Applied Superconductivity:

Josephson Effect and Superconducting Electronics

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Superconducting Quantum Bits

In section 3.5 we already have discussed the quantum consequences of the small capacitance of Josephson junctions leading to interesting secondary quantum macroscopic effects. The interest in macroscopic quantum effects in small capacitance Josephson junctions goes back to the 1980ies. The initial interest was to test whether or not the laws of quantum mechanics can be applied to macroscopic systems in a Hilbert space spanned by macroscopically distinct states.[1] The phase difference of the superconducting order parameter in a Josephson junction or the magnetic flux in a superconducting quantum interference devices were the degrees of freedom used in these studies. Although quantum phenomena such as macroscopic quantum tunneling and resonant tunneling of the phase difference could be demonstrated,[2,3,4] it was impossible to observe coherent quantum oscillations between two macroscopically different (e.g. flux) states, i.e. macroscopic quantum coherence.[5]

The field of macroscopic quantum coherence in superconducting systems received new attention in the last years, when it became obvious that Josephson circuits are interesting candidates for the realization of quantum bits (qubits). The vision that superconducting devices may serve as qubits in quantum information processing and that quantum logic operations could be performed by controlling gate voltages or magnetic fields stimulated an intensive research effort.[6,7,8,9,10,11,12] Meanwhile there has been a tremendous progress in quantum state engineering in superconducting and other solid state systems. In particular, the quantum superposition of macroscopically distinct states, coherent oscillations and entangled states of several qubits have been observed. Superconducting Josephson systems are very promising,

[1] A. J. Leggett, in *Chance and Matter*, edited by J. Souletie, J. Vannimenus, and R. Stora, Elsevier, Amsterdam (1987), p. 395.

[2] R. F. Voss, R. A. Webb, *Macroscopic quantum tunneling in a* $1\,\mu m$ *Nb Josephson junction*, Phys. Rev. Lett. **47**, 265 (1981).

[3] J. M. Martinis, M. H. Devoret, J. Clarke, *Experimental tests of the quantum behavior of a macroscopic degree of freedom: the phase difference across a Josephson junction*, Phys. Rev. **B 35**, 4682 (1987).

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[10] Y. Makhlin, G. Schön, A. Shnirman, *Josephson junction qubits with controlled couplings*, Nature **386**, 305 (1999).

[11] J.E. Mooij, T.P. Orlando, L. Levitov, L. Tian, C.H van der Wal, S. Lloyd, *Josephson persistent current qubit*, Science **285**, 1036 (1999).

[12] Y. Nakamura, Y.A. Pashkin, J.S. Tsai, *Coherent control of macroscopic quantum states in a single Cooper pair box*, Nature **398**, 786 (1999).

since they can be fabricated by established technologies and their control and measurement techniques are far advanced. Furthermore, superconducting qubits exploit the coherence of the superconducting state allowing the achievement of sufficiently long phase coherence times.

After giving a brief introduction to quantum information processing, in this chapter we discuss the realization of quantum bits by using superconducting Josephson junction devices.

9.1 Quantum Bits and Quantum Computers

9.1.1 Quantum Bits

Our today's classical computers represent the culmination of years of technological advancements beginning with the early ideas of **Charles Babbage** (1791-1871) and the creation of the first computer by the German engineer **Konrad Zuse** in 1941. Surprisingly however, the high speed modern computer is fundamentally not different from its gargantuan 30 ton ancestors, which were equipped with some 18 000 vacuum tubes and 500 miles of wiring. Although computers have become more compact and considerably faster in performing their task, the basic task remains the same: to manipulate and interpret an encoding of binary bits into a useful computational result. A bit is a fundamental unit of information, classically represented as a "0" or "1" in our digital computers. Each classical bit is physically realized through a macroscopic physical system, such as the magnetization on a hard disk or the charge on a capacitor. A document, for example, comprised of n-characters stored on the hard drive of a typical computer is accordingly described by a string of $8n$ zeros and ones. Herein lies a key difference between our todays classical computer and a quantum computer. Whereas a classical computer obeys the well understood laws of classical physics, a quantum computer is a device that uses physical phenomena unique to quantum mechanics (especially quantum interference) to realize a fundamentally new mode of information processing.[13,14,15]

Figure 9.1: Konrad Zuse 1945: Konrad Zuse was building the first binary digital computer Z1 in 1938. The first programmable electromechanical computer Z3 was completed in 1941. Zuse also developed the first algorithmic programming language called "Plankalkül".

Whereas classical computers operate with ***classical (c-) bits*** usually represented by "0" and "1", quantum computers operate with ***quantum (qu-) bits*** usually denoted as ***qubits***. Physically, a qubit can be represented by every two level quantum system. The basic properties of such systems are discussed in detail in Appendix F. With the basis states of a two level quantum system (e.g. a spin-1/2 system, see Appendix G)

$$|\phi_1\rangle = |0\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (9.1.1)$$

$$|\phi_2\rangle = |1\rangle = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (9.1.2)$$

we can define a single-qubit state in the following way:

[13] T. Beth and G. Leuchs, *Quantum Information Processing*, Wiley-VCH, Berlin (2003).

[14] M.A. Nielsen, I.L. Chuang, *Quantum Information and Quantum Information*, Cambridge University Press, Cambridge (2000).

[15] D. Bouwmeester, A. Ekert, A. Zeilinger eds., *The Physics of Quantum Information*, Springer, Berlin (2000).

