included 11 cells made in 1906, seven cells made in 1932, and 26 cells made in 1948. Long-term stability of the volt reference was also maintained by comparisons of neutral and acid cells, preparing and characterizing new cells, and through international comparisons and absolute ampère and ohm experiments.[8] The use of the Weston cell as the national standard of voltage was supported by a considerable amount of research in electrochemistry and related fields. However, there were still problems with the standard cells. In the late 1960s the relative deviation of the output voltages of the individual national standards could be determined with an accuracy of less than 1 μV. However, the values deviated from each other by more than 10 μV. This clearly demonstrated

[1] L. Galvani, *De Virebus Electricitatis in Motu Musculari Commentarius* (1791).

[2] W. Ostwald, *Elektrochemie, ihre Geschichte und Lehre* (1896).

[3] **John Frederic Daniell**, British chemist and meteorologist, born March 12, 1790 in London, England; died March 13, 1845 in London, England.

[4] J.F. Daniell, Philosophical Transactions of the Royal Society (1986).

[5] **Edward Weston**, born May 9, 1850, died August 20, 1936, English chemist noted for his achievements in electroplating and his development of the battery, named the Weston cell, for the voltage standard.

[6] F. B. Silsbee, *Establishment and Maintenance of the Electrical Units*, NBS Circular 475, National Bureau of Standards, Washington, DC (1949).

[7] Walter J. Hamer, *Standard Cells – Their Construction, Maintenance, and Characteristics*, NBS Monograph 84, National Bureau of Standards, Washington, DC (1965).

[8] R. L. Driscoll and P. T. Olsen, *Application of nuclear resonance to the monitoring of electrical standards*, in *Precision Measurement and Fundamental Constants*, D. N. Langenberg and B. N. Taylor (eds.), NBS Special Publication 343, National Bureau of Standards, Washington, DC (1971), pp. 117-121.

Figure 6.1: The Weston cell. Weston invented and patented the saturated cadmium cell in 1893. It had the advantage of being less temperature sensitive than the previous standard, the Clark cell. It also had the advantage of producing a voltage very near to one volt: 1.0183 V.

the necessity of a new voltage standard providing better accuracy and less deviations between of the various national standards.

We note that in 1948 the ***International Units*** were replaced by the ***Absolute Units – meter, kilogram, second, ampère (MKSA)***. In this system, the voltage is a derived quantity and its unit must be determined by an experiment, for example, by a voltage balance, which links it to the MKSA system. Accordingly, a standard cell became a laboratory realization of the unit volt. The system of absolute units was adopted unchanged by the ***Système International (SI)***, which was introduced by the Conférence Génerale des Poids et Mesures in 1960.

In the late 1950s, research in solid-state physics stimulated the growth of the semiconductor industry. A new type of voltage standard based on a solid-state device, the ***Zener diode***, appeared in the early 1960s. **W. G. Eicke** at NBS first reported the possibility of using Zener diodes as transport standards.[9] In the following years, after several manufacturers started making commercial Zener voltage standards, these references began to replace standard cells in commercial use. Although Zener voltage standards exhibit higher noise characteristics than standard cells and are affected by environmental conditions such as temperature, atmospheric pressure, and relative humidity, they are now widely used in many metrology laboratories because of their robust transportability.

6.1.2 Quantum Standards for Electrical Units

The realization of physical units by systems like the standard cells implies a number of problems such as (i) damaging by improper handling, (ii) dependence on external parameters, and (iii) difficult comparison due to transport problems. In contrast, devices based on quantum effects could reduce the realization of a physical unit to the determination of fundamental constants, which are – as far as we know – independent of time and space. In the case of a quantum representation of a physical unit, the calibration can be decentralized because the unit can be reproduced with fundamental accuracy in every laboratory. Then, a regular comparison of standards is no longer necessary.

Due to the obvious advantages of quantum standards, there have been considerable efforts over the past decades to realize the physical units by quantum devices. For example, much progress has been made in the laser supported realization of the unit of length and the realization of the electrical units on the basis of the following quantum effects:

[9] W. G. Eicke, *Commentaires sur l'utilisation des diodes de zener comme etalons de tension*, Comité Consultatif d'Électricité, 11e Session, BIPM, Paris, France (1963), pp. 1874-1877.

- ***Josephson Effect***[10]
- ***Quantum Hall Effect***[11,12]
- ***Single Charge Tunneling Effect***[13,14]

According to the second Josephson equation

$$V = \frac{h}{2e} f = \Phi_0 f \ , \tag{6.1.1}$$

the Josephson effect reduces the determination of a voltage V to the precise counting of the number of flux quanta Φ_0 crossing the junction. To ensure that all standard laboratories can take full advantage of the small uncertainty in the realization of the "quantum volt", the ***Consultive Committee of Electricity (CCE)***, recommended the following value for the Josephson constant in 1989[15]

$$\boxed{K_{J-90} \equiv \left(\frac{h}{2e}\right)^{-1} = \frac{1}{\Phi_0} = 483\,597.9 \text{ GHz/V} \ .} \tag{6.1.2}$$

This value is in use since January 1990. In the same way, according to

$$I = e f \tag{6.1.3}$$

the single charge tunneling effect reduces the determination of a current to the precise counting of single electrons tunneling from one electrode to the other. Although the determination of a current by counting electrons is studied intensively since about 1990, this method still suffers from the difficulties in fabricating the required nano-circuits.

Based on the quantum Hall effect also the ohm can be established based on a quantum effect. The von Klitzing constant $R_K = \frac{h}{e^2}$ was defined by the CCE in 1990 to

$$\boxed{R_{K-90} \equiv \frac{h}{e^2} = 25\,812.802\ \Omega \ .} \tag{6.1.4}$$

The Quantum Triangle for Electrical Units

The relations between the electrical units according to the quantum definition are shown in Fig. 6.2. They are interrelated by the so-called ***quantum mechanical triangle***. We note that the analogy between the determination of voltage and current – the counting of flux and charge quanta – results from the fact

[10]B.D. Josephson, *Possible new effects in superconducting tunneling*, Phys. Lett. **1**, 251-253 (1962).

[11]K. von Klitzing, G. Dorda, M. Pepper, *New method for high accuracy determination of the fine-structure constant based on quantized Hall resistance*, Phys. Rev. Lett. **45**, 494-497 (1980).

