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München

Superconductivity and Low Temperature Physics II


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Lecture Notes
Summer Semester 2024

R. Gross

Chapter 3

Cryogenic Techniques

Generation and Measurement of Low Temperatures

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Lecture No. 11

R. Gross
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Chapter III: Cryogenic Techniques

Contents:

Chapter III: Cryogenic Techniques

Literature:

1. **Tieftemperaturphysik**
   Enss, Hunklinger
   Springer (2000)

   Low-Temperature Physics
   Enss, Hunklinger
   Springer (2005)

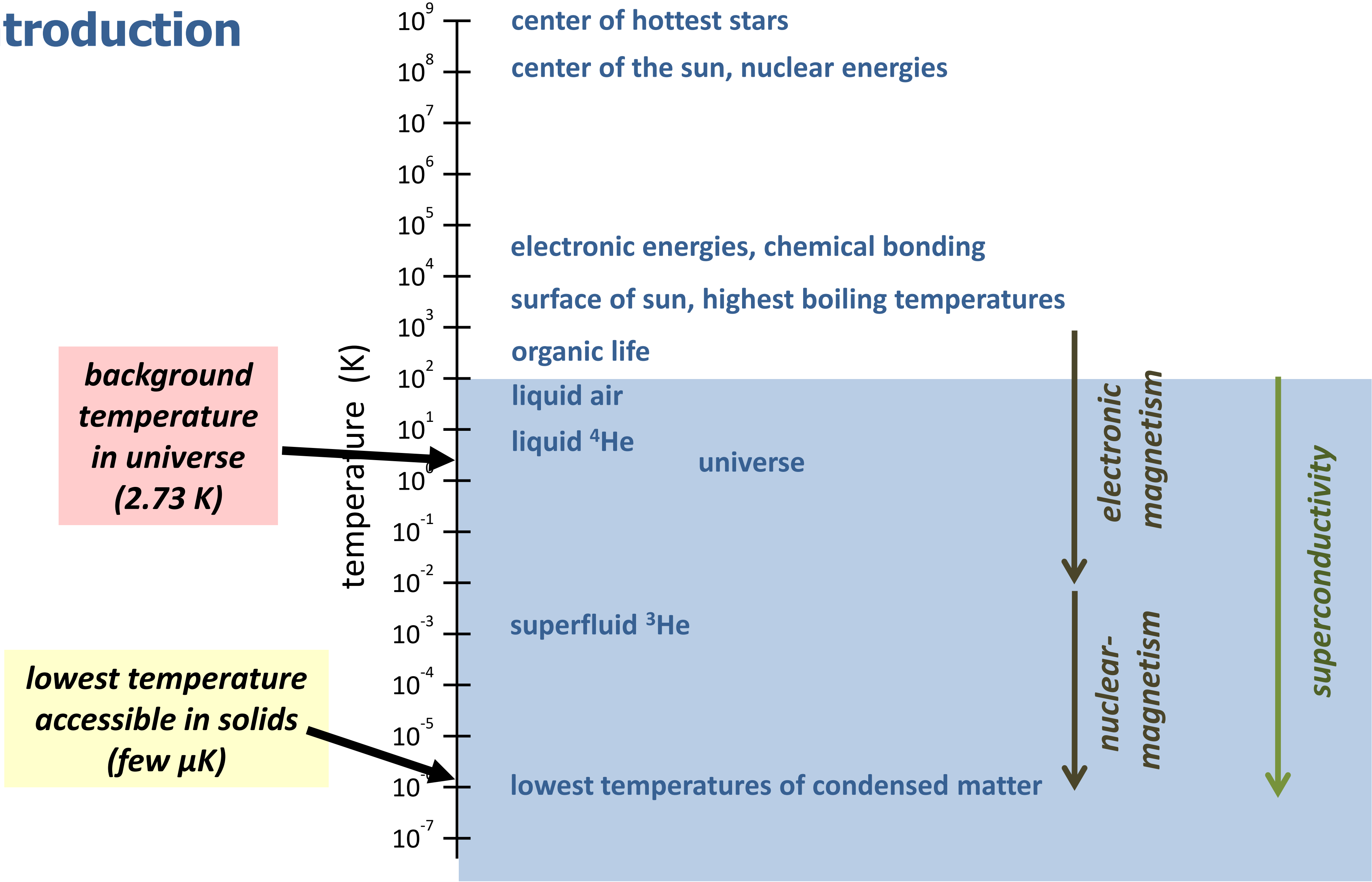
2. **Matter and Methods at Low Temperatures**
   F. Pobell
   Springer, 2nd edition (1996)

3. **Experimental Low-Temperature Physics**
   Anthony Kent
   American Institute of Physics (1993)

4. **Cryogenic Systems**
   Randall F. Barron
   Oxford University Press, Oxford (1985)

III.1 Generation of Low Temperatures

III.1.1 Introduction



background temperature in universe (2.73 K)

lowest temperature accessible in solids (few μK)

