Applied Superconductivity:

Josephson Effect and Superconducting Electronics

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Microwave Applications

Superconducting devices find brought applications in passive microwave devices such as filters or resonators. These applications are based on the very small losses due to the small microwave surface resistance of superconducting materials. Therefore, the quality factor of superconducting microwave resonators and filters is larger than for equally sized normalconducting devices. In the same way, at the same quality factor superconducting microwave resonators and filters can be made much smaller. This is in particular important for satellite or space applications. In this Chapter we briefly discuss the foundations of superconducting passive microwave devices and describe a few prominent devices structures and applications.

Superconducting devices based on Josephson junctions can serve as sources for microwave radiation. Here, the underlying principle is based on the voltage-frequency relation $V = f\Phi_0$, which immediately suggests that a Josephson junction can be used as a voltage controlled oscillator with $f/V = 483\,597.9$ GHz/V. In this Chapter we also present the foundations of superconducting microwave sources based on Josephson junction.

8.1 High Frequency Properties of Superconductors

8.1.1 The Two-Fluid Model

Already in 1934, that is long before the development of BCS theory, **Cornelius Gorter** and **H.B.G. Casimir** developed the two fluid model of superconductors.[1,2] The model is based on the concept that there are two fluids in superconductors, namely a superfluid with carrier density n_s and a normal fluid with carrier density n_n with the total carrier density given by

$$n \;=\; n_n + \frac{n_s}{2} \;. \tag{8.1.1}$$

Here, the factor $\frac{1}{2}$ arises from the fact that the carriers of the superfluid are pairs with charge $-2e$. We will use the two-fluid model together with Ohm's law

$$\frac{1}{\sigma_n}\mathbf{J}_n \;=\; \mathbf{E} \tag{8.1.2}$$

and the linearized first London equation (compare (1.1.70) and (1.1.71))

$$\frac{\partial}{\partial t}(\Lambda \mathbf{J}_s) \;=\; \frac{\partial}{\partial t}(\mu_0 \lambda_L^2 \mathbf{J}_s) \;=\; \mathbf{E} \tag{8.1.3}$$

to describe the relation of the normal and superfluid current density and the electric field. In (8.1.3)

$$\Lambda \;\equiv\; \frac{m_s^\star}{n_s^\star q^{\star 2}} \tag{8.1.4}$$

is the ***London coefficient*** and

$$\lambda_L \;\equiv\; \sqrt{\frac{m_s^\star}{\mu_0 n_s^\star q^{\star 2}}} \tag{8.1.5}$$

the ***London penetration depth*** (compare (1.1.63) and (1.1.64)). Here, $q^\star = -2e$ and $m_s^\star$ are the charge and the effective mass of the carrier forming the superfluid.

For a harmonic current with angular frequency ω (8.1.3) can be written as

$$\imath\omega\Lambda\,\mathbf{J}_s \;=\; \imath\omega\mu_0\lambda_L^2\,\mathbf{J}_s \;=\; \frac{1}{\sigma_s}\mathbf{J}_s \;=\; \mathbf{E} \tag{8.1.6}$$

with the purely imaginary conductivity of the superfluid

$$\boxed{\sigma_s \;=\; \frac{n_s^\star q^{\star 2}}{\imath\omega m_s^\star} \;=\; \frac{1}{\imath\omega\Lambda} \;=\; \frac{1}{\imath\omega\mu_0\lambda_L^2} \;.} \tag{8.1.7}$$

[1] D. Shoenberg, *Superconductivity*, Cambridge University Press, Cambridge (1965), pp. 194-196.
[2] T. van Duzer, *Principles of Superconducting Devices and Circuits*, Elsevier, New York, Amsterdam, London (1981), p. 124.