[12]K. von Klitzing, *The quantized Hall effect*, Rev. Mod. Phys. **58**, 519-531 (1986).

[13]L.J. Geerlings *et al.*, *Frequency locked turnstyle device for single electrons*, Phys. Rev. Lett. **64**, 2691-2694 (1990).

[14]K.K. Likharev, *Correlated discrete transfer of single electrons in ultra-small tunnel junctions*, IBM J. Res. Develop. **32**, 144-158 (1988).

[15]T.J. Quinn, *News from the BIPM*, Metrologia **26**, 69-74 (1989).

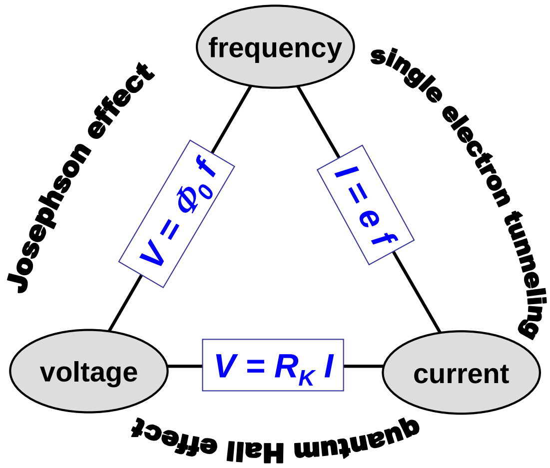

Figure 6.2: The metrological triangle for the electrical units. Josephson effect and single electron tunneling effect relate the units for voltage and current with that of the frequency. Voltage and current are linked via the quantum Hall effect.

that magnetic flux and electric charge are dual quantities from the viewpoint of quantum mechanics.[16] Fig. 6.2 demonstrates that it would be important to close the triangle experimentally by the precise determination of the current via charge counting. This could be important for the redefinition of the international system of physical constants and might result in the clarification of discrepancies in the determination of the fine structure constant.[17,18]

[16] A.J. Leggett, *Macroscopic Quantum Systems and the Quantum Theory of Measurement*, Suppl. Theor. Phys. **69**, 80-100 (1980).

[17] B.N. Taylor, *The possible role of the fundamental constants in replacing the kilogram*, IEEE Trans. Instrum. Meas. **IM-40**, 86-91 (1991).

[18] B.N. Taylor, E.R. Cohen, *How accurate are the Josephson and quantum Hall effects and QED*, Phys. Lett. **153 A**, 308-312 (1991).

6.2 The Josephson Voltage Standard

6.2.1 Underlying Physics

The physics underlying the Josephson voltage standard already has been discussed in Chapter 3. As we have seen in section 3.3, the dynamics of the phase difference φ across a Josephson junction can be locked to an external oscillator. In this case the supercurrent is forced to oscillate at the frequency $f_1 = 2\pi\omega_1$ of the external oscillator or at higher harmonics nf_1 over a considerable range of the applied dc current. This effect results in a series of constant voltage steps in the current-voltage characteristics (IVCs) of the junction, the so-called Shapiro steps,[19] at the voltages (compare (3.3.26) in section 3.3.3)

$$\boxed{V_n \;=\; n\,\frac{h}{2e}\,f_1 \;=\; n\,\Phi_0\,f_1 \qquad\qquad n \;=\; 1,2,3,\ldots\;.} \tag{6.2.1}$$

For a specific n the dc current range $|\langle I_s\rangle_n|$ of the n^{th} voltage step is given by the n^{th} order Bessel function $\mathscr{J}_n$ (compare (3.3.27) in section 3.3.3)

$$\boxed{|\langle I_s\rangle_n| \;=\; I_c\left|\mathscr{J}_n\left(\frac{2eV_1}{hf_1}\right)\right|\;.} \tag{6.2.2}$$

Here, V_1 is the amplitude of the external high-frequency voltage source. The maximum current width of the n^{th} step can be obtained by properly choosing the argument that maximizes the Bessel function, i.e. the amplitude V_1 of the microwave radiation. The dependence $\mathscr{J}_n\left(\frac{2eV_1}{hf_1}\right)$ has been shown in Fig. 3.11.

Equation (6.2.1) for V_n forms the physical basis of the Josephson voltage standard. It has been proven experimentally with very high precision.[20,21,22] Fig. 6.3 shows a sketch of an underdamped planar SIS-type Josephson junction together with the IVCs under microwave radiation. It is seen that the constant voltage steps are crossing the voltage axis leading to the so-called zero-current constant voltage steps.

6.2.2 Development of the Josephson Voltage Standard

Single Junction Standards

Soon after the prediction of the Josephson effect the voltage-frequency relation $V = \Phi_0 f$ was experimentally confirmed with high precision. One of the early issues was whether this relationship was material independent. In 1968 **Clarke** as well as **Parker, Langenberg, Denenstein**, and **Taylor** compared, via a potentiometer, the Josephson voltages of junctions consisting of five different superconducting materials and various combinations of thin-film tunnel junctions or point contacts with 1.018 V Weston saturated

[19] S. Shapiro, Phys. Rev. Lett. (1963).
[20] R.L. Kautz, L. Lloyd, *Precision of series array Josephson voltage standards*, Appl. Phys. Lett. **51**, 2043-2045 (1987).
[21] J. Niemeyer, L. Grimm, C.A. Hamilton, R.L. Steiner, *High precision measurement of a possible resistive slope of Josephson array voltage steps*, IEEE Electron Dev. Lett. **EDL-7**, 44-46 (1986).
[22] J.-S. Tsai, A.K. Jain, J.E. Lukens, *High precision test of the universality of the Josephson voltage-frequency relation*, Phys. Rev. Lett. **51**, 316-318 (1983).

