



Walther  
Meißner  
Institut

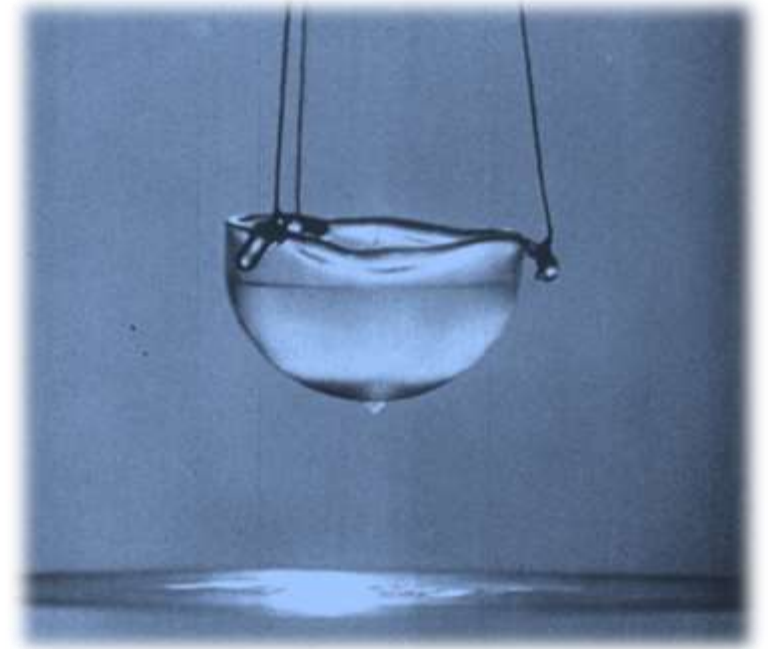
**BAaW**

BAYERISCHE  
AKADEMIE  
DER  
WISSENSCHAFTEN

Technische  
Universität  
München

**TUM**

# Superconductivity and Low Temperature Physics II



**Lecture Notes  
Summer Semester 2022**

**R. Gross  
© Walther-Meißner-Institut**



***Superconductivity,  
Superfluidity, Condensates, Quantum Liquids***





## 1. Superconductivity and Low Temperature Physics I + II

- Part I (WS 2021/2022): Foundations of Superconductivity
- Part II (SS 2022): Foundations of Low Temperature Physics and Techniques

This lecture

## 2. Applied Superconductivity I + II



- WS 2021/22 and SS 2022, 2 hrs lecture + 2 hrs exercises
- Josephson-Effect, Superconducting Electronics, Qubits, Quantum Circuits, ....

## 3. Seminars (SS 2022, WMI seminar room)

- Advances in Solid State Physics (TUE 10:15-11:30)
- Superconducting Quantum Circuits (TUE 14:30-16:00)



Further information: <https://www.wmi.badw.de/teaching>

- announcement of lectures
- downloads of lecture notes, handouts...
- seminar topics

# Nobel Prizes in Physics related to LT Physics

year	name	discovery
1913	<a href="#">Heike Kamerlingh Onnes</a>	"For his investigations on the properties of matter at low temperatures which led, inter alia, to the production of <a href="#">liquid helium</a> "
1972	<a href="#">John Bardeen</a> , <a href="#">Leon Neil Cooper</a> and <a href="#">Robert Schrieffer</a>	"for their jointly developed theory of <a href="#">superconductivity</a> , usually called the <a href="#">BCS-theory</a> "
1973	<a href="#">Brian David Josephson</a>	"for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the <a href="#">Josephson effect</a> "
1978	<a href="#">Pjotr Kapiza</a>	"for his basic inventions and discoveries in the area of low-temperature physics"
1985	<a href="#">Klaus von Klitzing</a>	"for the discovery of the <a href="#">quantized Hall effect</a> "
1987	<a href="#">Johannes Georg Bednorz</a> und <a href="#">Karl Alex Müller</a>	"for their important break-through in the discovery of <a href="#">superconductivity</a> in <a href="#">ceramic</a> materials"
1996	<a href="#">David M. Lee</a> , <a href="#">Douglas D. Osheroff</a> und <a href="#">Robert C. Richardson</a>	"for their discovery of <a href="#">superfluidity</a> in <a href="#">helium-3</a> "
1997	<a href="#">Steven Chu</a> , <a href="#">Claude Cohen-Tannoudji</a> and <a href="#">William D. Phillips</a>	"for development of methods to cool and trap atoms with laser light" See <a href="#">Laser cooling</a> .
1998	<a href="#">Robert B. Laughlin</a> , <a href="#">Horst Ludwig Störmer</a> and <a href="#">Daniel Chee Tsui</a>	"for their discovery of a new form of quantum fluid with fractionally charged excitations". See <a href="#">Quantum Hall effect</a> .
2001	<a href="#">Eric A. Cornell</a> , <a href="#">Wolfgang Ketterle</a> and <a href="#">Carl E. Wieman</a>	"for the achievement of <a href="#">Bose-Einstein condensation</a> in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"
2003	<a href="#">Alexei Abrikosov</a> , <a href="#">Witali Ginsburg</a> and <a href="#">Anthony James Leggett</a>	"for pioneering contributions to the theory of <a href="#">superconductors</a> and superfluids"
2016	<a href="#">David J. Thouless</a> , <a href="#">F. Duncan M. Haldane</a> , <a href="#">J. Michael Kosterlitz</a>	"for theoretical discoveries of topological phase transitions and topological phases of matter"

## Superconductivity and Low Temperature Physics I

### Introduction

1. Basic Properties of Superconductors
2. Phenomenological Models of Superconductivity :
  - London equations
  - macroscopic quantum model
  - Ginzburg-Landau theory
3. Thermodynamics
4. Microscopic (BCS) theory
5. Flux pinning and critical currents
6. High Temperature Superconductivity
7. Applications

## Superconductivity and Low Temperature Physics II

1. Bose-Einstein condensation
2. Superfluid Helium ( $^4\text{He}$  and  $^3\text{He}$ )
3. Quantum Interference Effects in Mesoscopic Conductors
4. Low Temperature Techniques  
(generation and measurement of low temperatures)

## I.1 Foundations and General Properties

- I.1.1 Quantum Fluids
- I.1.2 Helium
- I.1.3 Van der Waals Bonding
- I.1.4 Zero-Point Fluctuations
- I.1.5 Helium under Pressure
- I.1.6 pT-Phase Diagram of  $^4\text{He}$  and  $^3\text{He}$
- I.1.7 Characteristic Properties of  $^4\text{He}$  and  $^3\text{He}$
- I.1.8 Specific Heat of  $^4\text{He}$  and  $^3\text{He}$

## I.2 $^4\text{He}$ as an Ideal Bose Gas

- I.2.1 Bose-Einstein Condensation
- I.2.2 Ideal Bose Gas
- I.2.3 Bose Gas with Interactions
- I.2.4 Bose-Einstein Condensation of  $^4\text{He}$

## I.3 Superfluid $^4\text{He}$

- I.3.1 Two-Fluid Model
- I.3.2 Experimental Observations
- I.3.3 Two-Fluid Hydrodynamics
- I.3.4 Excitation Spectrum of  $^4\text{He}$

## I.4 Vortices

- I.4.1 Quantization of Circulation
- I.4.2 Experimental Study of Vortices

## I.5 $^3\text{He}$

- I.5.1 normal fluid  $^3\text{He}$
- I.5.2 solid  $^3\text{He}$  and Pomeranchuk effect
- I.5.3 superfluid  $^3\text{He}$

## I.6 $^3\text{He}$ / $^4\text{He}$ mixtures

## II.1 Introduction

- II.1.1 General Remarks
- II.1.2 Mesoscopic Systems
- II.1.3 Characteristic Length Scales
- II.1.4 Characteristic Energy Scales
- II.1.5 Transport Regimes

## II.2 Description of Electron Transport by Scattering of Waves

- II.2.1 Electron Waves and Waveguides
- II.2.2 Landauer Formalism
- II.2.3 Multi-terminal Conductors
- II.2.4 Statistics of Charge Transport

