Superconductivity and Low Temperature Physics I

Lecture Notes
Winter Semester 2023/2024

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Chapter 6

Flux Pinning and Critical Currents
Superconductivity and Low Temperature Physics I

Lecture No. 11

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Chapter 6

6  Flux Pinning and Critical Currents

6.1 Power Applications of Superconductivity
   6.1.1  Examples
   6.1.2  Materials Requirements
   6.1.3  Superconducting Wires and Tapes

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   6.2.2  Depinning Critical Current Density

6.3 Flux Line Pinning

6.4 Magnetization of Hard Superconductors
6.1 Power Applications of Superconductivity

- power applications require high $T_c$, $B_{c2}$, and $J_c$
6.1 Power Applications of Superconductivity

6.1.1 Examples

- energy transport and storage

fault current limiter

SMES (2 MJ)
superconducting magnetic energy storage
6.1 Power Applications of Superconductivity

6.1.1 Examples

- fault current limiter

Nexans fault current limiter delivered on site in Essen
the fault current limiter for AmpaCity is designed to limit a 38 kA peak short circuit current to about 10 kA.
6.1 Power Applications of Superconductivity

6.1.1 Examples

Comparison of the amount of space consumed by a superconducting cable (blue) with copper wires carrying the same amount of current.
6.1 Power Applications of Superconductivity

6.1.1 Examples

- superconducting magnets

magnetic resonance imaging

fusion

high energy physics
6.1 Power Applications of Superconductivity

6.1.1 Examples

45-T Hybrid Magnet

45 Tesla, 32 mm Bore Hybrid Magnet

Figure 1 – Cross section of the 45-T Hybrid Magnet. The magnet cryostat is about 8 feet in diameter and the large-diameter part is about 9 feet tall.
AMS-02 is the Alpha Magnetic Spectrometer, a superconducting particle physics experiment which will be launched on the Space Shuttle and installed on the International Space Station. The project is an international collaboration of 56 research institutes from 16 countries.
6.1 Power Applications of Superconductivity

6.1.1 Examples

High Field Magnets for NMR

1.02 GHz (24 T) NMR magnet (world record at 2015)
DI-BSCCO Type HT-CA/Insert coil (3.6 T)
6.1 Power Applications of Superconductivity

6.1.1 Examples

High Field Magnets for NMR
Bavarian NMR Center (BNMRZ)

AV Neo 1200 ("Charlie")
Magnet:
1200 MHz (1H), Bruker Karlsruhe 2022, pumped (2 K)
Console:
Bruker Avance III HD, channels: 1H, X, Y, Z
Workstation:
PC (Linux)
Probes:
3 mm cryo-TCI (1H, 13C, 15N) with z-gradient
TXI (1H, 13C, 15N) with z-gradient
MAS probes (not yet installed)
6.1.2 Materials Requirements

- high $T_c, B_{c2}, J_c$
- manufacturability
- low cost
- availability (sustainability)
# 6.1.2 Materials Requirements

**material parameters:**

- **important low** $T_c$ **superconductors** (application in commercial magnets)

<table>
<thead>
<tr>
<th>Material</th>
<th>NbTi</th>
<th>Nb$_3$Sn</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material:</strong></td>
<td>1:1 alloy</td>
<td>intermetallic compound</td>
</tr>
<tr>
<td>$T_c$</td>
<td>9.6 K</td>
<td>18 K</td>
</tr>
<tr>
<td>$B_{c2}(T=0)$</td>
<td>10.5 - 15 T</td>
<td>23 - 29 T</td>
</tr>
</tbody>
</table>

- **high** $T_c$ **superconductors**

<table>
<thead>
<tr>
<th>Material</th>
<th>BSCCO</th>
<th>YBCO</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material:</strong></td>
<td>powder in Ag-tube</td>
<td>thin film on metal tape</td>
</tr>
<tr>
<td>$T_c$</td>
<td>110 K</td>
<td>91 K</td>
</tr>
<tr>
<td>$B_{c2}(T=0)$</td>
<td>$\sim$ 1000 T</td>
<td>800 T</td>
</tr>
</tbody>
</table>

still under development
### 6.1.2 Materials Requirements

material parameters of type-II superconductors

<table>
<thead>
<tr>
<th>Material</th>
<th>Transition Temperature (K)</th>
<th>Upper Critical Field (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NbTi</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>PbMoS</td>
<td>14.4</td>
<td>6.0</td>
</tr>
<tr>
<td>V₃Ga</td>
<td>14.8</td>
<td>2.1</td>
</tr>
<tr>
<td>NbN</td>
<td>15.7</td>
<td>1.5</td>
</tr>
<tr>
<td>V₃Si</td>
<td>16.9</td>
<td>2.35</td>
</tr>
<tr>
<td>Nb₃Sn</td>
<td>18.0</td>
<td>24.5</td>
</tr>
<tr>
<td>Nb₃Al</td>
<td>18.7</td>
<td>32.4</td>
</tr>
<tr>
<td>Nb₃(AlGe)</td>
<td>20.7</td>
<td>44</td>
</tr>
<tr>
<td>Nb₃Ge</td>
<td>23.2</td>
<td>38</td>
</tr>
</tbody>
</table>

Blatt, Frank J., Modern Physics, McGraw-Hill, 1992
6.1.2 Materials Requirements

Advancing Critical Currents in Nb-Ti

- Improvement of critical current density of superconductors requires decades of materials engineering.