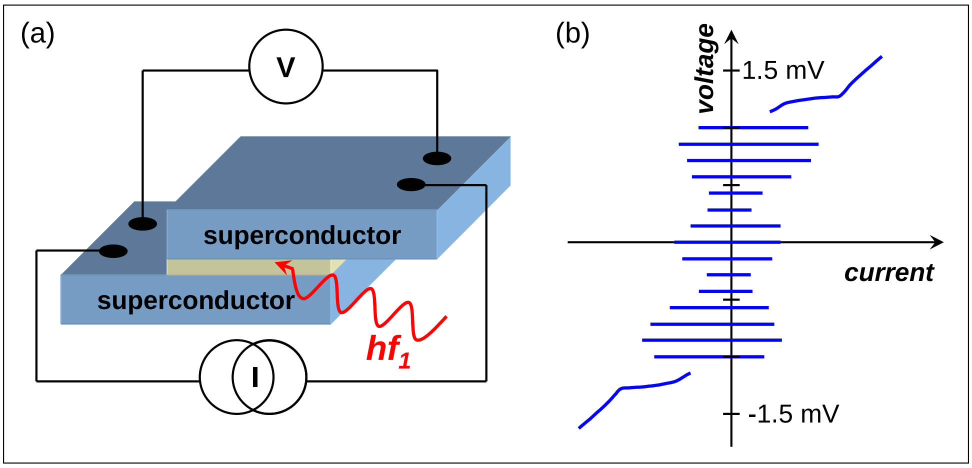

Figure 6.3: (a) Sketch of a planar SIS-type Josephson junction. (b) IVC of an underdamped Josephson junction under microwave irradiation ($f_1 = 35$ GHz). The Shapiro steps cross the voltage axis resulting in zero current constant voltage steps.

standard cells.[23,24,25] They obtained a value of $2e/h$ with a one-standard-deviation fractional uncertainty of 3.6×10^{-6}. The use of SQUID null detectors in the early 1970s allowed this to be tested to a few parts in 10^9, and thus the Josephson effect had obvious potential for use as a voltage standard.[26]

As indicated by Fig. 6.3, the maximum available voltage of the current steps is of the order of 1 mV for a single Josephson junction. Therefore, precise voltage dividers have to be developed to compare the Josephson voltage standard to the electrochemical standard cells. By the early 1970s, the mV-range Josephson junction voltages could be compared with 1.018 V standard cells to a few parts in 10^8.[27,28] International comparisons in 1971-72 among national metrology institutes found that the measured values of $2e/h$ agreed with each other to within 2×10^{-7}.[29] These results suggested the course of adopting a value of $2e/h$ for use in maintaining units of voltage. Consequently, in 1972 the CCE suggested to use a value of $K_{J-72} = 2e/h = 483\,594.0$ GHz/V for voltage comparison.[30] This value was corrected to $K_{J-90} = 483\,597.9$ GHz/V in 1990. Since that time the national voltage standards have been controlled by single Josephson junction voltage standards and the role of the Weston cell as the primary standard for the volt has ended. Besides the relative insensitivity against environmental influences, the new Josephson voltage standard had a better reproducibility of only a few parts in 10^8, which resulted in a considerable reduction of the spread of the standard voltages of the different national laboratories (cf. Fig. 6.4).[31]

[23]J. Clarke, *Experimental comparison of the Josephson voltage-frequency relation in different superconductors*, Phys. Rev. Lett. **21**, 1566-1569 (1968).

[24]W. H. Parker, D. N. Langenberg, A. Denenstein, and B. N. Taylor, *Determination of e/h using macroscopic quantum phase coherence in superconductors: I. Experiment*, Phys. Rev. **177**, 639-664 (1969).

[25]T.D. Bracken, W.O. Hamilton, *Comparison of the microwace induced constant voltage steps in Pb and Sn Josephson junctions*, Phys. Rev. **B 6**, 1603-2609 (1972).

[26]B. N. Taylor, W. H. Parker, D. N. Langenberg, and A. Denenstein, *On the use of the ac Josephson effect to maintain standards of electromotive force*, Metrologia **3**, 89-98 (1967).

[27]F. K. Harris, H. A. Fowler, and P. T. Olsen, *Accurate Hamon-pair potentiometer for Josephson frequency-to-voltage measurements*, Metrologia **6**, 134-142 (1970).

[28]B. F. Field, T. F. Finnegan, and J. Toots, *Volt maintenance at NBS via $2e/h$: a new definition of the NBS volt*, Metrologia **9**, 155-166 (1973).

[29]Woodward G. Eicke and Barry N. Taylor, *Summary of international comparisons of as-maintained units of voltage and values of $2e/h$*, IEEE Trans. Instrum. Meas. **IM-21**, 316-319 (1972).

[30]J. Terrien, *New from the Bureau International des Poids et Mesures*, Metrologia **9**, 40-43 (1973).

[31]B.W. Petley, *Quantum metrology and electrical standards: the measurement of $2e/h$*, Quantum Metrology and Fundamental Physical Constants, NATO ASI Series B, Vol. **98**, P.H. Cutler and A.A. Lucas eds., Plenum, New York (1983).

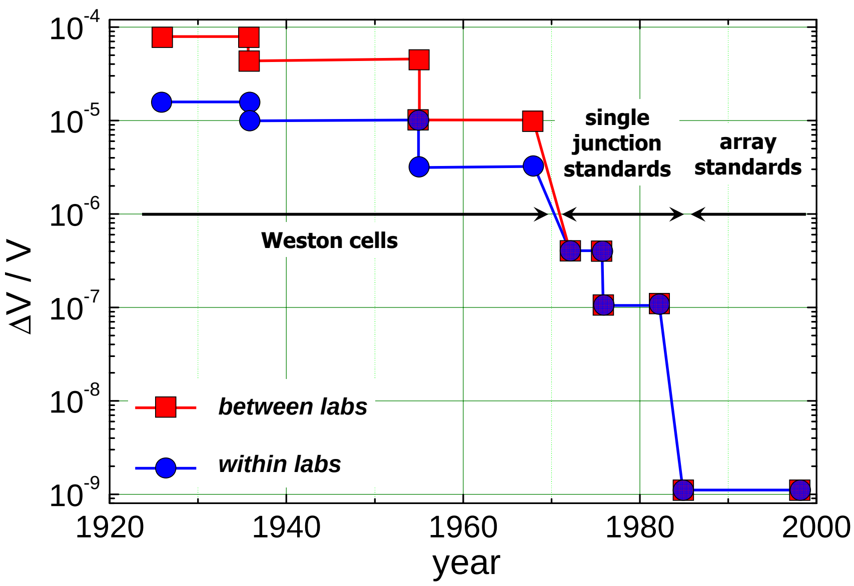

Figure 6.4: Results of the international voltage comparisons between 1920 and 2000 performed by several national laboratories. The relative accuracy $\Delta V/V$ has been improved from about 10^{-4} to 10^{-9} going from Weston cells to series array Josephson voltage standards (data from H. Bachmair, PTB Bericht **E-24**, 1 (1988), and C. A. Hamilton and Y. H. Tang, Metrologia **36**, 53-58 (1999)).

