Superconductivity and Low Temperature Physics 1
typical exam questions

History of Superconductivity

1. When (and by whom) superconductivity (perfect conductivity) and the Meißner effect (perfect diamagnetism) have been discovered?
2. What was the basic technical development leading to the discovery of superconductivity?
3. Did scientists search for superconductivity or was it discovered accidentally?
4. When (and by whom) the first microscopic theory of superconductivity has been developed?
5. When (and by whom) flux quantization has been demonstrated?
6. When (and by whom) the Josephson effect has been predicted?
7. In which year (and by whom) high temperature superconductivity has been discovered?

Basic Properties of Superconductors

8. Which basic classes of superconducting materials do you know?
9. Considering the chemical elements: Is superconductivity a quite rare or common low temperature property of chemical elements? If superconductivity cannot be observed, what is usually the competing ground state?
10. What are the fundamental phenomena show by superconductors? Describe these phenomena qualitatively.
11. Perfect conductivity: Can we measure $R = 0$ in an experiment? Explain why we can or can't. If we cannot measure $R = 0$, how can we make the threshold as small as possible?
12. How can the magnetization of a superconductor be measured quantitatively?
13. If we use Faraday's law, what can we say about the magnetic flux in a cylinder consisting of a material with perfect electrical conductivity?
14. What is the fundamental difference between a perfect conductor and a superconductor?
15. Suppose that we have a material with perfect conductivity but not showing the Meißner effect (perfect diamagnetism). Is the state with perfect conductivity a true thermodynamic phase? Provide arguments for your answer.
16. Provide arguments for the existence of a thermodynamic critical field for superconductor. Which information can we obtain by measuring the thermodynamic critical field?
17. What is the empirical temperature dependence of the thermodynamic critical field found for superconductors?
18. Sketch the field-temperature phase diagram of a superconductor.
19. Why can we levitate a superconductor above a permanent magnet?
20. Sketch the typical field-temperature phase diagram of a type-I and a type-II superconductor. Discuss the different parts of the phase diagram and the appearing critical fields.
21. Describe the experiment set-up used by Doll and Näbauer to measure the flux quantization in superconductors. What was the key result? What was the important conclusion that could be drawn from the measured value of the flux quantum?

22. Which elementary superconductors do you know?

23. Do you know superconducting materials with $T_c > 30$ K? What are the highest transition temperatures observed until today?

Thermodynamic Properties of Superconductors

24. Discuss the thermodynamic various potentials. What is the difference between the free energy and the free enthalpy? Why do theoreticians usually calculate the free energy and experimentalist measure the free enthalpy?
25. Discuss the thermodynamic properties of a type-I superconductor in an applied magnetic field:
    a. What is the difference of the free enthalpies of the normal and superconducting state?
    b. Sketch the T-dependence of the free enthalpy density of a metal in the normal and the superconducting state.
    c. What is the T-dependence of the free enthalpy density difference between a metal in the normal and the superconducting state? What can we conclude from it?
    d. Sketch the temperature dependence of the entropy density of a normal metal and a superconductor? Interpret the differences and discuss the cases $T \to 0$ and $T \to T_c$.
    e. Sketch the temperature dependence $c_p(T)$ for a conventional superconductor with an isotropic gap.
    f. What is the T-dependence of the difference of the difference of the specific heat of a metal in the normal and the superconducting state?

Phenomenological Models of Superconductivity

26. How can we derive in a simplistic way the London equations from the equation of motion of charge carriers in a metal?
    a. What are the conclusions we can draw from the London equations?
    b. What is the characteristic screening length of magnetic fields in a superconductor? Which quantities determine this length scale?
    c. What kind of phenomena can we describe by combining the London equations with the Maxwell equations?
    d. Why is the simplistic derivation of the London equations from the equation of motion of charge carriers problematic?
27. Superconductivity as macroscopic quantum phenomenon:
    a. What is the basic hypothesis of the macroscopic quantum model of superconductivity?
    b. If we describe the superconducting state by a macroscopic wave function, what is the meaning of the absolute square of the wave function? What is the meaning of the absolute square of a wave function used to describe a microscopic particle such as an electron?
    c. What are the basic relations we can obtain by plugging in the macroscopic wave function into the Schrödinger equation of a charged particle (Madelung transformation)? Write down these basic relations and discuss their meaning.
    d. Interpret the basic relations obtain from the Madelung transformation. Can we use these relations also for other physical systems besides superconductors?