A qubit $|\Psi\rangle$ is the superposition of two computational basis states

$$|\Psi(t)\rangle \;=\; a(t)|0\rangle + b(t)|1\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} . \tag{9.1.3}$$

Here, $a(t)$ and $b(t)$ are complex amplitudes. If we are measuring the quantum state of a qubit, we obtain the result $|0\rangle$ with probability $|a(t)|^2$ and the result $|1\rangle$ with probability $|b(t)|^2$. Since the total probability must be unity, we have to satisfy the normalization condition

$$\langle\Psi(t)|\Psi(t)\rangle \;=\; |a(t)|^2 + |b(t)|^2 = 1 . \tag{9.1.4}$$

We see that the single-qubit exists in a continuum of states. It is a superposition of two basis states and therefore can be represented as a unit vector in a two-dimensional Hilbert space $\mathscr{H}_2$. The key fact is that a qubit can exist not only in a state corresponding to the logical state "0" or "1" as in a classical bit, but also in states corresponding to a blend or superposition of these classical states. In other words, a qubit can exist as a zero, a one, or simultaneously as both "0" and "1", with a numerical coefficient representing the probability for each state. This may seem counterintuitive because everyday phenomenon are governed by classical physics, not quantum mechanics – which takes over at the atomic level.

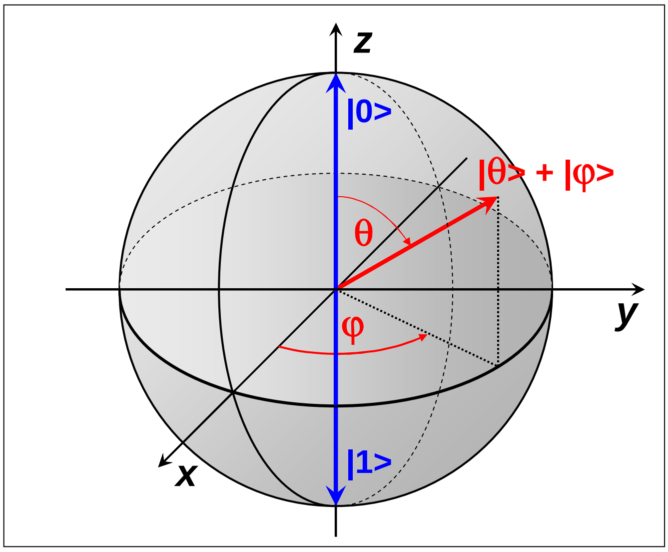


Figure 9.2: Representation of a qubit state as a vector on the Bloch sphere.

A convenient way of representing of a qubit state is as a unit vector on the Bloch sphere as shown in Fig. 9.2. With the angles θ and φ the general state can be expressed as (compare Appendix E)

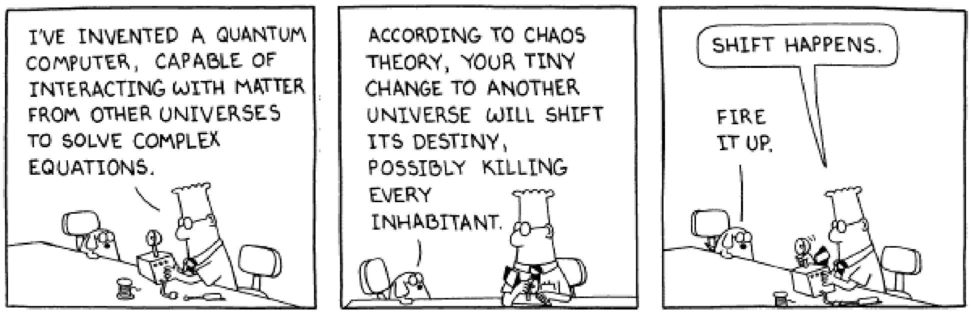
$$|\Psi\rangle \;=\; \cos\frac{\theta}{2}e^{-\imath\varphi/2}|0\rangle + \sin\frac{\theta}{2}e^{+\imath\varphi/2}|1\rangle . \tag{9.1.5}$$

Note that this state is equivalent to the general state of a spin-1/2-system (compare (G.II.20) in Appendix G). In a measurement process one has to get information on the angles θ and φ.

The foundations of single- and two-qubit states and their manipulation by unitary operators are discussed in detail in Appendix E. There, also an introduction to classical one- and two-bit gates and their quantum counterparts is given.

9.1.2 Quantum Computing

Over the last decades the performance of computers has increased tremendously. This could lead us to suppose that there is no problem that is too complicated to be solved with a classical computer. However, this is not the case. Let us consider the very simple problem of about 150 spin 1/2 particles (e.g. electrons). If we want to do a quantum mechanical description of the state of these electrons, we will exceed the capacity of every conceivable classical computer. The reason for that is that the Hilbert space of the collective spin state of these 150 electrons has the dimension $2^{150} \simeq 10^{45}$. The corresponding density matrix would have 10^{90} elements. Since to our present knowledge the number of protons in our universe is just about 10^{90}, it is just impossible to build a classical computer with the required capacity. In contrast, a quantum computer would require only a few hundred quantum bits for the simulation of the 150 electrons. This simple example shows, that for the simulation of the quantum mechanics of many particle system quantum computers would be highly desirable.



In a classical computer, information is encoded in a series of ***classical bits***, and these bits are manipulated via ***Boolean logic gates*** arranged in succession to produce an end result. In a binary systems, the bits can only have two values usually denoted as "0" and "1". Similarly, a quantum computer manipulates ***quantum bits*** by executing a series of ***quantum gates***, each a ***unitary transformation*** acting on a single qubit or pair of qubits (for details see Appendix III and IV). The quantum bits are realized by quantum mechanical two level system with the basis state denoted as $|0\rangle$ and $|1\rangle$ or $|\uparrow\rangle$ and $|\downarrow\rangle$ (see Appendix E to G). A basic difference between classical bits and quantum bits is the fact that the superposition principle allows superpositions of qubit values as entries into a register. An example for three qubits is given in Fig. 9.3. In general, for N qubits we can form 2^N superposition states, that is, the number of superposition states grows exponentially.