Series Array Standards

Despite the fact that the Josephson junctions provided undoubtedly better references than standard cells, some problems remained. The typical 5 mV to 10 mV reference output from early Josephson devices made from a few junctions required both very low-level voltage balances and scaling by a factor of 100, both of which seriously limited the accuracy of measuring 1.018 V standard cells. To overcome these difficulties attempts to use series arrays of Josephson junctions were made to obtain larger reference voltages.[32] However, that time the junction fabrication was not reliable enough to guarantee parameter spreads that would have allowed to bias a large series array of junctions with a single current source. Nevertheless, reference voltages up to 100 mV with an uncertainty of a few parts in 10^9 have been obtained.

Then in 1977, **M.T. Levinson** and colleagues showed that unbiased Josephson junctions spontaneously develop quantized dc voltages when irradiated with microwaves. Using the zero current steps, arrays can tolerate much larger spread in the junction parameters because they can be operated at zero current, where all the junctions have steps.[33] Stable 1 V zero-crossing arrays were operating at National Bureau of Standards (NBS)[34] and the Physikalisch-Technische Bundesanstalt (PTB)[35] by 1985, using about 1500 junctions and rf fields of 70 GHz to 90 GHz. Arrays with output voltages at the level of 1 V soon were used worlwide.[36] Note that for an operation frequency of 70 GHz Shapiro steps at multiples of about 145 μV are obtained. That is, in order to obtain a total output voltage of 1 V, about 7 000 junctions are

[32] T. Endo, M. Koyanagi, A. Nakamura, *High accuracy Josephson potentiometer*, IEEE Trans. Instrum. Meas. **IM-32**, 267-271 (1983).

[33] M.T. Levinsen, R.Y. Chiao, M.J. Feldman, B.A. Tucker, *An inverse ac Josephson effect voltage standard*, Appl. Phys. Lett. **31**, 226-778 (1977).

[34] C. A. Hamilton, R. L. Kautz, R. L. Steiner, and Frances L. Lloyd, *A Practical Josephson Voltage Standard at 1 V*, IEEE Electron Device Lett. **EDL-6**, 623-625 (1985).

[35] J. Niemeyer, L. Grimm, W. Meier, J. H. Hinken, and E. Vollmer, *Stable Josephson reference voltages between 0.1 and 1.3 V for high-precision voltage standards*, Appl. Phys. Lett. **47**, 1222-1223 (1985).

[36] Richard L. Steiner and Bruce F. Field, *Josephson array voltage calibration system: operational use and verification*, IEEE Trans. Instrum. Meas. **38**, 296-301 (1989).

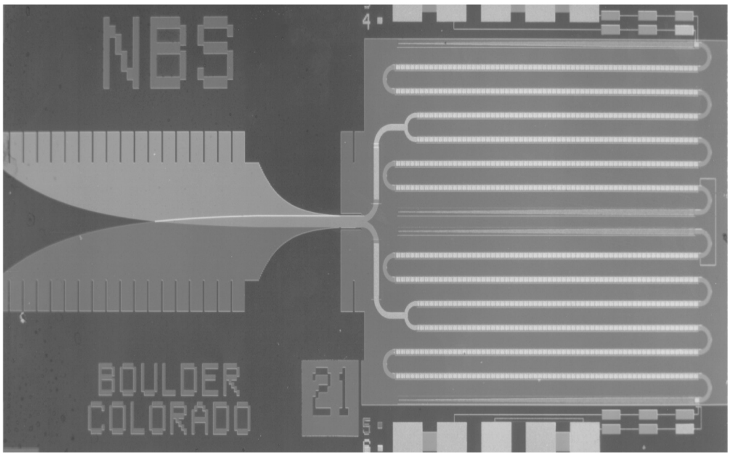

Figure 6.5: One-Volt Josephson junction array consisting of 3020 Josephson junctions (curtesy of NIST, Boulder).

required, if the junctions are operated at $n = 1$ Shapiro step. For $n = 5$, still about 1 400 junctions are required. Fig. 6.5 shows an optical micrograph of a 1 V series array consisting of 3020 junctions. The difficulties that have to be solved in the realization of series array Josephson voltage standards are the following:

- ***Fabrication of a large number of Josephson junctions with almost identical parameters***

  If there is for example a too large spread of the critical current values $I_{c,i}$ of the Josephson junctions, there is a certain probability that not all junctions are operated at the same Shapiro step, since they are biased with the same dc current resulting in different values for $I_{\rm bias}/I_{c,i}$. This problem has been solved by both improving the fabrication process, by optimizing the junction parameters (see section 6.2.3) and by using zero-current steps in arrays of underdamped junctions.

  Voltage standard chips are typically fabricated on silicon or glass substrates with the integrated circuit consisting of eight levels: (1) a 300 nm thick Nb ground plane, (2) a 2 μm layer of SiO_2 that forms the microstripline dielectric, (3) a 200 nm Nb film that forms the lower electrode of the Josephson junctions, (4) a 3 nm metal oxide layer that forms the Josephson tunneling barrier, (5) a 100 nm Nb junction counterelectrode, (6) a 300 nm SiO_2 film with windows for contacts to the counterelectrode, (7) a 400 nm film of Nb that connects the junction counterelectrodes, and (8) a 100 nm resistive film that forms the stripline terminations. This structure is shown in Fig. 6.6, however, with the superconducting groundplane on top instead at bottom.

- ***Homogeneous microwave irradiation***

  Each Josephson junction in the series array should be irradiated by the same microwave power. Therefore, the proper design of the microwave circuit is an important aspect of series array Josephson voltage standards. Usually, the Josephson junctions are arranged in a series array that is incorporated into a superconducting microwave stripline as shown in Fig. 6.6. In this way an effective and uniform coupling of the microwave to the single junctions could be obtained and large arrays with more than 15 000 junctions for reference voltages above 10 V could be realized.[37,38] Due to the finite damping of the microwave signal along the microwave stripline sketched in Fig. 6.6, a homogeneous microwave irradiation is still difficult to achieve for large series arrays of several

[37]F.L. Lloyd, C.A. Hamilton, J.A. Beall, D. Go, R.H. Ono, R.E. Harris, *A Josephson array voltage standard at 10 V*, IEEE El. Dev. Lett. **EDL-8**, 449-450 (1987).