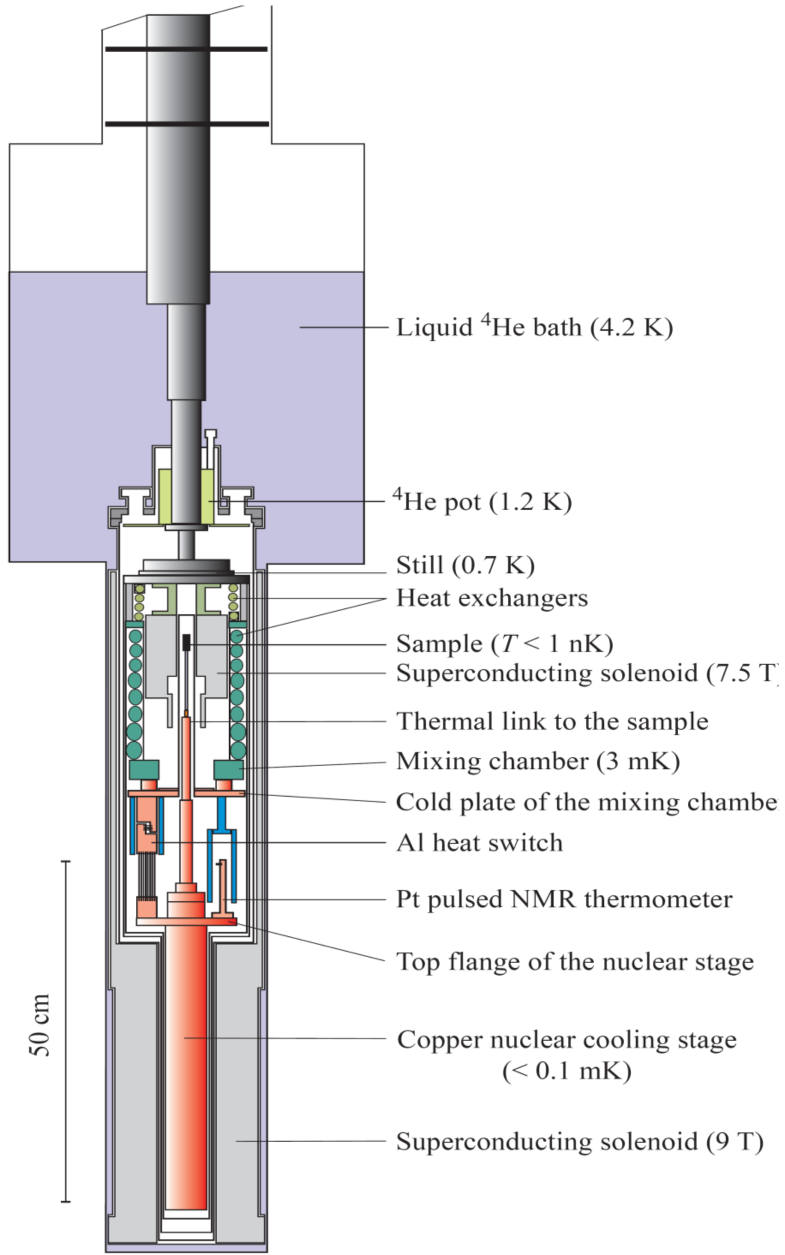
III.1.1 Introduction

- **lowest temperatures (in solid-state systems)**

  low temperature record
  for nuclear spin system:

  - experimental setup
    according to Tauno Knuuttila (2000)
  - lowest temperature:
    about **100 pK** by demagnetization of Rhodium nuclei *(„temperature of nuclear spins“)*

    PhD Thesis, Helsinki University of Technology
    (Espoo, Finland)
  - ***problem:***
    spin temperature cannot be transferred to lattice of solid (small spin-phonon coupling)



III.1.1 Introduction

- **generation of low temperatures by using cryo-liquids**

19th century: liquefaction of various gases by pressure except for “permanent gases” (O_2, H_2, He)

1877: liquefaction of O_2 by thermal expansion

(L. Cailletet, C.R. Acad. Sci. Paris 85, 1213 (1877); R. Pictet, C.R. Acad. Sci. Paris 85, 1214 (1877))

1884: liquefaction of H_2 (precooling with liquid O_2)

(K. Olszewski, Ann. Phys. u. Chem. 31, 58 (1887))

1898: significant amounts of LH_2 for physical experiments

(J. Dewar, Proc. R. Inst. Gt. Br. 15, 815 (1898))

1908: liquefaction of last “permanent gas” He by Kamerlingh Onnes

(H. Kammerlingh Onnes, Leiden Commun. 105, Proc. Roy. Acad. Sci. Amsterdam 11, 168 (1908))

1922: Kammerlingh Onnes reaches $T < 1$ K

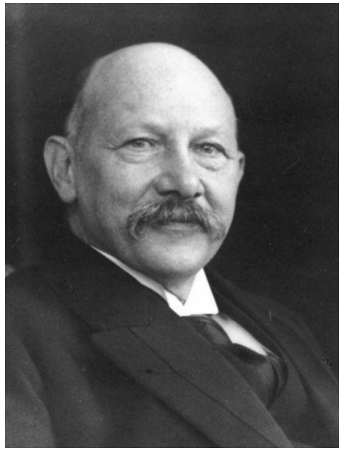
(H. Kammerlingh Onnes, Leiden Commun. 159, Trans. Faraday Soc. 18 (1922))

1926: adiabatic demagnetization of electron spins in paramagnetic salts by Debye and independently (P. Debye, Ann. Phys. 81, 1154 (1926)

1927: by Giauque (W.F. Giauque, J. Am. Chem. Soc. 49, 1864 (1927)

since 1950s: ^{3}He available
^{3}He cryostat
^{3}He-^{4}He dilution refrigerator