The second basic ingredient of a classical computer are ***Boolean logic gates***. In a quantum computer these classical gates are replaced by ***unitary operators***. It is well known that for a classical computer there exist universal sets of gates, that are sufficient to form all other possible gates. As discuss also in Appendix IV, universal sets for classical gates are (NOT, AND) or the NAND gate alone. In complete analogy there exist sets of unitary operations allowing for the realization of all possible unitary operations. Usually, such set consists of the single bit rotation $U(\theta,\varphi)$ with $U(\theta,\varphi) = \cos\theta e^{-\imath\varphi/2}|0\rangle + \sin\theta e^{+\imath\varphi/2}|0\rangle$ (compare Appendix III), and a two-qubit-operation. Here, a common example is the ***controlled-NOT gate*** (compare (E.IV.58) in Appendix IV). With the single qubit rotation and the CNOT gate we can form every arbitrary unitary operation on N qubits allowing for the implementation of any algorithm. That is, in applying these gates in succession, a quantum computer can perform a complicated unitary transformation to a set of qubits in some initial state. The qubits can then be measured, with this measurement serving as the final computational result. A simple scheme how a quantum computer works is shown in Fig. 9.3.

Fig. 9.3 suggests that the operational principle of a classical and a quantum computer are quite similar: the successive application of gates (quantum gates) to a set of bits (qubits). This similarity in calculation between a classical and quantum computer affords that in theory, a classical computer can accurately

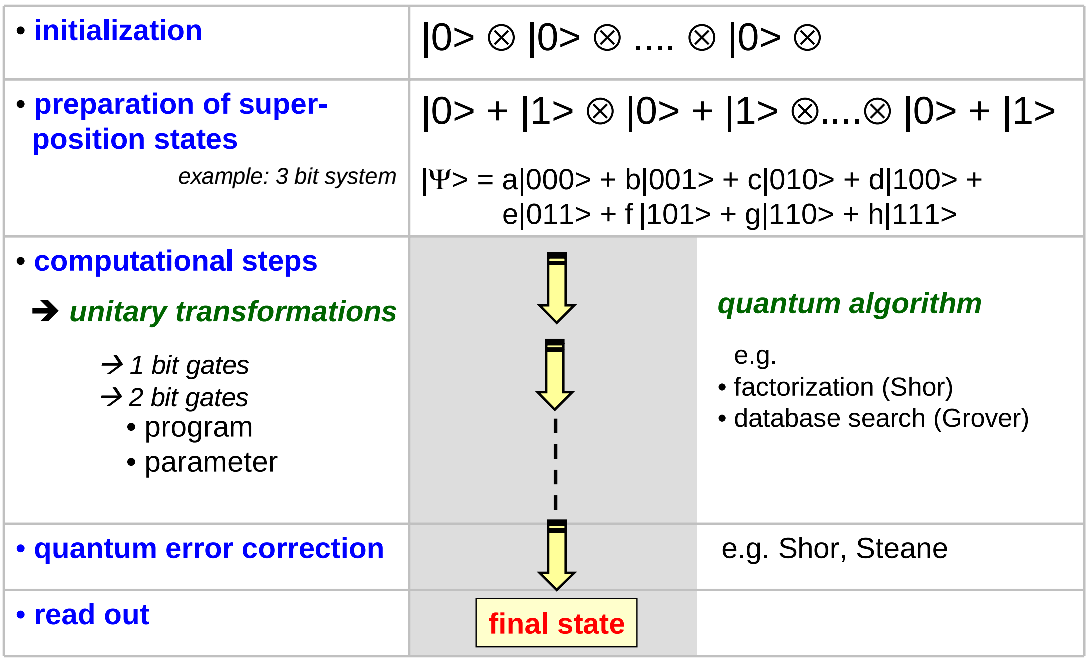


Figure 9.3: Simplified operational scheme of a quantum computer.

simulate a quantum computer. In other words, a classical computer would be able to do anything a quantum computer can. So why bother with quantum computers? Although a classical computer can theoretically simulate a quantum computer, it is incredibly inefficient, so much that a classical computer is effectively incapable of performing many tasks that a quantum computer could perform with ease. The simulation of a quantum computer on a classical one is a so-called computationally hard problem, since the correlations among quantum bits are qualitatively different from correlations among classical bits. This was first pointed out by **John Bell**. Take for example a system of only 300 qubits. This systems exists in a Hilbert space of dimension $2^{300} \simeq 10^{90}$ (quantum complexity, see Appendix VI) that in simulation would require a classical computer to work with exponentially large matrices (to perform calculations on each individual state, which is also represented as a matrix), meaning it would take an exponentially longer time than even a primitive quantum computer.

Richard Feynman was among the first to recognize the problem of handling the simulation of the quantum mechanics of many particle systems by a classical computer (already in 1981). At the same time he was pointing to the potential of quantum superposition for solving such problems much much faster. As mentioned already above, a system of 300 qubits, which is impossible to simulate classically, represents a quantum superposition of as many as $2^{300} \simeq 10^{90}$ states. Each state would be classically equivalent to a single list of 300 ones and zeros. Any quantum operation on that system – a particular pulse of radio waves, for instance, whose action might be to execute a controlled-NOT operation on the 100^{th} and 101^{st} qubits – would simultaneously operate on all 2^{300} states. Hence, with one fell swoop, one tick of the computer clock, a quantum operation could compute not just on one machine state, as serial computers do, but on 2^{300} machine states at once! Eventually, however, observing the system would cause it to collapse into a single quantum state corresponding to a single answer, a single list of 300 ones and zeros, as dictated by the measurement axiom of quantum mechanics. The reason this is an exciting result is because this answer, derived from the massive quantum parallelism achieved through superposition, is the equivalent of performing the same operation on a classical super computer with $\sim 10^{90}$ separate processors, what is of course impossible. That is, by exploiting the massive parallelism of the coherent evolution of superpositions of states, quantum computers can perform certain tasks that no classical computer could do in an acceptable time.[16,17]

Already between 1982 and 1985 **David Deutsch** provided the theoretical basis of the quantum computer by his work on quantum Turing machines. Although the early investigators in this field were naturally excited by the potential of the immense computing power, only very few people took that serious. Only when **Peter Shor**, a research and computer scientist at AT&T's Bell Laboratories in New Jersey, provided a specific application of a quantum computer by devising the first quantum computer algorithm, the field was widely recognized and then a very active hunt was on to find something interesting for a quantum computer to do. ***Shor's algorithm*** uses the power of quantum superposition to rapidly factorize very large numbers (on the order $\sim 10^{200}$ digits and greater) in a matter of seconds. The premier application of a quantum computer capable of implementing this algorithm lies in the field of ***encryption***, where one common encryption code, known as RSA, relies heavily on the difficulty of factoring very large composite numbers into their primes. A computer which can do this easily is naturally of great interest to numerous government agencies that use RSA – previously considered to be "uncrackable" – and anyone interested in electronic and financial privacy.