[38]R. Pöpel, J. Niemeyer, R. Frommknecht, W. Meier, L. Grimm, *1- and 10-V series array Josephson voltage standard in Nb/AlO$_x$/Nb technology*, J. Appl. Phys. **68**, 4294-4303 (1990).

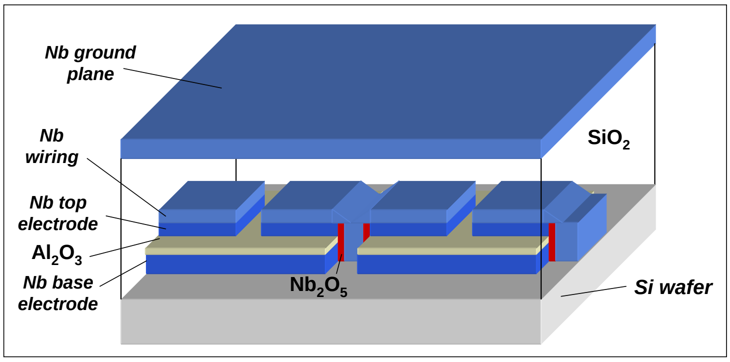

Figure 6.6: Sketch of a small part of a Josephson series array embedded into a microwave stripline.

thousands of junctions. This problem has been solved by using a meander-shaped design of the array. For the microwave signal, the different parts of the array are in parallel, whereas for the dc signal they are still in series. The microwave design and an optical micrograph of a chip with a series array Josephson voltage standard is shown in Fig. 6.7. A fin-line taper inserted into a slit in a waveguide is often used as microwave antenna. The antenna structure is then connected to a microwave divider and blocking capacitors. Each microwave path is terminated by a lossy line to prevent microwave reflections from the end, which would cause a standing wave pattern and hence an inhomogeneous microwave distribution. With respect to the dc connections, the four lines in Fig. 6.7a are connected in series. With some variations this design is used by most manufactures worldwide.[39,40]

In the 1980s series arrays with a continuously growing number of junctions have been developed mainly at NIST, Boulder, and PTB, Braunschweig.[41,42] With arrays containing about 15 000 Josephson junctions about 150 000 quantized voltage values spanning the range from -10 V to +10 V could be realized. By 1989, all of the hardware and software for a complete voltage metrology system were commercially available. The widespread use of Josephson junction arrays in national standards laboratories, and better SI determinations of $2e/h$, led the CCE to recommend a new exact conventional value for the Josephson constant: $K_{J-90} = 483\,597.9$ GHz/V, which is fractionally larger by 8×10^{-6} than the 1972 conventional value. The new value was adopted worldwide on January 1, 1990. This definition of K_{J-90} is the present volt representation, based on an ideal Josephson voltage standard. The conventional value was assumed by the CCE to have a relative standard uncertainty of $0.4\,\mu$V/V. By convention, this uncertainty is not included in the uncertainties of the representation of the volt, since any offset from the SI volt will be consistent among different laboratories using the Josephson effect standard. Today, there are Josephson array voltage standards in more than 50 national, industrial, and military standards laboratories around the world. A program of international comparisons carried out by the Bureau International des Poids et Mesures (BIPM) has measured differences between a traveling Josephson standard and those of the National Measurement Institutes that are typically less than 1 part in 10^9.[43,44,45,46]

[39] C.A. Hamilton, C.J. Burroughs, R.L. Kautz, *The performance and reliability of the NIST 10 V Josephson array*, IEEE Trans. Instrum. Meas. **IM-44**, 233-241 (1995).

[40] S.B. Kaplan, *Technology transfer and the Josephson voltage standard*, Supercond. Industry **8**, 25-30 (1995).

[41] C.A. Hamilton, *Josephson voltage standards*, Rev. Sci. Instrum. **71**, 3611-3623 (2000).

[42] J. Niemeyer, PTB Mitteilungen **110**, 169 (2000).

[43] D. Reymann and T. J. Witt, IEEE Trans. Instrum. Meas. **42**, 596 (1993).

[44] J. P. Lo-Hive, D. Reymann, and G. Geneve's, IEEE Trans. Instrum. Meas. **44**, 230 (1995).

[45] D. Reymann, T. J. Witt, G. Eklund, H. Pajander, and H. Nilsson, IEEE Trans. Instrum. Meas. **46**, 220 (1997).

[46] D. Reymann, T. J. Witt, G. Eklund, H. Pajander, H. Nilsson, R. Behr, T. Funck, and F. Müller, IEEE Trans. Instrum. Meas. **48**, 257 (1999).

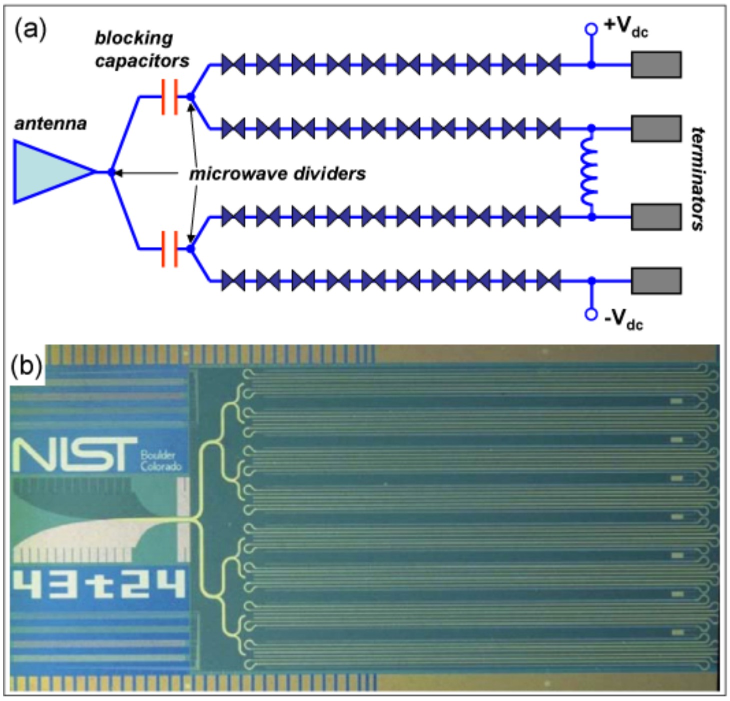

Figure 6.7: (a) Schematics of the microwave design of a series array Josephson voltage standard. (b) Optical micrograph of a NIST series array Josephson voltage standard for 10 V. The array consist of 20 208 Josephson junctions operated at 75 GHz. On the left the fin-line taper is seen. The series array on the right is split up into 16 parallel parts with 1 263 junctions each (curtesy of NIST, Boulder).