To derive the equivalent relations for the normal fluid we write the normal current as $\mathbf{J}_n = n_n e \mathbf{v}_n$, where $\mathbf{v}_n$ is the average velocity of the normal carriers, and assume that the normal carriers have to satisfy Newton's law[3]

$$m_n^\star \left(\frac{d\mathbf{v}_n}{dt} + \frac{\mathbf{v}_n}{\tau} \right) = e\mathbf{E} \ . \tag{8.1.8}$$

Here, τ is the scattering time of the normal carriers and $m_n^\star$ and e the effective mass and charge of the normal carriers. Again, for a sinusoidal current with angular frequency ω we obtain

$$\mathbf{J}_n = \left(\frac{n_n e^2}{m_n^\star} \right) \frac{\tau}{1 + \imath\omega\tau} \mathbf{E} = \sigma_n \mathbf{E} \ . \tag{8.1.9}$$

The complex conductivity of the normal current can be expressed as

$$\boxed{\sigma_n = \sigma_{n1} - \imath\sigma_{n2} = \left(\frac{n_n e^2 \tau}{m_n^\star} \right) \frac{1 - \imath\omega\tau}{1 + (\omega\tau)^2} = \sigma_0 \frac{n_n}{n} \frac{1 - \imath\omega\tau}{1 + (\omega\tau)^2} \ .} \tag{8.1.10}$$

Here, $\sigma_0 = n_n e^2 \tau / m_n^\star$ is the usual normal state Drude conductivity. With (8.1.2), (8.1.3) and (8.1.6) together with the expressions for the conductivities and Maxwell's equations we can derive the high-frequency properties of superconductors.

Note that the conductivities σ_n and σ_s show a strong temperature dependence below T_c due to the temperature variation of the normal and superfluid density. At $T = T_c$ we have $n_s = 0$ and $n_n = n$. Below T_c the superfluid density increases and the normal fluid density decreases as

$$\frac{n_n}{n} = \left(\frac{T}{T_c} \right)^4 \tag{8.1.11}$$

$$\frac{n_s}{2n} = 1 - \left(\frac{T}{T_c} \right)^4 \ . \tag{8.1.12}$$

At $T = 0$ all the carriers are condensed into the superfluid and we have $n_s = n/2$ and $n_n = 0$.

In an electrotechnical language the two-fluid model can be visualized by the equivalent circuit shown in Fig. 8.1. The superfluid channel which does not contribute to the loss and has purely imaginary conductivity can be modeled by an inductor $L_s(T)$. The normal channel has a conductivity composed of a imaginary and a real part. The former, represented by the inductor $L_n(T)$, is due to the inertia of the charge carriers and the latter, represented by the resistor $R_n(T)$, due scattering induced loss. Note that the inductor L_n in the normal channel is often neglected which is similar to modeling the normal channel as nondispersive (frequency independent).

We can use the equivalent circuit to classify different frequency regimes. Evidently at $\omega = 0$ all the current is carried by the nondissipative superconducting channel. However, increasing the frequency the conductivity of the superfluid density becomes finite and decreases with frequency. Therefore, the contribution of $\mathbf{J}_s$ decreases with increasing frequency and becomes equal to $\mathbf{J}_n$ at the cross-over frequency $\omega_{ns} = R_n/L_s$. That is, the superfluid dominates in the low-frequency regime $0 \leq \omega \leq \omega_{ns}$. In the high-frequency regime the normal channel dominates. We further can discuss the question at which frequency there is a cross-over between an ohmic (nondispersive) response to an inductive (dispersive) response in

[3]The quantity e represents the unit of the electric charge with an electron having the charge $-e$.

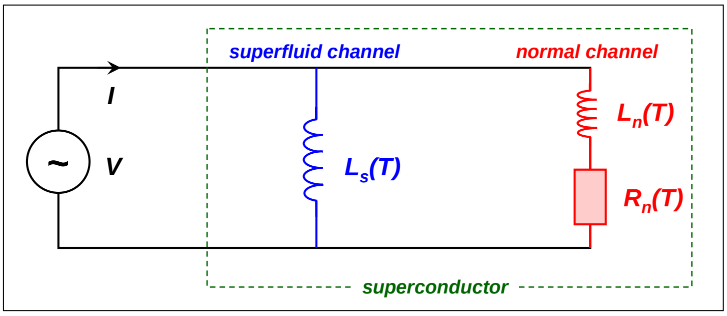

Figure 8.1: Equivalent circuit for the two-fluid model of a superconductor.

the normal channel. Evidently this occurs at the frequency $\omega_\tau = R_n/L_n = 1/\tau$. That is, in the frequency regime $\omega_{ns} \leq \omega \leq 1/\tau$ the ohmic response and for $\omega \geq 1/\tau$ the inductive response of the normal channel is dominant. In the high-frequency regime we restrict our discussion to $\omega < \omega_\Delta = \Delta/\hbar$. Above the gap frequency ω_Δ the microwave photons can break up Copper pairs and the situation becomes more complicated. For a superconductor with $\Delta = 1$ meV the gap frequency ω_Δ is about 1 THz. Note that $\tau_{ns} = L_s/R_n$ increases strongly with decreasing temperature due to the increase of L_s and the decrease of R_n associated with the temperature variation of n_n and n_s. Therefore, the cross-over frequency ω_{ns} increase with decreasing temperature and typically becomes larger than ω_τ as shown in Fig. 8.2. In this case there is no frequency regime where the ohmic normal channel dominates.