Figure 9.4: Quantum Computing: What's it good for?

Encryption, however, is only one application of a quantum computer. In addition, Shor has put together a toolbox of mathematical operations that can only be performed on a quantum computer, many of which he used in his factorization algorithm. Furthermore, Feynman asserted that a quantum computer could function as a kind of simulator for quantum physics, potentially opening the doors to many discoveries in the field. Currently the power and capability of a quantum computer is primarily theoretical speculation; the advent of the first fully functional quantum computer will undoubtedly bring many new and exciting applications.

A Brief History of Quantum Computing

The idea of a computational device based on quantum mechanics was first explored in the 1970's and early 1980's by physicists and computer scientists such as **Charles H. Bennett** of the IBM Thomas J. Watson Research Center, **Paul A. Benioff** of Argonne National Laboratory in Illinois, **David Deutsch** of the University of Oxford, and the late **Richard P. Feynman** of the California Institute of Technology (Caltech). The idea emerged, when scientists were pondering the fundamental limits of computation. They understood that if technology continued to abide by Moore's Law, then the continually shrinking size of circuitry packed onto silicon chips would eventually reach a point, where individual elements would be no larger than a few atoms. Here, a problem arose because at the atomic scale the physical laws that govern the behavior and properties of the circuit are inherently quantum mechanical in nature, not classical. This then raised the question of whether a new kind of computer could be devised based on the principles of quantum physics.

[16] C, Bennett, *Quantum information and computation*, Physics Today **48**, 24 (1995).
[17] D. DiVincenzo, *Quantum Computation*, Science **270**, 255 (1995).

Feynman[18] was among the first to attempt to provide an answer to this question by producing an abstract model in 1982 that showed how a quantum system could be used to do computations. He also explained how such a machine would be able to act as a simulator for quantum physics. In other words, a physicist would have the ability to carry out experiments in quantum physics inside a quantum mechanical computer. Later, in 1985, **Deutsch**[19] realized that Feynman's assertion could eventually lead to a general purpose quantum computer and published a crucial theoretical paper showing that any physical process, in principle, could be modeled perfectly by a quantum computer. Thus, a quantum computer would have capabilities far beyond those of any traditional classical computer. After Deutsch published this paper, the search began to find interesting applications for such a machine.

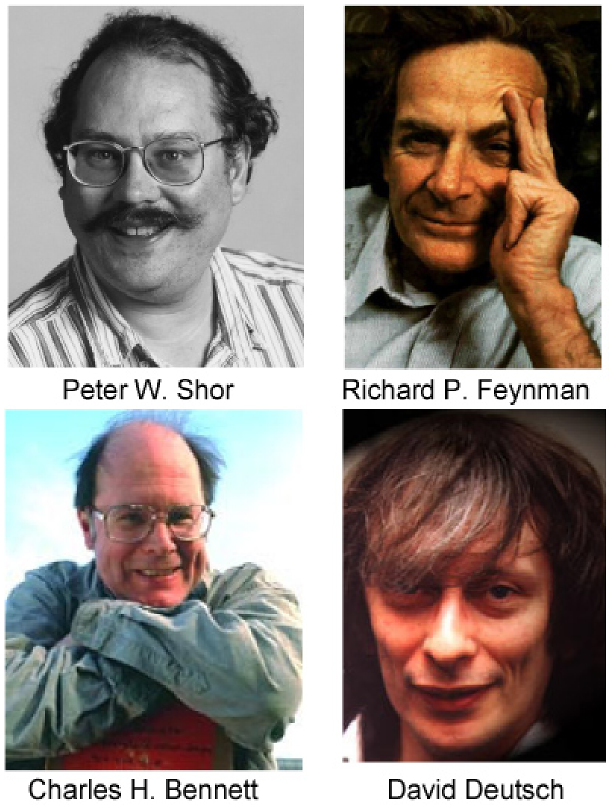


Figure 9.5: Some basic players in quantum computing.

Unfortunately, all that could be found were a few rather contrived mathematical problems, until **Shor**[20] circulated in 1994 a preprint of a paper in which he set out a method for using quantum computers to crack an important problem in number theory, namely factorization. He showed how an ensemble of mathematical operations, designed specifically for a quantum computer, could be organized to enable such a machine to factor huge numbers extremely rapidly, much faster than is possible on conventional computers. With this breakthrough, quantum computing transformed from a mere academic curiosity directly into a national and world interest.[21]

9.1.3 Quantum Error Correction

Error correction is a well known process in classical information processing. For example, a parity bit is added to each data packet in protocols used for data transmission, where parity 0 and 1 state that the data packet has an even or odd number of "1", respectively. In this way one can check whether or not a single data bit or the parity bit has changed during transmission. If for example after transmission a single bit has changed from "1" to "0" or vice versa, the parity of the data packet does no longer correspond to the parity bit attached to the packet and one has to send the whole packet again. Obviously, such simple error correction protocol only protects against the change of an odd number of bits, whereas the change of an even number of bits remain undetected. More complicated protocols, for example the ***Hamming protocol*** allow for a far more extensive protection.[22] In general, classical error correction operates by the judicious

[18]R. P. Feynman, Int. J. Theor. Phys. **21**, 467 (1982).
[19]D. Deutsch, Proc. Roy. Soc. London, Ser. A **400**, 97 (1985).
[20]Shor, P. W., *Algorithms for quantum computation: Discrete logarithms and factoring*, in Proceedings of the 35th Annual Symposium on Foundations of Computer Science, IEEE Computer Society Press (1994).
[21]D. Deutsch, A. Ekert, *Quantum Computation*, Physics World, March (1998).
[22]F. J. MacWilliams and N. J. A. Sloane, The Theory of Error-Correcting Codes, North Holland, Amsterdam (1977).

use of redundancy, that is, sending the same information many times. In this sense it is akin to making the system larger in order to make it more resistent to perturbations. However, the precise way in which the redundancy is introduced is very important. The type of redundancy, or encoding, employed must be carefully matched to the type of noise in the channel. Typically, one considers the case of random noise, which affects different bits independently, but this is not the only possible case. The encoding enables the most likely errors in the information to be identified and corrected. This corrective procedure is akin to active stabilization, and brings the associated benefits of powerful noise suppression.