## II.3 Quantum Interference Effects

- II.3.1 Double Slit Experiment
- II.3.2 Two Barriers – Resonant Tunneling
- II.3.3 Aharonov-Bohm Effect
- II.3.4 Weak Localization
- II.3.5 Universal Conductance Fluctuations

## II.4 From Quantum Mechanics to Ohm's Law

## II.5 Coulomb Blockade



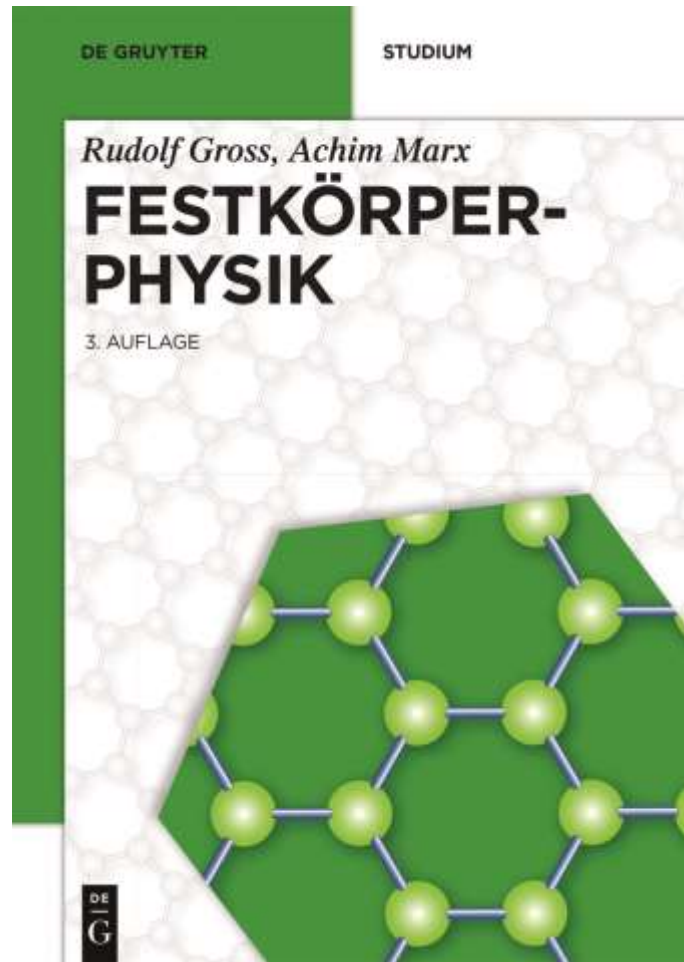
### **III.1 Generation of Low Temperatures**

- III.1.1 Introduction
- III.1.2 Expansion Machine
- III.3 Regenerative Machine
- III.1.4 Joule-Thomson Cooling
- III.1.5 Summary
- III.1.6 Evaporation Cooling
- III.1.7 Dilution Cooling
- III.1.8 Pomeranchuk Cooling
- III.1.9 Adiabatic Demagnetization

### **III.2 Thermometry**

- III.2.1 Introduction
- III.2.2 Primary Thermometers
- III.2.3 Secondary Thermometers

- F. Pobell  
**Matter and Methods at Low Temperatures**, Springer 1996
- D.R. Tilley and J. Tilley  
**Superfluidity and Superconductivity**, Adam Hilger 1990
- C. Enss and S. Hunklinger  
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- P.V.E. McClintock, D.J. Meredith, J.K. Wigmore  
**Matter at Low Temperatures**, Blackie 1984
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**Introduction to Liquid Helium**, Oxford 1987
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**Experimental Low Temperature Physics**, MacMillan, New York
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**Experimental Techniques in Low Temperature Physics**, Oxford University Press, 2002
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**The Physics of Liquid and Solid Helium I and II**, Wiley 1978
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- Supriyoto Datta  
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- Thomas Heinzel  
**Mesoscopic Electronic in Solid State Nanostructures**, Wiley VCH, Weinheim (2003)
- Thomas Ihn  
**Semiconductor Nanostructures**, Oxford University Press (2010)