6.2.3 Junction and Circuit Parameters for Series Arrays

The idea of using series arrays for the realization of Josephson voltage standards with higher reference voltage is simple. By the use of N junctions in series the output voltage can be increased to N times the value available for a single junction. However, a prerequisite for the generation of stable zero current steps is that the phase lock of the phase dynamics of the nonlinear Josephson junction and the external microwave oscillator must be maintained during the calibration process. This condition must now be satisfied for a large series array and not only a single junction. In order to achieve this goal, reduction of external noise sources is required and one also has to avoid to operate the Josephson junctions in a regime where chaotic phenomena can occur.

The parameter regime of single Josephson junctions where optimum phase locking is guaranteed has been analyzed by **Kautz** as well as **Nöldecke** and coworkers.[47,48] They found the following conditions for the frequency $f_1 = 2\pi\omega_1$ of the external microwave oscillator:

$$\omega_1 \gg \omega_p = \sqrt{\frac{2eI_c}{\hbar C}} = \sqrt{\frac{2eJ_c}{\hbar C_s}} . \tag{6.2.3}$$

Here, ω_p is the plasma frequency, $J_c = I_c/A$ is the critical current density and $C_s = C/A$ the specific junction capacitance. We see that one has to use junctions with low plasma frequency, i.e. low critical current density and large specific capacitance and/or high microwave frequencies.

[47] R.L. Kautz, *Noise, chaos and the Josephson effect*, Rep. Prog. Phys. **59**, 935-992 (1996).
[48] Ch. Noeldeke, R. Gross, M. Bauer, G. Reiner, H. Seifert, *Experimental Survey of Chaos in the Josephson Effect*, J. Low Temp. Phys. **64**, 235 (1986).

In order to obtain a homogeneous distribution of the rf current and in order to avoid large junction effects, the junction length L and width W must satisfy the conditions:

$$L < \frac{3}{\omega_1}\frac{1}{\sqrt{n\mu_0 C_s(t+2\lambda_L)}} = L_{\max} \tag{6.2.4}$$

$$L,W < \frac{\pi}{\omega_1}\frac{1}{\sqrt{\mu_0 C_s(t+2\lambda_L)}} = W_{\max} \ . \tag{6.2.5}$$

Here, t is the thickness of the tunneling barrier, λ_L the London penetration depth of the electrode material and n the number of the constant voltage step.[49]

Detailed simulations have shown that the microwave frequency ω_1 should be larger than about $3\omega_p$ in order to avoid the regime of chaotic phase dynamics. This results in an upper limit of

$$J_{\mathrm{c,max}} = \frac{9\omega_1^2\hbar C_s}{2e} \ . \tag{6.2.6}$$

Together with the limitations on the junction length and width this leads to a maximum value of the critical current $I_{\mathrm{c,max}} = J_{\mathrm{c,max}}W_{\max}L_{\max}$ and thus according to (6.2.2) to a maximum width of the constant voltage steps, over which a stable operation of the voltage standard can be achieved. For the commonly used Nb/AlO$_x$/Nb junction technology we have $C_s \simeq 6\,\mu$F/cm^2 and $\lambda_L \simeq 80$ nm. At $f_1 = 70$ GHz this results in $J_{\mathrm{c,max}} \simeq 40$ A/cm^2, $W_{\max} \simeq 65\,\mu$m, $L_{\max} \simeq 20\,\mu$m, and hence $I_{\mathrm{c,max}} \simeq 500\,\mu$A. The maximum step width for the 7^{th} voltage step is then about $350\,\mu$A.[50]

[49] R.L. Kautz, G. Costabile, *A Josephson voltage standard using a series array of 100 junctions*, IEEE Trans. Magn. **MAG-17**, 780-783 (1981).
[50] J. Niemeyer, *Josephson voltage standards*, in *Handbook of Applied Superconductivity 2*, B. Seeber ed., Institute of Physics Publishing, Bristol (1998), pp.1813-1834.

6.3 Programmable Josephson Voltage Standard

In the previous section we have discussed Josephson voltage standards based on zero-current voltage steps. These steps are obtained for series arrays or single underdamped junctions. It is obvious from Fig. 6.3 that a particular problem with these steps is their strong overlapping making the rapid switching between the steps problematic. This problem is sketched in Fig. 6.8. In order to access a certain Shapiro step, that is a specific output voltage $V_{\rm out}$, one could simply think of applying a dc voltage $V_{\rm b} \simeq V_{\rm out}$ using e.g. a battery. However, due to the finite resistance of the leads to the chip, the voltage is split up between the array and the leads following the loadline as shown in Fig. 6.8. Since the loadline is crossing several Shapiro steps, it is difficult to select a specific step in a well defined way. It is evident that there are different ways of splitting up the voltage between the leads and the array. Once a specific step is adjusted, the step voltage has to be measured by a voltmeter that has an accuracy sufficient to distinguish between neighboring steps (about 145 μV for 70 GHz).

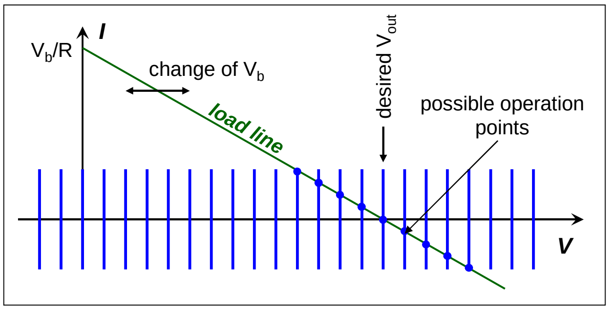

Figure 6.8: Sketch of the adjustment procedure for the quantized Shapiro step voltages in a series array of underdamped Josephson junctions. The external battery voltage $V_{\rm b}$ can be distributed along the load line $V_{\rm b}/R$ in different ways on the leads and the series array.