Since qubits are representing superposition states $|\Psi\rangle = a|0\rangle + b|1\rangle$, it is not obvious that there are error correction protocols also for qubits. However, this is indeed the case. As has been shown by **Calderbank, Shor** and **Steane** that there is even a direct relationship between classical and quantum error correction protocols.[23,24,25,26]

To understand the application of the classical ideas to the quantum regime, it is best to start with a simple example. Thus, suppose we have a collection of spin-half particles, each of which is subject independently to random "flips" or amplitude errors $|0\rangle \to |1\rangle$, but which otherwise is stable (in particular, the precession is free of phase error). Whenever such a flip occurs, the relevant two-state system may become entangled with its environment. In order to stabilize a single qubit, in the general state $a|0_L\rangle + b|1_L\rangle$, we express it by means of three two-state systems, with the "encoding" $|0_L\rangle = |000\rangle$, $|1_L\rangle = |111\rangle$. Thus, the total initial state of the three spins is $a|000\rangle + b|111\rangle$. After a period of time, during which random flips may occur, the three-spin system is measured twice. The first measurement is a projection onto the two-state basis

$$\{\ |000\rangle + |111\rangle + |001\rangle + |110\rangle, \quad |010\rangle + |101\rangle + |100\rangle + |011\rangle\ \}$$

The second measurement is a projection onto the two-state basis

$$\{\ |000\rangle + |111\rangle + |010\rangle + |101\rangle, \quad |001\rangle + |110\rangle + |100\rangle + |011\rangle\ \}$$

Each measurement has two possible results, which we will call 0 and 1. Depending on which results R are obtained, an appropriate action is carried out: if $R = 00$, do nothing; if $R = 01$, flip the rightmost spin; if $R = 10$, flip the middle spin; if $R = 11$, flip the leftmost spin. If, during the time interval when the system was left to evolve freely, no more than one spin flipped, then this procedure will return the three-spin state to $a|000\rangle + b|111\rangle$. It is remarkable that this can be done without gaining information about the values of a and b and thus disturbing the stored quantum information. During the correction procedure, the entanglement between the system and its environment is transferred to an entanglement between the measuring apparatus and the environment. The qubit is actively isolated from its environment by means of this carefully controlled entanglement transfer. The above error correction technique is based on the simplest classical error correcting code. More advanced techniques can be deduced from more advanced known classical codes.

Before this discovery it seemed to be impossible to carry out a longer quantum algorithm in a reliable way, since already tiny errors would spread in such way that the final result would no longer have any meaning. It now has been discussed that quantum error correction could be even more powerful than classical error correction. So called interlinked quantum error correction protocols have been shown to allow for the implementation of arbitrary quantum algorithms, since in this case the probability for a wrong result is independent of the actual length of the algorithm.

In order to achieve an error tolerant operation of a quantum computer, the error probability per gate or measurement operation has to stay below a certain level. At present this threshold level is estimated to be about 10^{-4} per memory unit, gate or read out process.

[23] P. W. Shor, Phys. Rev. **A 52**, R2493 (1995).
[24] A. M. Steane, Phys. Rev. Lett. **77**, 793 (1996).
[25] A. R. Calderbank and P. W. Shor, Phys. Rev. **A 54**, 1098 (1996).
[26] A. M. Steane, Proc. Roy. Soc. **A 452**, 2551 (1996).

9.1.4 What are the Problems?

The field of quantum information processing has made numerous promising advancements since its conception. However, a few potentially large obstacles still remain that prevent us from just building a quantum computer that can rival today's modern digital computer. Among these difficulties, ***error correction***, ***decoherence***, and***hardware architecture*** are probably the most formidable. Error correction is rather self explanatory, but what errors need correction? The answer is primarily those errors that arise as a direct result of decoherence, or the tendency of a quantum computer to decay from a given quantum state into an incoherent state as it interacts, or entangles, with the state of the environment. These interactions between the environment and qubits are unavoidable, and induce the breakdown of information stored in the quantum computer, and thus errors in computation. Before any quantum computer will be capable of solving hard problems, research must devise a way to maintain decoherence and other potential sources of error at an acceptable level. Thanks to the theory of quantum error correction, first proposed in 1995 and continually developed since, small scale quantum computers have been built. Probably the most important idea in this field is the application of error correction in phase coherence as a means to extract information and reduce error in a quantum system without actually measuring that system.

Today only a few of the benefits of quantum computation and quantum computers are readily obvious, but before more possibilities are uncovered theory must be put to the test. In order to do this, devices capable of quantum computation must be constructed. Unfortunately, quantum computing ***hardware*** is still in its infancy, whereas the theoretical concepts of quantum computing, the ***software***, are already rather advanced. A solid state based hardware concept seems promising, since it allows scaling and the used of well developed fabrication techniques. However, it may be that the future of quantum computer hardware architecture is very different from what we know today. Nevertheless, the current research helps to provide insight as to what obstacles the future will hold for these devices.

9.2 Implementation of Quantum Bits

Quantum bits can be implemented with every two level quantum system as shown in Fig. 9.6. The properties of such systems are summarized in Appendix F and G. In practice, quantum information processing also requires the coherent manipulation of suitable quantum systems. The coherent manipulations of the qubits can be performed, if we have sufficient control over the fields and interaction terms in the associated Hamiltonian and if the decoherence in the considered quantum systems is small enough.