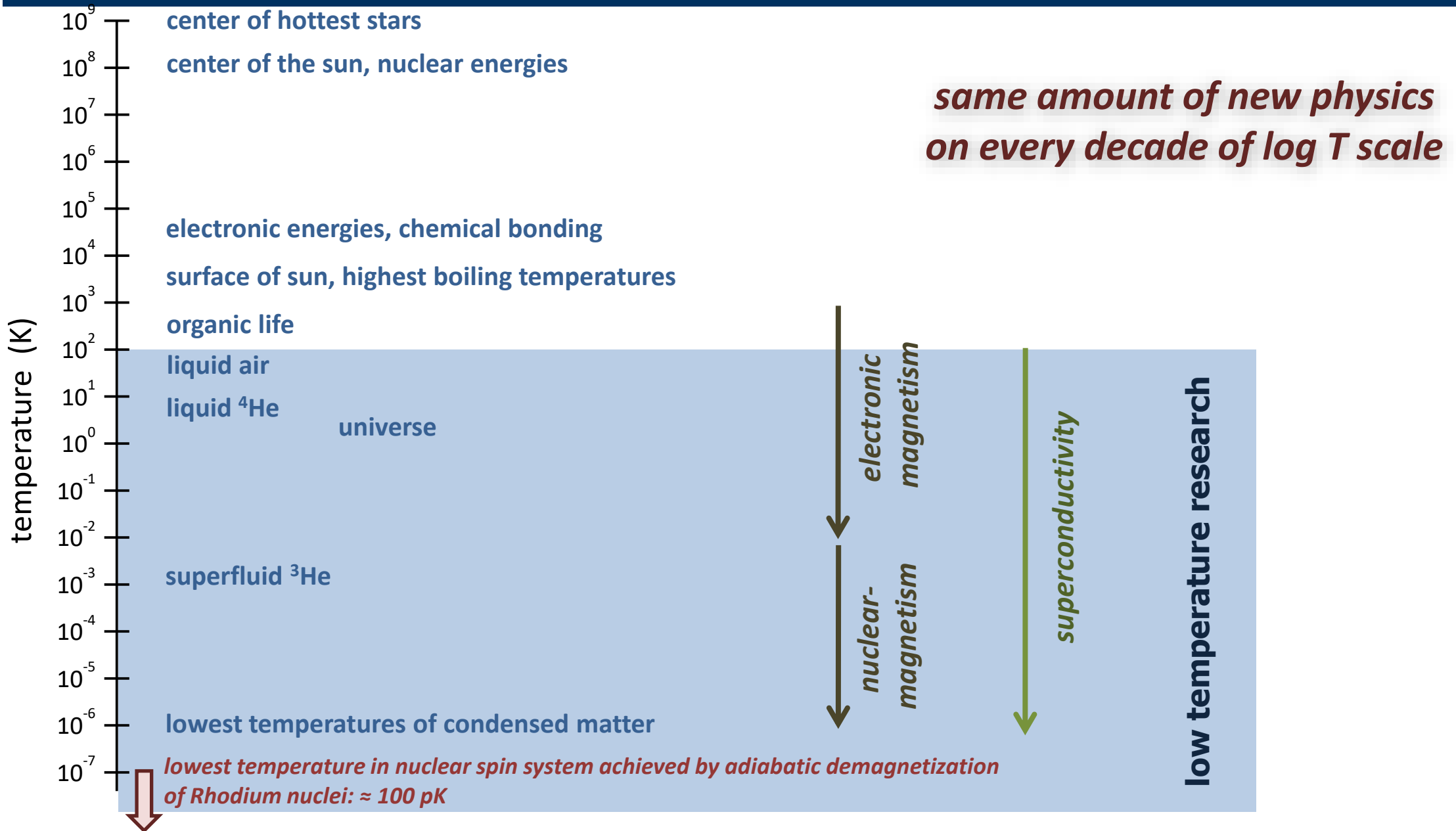
## Rudolf Gross, Achim Marx

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13	Supraleitung	769
13.1	Geschichte und grundlegende Eigenschaften	772
13.1.1	Geschichte der Supraleitung	772
13.1.2	Supraleitende Materialien	780
13.1.3	Sprungtemperaturen	783
13.1.4	Grundlegende Eigenschaften	784
13.2	Thermodynamische Eigenschaften von Supraleitern	792
13.2.1	Typ-I Supraleiter im Magnetfeld	792
13.2.2	Typ-II Supraleiter im Magnetfeld	797
13.3	Phänomenologische Modelle	798
13.3.1	London-Gleichungen	798
13.3.2	Verallgemeinerte London Theorie – Supraleitung als makroskopisches Quantenphänomen	801
13.3.3	Die Ginzburg-Landau-Theorie	813
13.4	Typ-I und Typ-II Supraleiter	825
13.4.1	Mischzustand und kritische Felder	826
13.4.2	Supraleiter-Normalleiter Grenzflächenenergie	827
13.4.3	Vertiefungsthema: Zwischenzustand und Entmagnetisierungseffekte	829
13.4.4	Kritische Felder	830
13.4.5	Vertiefungsthema: Nukleation an Oberflächen	834
13.4.6	Vertiefungsthema: Shubnikov-Phase und Flussliniengitter	835
13.4.7	Vertiefungsthema: Flusslinien in Typ-II Supraleitern	838
13.4.8	Kritische Stromdichte	843
13.5	Mikroskopische Theorie	847
13.5.1	Attraktive Elektron-Elektron-Wechselwirkung und Cooper-Paare	849
13.5.2	Der BCS-Grundzustand	858
13.5.3	Energielücke und Anregungsspektrum	873
13.5.4	Quasiteilchentunneln	875
13.5.5	Thermodynamische Größen	879
13.6	Josephson-Effekt	883
13.6.1	Die Josephson-Gleichungen	883
13.6.2	Josephson-Kontakt mit Wechselspannung	888
13.6.3	Josephson-Kontakt im Magnetfeld	889
13.6.4	Supraleitende Quanteninterferometer	893
13.7	Kritische Ströme in Typ-II Supraleitern	895
13.7.1	Stromtransport im Mischzustand	896
13.7.2	Lorentz-Kraft	898
13.7.3	Reibungskraft	900
13.7.4	Haftkraft	901
13.8	Unkonventionelle Supraleitung	903
13.9	Kuprat-Supraleiter	905
13.9.1	Strukturelle Eigenschaften	906
13.9.2	Elektronische Eigenschaften	907
13.9.3	Supraleitende Eigenschaften	912
	Literatur	921

# Introduction

# Temperature Scale

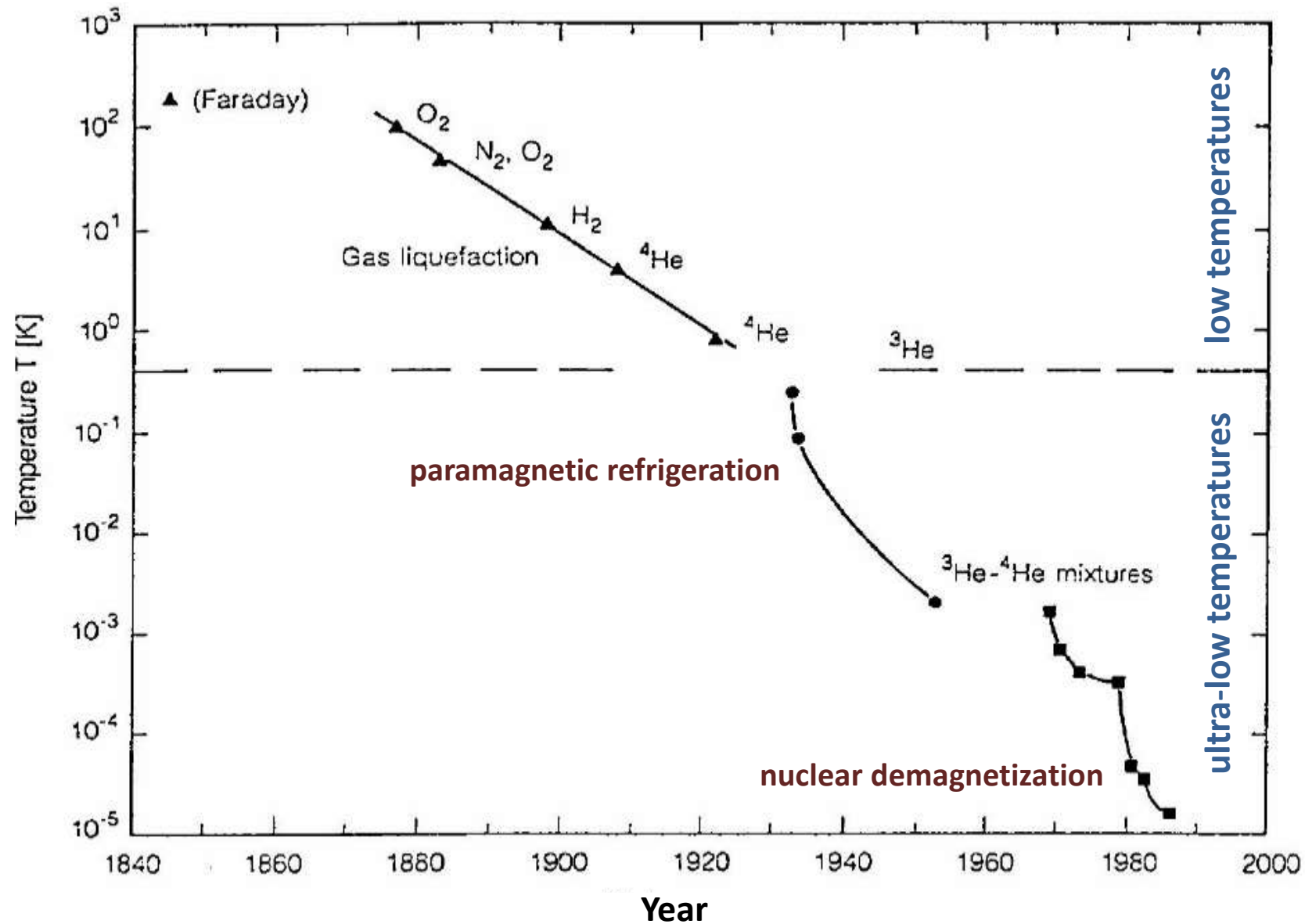




# Low Temperature Phenomena

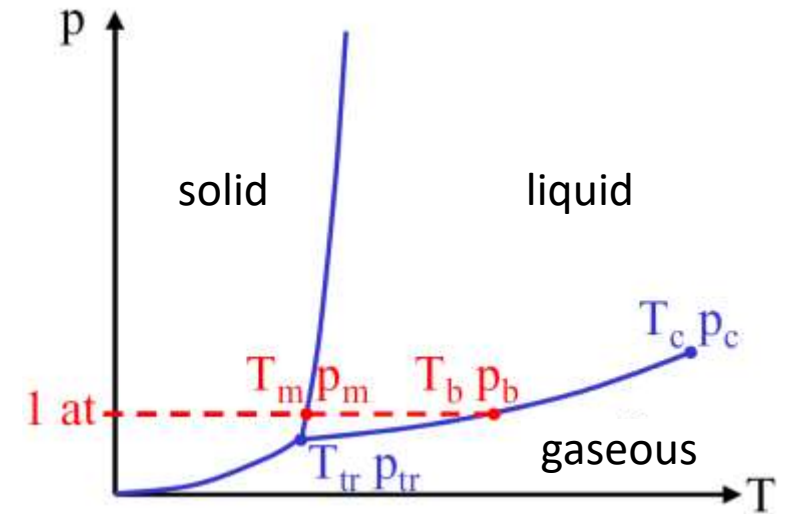
- many direct implications of quantum mechanics only show up at low temperatures  
e.g. magnetism, superconductivity
- particularly interesting are quantum phenomena manifesting themselves on a macroscopic length scale  
→ *macroscopic quantum phenomena*
  - Superconductivity
  - **Superfluidity**
  - **Bose-Einstein Condensation** } this lecture