Sir **James Dewar**,
(1842-1923)

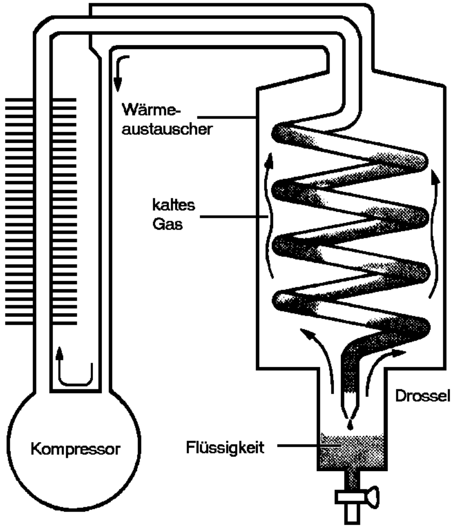


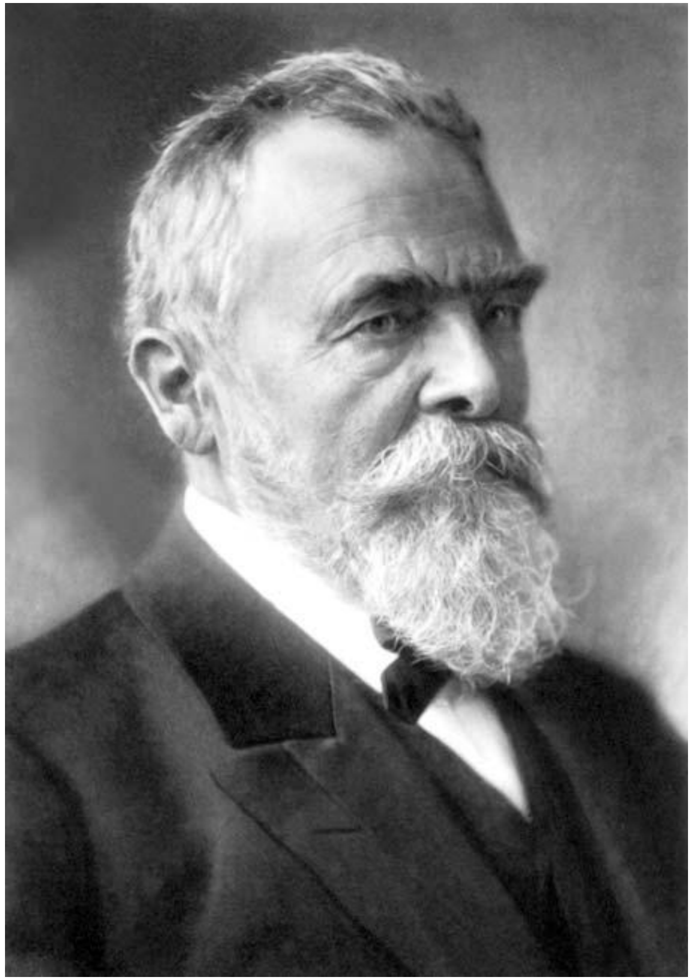
Heike Kammerlingh Onnes
(1853 – 1926)
Nobelpreis für Physik: 1913

Peter J. Debye
1884 - 1966

III.1.1 Introduction

1868	offer of chair at the Polytechnische Schule München (now TUM)
1873	development of cooling machine allowing the temperature stabilization in beer brewing
21. 6. 1879	foundation of „***Gesellschaft für Linde's Eismaschinen AG***" together with two beer brewers and three other co-founders
1892 – 1910	re-establishment of professorship
12.5.1903	patent application: „***Lindesches Gegenstrom-verfahren***" liquefaction of oxygen (-182°C = 90 K)