1997

https://nationalmaglab.org/
6.1.2 Materials Requirements

Bulk Pinning Force Comparison

- Nb-Ti: APC strand Nb-47wt.%Ti with 24vol.%Nb pins (24nm nominal diam.) — Heusserer et al. (UW-ASC)
- Nb-Ti: Aligned ribbons, Bi ribbons — Cooley et al. (UW-ASC)
- Nb-Ti: Best Heat Treated UW Mone-Filament. (Li and Larbalestier, '77)
- Nb-Ti: Nb-Nb (21%) 300 nm multilayer '95 (E'), 90 pV/cm, McGrawbridge et al. (Yale)
- Nb-Sn: Sn plated Cu APC, 40x@650°C, R. Zhou PhD Thesis (OST), '94
- Nb-Sn: Sn Plated Cu APC 200hr@650°C, Zhou et al. (OST), '93
- Nb-Sn: Mitsubishi ITER BM3 Internal Sn
- Nb-Sn: High Jc, Internal Sn RSRP (Parrell et al ASC '02)
- Nb-Nb: 13-nm NbN/2nm AlN multi-layer — B. Gray et al. (AFL) Physics C, 152 '98
- YBCO: ANVYS - < 1 µm thick microbridge, Hjul et al. (LANL) '96
- B2212: 16 filament tape Bi2212 tape - Okada et al (Hitachi) '96
- Bi2223: Rolled 85 Ft. Tape (AmSC) Bi, UW/666
- MgB2: "low oxygen" film 3, Eom et al. (UW) Nature 2002
- MgB2: 10% SiC doped (Dou et al APL 2002, UW measurements)

2003

Improvement of pinning force by defect engineering

https://nationalmaglab.org/
6.1.2 Materials Requirements

Improvement of pinning force by defect engineering

https://nationalmaglab.org/
6.1.2 Materials Requirements

Non-stabilizer Critical Current Density vs. Applied Field for Superconductors Available in Long Lengths
April 16, 2018

https://nationalmaglab.org/
6.1.2 Materials Requirements

References (updated 4/2021)

- YBCO: Tape, || Tape-plane, SuperPower. REBCO: SP26 tape, 50 μm substrate, 7.5%Zr. Measured at NHMFL by Valeria Braccini, Jan Jaroszynski and Aixia Xu: DOI: 10.1088/0953-2048/24/3/035001
- YBCO: Tape, ⊥ Tape-plane. REBCO: SP26 tape, 50 μm substrate, 7.5%Zr. Measured at NHMFL by Valeria Braccini, Jan Jaroszynski and Aixia Xu: DOI: 10.1088/0953-2048/24/3/035001
- Bi-2223: B || Tape plane: Sumitomo Electric Industries. Measured at NHMFL (D. Abramov) unpublished
- Bi-2223 (Carrier Controlled): B ⊥ Tape-plane "DI" BSCCO "Carrier Controlled" Sumitomo Electric Industries (MEM’13 presented by Kazuhiko Hayashi).

- Nb-47Ti 1.8 K 5-8 T Maximal for whole LHC NbTi strand production (CERN-T. Boutboul ‘07)

- MgB2: 18 Filament - The OSU/HTRI C 2 mol% AIMI (“Advanced Internal Mg Infiltration”) 33.8 Filament to strand ratio, 39.1% MgB2 in filament. (DOI: 10.1088/0953-2048/25/11/115023)
6.1.2 Materials Requirements

6.1.2 Materials Requirements

Engineering Critical Current Density vs. Applied Field for Superconductors Available in Long Lengths
April 11, 2018

https://nationalmaglab.org/
6.1.2 Materials Requirements

References (updated 4/2021)

- YBCO: Tape, || Tape-plane, SuperPower. REBCO: SP26 tape, 50 µm substrate, 7.5%Zr. Measured at NHMFL by Valeria Braccini, Jan Jaroszynski and Aixia Xu: DOI: 10.1088/0953-2048/24/3/035001
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- MgB₂: 18 Filament - The OSU/HTRI C 2 mol% AIMI ("Advanced Internal Mg Infiltration") 33.8 Filament to strand ratio, 39.1% MgB₂ in filament. (DOI: 10.1088/0953-2048/25/11/115023)
critical current density of Fe-based superconductors:

![Graph showing critical current density vs. applied magnetic field for various Fe-based superconductors.](https://nationalmaglab.org/)

**References**

  [https://doi.org/10.1103/PhysRevLett.106.137001](https://doi.org/10.1103/PhysRevLett.106.137001)
  [https://doi.org/10.1016/j.physc.2015.03.022](https://doi.org/10.1016/j.physc.2015.03.022)
  [https://doi.org/10.1088/1361-6668/aaa821](https://doi.org/10.1088/1361-6668/aaa821)
  [https://doi.org/10.1038/srep02139](https://doi.org/10.1038/srep02139)
  [http://dx.doi.org/10.1063/1.4973522](http://dx.doi.org/10.1063/1.4973522)
  [http://dx.doi.org/10.1063/1.4875956](http://dx.doi.org/10.1063/1.4875956)
  [https://doi.org/10.1038/srep07305](https://doi.org/10.1038/srep07305)
  [http://dx.doi.org/10.1088/0953-2048/30/2/025001](http://dx.doi.org/10.1088/0953-2048/30/2/025001)
  [http://dx.doi.org/10.1088/0953-2048/29/3/035013](http://dx.doi.org/10.1088/0953-2048/29/3/035013)
6.1.3 Superconducting Wires and Tapes

Fabrication technology for superconducting wires:

- **Hexagonal Stab**
  - Chem. cleaning
  - Stacking in Cu-hülse

- **Nb Ti Stab**
  - Evakuieren und dichtschweißen
  - Aufheizung (≈ 550 °C)
  - Extrudieren
  - Querschnittsfaktor ≤ 20

- **Vielkernstab**
  - Kaltziehen
  - Wärmebehandlung und Extrudieren

- **Cu Tube**
  - Wire drawing
  - Bundling

- **Cu + Sn**
  - NiW tube
  - Wire drawing
  - Monofilament

- **(Nb, Ti)Sn**
  - Not reacted NiW's
  - Cu tube
  - Wire drawing

- **Cu**
  - Bundling of the 271 monofilaments

- **Magnetization of the cross section**

- **Magnetization of the final wire**
6.1.3 Superconducting Wires and Tapes

fabrication technology for wires of Fe-based superconductors:

Polycrystalline powder

Ag tube (φ 4.5 mm)

Square roller ("Rolled")

Cu tube (1/8 inch = 3.175 mm)

Rolling (φ 1.2 mm)

Circular die ("Drawn")

Rotary swager ("Swaged")

Arc melting

HIP (Hot Isostatic Press)

Sunseng Pyon et al.,
Recent Progress of Iron-Based Superconducting Round Wires,
DOI: 10.1088/1742-6596/1293/1/012042
6.1.3 Superconducting Wires and Tapes

Superconducting wires:
NbTi, Nb₃Sn in Cu-matrix

cable from high-\(T_c\) superconductor

Multiple Traditional Copper Power Cables...