# Generation of Low Temperatures



# Generation of Low Temperatures

Substance	$T_b$ [K]	$T_m$ [K]	$T_{tr}$ [K]	$P_{tr}$ [bar]	$T_c$ [K]	$P_c$ [bar]
	@ 1 bar					
H <sub>2</sub> O	373.15	273.15	273.16	0.06	647.3	220
Xe	165.1	161.3	161.4	0.82	289.8	58.9
Kr	119.9	115.8	114.9	0.73	209.4	54.9
O <sub>2</sub>	90.2	54.4	54.36	0.016	154.3	50.4
Ar	87.3	83.8	83.81	0.67	150.9	48.7
N <sub>2</sub>	77.4	63.3	63.15	0.12	126.0	33.9
Ne	27.1	24.5	24.56	0.43	44.5	27.2
D <sub>2</sub>	23.7	18.7	18.72	0.17	38.3	16.6
H <sub>2</sub>	20.3	14.0	13.80	0.07	33.3	13.0
<sup>4</sup> He	4.21	--	--	--	5.20	2.28
<sup>3</sup> He	3.19	--	--	--	3.32	1.16



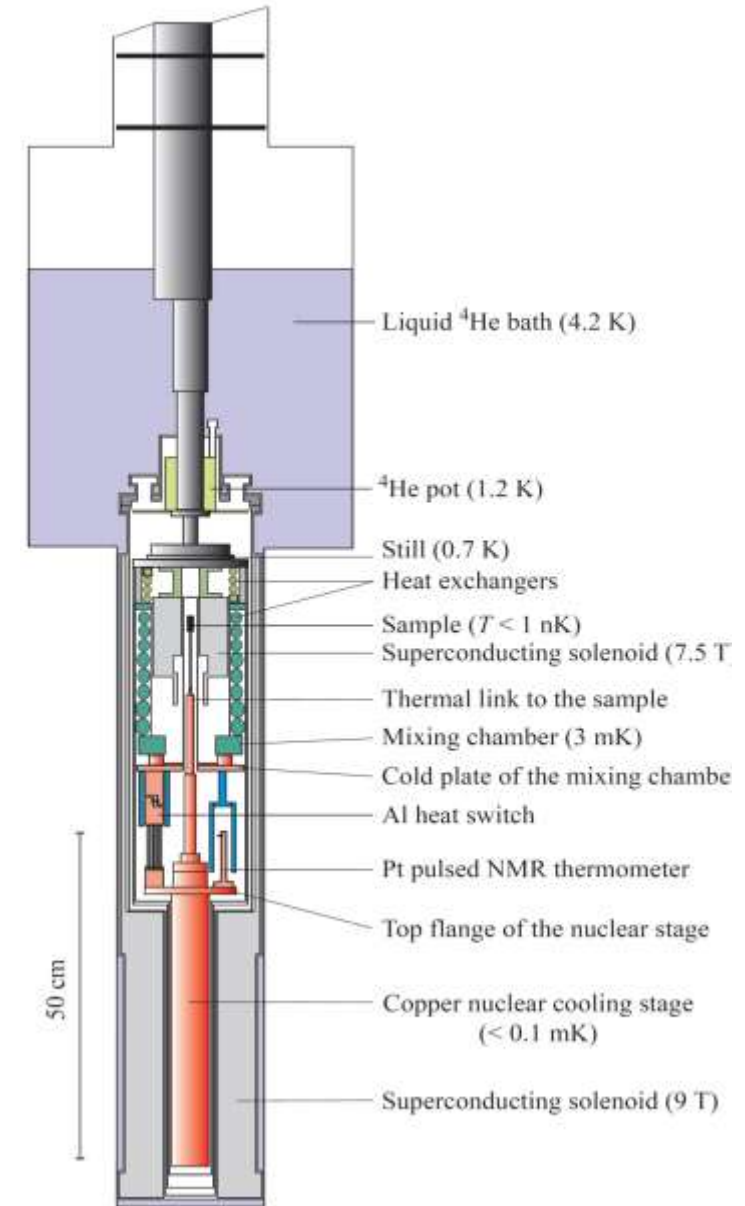
# Generation of Low Temperatures

- nuclear demagnetization

experimental setup according to  
Tauno Knuutila (2000)

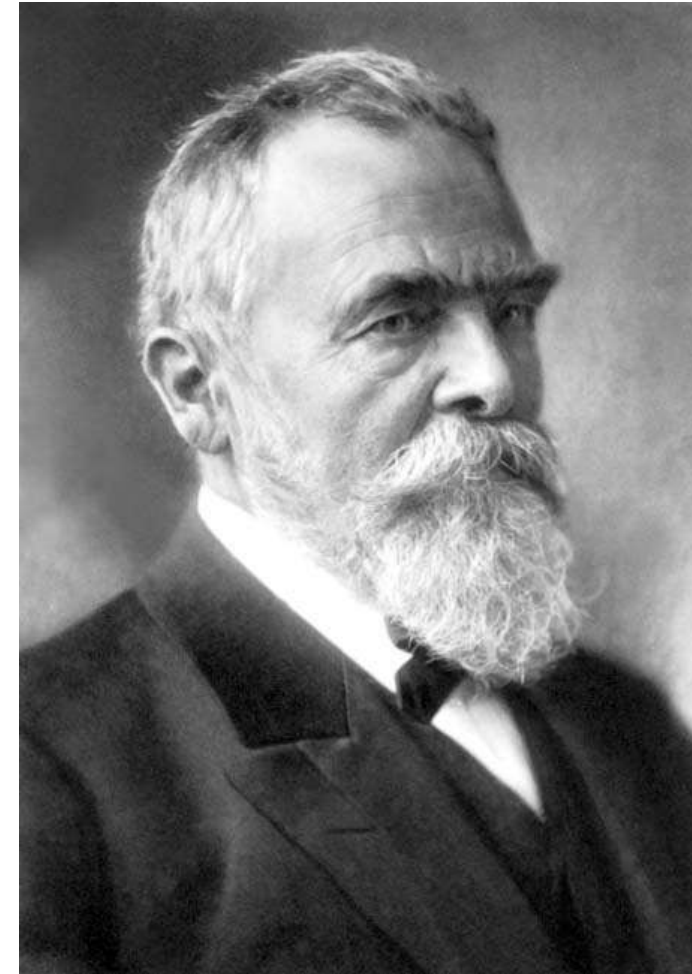
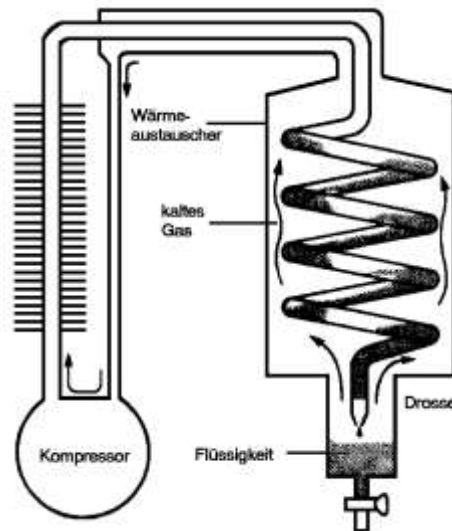
lowest temperature of nuclear spin system:  
about **100 pK**  
by demagnetization of Rhodium nuclei

PhD Thesis,  
Helsinki University of Technology  
(Espoo, Finland)



- 1868** offer of chair at the Polytechnische Schule München (now TUM)
- 1873** development of cooling machine allowing the temperature stabilization in beer brewing
- 21. 6. 1879** foundation of „*Gesellschaft für Linde's Eismaschinen AG*“ together with two beer brewers and three other co-founders
- 1892 – 1910** re-establishment of professorship

**12.5.1903** patent application:  
 „*Lindesches Gegenstromverfahren*“  
 liquefaction of oxygen  
 ( $-182^{\circ}\text{C} = 90\text{ K}$ )



***Carl Paul Gottfried von Linde***

\* 11. Juni 1842 in Berndorf, Oberfranken  
 † 16. November 1934 in Munich



**1930** Linde AG uses the first cooling turbines for the generation of low temperatures

turbines have higher efficiency and therefore are used today in all larger liquefaction machines

e.g. He liquefier at the Walther-Meißner-Institut

**1947** first commercial He liquefier (design by engineer Collins, therefore denoted as „Collins“ machine)

***Arthur D. Little*** Inc. (today CTI)

**1966** ***Hall*** et al. and ***Neganov*** et al. develop  $^3\text{He}/^4\text{He}$  dilution refrigerators, generation of temperatures down to 2 mK