28. Consequences of the relations derived by the macroscopic quantum model of superconductivity (current-phase and energy-phase relation):
    a. How can we derive London's equations from the current-phase and energy-phase relation?
    b. How can we derive fluxoid quantization form the current-phase relation?
    c. What is the difference between fluxoid and flux quantization? Under which experimental conditions can we observe flux quantization?
    d. Can we use the macroscopic quantum model for situations with spatially varying density of the superconducting electrons? What about time-dependent phenomena?
29. Ginzburg-Landau theory:
    a. What is the starting point of the GL-theory of superconductivity? Which basic assumptions are made? What is the relation to the Landau theory of phase transitions?
    b. How can we describe the phase transition into the superconducting state (homogeneous superconductor in zero applied magnetic field) within the Landau theory of phase transitions? How does the free enthalpy density look like?
    c. How can we calculate the free enthalpy density (or free energy density) of a spatially homogeneous superconductor in zero applied magnetic field?
    d. How does the free enthalpy density look like as a function of the magnitude and the phase of the order parameter? What kind of symmetry is broken on going from the normal to the superconducting state?
    e. How can we describe a superconductor with spatially inhomogeneous order parameter in a non-zero applied magnetic field?
    f. What are the additional terms which we have to take into account in the free enthalpy density? Sketch in a qualitative way how we can derive the additional terms. What is the physical interpretation of these terms?
    g. Discuss in a qualitative way, how we can derive the two GL equations from the expression of the free enthalpy density.
    h. Compare the GL equations with the current-phase and energy-phase relation obtain from the macroscopic quantum model. What are the analogies and differences?
    i. What are the characteristic length scales related to the GL equations?
    j. What is the meaning of the GL coherence length?
    k. What is the definition of the GL parameter? How does the Ginzburg-Landau parameter change if we add impurities to a clean metallic superconductor?
    l. What kind of temperature dependence do we expect for the coherence length, the field penetration depth and the thermodynamic critical field according to GL theory?
30. Discuss the variation of the order parameter and the magnetic flux density at a superconductor-normal metal-interface for a magnetic field applied parallel to the interface.
31. Discuss the energy per unit area associated with an interface between a superconductor and a normal metal:
    a. What is the sign of the interface energy?
    b. How can we use the interface energy to distinguish between type-I and type-II superconductors?
32. Type-I and type-II superconductors:

a. Sketch the magnetic flux density inside of a type-I and type-II superconductor as a function of the applied magnetic field. How does the variation of the magnetization look like?
b. What is the condition for the integral of the $M(H_{\text{ext}})$ curve of a type-I and type-II superconductor with the same condensation energy?
c. Discuss qualitatively the magnitude and meaning of the upper and lower critical field of a type-II superconductor. How is their magnitude related to the characteristic length scales of the superconductor?
d. What is the spatial distribution of the magnetic flux density in the mixed state of a type-II superconductor? How can we measure the flux distribution?
e. Under which experimental conditions can we observe an imperfect flux expulsion (intermediate state) of a type-I superconductor?

33. Compare the GL theory and the macroscopic quantum model of superconductivity: What can and what cannot be explained by these models? Can these models give insight into the microscopic origin of superconductivity?

Microscopic (BCS) Theory of Superconductivity

34. How would you describe the interaction of two conduction electrons by a Feynman diagram? What kind of exchange bosons could contribute to an interaction in a solid?
35. What is the isotope effect in superconductors? What kind of conclusions can be drawn from the observation of the isotope effect (e.g. in Sn)? Do all metallic superconductors show an isotpe effect?
36. How can we qualitatively describe the attractive interaction between two conduction electrons via the exchange of a virtual phonon? What are the basic requirements for making this attractive interaction efficient?
37. Could you derive an interaction range for the attractive interaction?
38. Cooper pairs:
    a. Which electrons can participate in the phonon-mediated attractive interaction?
    b. Provide a qualitative argument why the phonon-mediated attractive interaction between two conduction electrons is most efficient for two electrons with opposite momentum.
    c. How could we do an estimate of the interaction energy? What kind of basic assumptions are usually made to make the estimate of the interaction energy as simple as possible?
    d. What is the order of magnitude of the interaction energy and what are the parameters it depends on?
    e. How many Cooper pairs do we have in a volume given by the interaction range cubed? What do we expect from this number for the state formed by the whole entity of Cooper pairs?
39. Discuss the symmetry of the pair wave function. What kind of combinations of spin and orbital wave function are possible? What kind of combination do we have for the classical metallic superconductors and what for the cuprate (high T_c) superconductors?
40. Diskutieren sie die Symmetrie der Paar-Wellenfunktion? Welche Kombinationen aus Spin- und Orbital-Wellenfunktionen sind möglich?
41. BCS ground state:

a. How can we express the Hamiltonian of an interacting electron systems in the language of second quantization?
b. What is the Gorkov or pairing amplitude, what is the pairing potential? What assumptions can we make if we discuss a weak isotropic interaction of the conduction electrons?
c. Qualitatively discuss how we could construct a coherent many-body ground state of fermions? Why does one have to use a mean-field approach to handle the problem? What was the approach mage by BCS for the ground state wave function?
d. Discuss a coherent state of bosons: What is the statistical probability to have N bosons in a coherent state? What is the variance?
e. Discuss a coherent state of fermions: What is the key difference to a bosonic coherent state? Compare the fermionic coherent state to the BCS mean-field ansatz for the ground state wave function.
f. Sketch the probabilities for a pair state $(k, -k)$ being occupied and empty close to the Fermi energy at $T = 0$.
g. What are the expectation values for the total pair number and its variance in the BCS ground state?