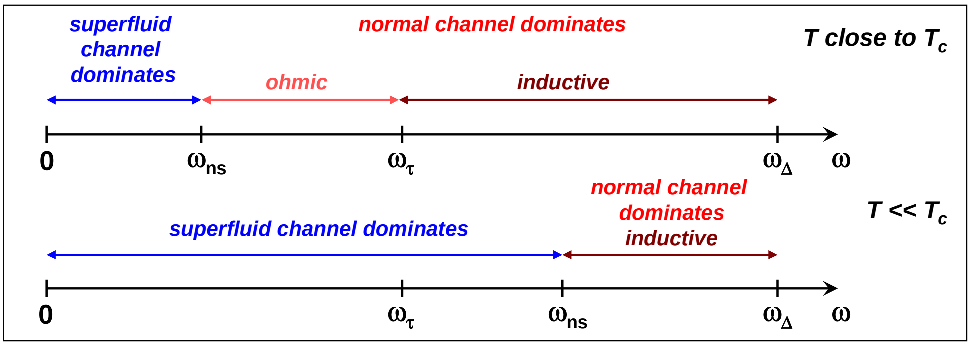

Figure 8.2: Characteristic frequency regimes for a superconductor for different temperatures.

Using equations (8.1.7) and (8.1.10) we obtain the total conductivity of a superconductor to

$$\boxed{\sigma = \sigma_s + \sigma_n = \frac{n_n e^2 \tau}{m_n^\star} \frac{1}{1+(\omega\tau)^2} - \imath \frac{n_n e^2 \tau}{m_n^\star} \frac{\omega\tau}{1+(\omega\tau)^2} - \imath \frac{1}{\omega\mu_0\lambda_L^2} .} \qquad (8.1.13)$$

At frequencies $\omega\tau \ll 1$ this can be simplified to

$$\sigma = \sigma_1 - \imath\sigma_2 = \frac{n_n e^2 \tau}{m_n^\star} - \imath \frac{1}{\omega\mu_0\lambda_L^2} , \qquad (8.1.14)$$

where σ_1 and σ_2 are the real and imaginary components of the complex conductivity. The real part represents the loss from the normal carriers, whereas the imaginary part represents the kinetic energy of the superconductive carriers.

8.1.2 The Surface Impedance

Normal Metals

The surface impedance is defined as the characteristic impedance seen by a plane wave incident perpendicular upon a flat surface of a conductor. It is given by the ratio of the electric and the magnetic field at the surface. For thick normalconducting materials the surface impedance is equal to the bulk wave impedance Z which can be derived from Maxwell's equations to

$$Z_s = R_s + \imath X_s = \sqrt{\frac{\imath\mu_0\omega}{\sigma_0}} = (1+\imath)\sqrt{\frac{\mu_0\omega}{2\sigma_0}} = (1+\imath)\frac{\mu_0\omega\delta_0}{2} . \tag{8.1.15}$$

Here, $\sigma_0 = ne^2\tau/m^\star$ is the normal state conductivity and

$$\delta_0 = \sqrt{\frac{2}{\mu_0\omega\sigma_0}} \tag{8.1.16}$$

is the normal state field penetration depth. For a normal metal at frequencies $\omega\tau \ll 1$, i.e. in the ohmic regime, the conductivity is a real number and according to (8.1.16) the surface resistance R_s and the surface reactance X_s are equal:

$$R_s = X_s = \sqrt{\frac{\mu_0\omega}{2\sigma_0}} = \frac{\mu_0\omega\delta_0}{2} . \tag{8.1.17}$$

We see that for normal metals both R_s and X_s are proportional to $\sqrt{\omega}$.