# Discovery of Superconductivity (1911)

Heike Kammerlingh Onnes (1853-1926)

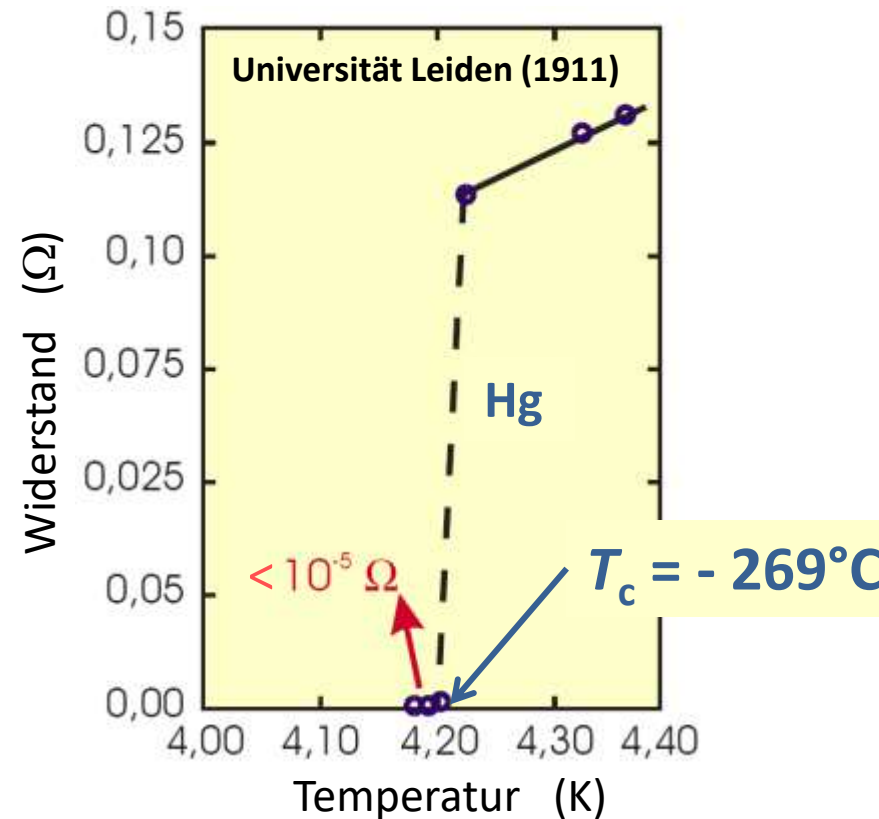


- Helium liquefaction: 1908
- discovery of superconductivity: 1911

## Nobel Price in Physics 1913

choice of name:

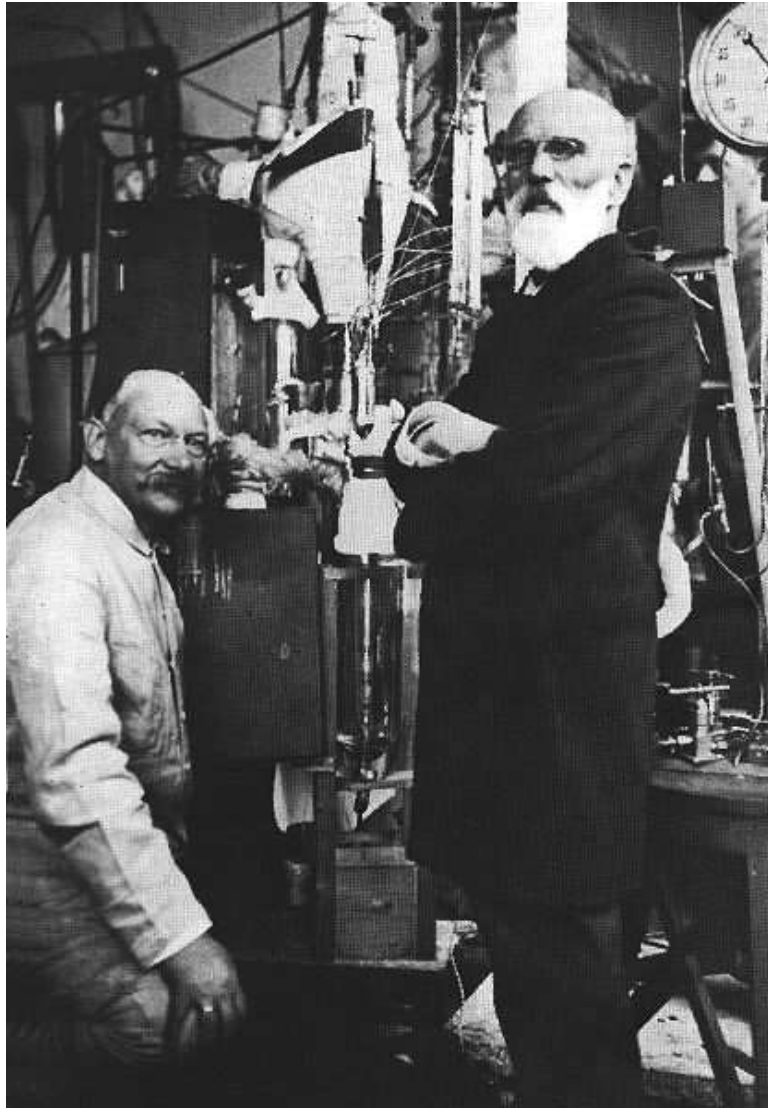
*infinite electrical conductivity* → **superconductivity**



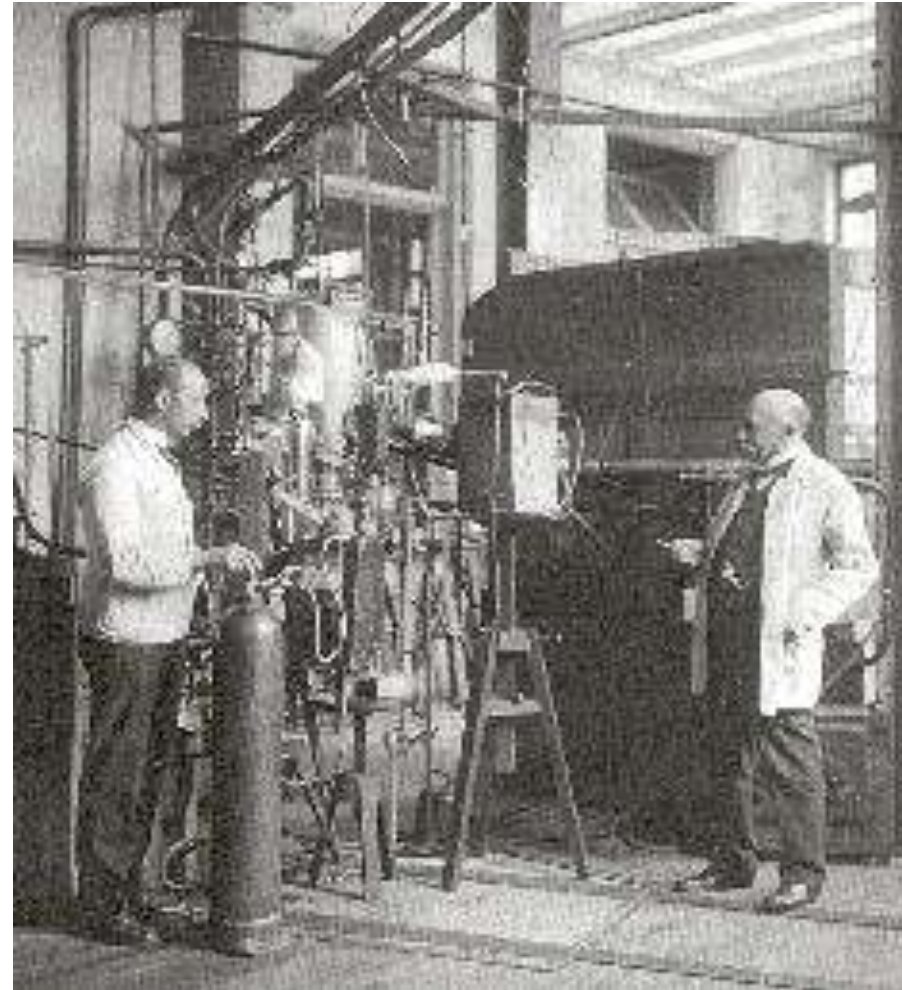
*H. K. Onnes, Comm. Leiden 120b (1911)*

*"for his investigations on the properties of matter at low temperatures which led, inter alia to the production of liquid helium"*