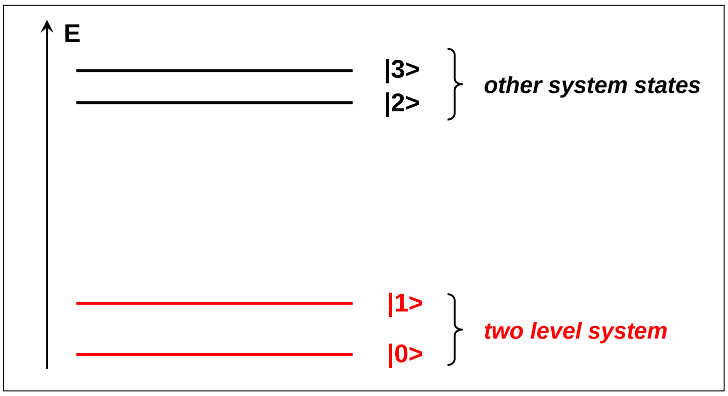


Figure 9.6: A qubit can be realized by a quantum mechanical two level system with the qubit representing the coherent superposition of the two discrete states: $|\Psi\rangle = a|0\rangle + b|1\rangle$. The only requirement is that all other states of the system are well separated from the states $|0\rangle$ and $|1\rangle$ in order to have an effective two-level system. Note that this requirement cannot be achieved for a harmonic potential, where all the states are equidistant.

As has been stressed by **DiVincenzo**, [27,28] any physical system that is considered as a candidate for the implementation of quantum bits should satisfy the following criteria (DiVincenzo checklist):

1. **Qubits:** The system has to provide a well defined two-level quantum system. This implies that higher level states that are present in most real systems are not excited during qubit manipulations.
2. **Preparation of initial state:** It must be possible to prepare the initial state with sufficient accuracy.
3. **Decoherence:** The phase coherence time must be long enough to allow for a sufficiently large number (typically 10^4) of coherent manipulations. That is, the superposition states of the qubits are allowed to dephase only on time scales much longer than the elementary gate time.
4. **Quantum gates:** There must be sufficient control over the qubit Hamiltonian to perform the necessary unitary transformations, i.e. single- and two-qubit operations (see Appendix III). For this purpose it should be possible to control the fields at the sites of the qubits separately and to couple the qubits in a controlled way (e.g. by switching on and off the inter-qubit interactions). Then, the single- and two-qubit operations allow for the generation of arbitrary superpositions and nontrivially coupled states such as entangled states (see Appendix III and IV).
5. **Quantum measurement:** For read out of the quantum information a quantum measurement is needed. This can be either at the final stage or during the computation for the purpose of error correction.
6. **Scalability:** There should be the possibility to increase to number of qubits (scalability).

[27] D. DiVincenzo, *The physical implementation of quantum computation*, Fortschr. Phys. **48**, 771 (2000).
[28] D. DiVincenzo, in*Mesoscopic Electron Transport*, edited by L. Kouwenhoven, G. Schön, and L. Sohn, NATO ASI Series E: Applied Sciences No. 345, Kluwer Academic, Dordrecht (1997), p. 657.

With respect to requirement 1 we can state that there is a large number of physical systems that have been suggested as possible realizations of qubits and gates.[29] They are usually split up into non-solid state systems (e.g. ions in electromagnetic traps,[30,31] nuclear magnetic resonance on ensembles of molecules in liquids,[32,33] cavity QED systems,[34] and neutral atoms in optical lattices[35,36,37]) and solid state systems. Solid state devices including the above mentioned Josephson systems have the advantage of being more easily embedded into electronic circuits and scaled up to a larger number of qubits (requirement 6). Besides the Josephson systems, electronic states and spin states in quantum dots as well as impurity spins in semiconductors are further candidates. They can be manipulated by tuning potentials and barriers.[38,39] Finally, electrons floating on liquid helium are discussed.

Besides the advantages of solid state systems with respect to ***scalability*** and embedding into electronic circuits, ***decoherence*** is a severe problem for solid state systems (requirement 3). Unavoidable for devices that have to be controlled externally are interactions with the environment. Due to the coupling to the environment the quantum state of the qubit gets entangled (see Appendix II) with the environmental degrees of freedom. As a consequence the phase coherence is destroyed after a time scale called the ***dephasing time***. Due to the large number of environmental degrees of freedom in solid state systems, decoherence is an important issue. Maintaining coherence of a quantum device throughout the manipulation processes is therefore the major challenge for practical quantum computing. We also note that the time evolution of the quantum state may be perturbed also by other sources such as inaccuracies in the preparation of the initial state, inaccuracies in the manipulations and uncontrolled couplings between qubits.

Quantum state engineering requires the ***coherent manipulation of quantum systems***. The manipulations can be performed, if we have sufficient control over the fields and interaction terms in the Hamiltonian. In order to discuss the requirements 4 and 5 we use a model Hamiltonian of a two-state quantum system (e.g. a spin system). We will see later that under certain conditions other systems such as “charge in a box” or “flux in a SQUID loop” effectively reduce to two-state systems. Since any single two-state quantum systems can be represented as a spin-1/2 system, in the following we write down the model Hamiltonian for this system. With the effective magnetic field **B** the Hamiltonian for the manipulation can be written as (compare Appendix III)

$$\mathscr{H}_{\mathrm{man}}(t) \quad = \quad -\frac{\hbar}{2}\gamma\mathbf{B}(t)\,\vec{\sigma} \;. \tag{9.2.1}$$

Here, $\vec{\sigma} = (\mathbf{X},\mathbf{Y},\mathbf{Z})$ are the Pauli spin matrices in the space of states $|\uparrow\rangle$ and $|\downarrow\rangle$ (compare (E.III.25) in Appendix III) and γ the gyromagnetic ratio. These states form the basis states of a physical quantity (spin, charge, flux, ...) that has to be manipulated. Full control of the quantum dynamics of the spin is achieved,