Superconductors

In order to derive the surface impedance of a superconductor we the normal state conductivity σ_0 in (8.1.16) by the total conductivity of a superconductor and obtain

$$\begin{aligned} Z_s = R_s + \imath X_s &= \sqrt{\frac{\imath\mu_0\omega}{\sigma}} = \left[\frac{\sigma_n\frac{1}{1+(\omega\tau)^2} - \imath\sigma_n\frac{\omega\tau}{1+(\omega\tau)^2} - \imath\frac{1}{\omega\mu_0\lambda_L^2}}{\imath\omega\mu_0}\right]^{-1/2} \\ &= \imath\omega\mu_0\left[\imath\omega\mu_0\sigma_n\frac{1}{1+(\omega\tau)^2} + \omega\mu_0\sigma_n\frac{\omega\tau}{1+(\omega\tau)^2} + \frac{1}{\lambda_L^2}\right]^{-1/2} . \end{aligned} \tag{8.1.18}$$

At frequencies $\omega\tau \ll 1$ this can be simplified to

$$Z_s = \frac{\imath\omega\mu_0}{\lambda_L}\left[1 + \imath\omega\mu_0\lambda_L^2\sigma_n\right]^{-1/2} = \imath\sqrt{\frac{\omega\mu_0}{\sigma_2}}\left[1 + \imath\frac{\sigma_1}{\sigma_2}\right]^{-1/2} , \tag{8.1.19}$$

where we have used $\sigma = \sigma_1 + \imath\sigma_2$ with $\sigma_1 = \sigma_n$ and $\sigma_2 = 1/\omega\mu_0\lambda_L^2$. We can now further simplify (8.1.19) by taking into account that $\sigma_1 \ll \sigma_2$ for temperatures not to close to T_c. With $(1+x)^{-1/2} \simeq 1 - \frac{1}{2}x$ we obtain

$$Z_s = \imath\sqrt{\frac{\omega\mu_0}{\sigma_2}}\left(1 - \imath\frac{\sigma_1}{2\sigma_2}\right) = \sqrt{\frac{\omega\mu_0\sigma_1^2}{2\sigma_2^3}} + \imath\sqrt{\frac{\omega\mu_0}{\sigma_2}} . \tag{8.1.20}$$

Table 8.1: Conductivity σ, penetration depth δ_0 due to the normal skin effect, London penetration depth λ_L, surface resistance R_s and surface reactance X_s of normal conductors and superconductors for $\omega\tau \ll 1$ and temperatures $T \ll T_c$.

	normal conductor	superconductor
conductivity	$\sigma_0 = \frac{ne^2\tau}{m_n^\star}$	$\sigma_1 + \imath\sigma_2 = \frac{ne^2\tau}{m_n^\star}\left(\frac{n_n}{n}\right) - \imath\frac{1}{\omega\mu_0\lambda_L^2}$
field penetration depth	$\delta_0 = \sqrt{2/\omega\mu_0\sigma_0}$	$\delta_s = \lambda_L$
surface resistance	$R_s = \frac{1}{2}\omega\mu_0\delta_0 = \sqrt{\frac{\omega\mu_0}{2\sigma_0}}$	$R_s = \frac{1}{2}\omega^2\mu_0^2\lambda_L^3\sigma_0\left(\frac{n_n}{n}\right)$
surface reactance	$X_s = \frac{1}{2}\omega\mu_0\delta_0 = \sqrt{\frac{\omega\mu_0}{2\sigma_0}}$	$X_s = \omega\mu_0\lambda_L$

Using expression (8.1.14) for σ_1 and σ_2 we finally obtain

$$\boxed{Z_s \;=\; R_s + \imath X_s \;=\; \frac{\omega^2\mu_0^2\lambda_L^3 n_n e^2\tau}{2m_n^\star} + \imath\omega\mu_0\lambda_L \;=\; \frac{1}{2}\omega^2\mu_0^2\lambda_L^3\sigma_0\left(\frac{n_n}{n}\right) + \imath\omega\mu_0\lambda_L\ .} \tag{8.1.21}$$

We see that the surface resistance, the real part of Z_s, increases proportional to ω^2 in contrast to normal conductors, where $R_s \propto \sqrt{\omega}$. Furthermore, it increases proportional to λ_L^3 and the conductivity $\sigma_0 n_n/n$ of the normal fluid. In Table 8.1 the most main characteristics of superconductors are compared to those of normal metals.