The problem sketched in Fig. 6.8 can be avoided by using overdamped Josephson junctions. In principle, overdamped Josephson junctions can be obtained by shunting underdamped SIS-type junctions with an external resistor. However, this makes the fabrication process more complicated. More appropriate are SNS-type (superconductor/normal/superconductor) or SINIS-type (superconductor/insulator/normal/insulator/superconductor) Josephson junctions. As shown by Fig. 6.9, in these overdamped junctions the steps do not overlap and one can rapidly select a specific step by adjusting the bias current.

Series arrays of overdamped junctions can be used to realize programmable Josephson voltage standards. Such systems use a series array of nonhysteretic junctions with IVCs as shown in Fig. 6.9, which are divided into a binary sequence as schematically shown in Fig. 6.10. The microwave excitation for each junction is set to roughly equalize the amplitude of the $n = 0$ and $n = 1$ constant voltage steps. Each segment of the array can be set to the $n = 1$, 0, or $+1$ steps by applying a bias current ($-I_1$, $I_0 = 0$, $+I_1$) corresponding to the middle of the corresponding steps (cf. Fig. 6.9). The combined step number N for the whole array can thus be set to integer values between $-M$ and $+M$, where M is the total number of junctions in the array.[51,52,53] The rapid settling time, the inherent step stability, and the large operating current margins of the Josephson voltage standard sketched in Fig. 6.10 make it superior to a conventional Josephson voltage standard for many dc measurements. The improved performance of

[51] C.A. Hamilton, C.J. Burroughs, and R.L. Kautz, *Josephson D/A Converter with Fundamental Accuracy*, IEEE Trans. Instrum. Meas. **44**, 223-225 (1995).

[52] C.A. Hamilton, C.J. Burroughs, R.L. Kautz, *Digital-to-Analog Converter with Voltage defined by Josephson Frequency Voltage Relation*, U.S. Patent No. 5565866, Oct. 1996.

[53] C.A. Hamilton, C.J. Burroughs, S.P. Benz, and J.R. Kinard, *AC Josephson Voltage Standard: A Progress Report*, IEEE Trans. Instrum. Meas. **46**, 224-227 (1997).

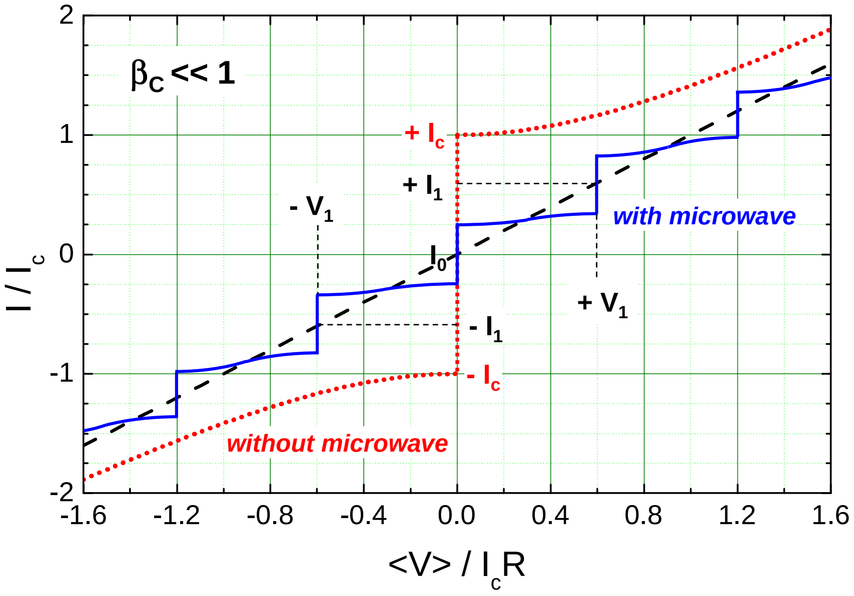

Figure 6.9: IVC of an overdamped ($\beta_C \ll 1$) Josephson junction with (solid line) and without (dotted line) microwave irradiation. The dashed curve shows the ohmic line.

programmable Josephson voltage standards has been made possible by a new integrated-circuit technology using intrinsically shunted SNS- or SINIS-type Josephson junctions. These junctions have been fabricated by extending the Nb/AlO$_x$/Nb technology. However, also intrinsically overdamped junctions of the high temperature superconductors could be used, although at present the parameter spread of these junctions is still too large.[54] An important fact is that the overdamped junctions operate at lower excitation frequencies (10 to 20 GHz)[55] than those of Josephson voltage standards based on hysteretic SIS junctions and have about 100 times larger step amplitudes (typically 2 to 4 mA).[56] The only disadvantage of the programmable Josephson voltage standard is the required larger number of Josephson junctions, because only the first voltage step is contributing to the sum voltage. Typically, the number of junctions must be increased by a factor of five to seven.

For a microwave frequency of 20 GHz the voltage generated by each step is $V = f/K_{J-90} \simeq 41.3\,\mu$V. Then, about 24 000 junctions are required to achieve a total output voltage of about 1 V. For example, a binary array consisting of segments from 2^0 to 2^{14} junctions would consist of $2^{15} - 1 = 32\,367$ individual junctions and would yield a maximum output voltage of about 1.35 V at a driving frequency of 20 GHz. Programmable voltage standards for the 1 V level have been successfully fabricated by NIST, Boulder, using Josephson junctions with a normal conducting barrier of AuPd alloy,[57] and by PTB using Josephson junctions with a superconductor-insulator-normal conductor-insulator-superconductor (SINIS) structure.[58]

Since the Zener reference secondary voltage standards commonly used provide reference voltages of 1 and 10 V, there is a demand for an intrinsically stable and rapidly programmable voltage standard for the 10 V level. If microwaves in the frequency range of 70 GHz are used, a series array of about 70 000 junctions is required, which must be fabricated with low parameter spread and small defect rate. The

[54] A.M. Klushin, S.I. Borovitskii, C. Weber, E. Sodtke, R. Semerad, W. Prusseit, V.D. Gelikonova, H. Kohlstedt, Appl. Supercond. **158**, 587 (1997).

[55] R.L. Kautz, *Quasipotential and the stability of phase lock in nonhysteretic Josephson junctions*, J. Appl. Phys. **76**, 5538-5544 (1994).

[56] S.P. Benz, *Superconductor-normal-superconductor Junctions for Programmable Voltage Standards*, Appl. Phys. Lett. **67**, 2714-2716 (1995).

[57] S.P. Benz, C.A. Hamilton, C.J. Burroughs, T.E. Harvey, and L.A. Christian, Appl. Phys. Lett. **71**, 1866 (1997).