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if the field $\mathbf{B}(t)$ can be switched arbitrarily. Actually, as shown in Appendix E, full control is already achieved, if only two field components[40] can be controlled, e.g. (compare (E.III.24) in Appendix III)

$$\mathscr{H}_{\rm man}(t) = -\frac{\hbar}{2}\gamma B_z\,\mathbf{Z} - \frac{\hbar}{2}\gamma B_x\,\mathbf{X}\ . \tag{9.2.2}$$

If we want to manipulate a many-qubit system in order to perform quantum computing, we have to control the field at the sites of each spin separately. Furthermore, in addition to single-qubit operations we need two-qubit unitary operations (see Appendix III and IV). The latter require the coupling of two qubits. Including this coupling, the following model Hamiltonian seems to be suitable for a N-qubit system:

$$\mathscr{H}_{\rm man}(t) = -\sum_{i=1}^{N}\frac{\hbar}{2}\gamma\mathbf{B}^i(t)\,\vec{\sigma}^i + \sum_{i\neq j} J^{ij}_{\alpha\beta}(t)\,\vec{\sigma}^i_\alpha\vec{\sigma}^j_\beta\ . \tag{9.2.3}$$

Here, the summation over the spin indices $\alpha,\beta = x,y,z$ is implied. Note that in this model Hamiltonian we have assumed a general form of the coupling between the qubits. In many cases simpler forms such as the pure Ising (ZZ), the XY (see Appendix III) or the Heisenberg coupling are sufficient.

In the model Hamiltonian we so far have neglected the measurement system and the coupling with the environment. This can be accounted for by the two extra terms $\mathscr{H}_{\rm meas}$ and $\mathscr{H}_{\rm envir}$, respectively, resulting in the total Hamiltonian

$$\mathscr{H}(t) = \mathscr{H}_{\rm man}(t) + \mathscr{H}_{\rm meas}(t) + \mathscr{H}_{\rm envir}\ , \tag{9.2.4}$$

where we have assumed that the residual coupling to the environment is time independent. During the manipulation of the qubits the measurement device should be in the off-state, i.e. $\mathscr{H}_{\rm meas} = 0$. Furthermore, the interaction with the environment should be as small as possible, since it results in dephasing and relaxation processes.

The ***preparation of the initial state*** (requirement 2) can be achieved by keeping the system at low temperatures so that it relaxes to the ground state. For a spin system this can can achieved for example by switching on a large field $B_z \gg k_BT$ for a sufficiently long time, while $B_x(t) = B_y(t) = 0$. Then, due to the residual interaction with the environment, each qubit relaxes into its ground state, e.g. $|\uparrow\rangle$ for a spin system. If we then switch off B_z we are left with the system in a well-defined pure ground state.

A typical experiment performed with qubits involves the preparation of the initial state, the switching of the fields $\mathbf{B}(t)$ and the coupling $J^{ij}_{\alpha\beta}(t)$ to achieve a specific unitary evolution of the qubit state, and the measurement of the final state.

[40]If all three field components can be controlled, the topological or Berry phase of the systems can be manipulated as well.

9.3 Why Superconducting Qubits

On the first sight, microscopic systems seem to be ideal candidates for qubits, since they can be easily isolated from the environment thereby avoiding decoherence. However, the disadvantage of microscopic systems (e.g. ions in an electromagnetic trap) is usually scalability and the lack of simple embedding into other electronic circuits. That is, it is difficult to integrate many qubits into a more complex circuit in order to approach the vision of a practical quantum computer. Therefore, macroscopic quantum systems such as superconductors are attractive, since they offer more flexibility in scaling using standard integrated circuit technology. Until now several “macroscopic” qubits have been proposed that are based on nanostructured solid state electronic circuits, which are based either of semiconductor quantum dots or superconducting Josephson junctions.

As already mentioned above the large number of microscopic degrees of freedom in solid state devices makes it more difficult to achieve sufficiently long dephasing times. This is in particular a problem for charge based qubits, since the charge degree of freedom strongly couples to environmental degrees of freedom. Therefore, the use of isolated spins on quantum dots[41] or through the deliberate doping of semiconductors[42] seems more promising.

Quantum bits based on superconducting materials have particular advantages. First, the superconducting ground state is separated by an energy gap Δ of the order of meV from the quasiparticle excitation spectrum (see Fig. 9.7). Second, the superconducting state represents a non-degenerate macroscopic ground state and finally, superconducting metals have a large electron density resulting in a short screening length for perturbing background charges.

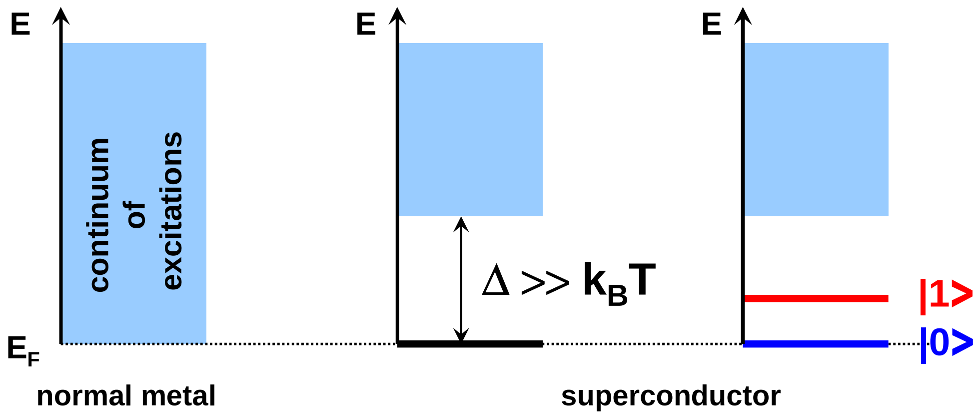

Figure 9.7: Advantage of superconductors for constructing solid state based quantum bits.