...Replaced by One Power Equivalent HTS Cable
6.1.3 Superconducting Wires and Tapes

Figure 2 – Conductors for the three subcoils of the Superconducting Outsert Magnet (A, B, and C) were jacketed in special stainless-steel alloys at Gibson Tube. More than 6 km of conductor were used in these coils.
6.1.3 Superconducting Wires and Tapes

6.1.3 Superconducting Wires and Tapes

cross-sections of powder-in-tube \( \text{Nb}_3\text{Sn} \) wires of different designs (courtesy of SMI and Bruker-EAS). The numbers represent the total number of superconducting tubes.

Barzi E., Zlobin A.V. (2019)
\( \text{Nb}_3\text{Sn} \) Wires and Cables for High-Field Accelerator Magnets.
In: Schoerling D., Zlobin A. (eds) \( \text{Nb}_3\text{Sn} \) Accelerator Magnets. Particle Acceleration and Detection. Springer, Cham.
https://doi.org/10.1007/978-3-030-16118-7_2
6.1.3 Superconducting Wires and Tapes

AMSC's 344 Superconductors

https://sumitomoelectric.com/

https://www.nexans.com/

https://www.hts-powercables.nl/
6.1.3 Superconducting Wires and Tapes

high-$T_c$ superconducting wires and tapes based on powder-in-tube technique:

*can be made*

- Ag is too expensive and too soft
- HTS have higher critical fields

But it’s 70% silver!
6.1.3 Superconducting Wires and Tapes

preparation of multi-filamentary BiSrCaCu-oxid (2223) tapes in Ag/AgMg-sheath by the powder in tube method
Investigation of the structural and superconducting properties

cross-section of 61 filamentary tape

600 m tape on a coil for $J_c$-measurement
6.1.3 Superconducting Wires and Tapes

HTS cable
6.1.3 Superconducting Wires and Tapes

10kV, 2300 A HTS cable that does the same job as a 100 kV conventional cable. This cable has now been in continuous use for six years (2021).

19. January 2012

The "AmpaCity" project has been kicked off:
The RWE Group and its partners are just about to replace a 1-kilometre-long high-voltage cable connecting two transformer stations in the Ruhr city of Essen with a state-of-the-art superconductor solution. This will mark the longest superconductor cable installation in the world. As part of this project, the Karlsruhe Institute of Technology will analyse suitable superconducting and insulating materials.

The three-phase, concentric 10 kV cable will be produced by Nexans and is designed for a transmission capacity of 40 megawatts.
October 23, 2020:
Stadtwerke München and five cooperation partners have the green light to start development and testing of the components for a 12-kilometer superconductor cable in Munich as part of the SuperLink project.

In order to ensure that all components are optimally matched from the outset, the three companies Linde for the cooling technology, NKT for the cable and THEVA for the superconductor are involved.
6.1.3 Superconducting Wires and Tapes

Contactless High Performance Power Transmission

12 March 2021

Superconducting coils boost performance of contactless power transmission

A team led by Technical University of Munich (TUM) physicists Christoph Utschick and Prof. Rudolf Gross has succeeded in making a coil with superconducting wires capable of transmitting power in the range of more than five kilowatts contactless and with only small losses. The wide field of conceivable applications include autonomous industrial robots, medical equipment, vehicles and even aircraft.

Contactless power transmission has already established itself as a key technology when it comes to charging small devices such as mobile telephones and electric toothbrushes. Users would also like to see contactless charging made available for larger electric machines such as industrial robots, medical equipment and electric vehicles.

Such devices could be placed on a charging station whenever they are not in use. This would make it possible to effectively utilize even short idle times to recharge their batteries. However, the currently available transmission systems for high performance recharging in the kilowatt range and above are large and heavy, since they are based on copper coils.

Working in a research partnership with the companies Württh Elektronik eSos and superconductor coating specialist Theva Dünnschichttechnik, a team of physicists led by Christoph Utschick and Rudolf Gross have succeeded in creating a coil with superconducting wires capable of contactless power transmission in the order of more than five kilowatts (kW) and without significant loss.

https://www.wmi.badw.de/news-1/contactless-high-performace-power-transnmission#c126
6.1.3 Superconducting Wires and Tapes

"Garching-technology", for HTS tapes:

- low cost process for large area deposition of HTS films
- high quality and reproducibility

http://www.theva.com/
6.1.3 Superconducting Wires and Tapes

THEVA coated conductors

coated conductor:

superconducting film on flexible steel tape
6.1.4 Superconducting Bulk Material

bulk superconductors as “permanent magnets”
6.1.4 Superconducting Bulk Material
6.1.4 Superconducting Bulk Material

[Images of superconducting bulk material and a sumo wrestler standing on top of a superconducting material]
6.1.4 Superconducting Bulk Material