[58] R. Behr, H. Schulze, F. Müller, J. Kohlmann, and J. Niemeyer, IEEE Trans. Instrum. Meas. **48**, 270 (1999).

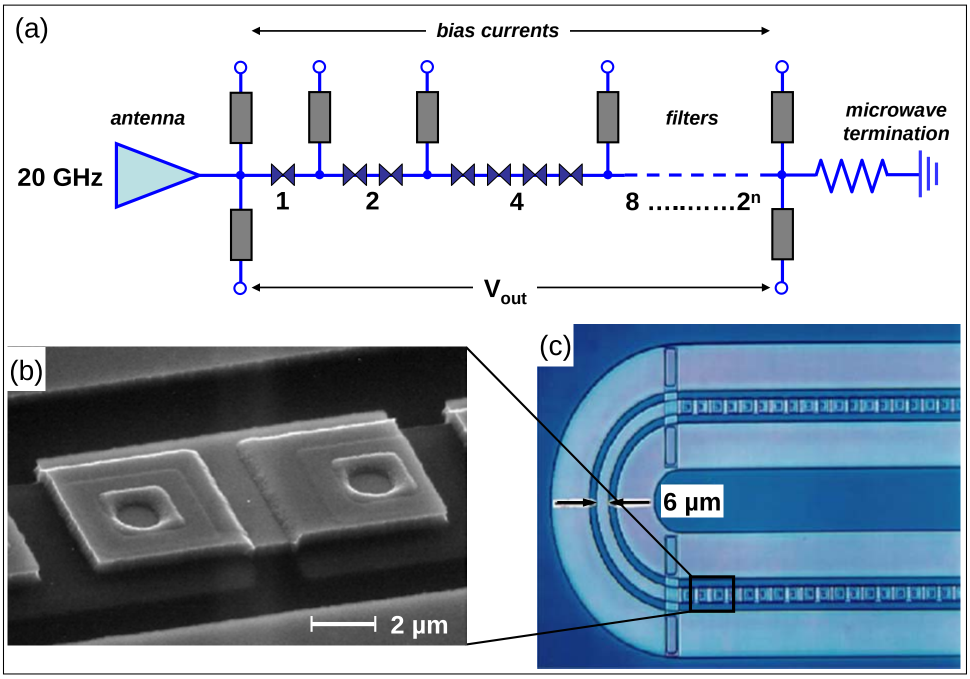

Figure 6.10: (a) Sketch of the arrangement and bias schematic of a programmable Josephson voltage standard. (b) Scanning electron microscopy image showing the $2 \times 2\,\mu m^2$ Nb/PdAu/Nb Josephson junctions of a 32 768 Josephson junction series array. The series array is arranged in several segments connected with coplanar waveguides. In (c) a small section of the coplanar waveguide with the distribution of junctions along the center conductor is shown (images by curtesy of NIST, Boulder).

whole array must be fed homogeneously with sufficient microwave power. Due to the larger number of junctions required to obtain 10 V using programmable voltage standard arrays, a higher integration density is required to avoid an increase in the chip size. This can be achieved by moderate reduction of the junction area and by a modified microwave design, which is possible because of the different microwave properties of SINIS arrays. Chips with 69 120 SINIS-type Josephson junction series arrays for a programmable 10 V Josephson dc voltage standard have been successfully fabricated and operated with 70 GHz microwave irradiation. The current-voltage characteristics exhibits non-hysteretic voltage steps with a step width of $200\,\mu$A.[59,60]

6.3.1 Pulse Driven Josephson Arrays

Although the programmable Josephson voltage standards are quite useful in several applications, binary programmable arrays have not been very successful in the synthesis of ac wave forms because the undefined voltage during transitions between steps adds an unacceptable level of uncertainty. To solve this problem, **Benz** and **Hamilton** have developed another approach that biases the array with pulses.[61] So far we only discussed to program the voltage of a Josephson array by changing the step number n resulting in a different voltage $V_n = nf\Phi_0$. It is clear that the same result might be achieved by changing f. Unfortunately, in the case of a sine-wave excitation, the step amplitudes collapse rapidly to zero as the

[59] H. Schulze, R. Behr, J. Kohlmann, F. Müller, and J. Niemeyer, *Design and fabrication of 10 V SINIS Josephson arrays for programmable voltage standards*, Supercond. Sci. Technol. **13**, 1293-1295 (2000).
[60] Yonuk Chong, C. J. Burroughs, P. D. Dresselhaus, N. Hadacek, H. Yamamori, and S. P. Benz, *Practical High-Resolution Programmable Josephson Voltage Standards Using Double- and Triple-Stacked $MoSi_2$-Barrier Junctions*, IEEE Trans. Appl. Supercond. **AS-15**, 461-464 (2005).
[61] S. P. Benz and C. A. Hamilton, Appl. Phys. Lett. **68**, 3171 (1996).

frequency decreases. This means that it is practical to control the voltage via the frequency only over a range of frequency within about a factor of 2 of the optimum frequency $f_c = I_c R/\Phi_0$. However, simulations show that if the sine-wave excitation is replaced with a pulse excitation, then the step amplitude is independent of the pulse repetition frequency for all frequencies below f_c.[62,63,64] The optimum pulse width is $\tau = 1/\omega_c = 1/2\pi f_c$.

A programmable voltage source based on this idea consists of a single large array of N junctions distributed along a transmission line with wide bandwidth. A pulse train at frequency f propagating down the line generates an average voltage $N f \Phi_0$ across the ends of the array. A complex output wave form can be generated by modulating the pulse train with a digital word generator. For example, using a clock frequency of $f_c = I_c R/\Phi_0 = 10\,$GHz, the pulse sequence 11111000001111100000 creates an output square wave with amplitude $N f_c \Phi_0$ and frequency 1 GHz.

[62] R. Monaco, J. Appl. Phys. **68**, 679 (1990).

[63] S. P. Benz, C. A. Hamilton, and C. J. Burroughs, IEEE Trans. Appl. Supercond. **AS-7**, 2653 (1997).

[64] F. Liefrink, G. de Jong, P. Teunissen, J. W. Heimeriks, A. Royset, A. A. Dyrseth, H. Schulze, R. Behr, J. Kohlmann, E. Volmer, and J. Niemeyer, Proc. BEMC (1999).