9.3.1 Superconducting Island with Leads

In order to get a feeling for the relevant energy scales for Josephson junction devices used for the realization of superconducting quantum bits we consider a superconducting island coupled to a superconducting lead via a Josephson junction that is characterized by an ideal Josephson element with Josephson inductance $L_J = \Phi_0/2\pi I_c$ (see (2.1.22)), normal resistance R_N and capacitance C as shown in Fig. 9.8. We can now consider the energy required to change the number N of Cooper pairs on the island by one and on the other hand the energy required to change the phase ϕ of the superconducting wave function by 2π.

[41] D. Loss, D.P. DiVincenzo, *Quantum computation with quantum dots*, Phys. Rev. A **57**, 120 (1998).
[42] B.E. Kane, *A silicon based nuclear spin quantum computer*, Nature **393**, 133 (1998).

The first is given by the charging energy

$$E_C = \frac{q^2}{2C} \tag{9.3.1}$$

with $q = 2e$. The latter is just the energy required to change the phase difference $\varphi = \phi_{\rm island} - \phi_{\rm reservoir}$ across the Josephson junction by 2π or, equivalently, to move a single flux quantum Φ_0 across the Josephson junction. This energy is given by the Josephson coupling energy $E_J = E_{J0}(1-\cos\varphi)$ with the maximum value for $\varphi = \pi$ given by

$$2E_{J0} = \frac{\Phi_0 I_c}{\pi} = \frac{(\Phi_0/\pi)^2}{2L_J} . \tag{9.3.2}$$

As already discussed in section 3.5, there is an uncertainty relation $\Delta\phi \cdot \Delta \geq 1$ for the number N of Cooper pairs and the phase ϕ. Considering the two characteristic energy scales E_C and E_{J0} we can conclude the following:

- $E_C \gg E_{J0}$:
  In this case large energy is required to change N. That is, the number N of Cooper pairs or the charge state of the island is well defined, whereas according to $\Delta\phi \cdot \Delta \geq 1$ the phase ϕ is completely smeared out.

- $E_C \ll E_{J0}$:
  In this case a large energy is required to change the phase ϕ, whereas the energy for changing N is small. Then, Cooper pairs easily can enter and leave the island resulting in large fluctuation of N what, in turn, causes small fluctuations of ϕ and hence in a well defined phase.

Note that in this discussion we only considered the Cooper pairs and have completely neglected the quasiparticle degrees of freedom.

We also have to consider the effect of thermal and quantum fluctuations. Thermal fluctuation do not play any role as long as

$$E_{J0}, E_C \gg k_B T . \tag{9.3.3}$$

This condition can easily satisfied with respect to E_{J0}. For example, a Josephson junction with a maximum Josephson current $I_c = 100\,\mu$A (corresponding to a junction area of $10 \times 10\,\mu$m^2 at a typical current density of $J_c = 100$ A/cm^2) has a coupling energy $E_{J0} \simeq 3 \times 10^{-18}$J corresponding to $T \simeq 2300$ K. For the charging energy this is more difficult. In order to have a charging energy corresponding a temperature of only 1 K, the capacitance has to be as small as only 1 fF. With a specific capacitance of Josephson tunnel junctions of typically 100 fF/μm^2, the junction area has to be as small as $0.1 \times 0.1\,\mu$m^2 requiring advanced fabrication technology.

The effect of quantum fluctuations can be estimated from energy-time uncertainty relation $\Delta E \Delta t \geq \hbar$. For the charge and the phase channel the characteristic time scales are $R_N C$ and L_J/R_N, respectively. With the condition $\Delta E \ll E_C$ and $\Delta E \ll E_{J0}$ we obtain the conditions

$$R_N \gg h/q^2 \qquad \text{for } \Delta E \ll E_C \tag{9.3.4}$$

$$R_N \ll h/q^2 \qquad \text{for } \Delta E \ll E_J \tag{9.3.5}$$

respectively, with $q = 2e$. We see that the resistance $R_Q/4$, where $R_Q = h/e^2$ is the quantum resistance, separates the regimes, where quantum fluctuations of the charge and the phase are dominant.

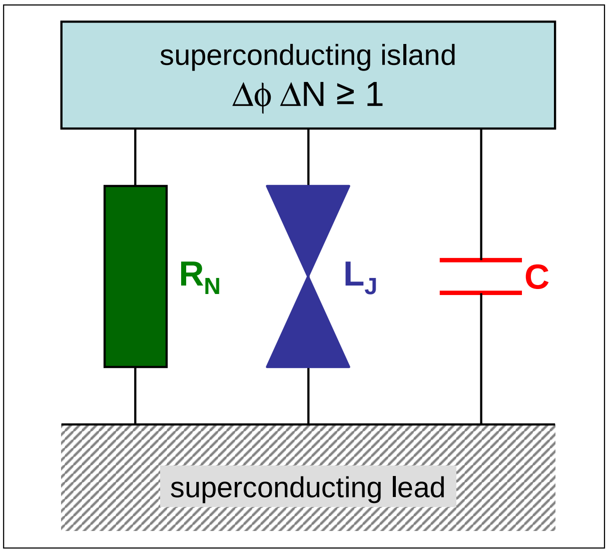

Figure 9.8: A superconducting island coupled to a superconducting lead (reservoir) via a Josephson junction characterized by an ideal Josephson element of Josephson inductance L_J, normal resistance R_N and a capacitance C.

A further characteristic energy scale of superconductors is the energy gap Δ. With the fact that the characteristic energy $eI_cR_N = eV_c \simeq \Delta$ we obtain

$$E_{J0} \simeq \frac{\hbar I_c}{2e} = \frac{\hbar}{2e^2 R_N} eI_cR_N \simeq \frac{R_Q}{4R_N}\Delta \; . \tag{9.3.6}$$

We see that for junctions with $R_N \gg R_Q/4$ we have $E_{J0} \ll \Delta$. Then one can always find a regime for which the two inequalities $E_{J0} \ll E_C \ll \Delta$ hold. The fact that Δ is the largest energy scale is often used in theoretical treatments. It allows to restrict to states of the island containing only an even number of electrons, which form Cooper pairs. The net charge Q on the island can then be written as $qN = 2eN$.