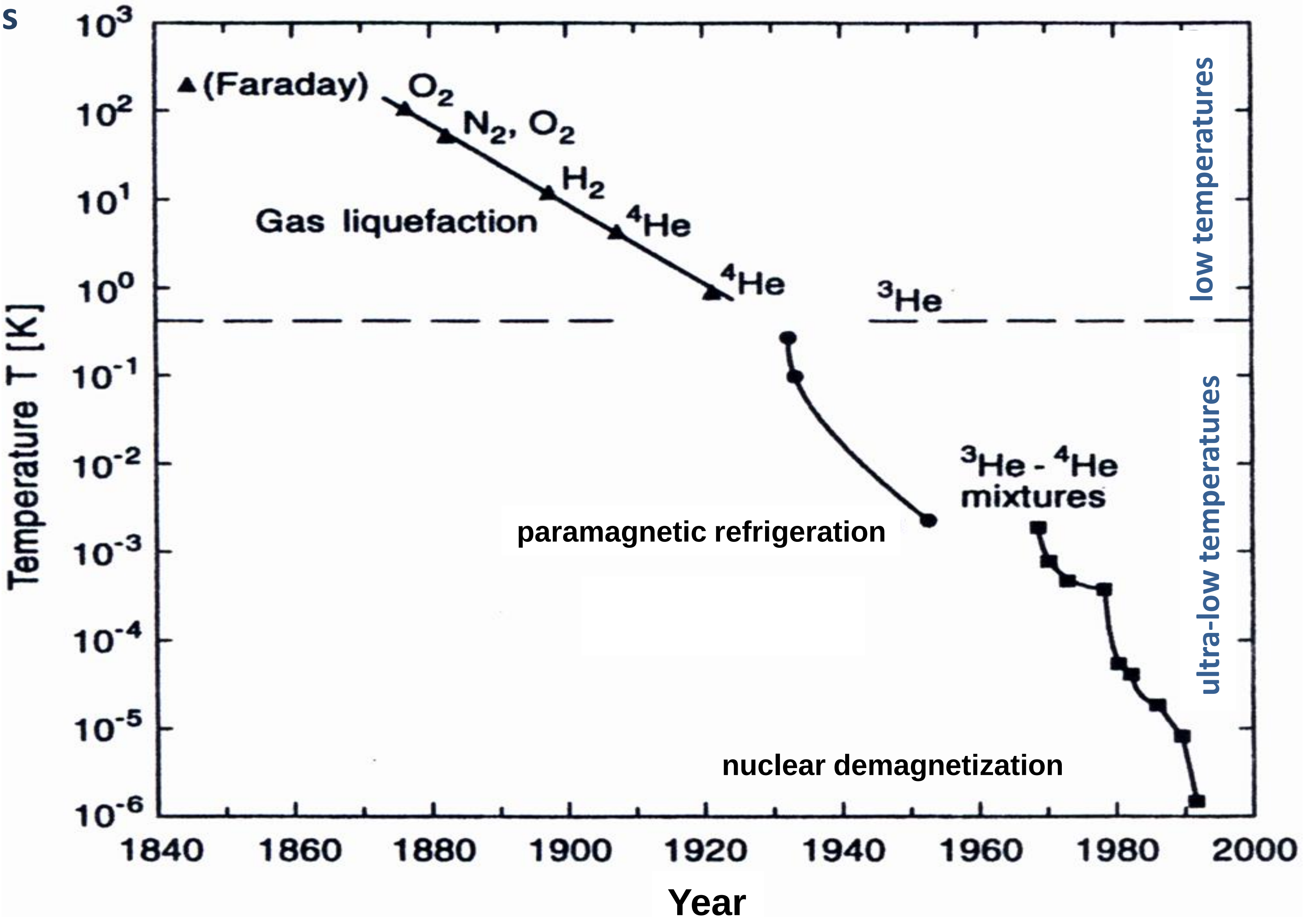


Carl Paul Gottfried von Linde

* 11. Juni 1842 in Berndorf, Oberfranken
† 16. November 1934 in Munich

III.1.1 Introduction

- **refrigeration techniques**



III.1.1 Introduction

- **refrigeration techniques**

temperature range	**refrigeration technique**	**available since**	**typical T_{min}**	**record T_{min}**
Kelvin	universe ^{4}He evaporation ^{3}He evaporation	 1908 1950	 1.3 K 0.3 K	2.73 K 0.7 K 0.25 K
millikelvin	^{3}He-^{4}He dilution Pomeranchuk cooling electron spin demagnetization	1965 1965 1934	10 mK 3 mK 3 mK	2 mK 2 mK 1 mK
microkelvin	nuclear spin demagnetization	1956	50 µK	100 pK

III.1.1 Introduction

- **cooling techniques**
  - expansion of an ideal gas
    - expansion machine
    - regenerative machine
    - ***work against outside world***
  - expansion of a real gas
    - Joule Thomson cooler
    - ***work against internal interactions***
  - evaporation of a real gas:
    - ***work against internal interactions***
  - dilution cooling (^{3}He/^{4}He)
    - ***work against ordering in momentum space***
  - adiabatic demagnetization (electronic/nuclear moments)
    - ***work against magnetic ordering***

III.1.1 Introduction

- **liquefaction of gases**

three useful methods:

1. direct liquefaction by ***isothermal compression***
2. letting the gas perform ***work against external forces*** at the expense of its internal energy

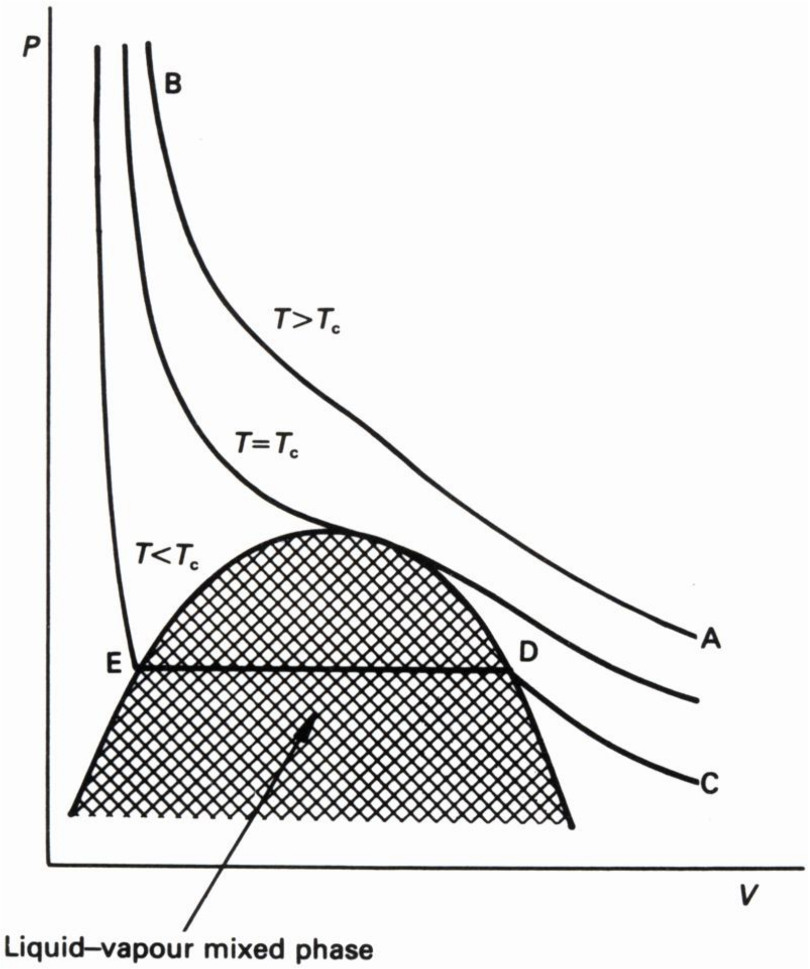
   ➔ cooling and eventual liquefaction
3. making the gas perform ***work against its own internal forces*** by Joule-Kelvin or Joule-Thomson expansion