MLX01-901

Jap. Yamanashi MAGLEV-System
(42.8 km long test track between Sakaigawa and Akiyama)

maximum velocity: 581 km/h (02. 12. 2003)
6.1.4 Superconducting Bulk Material

Magnetization Curve for a Superconducting Slab

\[ M(H_{\text{ext}})/Lw^2J_c \]

\[ -M(H_{\text{ext}})/Lw^2J_c \]

\[ H_{\text{ext}}/H^* \]

\[ \text{Bi}_{1.8}\text{Pb}_{0.2}\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10} \]

\[ T=4.2 \text{ K} \]

https://commons.wikimedia.org/
Chapter 5

6 Flux Pinning and Critical Currents

6.1 Power Applications of Superconductivity
   6.1.1 Examples
   6.1.2 Materials Requirements
   6.1.3 Superconducting Wires and Tapes

6.2 Critical Current of Superconductors
   6.2.1 Depairing Critical Current Density
   6.2.2 Depinning Critical Current Density

6.3 Flux Line Pinning

6.4 Magnetization of Hard Superconductors
6.2 Critical Current of Superconductors

critical current density of type-II superconductors is limited by different physical effects

- increase of supercurrent density results in increase of velocity of superconducting electrons
  
  critical current density: \textit{kinetic energy} = \textit{binding energy of Cooper pairs}

\textbf{depairing critical current density}

- increase of supercurrent results in Lorentz force on flux lines in mixed state of type-II superconductors

  critical current density: \textit{Lorentz force} = \textit{pinning force}

\textbf{depinning critical current density}
6.2.1 Depairing Critical Current Density

revision: Ginzburg-Landau theory (cf. 3.3)

- minimization of free enthalpy of superconductor:
  \( \rightarrow \) integration of enthalpy density over whole volume of superconductor
  \( \rightarrow \) minimization by variation of \( \Psi(\mathbf{r}) \) and \( \mathbf{A}(\mathbf{r}) \)

Ginzburg-Landau equations:

\[
\frac{1}{2m_s} \left( \frac{\hbar}{i} \nabla - q_s A(\mathbf{r}) \right)^2 \Psi(\mathbf{r}) + \alpha \Psi(\mathbf{r}) + \frac{1}{2} \beta |\Psi(\mathbf{r})|^2 \Psi(\mathbf{r}) = 0
\]

1st Ginzburg-Landau equation

\[
J_s = \frac{q_s \hbar}{2m_s l} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) - \frac{q_s^2}{m_s} |\Psi|^2 A
\]

2nd Ginzburg-Landau equation

\[
\lambda_{GL} = \frac{m_s}{\mu_0 n_s q_s^2} \quad \text{GL penetration depth} \quad \xi_{GL} = \frac{\hbar^2}{\sqrt{2m_s |\alpha|}} \quad \text{GL coherence length}
\]
6.2.1 Depairing Critical Current Density

derivation of the depairing critical current density from the GL equations

- simplifying assumptions:
  - consider a thin wire with diameter $d \ll \xi_{GL}$  
  - superconducting material is assumed homogeneous
  - no amplitude variation of order parameter $\Psi$ across wire
  - same current density along the wire
  - no amplitude variation of order parameter $\Psi$ along the wire
  - $\Psi(r) = \Psi_0 e^{i\theta(r)}$

- we use 1. and 2. GL equation:

\[
J_s = \frac{q_s h}{2m_s l} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) - \frac{q_s^2}{m_s} |\Psi|^2 A_s 
\Rightarrow
J_s = q_s n_s \left\{ \frac{h}{m_s} \nabla \theta(r) - \frac{q_s}{m_s} A(r) \right\} = q_s n_s v_s \quad \text{with} \quad n_s = |\Psi|^2
\]

\[
\frac{1}{2m_s} \left( \frac{h}{l} \nabla - q_s A(r) \right)^2 \Psi(r) + \alpha \Psi(r) + \frac{1}{2} \beta |\Psi(r)|^2 \Psi(r) = 0
\Rightarrow
0 = -\xi_{GL}^2 \left( \frac{h}{l} \nabla - q_s A(r) \right)^2 \bar{\Psi} + \bar{\Psi} - |\bar{\Psi}|^2 \bar{\Psi}
\]

\[
0 = -\frac{\xi_{GL}^2 m_s^2}{h^2} \left\{ \frac{h}{m_s} \nabla \theta(r) - \frac{q_s}{m_s} A(r) \right\}^2 \bar{\Psi} + \bar{\Psi} - |\bar{\Psi}|^2 \bar{\Psi}
\]

with $|\Psi|^2 = -\frac{\alpha}{\beta}$

\[
\xi_{GL}^2 = \frac{h^2}{2m_s |\alpha|}
\]
6.2.1 Depairing Critical Current Density

- resolving for $|\Psi|^2$ yields

$$|\Psi|^2 = 1 - \frac{\xi_{GL}^2 m_s^2}{\hbar^2 v_s^2} = \frac{1}{\hbar^2 \xi_{GL}^2} - \frac{1}{\hbar^2 |\alpha|}$$

condensation energy per Cooper pair

$\Rightarrow$ reduction of $|\Psi|^2$ is just proportional to ratio of kinetic and condensation energy

$\Rightarrow$ order parameter decreases due to additional kinetic energy of pairs

- expression for current density:

$$J_s = q_s n_s v_s = q_s |\Psi|^2 v_s = q_s |\Psi|^2 |\Psi_0|^2 v_s = q_s |\Psi_0|^2 \left(1 - \frac{\xi_{GL}^2 m_s^2}{\hbar^2 v_s^2}\right) v_s$$

- determine maximum of $J_s$ by setting $\partial J_s / \partial v_s = 0$:

$$J_c^{GL} = \frac{2}{3\sqrt{3}} \frac{\hbar q_s^2}{m_s \xi_{GL}} |\Psi_0|^2 = \frac{\Phi_0}{3\pi \sqrt{3} \mu_0 \lambda_L^2 \xi_{GL}}$$

GL depairing critical current density

$\Phi_0 = h/2e$

$\lambda_L^2 = m_s / \mu_0 |\Psi_0|^2 q_s^2$
6.2.1 Depairing Critical Current Density