42. BCS ground state energy at $T = 0$:
    a. What is the general procedures for calculating the BCS ground state energy at $T = 0$?
    b. What are the key results we obtain by minimizing the ground state energy?
    c. How do the probabilities $|u_k|^2$ and $|v_k|^2$ for a pair state $(k, -k)$ being occupied and empty and the Gorkov amplitude look like around the Fermi energy and which are the parameters they depend on?
    d. Where do we get the BCS gap equation get from and how can we solve it? What is the key result?
    e. How can we calculate the expectation value of the BCS ground state energy for $T = 0$ and what is the key result? How can we interpret the result?
    f. If we compare the BCS result with the result obtained from thermodynamics for the condensation energy, what can we conclude for the thermodynamic critical field?
43. Bogoliubov-Valatin transformation
    a. How can we rewrite the pair creation and annihilation operators in the BCS Hamiltonian by using the expectation values of the Gorkov amplitudes?
    b. How does the $2x2$ matrix connecting electron creation and annihilation operators look like? What do we have to do to diagonalize this matrix?
    c. By performing a Bogoliubov-Valatin transformation we are defining new fermionic operators for quasiparticles. How are the creation and annihilation operators for Bogoliubov quasiparticles related to the creation and annihilation operators of the non-interacting electrons?
    d. How do the probabilities $|u_k|^2$ and $|v_k|^2$ for a pair state $(k, -k)$ being occupied and empty and the Gorkov amplitude look like obtained for $T = 0$ by the Bogoliubov transformation?
    e. Sketch the energy of the Bogoliubov quasiparticles as a function of energy around the Fermi level and compare it to that of non-interacting electrons (holes).
    f. Are the Bogoliubov quasiparticles part of the BCS ground state?
44. BCS theory for $T > 0$:

a. How can we express the BCS Hamiltonian at finite T by using the energies of the Bogoliubov quasiparticles?
b. What is the general procedure for minimizing the free energy of BCS state at finite T?
c. What do we obtain for $T \rightarrow 0$ and $T \rightarrow T_c$? What is the BCS prediction for the relation between the energy gap $\Delta(T = 0)$ and the critical temperature T_c of a superconductor?
d. How does the excitation spectrum of the Bogoliubov quasiparticle change when we are increasing T from zero to T_c.

45. How does the density of states in a superconductor look like? How could we measure the DOS?
46. How can we derive the entropy and the specific heat of a superconductor from the basic results of BCS theory?
    a. How does the specific heat very with temperature at very low T? Which quantity can we determine by measuring $c_p(T)$ at low T?
    b. What does BCS theory predict for the jump of the specific heat at $T = T_c$? Is there a difference to the result obtain from simple thermodynamics based on the phenomenological $B_{\text{cth}}(T)$ dependence?
47. Tunneling between superconductors:
    a. Why can we describe tunneling between superconductors within a simple “semiconductor-lie” picture?
    b. How can we measure the DOS in a superconductor by measuring the tunneling conductance of a NIS tunnel junction? How does the current-voltage characteristic (IVC) and its voltage-derivative look like?
    c. Sketch the IVC of a SIS tunnel junction. How can we derive the energy gaps of the superconducting electrodes from such IVC?
48. Why do we have to take into account coherence effects when we treat the response of superconductors to external perturbations (e.g. ultrasound attenuation, electromagnetic absorption)?

Josephson Effects

49. How can we derive the Josephson equations from the expression for the current-phase and energy phase relation in a bulk superconductor?
50. Write down and discuss the Josephson equations.
51. What is the Josephson coupling energy, how can it be derived and what is its meaning?
52. What are potential applications of the Josephson effect?
53. Discuss how the critical current density J_c and normal resistance R_n of a superconductor / insulator / superconductor Josephson junction depend on the thickness of the tunneling barrier.
54. What is the relation between the product $I_c R_n$ to the energy gap of the junction electrodes?

Flux Pinning and Critical Currents

55. What are typical power applications of superconductors?
56. What are the relevant materials parameters for power applications of superconductors?
57. How can we manufacture superconducting wires of metallic superconductors?
58. What is the critical current density of a type-I superconductor expected from London and GL theory? Are there differences? If yes, why?

59. Why can't we use type-I superconductors for superconducting wires and magnetic field coils?
60. How does the force acting on a flux line in the presence of a finite transport current density look like in a type-II superconductor?
61. How would the current-voltage characteristics of an ideal (defect-free) type-II superconductor look like?
62. How can we pin magnetic flux lines in a type-II superconductor?
    a. What is the relation between the pinning force and the shape of the pinning potential?
    b. How should an optimum pinning center (maximum pinning force) look like?
63. Sketch the magnetization curve for a type-II superconductor with pinning.
64. Sketch the magnetization and the magnetic flux density inside a type-I and type-II superconductor as a function of the applied magnetic field.
65. How does the magnetic flux distribution look like in the mixed state of a type-II superconductor with zero and finite flux pinning?

High Temperature Superconductivity (HTS)

66. Why has the discovery of HTS generated so much excitement?
67. What are the basic building blocks of the crystal structure of copper-based HTS?
68. How does the typical phase diagram of a cuprate superconductor look like?
69. How does the typical electronic structure of a cuprate superconductor look like?
70. What is the symmetry of the order parameter of YBaCuO and how can we measure it?