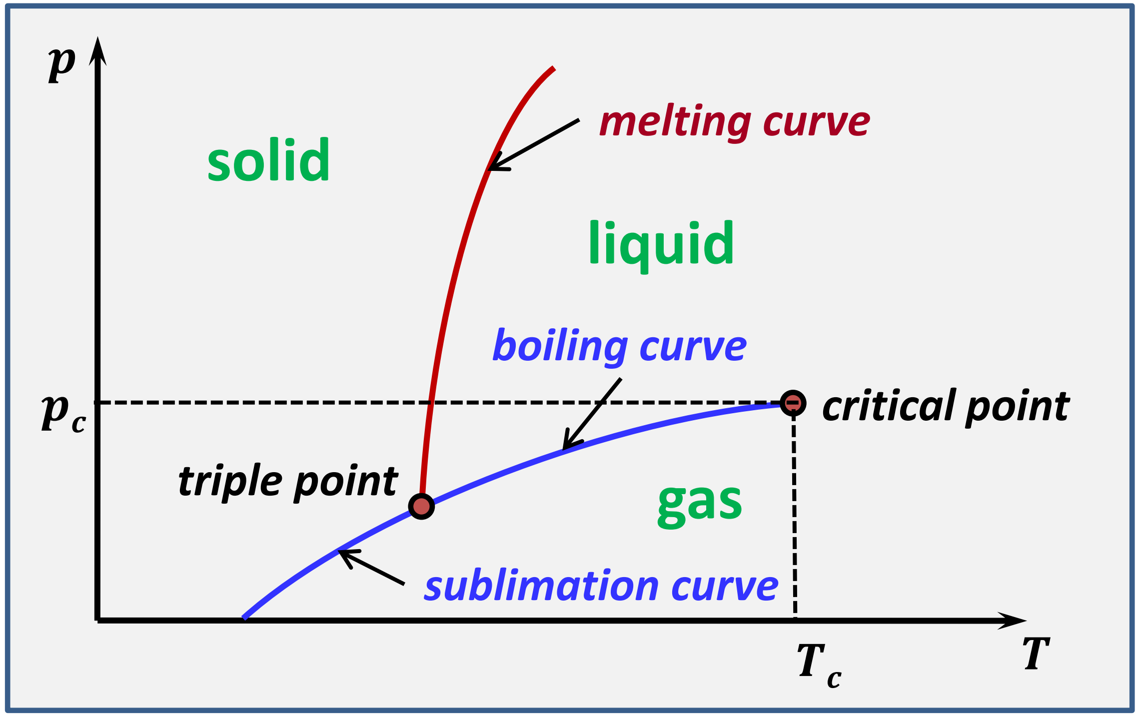
   ➔ cooling and eventual liquefaction

III.1.1 Introduction

- **direct liquefaction of gases by isothermal compression**

  → *starting temperature must be smaller than critical temperature T_c*



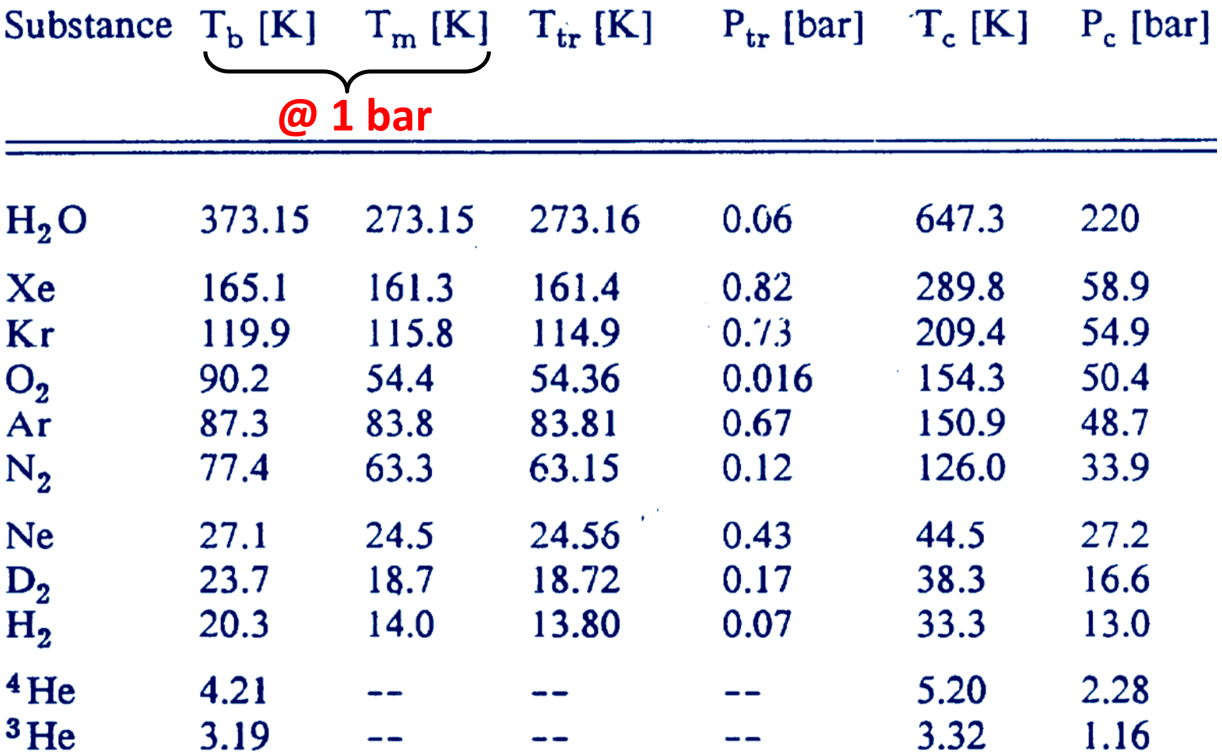


critical temperatures T_c in K of selected liquid cryogens

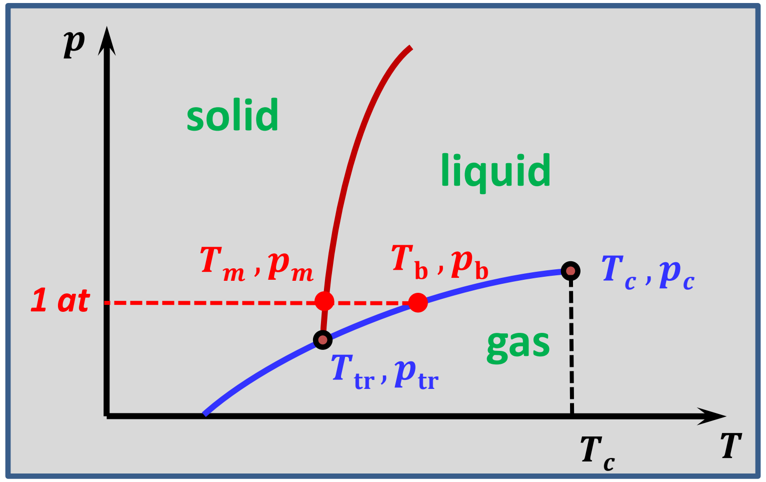
ammonia (NH_3)	406
O_2	154.5
N_2	126
H_2	33.2
^4He	5.2
^3He	3.32

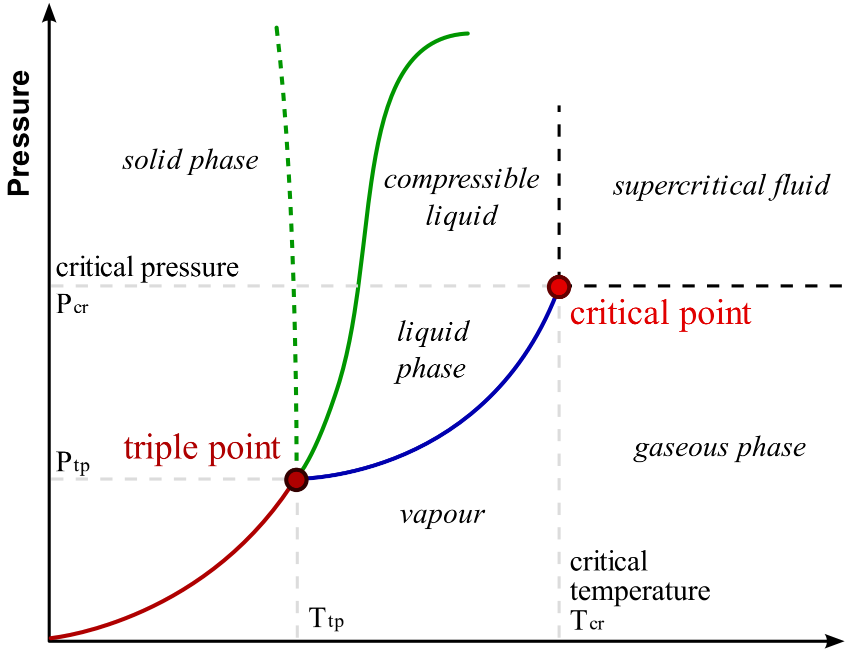
III.1.1 Introduction

- **properties of cryogenic liquids**



Substance	T_b [K]	T_m [K]	T_{tr} [K]	P_{tr} [bar]	T_c [K]	P_c [bar]
	$\underbrace{\qquad\qquad\qquad}_{\text{@ 1 bar}}$					
H_2O	373.15	273.15	273.16	0.06	647.3	220
Xe	165.1	161.3	161.4	0.82	289.8	58.9
Kr	119.9	115.8	114.9	0.73	209.4	54.9
O_2	90.2	54.4	54.36	0.016	154.3	50.4
Ar	87.3	83.8	83.81	0.67	150.9	48.7
N_2	77.4	63.3	63.15	0.12	126.0	33.9
Ne	27.1	24.5	24.56	0.43	44.5	27.2
D_2	23.7	18.7	18.72	0.17	38.3	16.6
H_2	20.3	14.0	13.80	0.07	33.3	13.0
^{4}He	4.21	--	--	--	5.20	2.28
^{3}He	3.19	--	--	--	3.32	1.16