# Discovery of Superconductivity (1911)



**Kammerlingh Onnes and van der Waals**



**Kammerlingh Onnes and Technician Flim**

# Discovery of the Meißner-Ochsenfeld Effect (1933)



**Robert Ochsenfeld**  
(1901 – 1993)



**perfect diamagnetism**

*W. Meißner, R. Ochsenfeld,  
Ein neuer Effekt bei Eintritt der Supraleitfähigkeit,  
Naturwissenschaften 21, 787 (1933).*



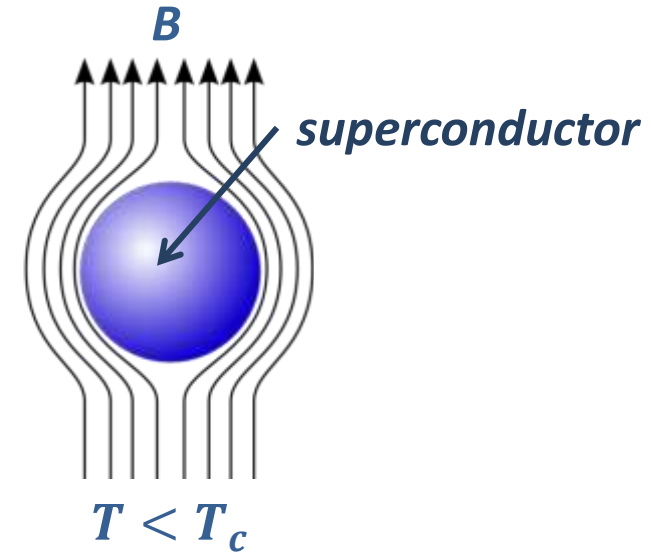
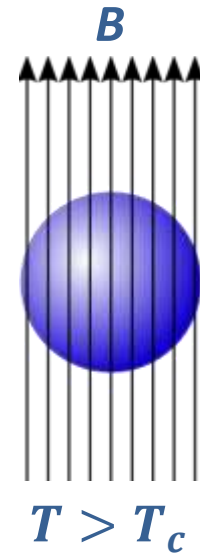
Dr. Walther Meißner  
a. Prof. für technische Physik. Präsident 1946-50

**Walther Meißner**  
(1882 – 1974)



# Discovery of the Meißner-Ochsenfeld Effect (1933)

Walther Meißner (1882 – 1974)



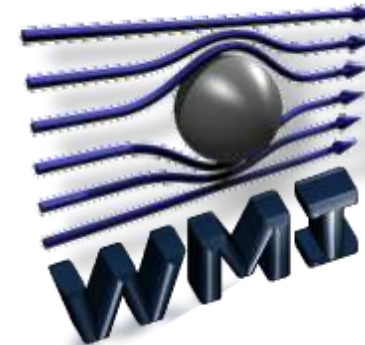
superconductors perfectly expel magnetic field

$$B_{\text{in}} = (1 + \chi) B_{\text{ext}} = 0 \quad \chi = \text{magnetic susceptibility}$$

➔ ideal diamagnetism,  $\chi = -1$

choice of name for perfect diamagnetism:

**Meißner-Ochsenfeld Effect**





# Walther Meißner (1882 – 1974)

- 1913 – 1934** building and heading of low temperature laboratory at the Physikalisch-Technischen-Reichsanstalt, **liquefaction of H<sub>2</sub> (20K)**
- 7.3.1925** **first liquefaction of He in Germany** (4.2 K, 200 ml), 3<sup>rd</sup> system world-wide besides Leiden and Toronto
- 1933** discovery of perfect diamagnetism of superconductors together with Ochsenfeld  
→ ***Meißner-Ochsenfeld Effect***
- 1934** offer of chair at the Technische Hochschule München (now TUM)
- 1946 – 1950** president of the Bavarian Academy of Sciences and Humanities
- 1946** foundation of the commission for Low Temperature Research  
→ ***Walther-Meißner-Institut***



***Walther Meißner***

\* 16. Dezember 1882 in Berlin

† 15. November 1974 in Munich

**Walther Meißner - der Mann, mit dem die Kälte kam**

W. Buck, D. Einzel, R. Gross, Physik Journal, Mai 2013

**Low Temperature Physics**

**at**

**WMI**

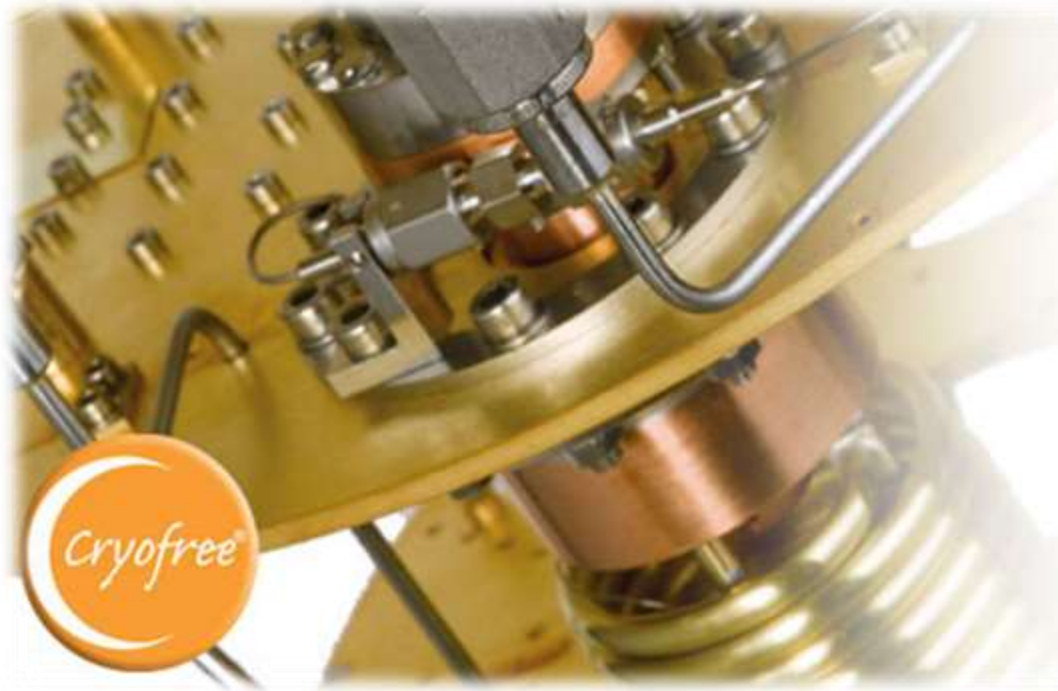
- **superconductivity and superfluidity**
- **magnetism and spintronics**
- **nanoscale superconducting and spintronic devices**
- **solid-state based quantum systems**
- **superconducting quantum circuits for quantum computing**
- **quantum microwave communication and sensing**
- **.....**