- \( T \)-dependence of \( J_c^{GL} \) is determined by \( T \)-dependence of \( \lambda_L \) and \( \xi_{GL} \):

\[
\lambda_{GL}(T) = \frac{\lambda_{GL}(0)}{\sqrt{1 - \frac{T}{T_c}}}
\]

\[
\xi_{GL}(T) = \frac{\xi_{GL}(0)}{\sqrt{1 - \frac{T}{T_c}}}
\]

close to \( T_c \):

\[
J_c^{GL} \propto \frac{1}{\lambda_L^2 \xi_{GL}} \propto \left( 1 - \frac{T}{T_c} \right)^{3/2}
\]

- we can use \( B_{cth}(T) = \frac{\Phi_0}{2\pi\sqrt{2} \xi_{GL}(T) \lambda_{GL}(T)} \)

\[
J_c^{GL} = \frac{2\sqrt{2}}{3\sqrt{3}} \frac{B_{cth}}{\mu_0 \lambda_L} = \frac{2\sqrt{2}}{3\sqrt{3}} \frac{H_{cth}}{\lambda_L} = 0.544 \frac{H_{cth}}{\lambda_L}
\]

- note: according London theory we would expect \( J_c^{GL} = H_{cth}/\lambda_L \)

(London theory does not take into account reduction of OP with increasing \( J_s \))

\[
J_c^{GL} = \frac{2}{3\sqrt{3}} \frac{h q_s^2}{m_s \xi_{GL}} |\Psi_0|^2 = \frac{\Phi_0}{3\pi\sqrt{3} \mu_0 \lambda_L^2 \xi_{GL}}
\]

**Pb:**

\( B_{cth} \approx 80 \text{ mT}, \lambda_L \approx 40 \text{ nm} \)

\( \Rightarrow J_c^{GL} \approx 8 \times 10^{11} \text{ A/m}^2 \)

**Nb:**

\( B_{cth} \approx 200 \text{ mT}, \lambda_L \approx 40 \text{ nm} \)

\( \Rightarrow J_c^{GL} \approx 2 \times 10^{12} \text{ A/m}^2 \)
6.2.1 Depairing Critical Current Density

- Gedanken experiment: what is the critical current of a Pb (type-I SC) rod with large diameter \( d \) ?

Critical current \( I_c = J_c^{GL} \cdot A = J_c^{GL} \cdot \pi \left( \frac{d}{2} \right)^2 \approx 6 \times 10^7 \text{ A} \)

London theory (cf. 3.1.1): 

\[ B_z(x) = B_z(0) \exp \left( -\frac{x}{\lambda_L} \right) \]

\[ J_{s,x}(x) = J_{s,x}(0) \exp \left( -\frac{x}{\lambda_L} \right) \]

Supercurrent flows only in thin surface layer of thickness \( \lambda_L \)
6.2.1 Depairing Critical Current Density

- Supercurrent in a type-I superconductor flows only within surface layer of thickness $\lambda_L$

\[ \lambda_L \approx 40 \text{ nm} \]

- Technical critical current density:

\[
J_{c,\text{tech}} = \frac{I_c}{\pi (d/2)^2} \approx 10 \text{ A/mm}^2
\]

- Possible solutions:
  - Use multifilament wire with $d < \lambda_L$ → difficult to fabricate
  - Use type-II superconductor

Supercurrent flow in mixed state is not limited to thin surface layer

Critical current:

\[
I_c = J_{c,\text{GL}} \cdot A = J_{c,\text{GL}} \cdot \pi d \lambda_L \approx 1 \times 10^3 \text{ A}
\]
power applications of superconductors require high $T_c, B_{c2}, J_c$
- examples: power transmission lines, fault current limiters, NMR/MRI magnets, magnets for fusion, magnetic levitation, high magnetic fields for research, ....

materials requirements for power applications
- high supercurrent densities at high operation temperatures and high magnetic fields
- only type-II superconductors are relevant
- simple and cheap manufacturing, abundant chemical elements

depairing critical current density
- kinetic energy = binding energy of Cooper pairs
- calculation by GL theory for 1D conductor: $J_{c, GL} = 0.544 \frac{H_{c2}}{\lambda_L}$ (for comparison, London theory: $J_{c, L} = \frac{H_{c2}}{\lambda_L}$)

depinning critical current density
- Lorentz force on flux lines = pinning force due to pinning potential
- high depinning critical current densities require defect engineering: large density, ideal size, columnar structure, ...
Chapter 6

6 Flux Pinning and Critical Currents

6.1 Power Applications of Superconductivity
   6.1.1 Examples
   6.1.2 Materials Requirements
   6.1.3 Superconducting Wires and Tapes

6.2 Critical Current of Superconductors
   6.2.1 Depairing Critical Current Density
   6.2.2 Depinning Critical Current Density

6.3 Flux Line Pinning

6.4 Magnetization of Hard Superconductors
6.2.2 Depinning Critical Current Density

type-II superconductors:

\[ \kappa = \frac{\lambda_{GL}}{\xi_{GL}} \leq \frac{1}{\sqrt{2}} \] type I superconductor

\[ \kappa = \frac{\lambda_{GL}}{\xi_{GL}} \geq \frac{1}{\sqrt{2}} \] type II superconductor

- partial magnetic flux penetration above \( B_{c1} \)
  \[ B_i > 0 \text{ for } B_{ext} > B_{c1} \]
  \[ \text{Shubnikov phase between } B_{c1} \leq B_{ext} \leq B_{c2} \]
  \[ \text{upper and lower critical fields } B_{c1} \text{ and } B_{c2} \]

- in the mixed state, magnetic flux penetrates the superconductor
  \[ \text{current flow is not restricted to thin surface layer} \]
  \[ \text{superconductivity survives up to high values of } B_{c2} \]
6.2.2 Depinning Critical Current Density

Extreme type-II superconductors ($\kappa \gg 1$) have very high $B_{c2} \rightarrow$ high field operation