https://en.wikipedia.org/wiki/Phase_diagram

III.1.1 Introduction

- **direct liquefaction of gases by expansion (Joule-Thomson-Effect)**

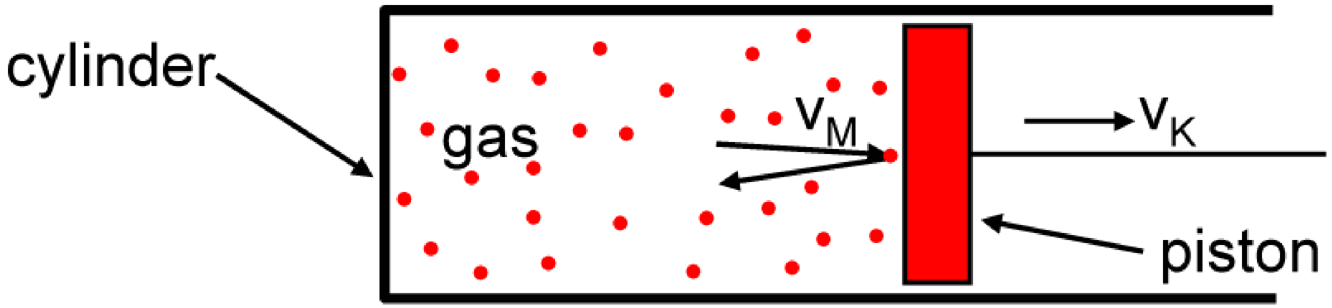
→ ***starting temperature must be smaller than inversion temperature*** (definition follows later)

cryogen	**boiling point [K]**	**liquefaction**	**latent heat [kJ/liter]**	**inversion temp. [K]**
oxygen	90.2	1877: Cailletet and Pictet	240	762
nitrogen	77.3	1883: Wroblewski and Olszewski	160	625
hydrogen	20.4	1898: Dewar	30	203
^{4}Helium	4.2	1908: Onnes	2.6	43.2
^{3}Helium	3.2		0.5	-

- **liquid oxygen and hydrogen have potential hazards**
- liquid nitrogen and ^{4}He are the most widely used cryogens
- **liquid ^{3}He is very expensive**

III.1.1 Introduction

- **liquefaction of gases by performance of external work**



gas molecules are reflected at the moving piston-surface:

incoming: laboratory system: v_M

piston system: $v_\mathrm{M} - v_\mathrm{K}$

outgoing: piston system: $-(v_\mathrm{M} - v_\mathrm{K})$

laboratory system: $-(v_\mathrm{M} - v_\mathrm{K}) + v_\mathrm{K} = 2v_\mathrm{K} - v_\mathrm{M} = v'_\mathrm{M}$

i.e.: $|v'_\mathrm{M}| = v_\mathrm{M} - 2v_\mathrm{K}$ **molecule is slower, i.e. colder**

average momentum transfer per time to piston = force, ***force · distance = work***

➔ external work at the expense of internal energy → cooling

III.1.1 Introduction

- **liquefaction of gases by performance of external work**

  – in practice,

    1. gas is **isothermally compressed** (involves removing of heat corresponding to work done on gas)
    2. and then allowed to **expand adiabatically** (resulting in cooling)

  – 1st law of thermodynamics $dU = dQ + dW = dQ - pdV = C_V dT$

$$C_V = \left.\frac{dQ}{dT}\right|_V = \left.\frac{dU}{dT}\right|_V$$

$$C_p = \left.\frac{dQ}{dT}\right|_p = \left.\frac{dH}{dT}\right|_p$$

$$H = U + pV$$

  - ➢ adiabatic expansion: $dQ = 0$ $\Rightarrow\ dQ = C_V dT + pdV = 0$
  - ➢ for ideal gas: $pV = RT$ $\Rightarrow\ pdV + Vdp = RdT$
  - ➢ elimination of dT

$$\Rightarrow\ 0 = C_V\left(\frac{pdV + Vdp}{R}\right) + pdV \underbrace{=}_{C_p - C_V = R} C_V(pdV + Vdp) + pdV\,(C_p - C_V)$$

$$\Rightarrow\ 0 = C_V\,Vdp + C_p\ pdV = C_V\frac{dp}{p} + C_p\frac{dV}{V} \underbrace{=}_{\frac{C_p}{C_V} = \kappa} \kappa\frac{dV}{V} + \frac{dp}{p}$$

  - ➢ integration yields $\kappa \ln V + \ln p = const.$ $\Rightarrow pV^{\kappa} = const.$

$\kappa =$ isentropic coefficient

III.1.1 Introduction

- ***Carnot process:***

  - counterclockwise: *heat pump* *(conversion of mechanical work into heat)*
  - clockwise: *heat engine* *(conversion of heat into mechanical work)*