**an appetizer**

**innovative cryoengineering**

*....WMI develops first dry dilution fridge*

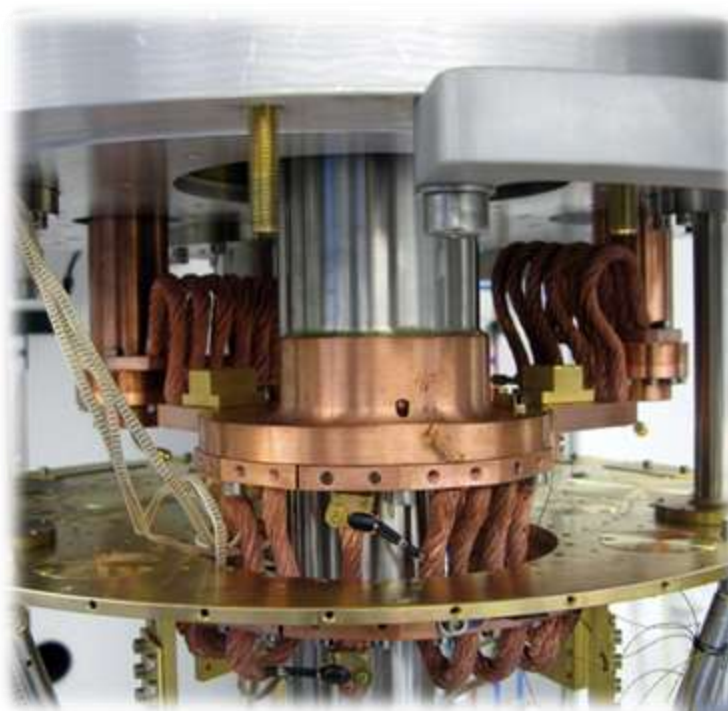
K. Uhlig, Cryogenics 42, 73 – 77 (2002)



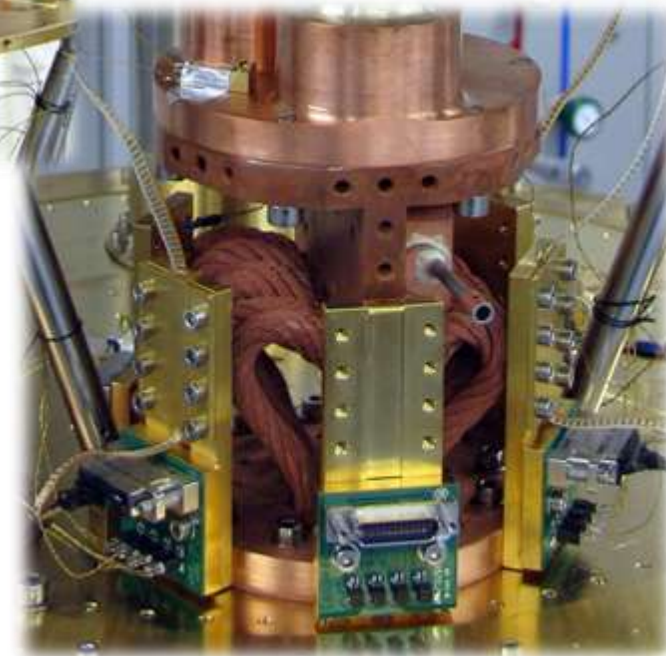
**Oxford Instruments Triton family**

*market share of dry dilution fridges: > 95 %*





*dry dilution fridge  
designed and fabricated  
at WMI*





- BlueFors XLD Dilution Fridge (2020)



- Oxford Instruments Dilution Fridge (2011)  
(with 3D vector magnet)

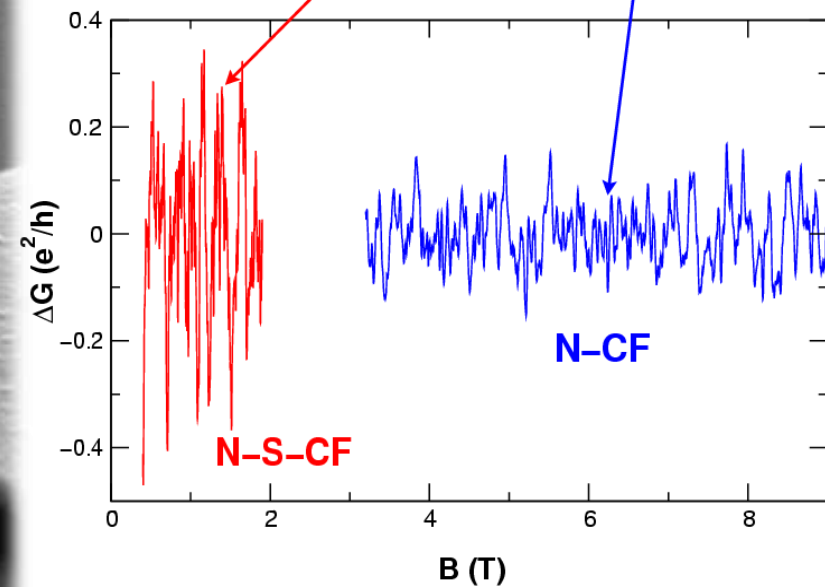
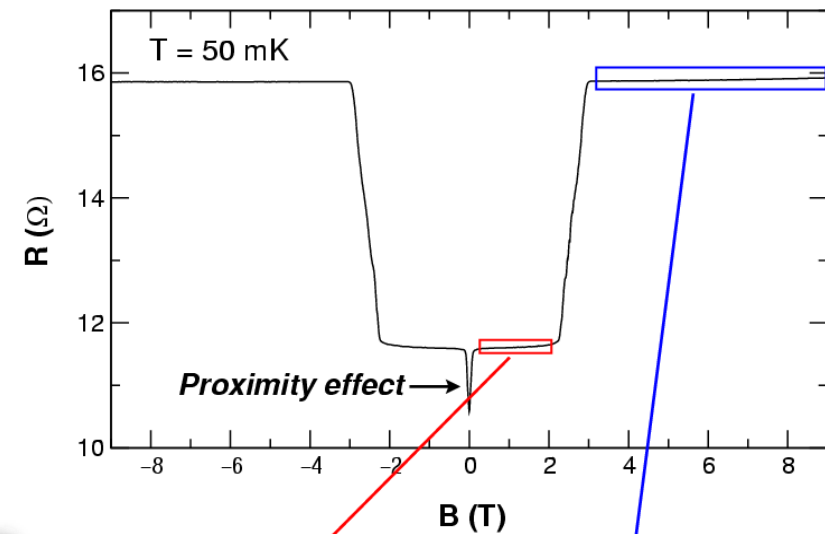
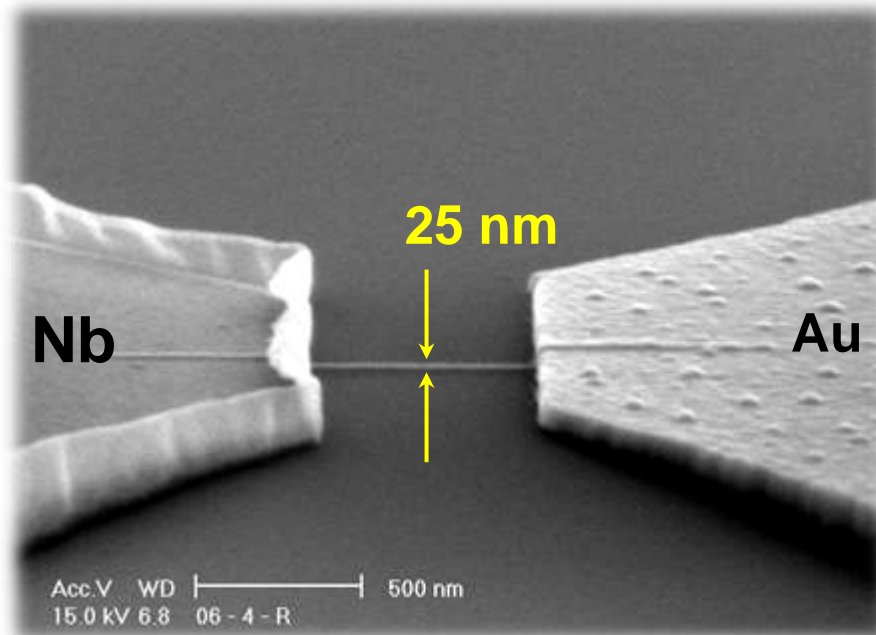




- **Helium liquefier at WMI:  
Linde TCF 20**
- **supply of LHe to both Munich  
Universities**
- **liquefaction power:  
> 150 000 l/year**
- **market price:  
about 2 Mio. €**