### $B_{\text{cth}}$ and $\lambda_L$ of type-I superconductors

<table>
<thead>
<tr>
<th>Element</th>
<th>Al</th>
<th>In</th>
<th>Nb</th>
<th>Pb</th>
<th>Sn</th>
<th>Ta</th>
<th>Tl</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{\text{cth}}$ [mT]</td>
<td>10.49</td>
<td>28.15</td>
<td>206</td>
<td>80.34</td>
<td>30.55</td>
<td>82.9</td>
<td>17.65</td>
<td>140</td>
</tr>
<tr>
<td>$\lambda_L(0)$ [nm]</td>
<td>50</td>
<td>65</td>
<td>32-45</td>
<td>40</td>
<td>50</td>
<td>35</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>$\kappa_{\infty}$</td>
<td>0.03</td>
<td>0.06</td>
<td>0.8</td>
<td>0.4</td>
<td>0.1</td>
<td>0.35</td>
<td>0.3</td>
<td>0.85</td>
</tr>
</tbody>
</table>

### $B_{c2}$ and $\lambda_L$ of type-II superconductors

<table>
<thead>
<tr>
<th>Verbindung</th>
<th>NbTi</th>
<th>Nb$_3$Sn</th>
<th>NbN</th>
<th>PbIn (2-30%)</th>
<th>PbIn (2-50%)</th>
<th>Nb$_3$Ge</th>
<th>V$_3$Si</th>
<th>YBa$_2$Cu$_3$O$_7$ (ab-Ebene)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$ [K]</td>
<td>$\approx$ 10</td>
<td>$\approx$ 18</td>
<td>$\approx$ 16</td>
<td>$\approx$ 7</td>
<td>$\approx$ 8.3</td>
<td>23</td>
<td>16</td>
<td>92</td>
</tr>
<tr>
<td>$B_{c2}$ [T]</td>
<td>$\approx$ 10.5</td>
<td>$\approx$ 23-29</td>
<td>$\approx$ 15</td>
<td>$\approx$ 0.1-0.4</td>
<td>$\approx$ 0.1-0.2</td>
<td>38</td>
<td>20</td>
<td>160±25</td>
</tr>
<tr>
<td>$\lambda_L(0)$ [nm]</td>
<td>$\approx$ 300</td>
<td>$\approx$ 80</td>
<td>$\approx$ 200</td>
<td>$\approx$ 150</td>
<td>$\approx$ 200</td>
<td>90</td>
<td>60</td>
<td>$\approx$ 140±10</td>
</tr>
<tr>
<td>$\kappa_{\infty}$</td>
<td>$\approx$ 75</td>
<td>$\approx$ 20-25</td>
<td>$\approx$ 40</td>
<td>$\approx$ 5-15</td>
<td>$\approx$ 8-16</td>
<td>30</td>
<td>20</td>
<td>$\approx$ 100±20</td>
</tr>
</tbody>
</table>

$B_{c2}(T) = \sqrt{2} \kappa B_{\text{cth}}(T)$
6.2.2 Depinning Critical Current Density

Superconducting transport current in the mixed state of a type-II superconductor

- how does the transport current $J_t$ interact with the vortex lattice and the associated circulating supercurrents??

\[
\frac{\partial b(r)}{\partial r} \propto J_s(r)
\]
6.2.2 Depinning Critical Current Density

Interaction of a transport current with the vortex lattice:

- Lorentz force on single flux line: 
  \[ F_L = L \int_A J_t \times b \ dA \]

- With homogeneous \( J_t \):
  \[ F_L = L J_t \times \int_A b \ dA = L J_t \times \Phi_0 \]

- In mixed state of type-II SC: many flux lines with areal density \( n_\Phi \) resulting in average flux density \( b = n_\Phi \Phi_0 \):

  \[ f_L = \frac{F_L}{L} n_\Phi = J_t \times n_\Phi \Phi_0 = J_t \times b \]

\( f_L \) results in force on charge carriers of transport current, which cannot leave conductor

\( \Rightarrow \) force on flux lines (actio = reactio) leads to flux motion perpendicular to applied transport current
6.2.2 Depinning Critical Current Density

**origin of force acting on a flux line (plausibility check)**

- **Bernoulli’s principle**: An increase in the speed of the fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy.

- **without transport current**: zero net force

- **contributions of circulating current cancel**

\[ \begin{align*}
\mathbf{F}_{\text{netto}} &= 0 \\
&= 0 \\
&= \mathbf{F}_{\text{netto}}
\end{align*} \]

\[ J_{t} \]

\[ J_{M} \]

\[ \mathbf{B} \]

\[ \text{small total superfluid velocity} \Rightarrow \text{small total } \mathbf{F}_{L} \]

\[ \text{large total superfluid velocity} \Rightarrow \text{large total } \mathbf{F}_{L} \]

\[ \text{core of flux line} \]

\[ \text{net force} \]

Analogy to Magnus force
6.2.2 Depinning Critical Current Density

Lorentz force on single flux line

- Lorentz force on single flux line: $\mathbf{F}_L = L \int_A \mathbf{j}_t \times \mathbf{b} \, dA$
  - homogeneous

  $$\mathbf{F}_L = LJ_t \times \int_A \mathbf{b} \, dA = LJ_t \times \Phi_0$$

- Force per unit length of flux line: $\frac{\mathbf{F}_L}{L} = J_t \times \Phi_0$

- $\mathbf{F}_L$ causes motion of the flux line
  - what is the velocity $v_L$ of the flux line (depends on damping)
  - what is the work done by the Lorentz force
6.2.2 Depinning Critical Current Density

**flux line motion**

- **Faraday's law of induction:**
  the induced electromotive force (EMF) in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit.