Fig. 8.3 shows the theoretically expected surface resistance as a function of frequency for the superconductor Nb and the normal metal Cu. We see that for frequencies below about 100 GHz the surface resistance of Nb is considerably lower than for Cu at 77 K. At high frequencies there is a cross-over due to the much weaker frequency dependence of the surface resistance of normal metals. Note that the surface resistance is expected to be further reduced by going to lower temperatures due to the strong decrease of n_n. At $T/T_c \ll 1$, $\lambda_L(T) \simeq const$ and $n_n \propto \exp(-2\Delta_0/k_BT)$. Therefore, an exponential decrease of R_s with decreasing T is expected. However, this behavior is usually not observed in experiment. Rather a temperature independent residual surface resistance is measured at very low T, which is attributed to material defects. For Nb this residual resistance is as low as $10^{-9}\Omega/\Box$ at 10 GHz, whereas it reaches only about $10^{-5}\Omega/\Box$ for $YBa_2Cu_3O_{7-\delta}$ films.

Kinetic Inductance

The surface reactance X_s, the imaginary part of the surface impedance, is purely inductive. The equivalent inductance L_k is denoted as ***kinetic inductance***

$$\boxed{L_k \;=\; \mu_0\lambda_L\ .} \tag{8.1.22}$$

The kinetic inductance reflects the kinetic energy of the carriers of the superfluid.

Finally we note that the phenomenological model used above id based on local theory, which is valid only as long as the coherence length ξ of a superconductor is much smaller than the London penetration

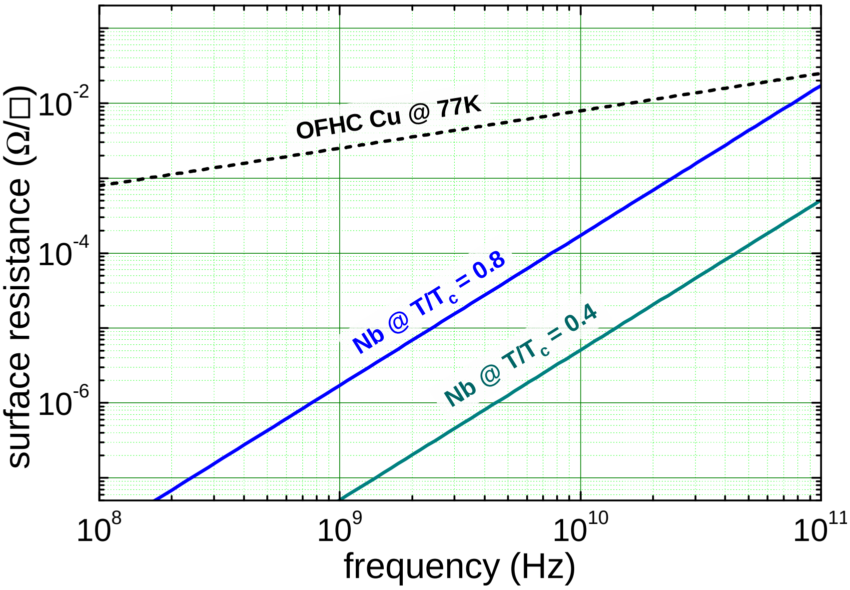

Figure 8.3: Intrinsic surface resistance versus frequency for Nb and oxygen free high conductance (OFHC) Cu. For Cu, $\sigma_0 = 10^8\Omega^{-1}\mathrm{m}^{-1}$, for Nb, $\lambda(0) = 85$ nm and $\sigma_0 = 10^8\Omega^{-1}\mathrm{m}^{-1}$ were used.

depth λ_L. This is for example the case for extreme type-II superconductors such as the high temperature superconductors. In contrast, for type-I superconductors we have $\xi > \lambda_L$. In this case the above treatment is non longer valid and we have to used a more complicated nonlocal theory.[4]

[4] A.B. Pippard, *An experimental and theoretical study of the relation between magnetic field and current in a superconductor*, Proc. Roy. Soc. (London), **A 216**, 547-568 (1963).

8.2 Superconducting Resonators and Filters

in preparation

8.3 Superconducting Microwave Sources

in preparation