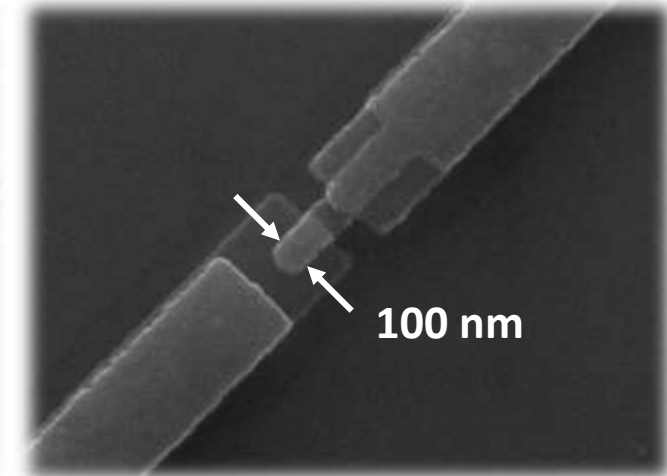
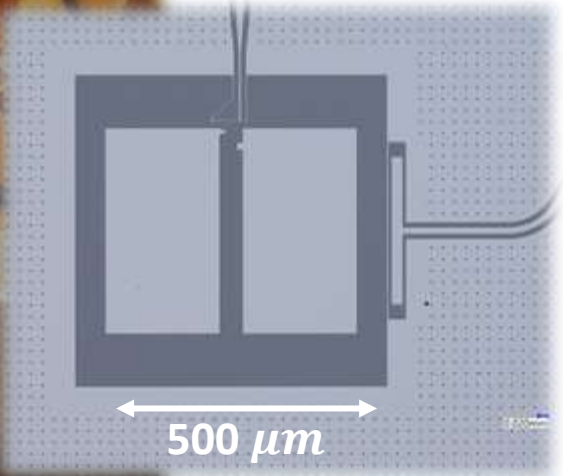
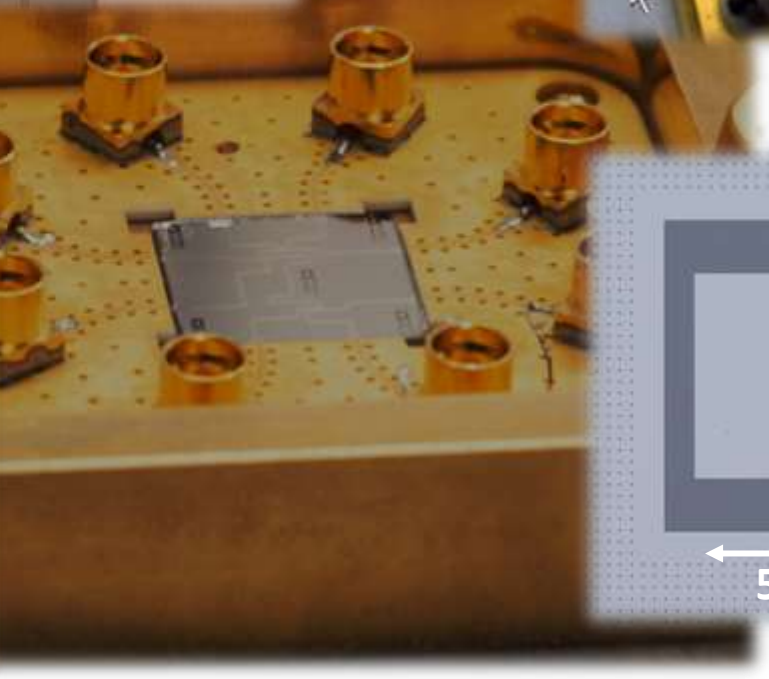
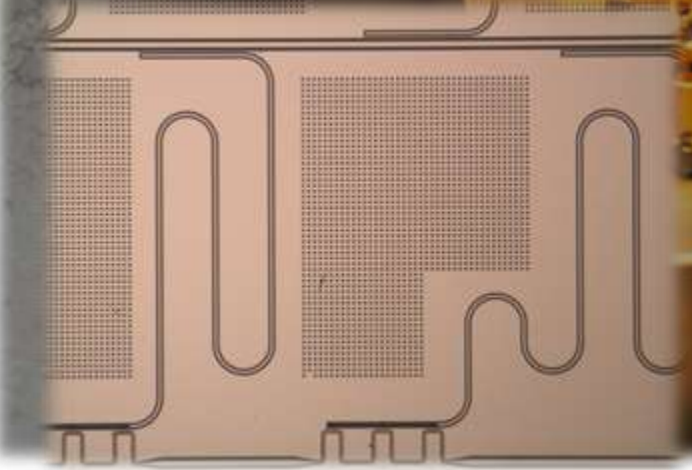
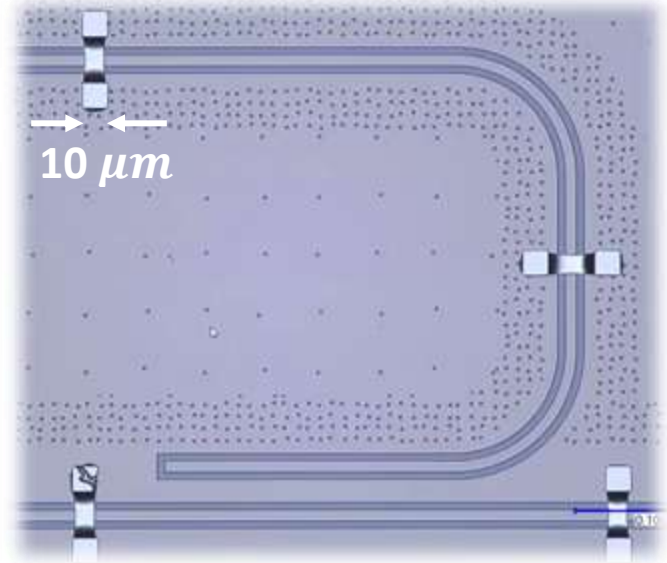
# Low Temperature Physics at WMI



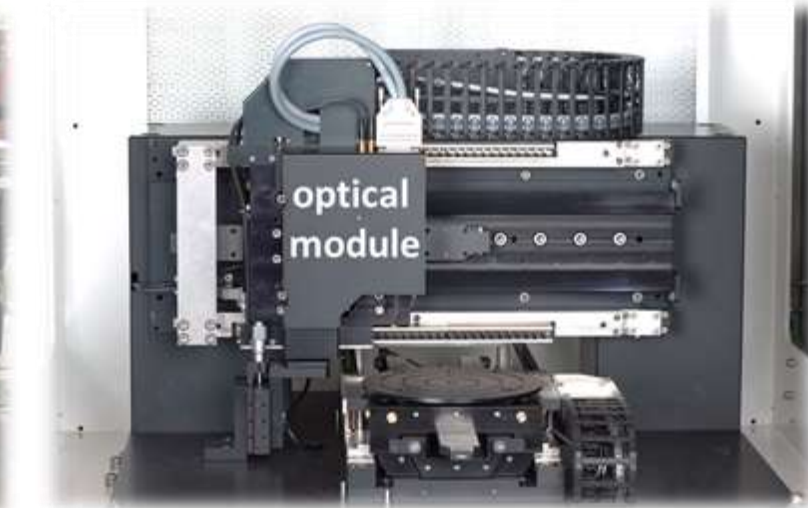




#we do technology







# Quantum Technology Projects @ WMI



Bayerisches Staatsministerium  
für Wissenschaft und Kunst

GeQCoS (BMBF)  
MQV-SQQC (StMWK Bayern)  
MUNIQC-SC (BMBF)



Bundesministerium  
für Bildung  
und Forschung

R. Gross © Walther-Meißner-Institut (2004 - 2022)



Bundesministerium  
für Bildung  
und Forschung



QuaMe

DFG Deutsche  
Forschungsgemeinschaft



BUILDING  
**QUANTUM  
COMPUTERS**

Demonstrator-QC  
~ 10 Qubits



NISQ-QC  
~ 100 Qubits



FT-QC  
~ 10<sup>6</sup> Qubits



kiutra



Funded by  
the European Union



**Have  
Fun !!**