- **electromotive force** (EMF):
  \[
  \text{EMF} = \frac{1}{e} \oint_{\partial A} \mathbf{F} \cdot d\mathbf{s} = \frac{1}{e} \oint_{\partial A} (eE + e\mathbf{v}_L \times \mathbf{B}) \cdot d\mathbf{s} = -\frac{d}{dt} \int_A \mathbf{B} \cdot \mathbf{n} \, dA
  \]
  
  \[
  = 0
  \]

  \[
  \text{EMF} = 0 \quad \text{no flux change}
  \]

  \[
  \mathbf{E} = -\mathbf{v}_L \times \mathbf{B} \quad \text{electric field } \parallel \text{ transport current}
  \]

- **dissipation by moving flux line:**
  - motion with velocity \( \mathbf{v}_L \) induces electric field \( \mathbf{E} = -\mathbf{v}_L \times \mathbf{B} = \mathbf{B} \times \mathbf{v}_L \)
  - \( \mathbf{E} \) is parallel to \( \mathbf{J}_t \) \( \rightarrow \) acts like “resistive” electric field

  \[
  \oint_A (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \cdot d\mathbf{s} = -\frac{d}{dt} \int_A \mathbf{B} \cdot \mathbf{n} \, dA
  \]

  \[
  \text{flux density inside loop does not change}
  \]
6.2.2 Depinning Critical Current Density

flux line motion – other point of view: power balance

- generated power for
  - single flux line: \( P_1 = F_L \cdot v_L \)
  - \( N \) flux lines: \( P_N = N F_L \cdot v_L \)

- power balance: \( P_N = N F_L \cdot v_L = V I_t \)

\[ E = \frac{V}{\ell} = \frac{N F_L \cdot v_L}{I_t \ell} \]

\[ \hat{j}_t = \frac{NL (\hat{j}_t \times \Phi_0) \cdot v_L}{I_t bL \ell} \]

\[ \hat{j}_t = \frac{N (\hat{j}_t \times \Phi_0) \cdot v_L}{bL \ell} \]

\[ \hat{j}_t = \left[ \left( \hat{j}_t \times \vec{B} \right) \cdot v_L \right] \hat{j}_t \]

\[ E || \hat{j}_t \Rightarrow \text{superconductor shows resistive behavior: flux-flow resistance} \]
6.2.2 Depinning Critical Current Density

phase change due to flux motion:

• assumption: single flux line in sample

• phase difference between the sample ends:

\[ \delta \theta_1 = \int_{\text{path 1}} \nabla \theta \, ds \]

• integration path 2:

\[ \delta \theta_2 = \int_{\text{path 2}} \nabla \theta \, ds = \int_{\text{path 2}} \nabla \theta \, ds - \int_{\text{path 1}} \nabla \theta \, ds + \int_{\text{path 1}} \nabla \theta \, ds = \oint \nabla \theta \, ds + \delta \theta_1 = 2\pi + \delta \theta_1 \]

• note: only the phase factor \( e^{i\theta} \) must be unambiguous
6.2.2 Depinning Critical Current Density

- crossing of single flux line changes phase difference by \( \varphi = \theta_2 - \theta_1 = 2\pi \)

- required time for crossing: \( \delta t = \frac{b}{v_L} \)

\[
\frac{\partial \varphi}{\partial t} = N \frac{2\pi}{\delta t} = N \frac{2\pi}{b/v_L} = \frac{\Phi}{\Phi_0} \frac{2\pi}{b} v_L = \frac{B b \ell 2\pi}{\Phi_0} v_L = B v_L \ell \frac{2\pi}{\Phi_0}
\]

\( N \)

\( \ell \)

\( v_L \)

temporal change of phase difference due to motion of flux lines
6.2.2 Depinning Critical Current Density

relation between temporal change of phase difference $\frac{\partial \varphi}{\partial t}$ and electric field $E$

$$\frac{\partial \varphi}{\partial t} = B v_L \ell \frac{2\pi}{\Phi_0} = B v_L \ell \frac{2e}{h}$$

- we make use of $E = -v_L \times B = B \times v_L \Rightarrow |E| = B v_L$

$$\frac{\partial \varphi}{\partial t} = \frac{2e}{h} B v_L \ell = \frac{2e}{h} E \ell = \frac{2eV}{\hbar}$$

**corresponds to 2$^{\text{nd}}$ Josephson Equation**
6.2.2 Depinning Critical Current Density

power dissipation during vortex motion:

a. pair breaking and recombination:

- in front of flux-line: $|\Psi|$ decreases
  \[ \Rightarrow \text{pairs have to break up} \]

- pair breaking due to absorption of phonons:

- behind the flux-line: $|\Psi|$ increases
  \[ \Rightarrow \text{recombination of pairs by phonon emission:} \]

- finite phonon lifetime delays thermal equilibrium:

  \textit{irreversible process } \rightarrow \textit{friction, viscous flow}

- electric energy is transferred to heat
6.2.2 Depinning Critical Current Density

power dissipation during vortex motion:

b. *eddy current losses:*

\[
-\nabla \times E = \frac{\partial B}{\partial t} = \frac{\partial B}{\partial x} \frac{\partial x}{\partial t} = \frac{\partial B}{\partial x} v_x
\]

core of the flux line is considered as normal conductor

⇒ *eddy current* with ohmic losses

• *both mechanisms lead to* \( E \propto v_L \)  ⇒ viscous damping

• balance between Lorentz and friction force: \( v_L \) becomes stationary

\[
v_L \propto J_t \quad \Rightarrow \quad v_L \propto E
\]

• we expect:

\[
J_t \propto E \quad \text{Ohm’s law} \quad \Rightarrow \quad \text{zero critical current density}
\]
6.3 Flux Line Pinning

experimental result:

- $I_c$ depends on defect density (and specific properties of defects)
- inhomogeneities pin flux-lines: *flux line pinning*
- $v_L = 0$ is caused by flux pinning $\Rightarrow$ *no work, no dissipation*  
  $\Rightarrow$ *no voltage drop*
6.3 Flux Line Pinning

- at precipitate: normal vortex core causes no additional loss in condensation energy
- motion of vortex core from left to right position costs energy (condensation energy)
- effective binding forced at precipitate → "pinning force"
- most effective, if defect size $\approx \xi_{GL}$
### 6.3 Flux Line Pinning

#### Pinning Potential

- **Pinning Force:**
  
  \[ F_p = -\frac{\partial V_p}{\partial r} \approx -\frac{E_{\text{cond}}}{r_p} \]

  - To increase \( F_p \), reduce \( r_p \).
  - \( E_{\text{cond}} = (E_{\text{cond}}/V)r_p^2L \) → increase \( L \) (columnar defects).
  - Optimum: \( r_p \approx \xi_{\text{GL}} \) (vortex size ≈ pin size).

- **For High-Temperature Superconductors:**
  
  \( \xi_{\text{GL}} \approx 1 \text{ nm} \) and large condensation energy \( E_{\text{cond}} \)

  - Very small pinning sites required.
  - Large pinning force.

- **Problem for HTS:** Thermally activated escape of flux line → **Thermally Activated Flux Flow (TAFF)**

\[
\frac{E_{\text{cond}}}{V} \xi_{\text{GL}}^3 \approx k_B T \quad \text{as} \quad \xi_{\text{GL}} \text{ small and operation } T \text{ large}
\]
6.3 Flux Line Pinning

pinning of the flux line lattice

• **so far:** pinning of a single flux line

• **now:** pinning of the complete flux line lattice

→ complicated problem:
  - flux line lattice is an elastic object
    
    *(stiffness of the lattice, flux lines can bend)*
  - pinning potential is usually highly disordered

• **example:** net pinning force of a completely stiff flux line lattice by statistical pinning potential vanishes
  ➢ flux line lattice has to deform to adopt flux line positions to pinning potential
  ➢ even if a single flux line does not sit in a potential well, it is pinned by the interaction with the other flux lines (rigidity of the lattice)

→ *collective pinning theory*

see e.g.  
*Vortices in high-temperature superconductors*  
G. Blatter, M. V. Feigel’man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur  
Rev. Mod. Phys. 66, 1125 (1994)
6.3 Flux Line Pinning

pinning by surface roughness

Important for superconducting thin films

→ relative length difference of flux line at different positions may be large
6.3 Flux Line Pinning

$J_c$ as a function of temperature and applied magnetic field:

- decrease of $J_c$ with increasing $B$: several flux lines per pinning site, not enough pinning sites
- decrease of $J_c$ with increasing $T$: thermally activated flux motion
  - reduced condensation energy
6.4 Magnetization of Hard Superconductors

flux line pinning in an external field:

- $B_{c1}$ and $B_{c2}$ stay the same
- pinning prevents flux motion, i.e. penetration and exit of flux lines $\Rightarrow$ hysteresis loop
- $B_{i}$ is inhomogeneous within the sample
- finite remanent flux density allows application of SC as permanent magnet for HTS: remanent flux density can exceed 15 T $\Rightarrow$ extremely strong permanent magnets

$hysteresis loop !$

$\text{magnetization curve}$

$\text{remanent flux}$

$\text{ideal SC}$

$\text{virgin curve}$
6.4 Magnetization of Hard Superconductors

magnetic flux distribution in sample (qualitative discussion):

- sample surface: jump ↔ ideal magnetization curve
- within superconductor: gradient of flux density
- flux lines repel each other: motion if repulsion > pinning force
- gradient of flux density decreases with increasing magnetic field
6.4 Magnetization of Hard Superconductors

Field distribution in sample (demagnetization):

- gradient changes sign
- $B_{\text{ext}} < -B_{c1}$: flux lines with opposite direction penetrate

_recombination with frozen-in flux lines inside the superconductor_
6.4 Magnetization of Hard Superconductors

Bean (critical state) model

- flux gradient ↔ shielding current
- macroscopic average:
  \[ \nabla \times B_i = \mu_0 J_{\text{scr}} \]
- here:
  \[ \frac{\partial B_{i,z}}{\partial x} = \mu_0 J_{\text{scr,y}} \]
- for small \( \frac{\partial B_{i,z}}{\partial x} \):
  \( J_{\text{scr}} < J_c \) ⇒ flux lines are pinned
- for large \( \frac{\partial B_{i,z}}{\partial x} \):
  \( J_{\text{scr}} > J_c \) ⇒ flux lines move
- motion until
  \[ J_{\text{scr}} = J_c \] “critical state”, (similar critical slope of pile of sand)
- note:
  measurement of \( \frac{\partial B_{i,z}}{\partial x} \) ⇒ \( J_c \)
  \( J_c \) decreases with increasing \( B_i \) ⇒ smaller slope
6.4 Magnetization of Hard Superconductors

magneto-optical imaging of flux distribution

imaging technique: Faraday rotation $\propto B_i(r)$

Meißner state

critical state

remanent state
**Summary of Lecture No. 12 (1)**

**Depinning critical current density**
- Lorentz force on flux lines = pinning force due to pinning potential
- High depinning critical current densities require defect engineering: large density, ideal size, columnar structure, ...
- Collective pinning: interaction of elastic flux line lattice with disordered pinning potential

**Flux line motion**
- Flux lines start to move if the Lorentz force exceeds the pinning force
- Flux lines move perpendicular to current direction
- Moving flux lines cause time change of phase difference and thereby a voltage drop $V \propto \phi$ in current direction $\Rightarrow$ no dissipationless current flow
- Dissipationless supercurrent only if flux line motion is avoided by flux pinning
- Engineering of pinning landscape: defects, surface roughness, ion bombardment, ...

**Critical state of superconductors**
- Flux pinning allows for finite gradient of the magnetic flux density
- Maximum possible gradient determines depinning critical current density (Bean critical state model)
- Flux pinning results in hysteretic magnetization curve with finite remanent flux density $\Rightarrow$ application of superconductors as permanent magnets