- $\boldsymbol{pV}$ **diagram (*heat engine*):**

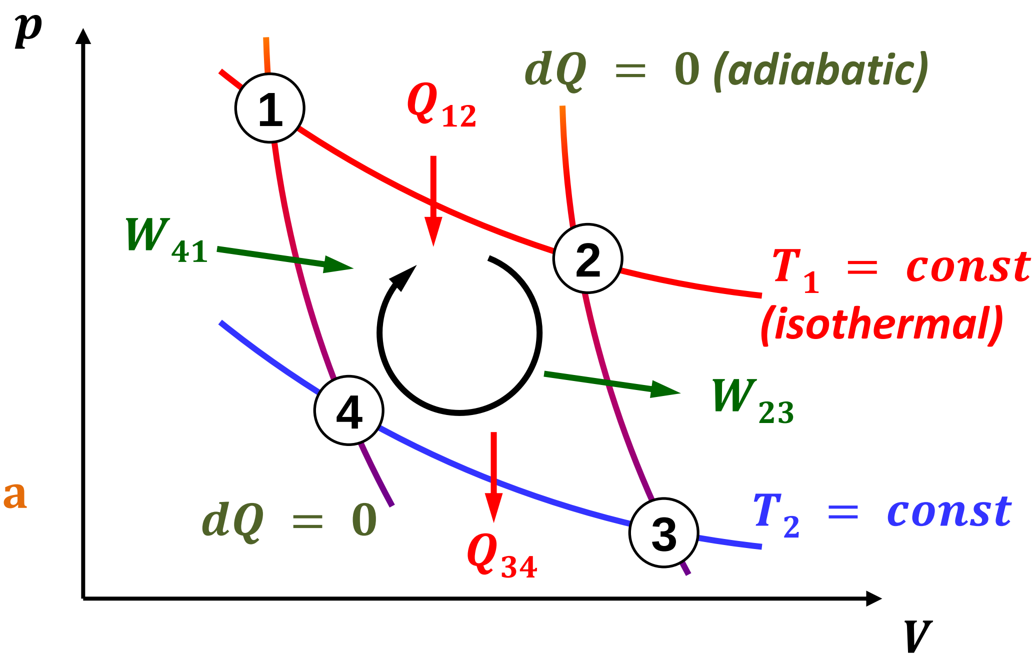
  expansion cooling: adiabats

  $(pV^\kappa = const, \quad \boldsymbol{dQ = 0} \qquad \kappa = \frac{c_p}{c_V} > 1)$

  heat exchange: isotherms

  $(pV = const, \quad \boldsymbol{dT = 0})$

  work per cycle: $W = -\oint p dV =$ **enclosed area**
  (negative since work done by the heat engine)



for complete cycle: $W + Q = 0 \quad \Rightarrow \quad W = -Q = -(Q_{34} + Q_{12}) = Q_{34} - Q_{12}$

$(Q_{12} > 0, Q_{34} < 0)$

III.1.1 Introduction

- counterclockwise: *heat pump (conversion of mechanical work into heat)*
- clockwise: *heat engine (conversion of heat into mechanical work)*

- **efficiency (*heat engine*):**

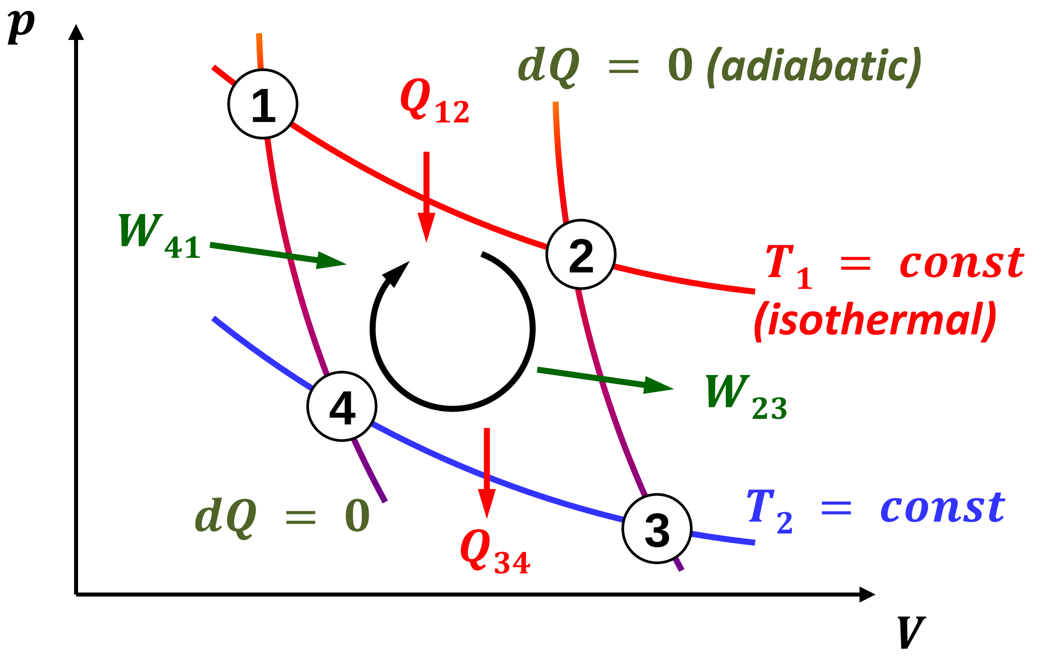
$$\eta \equiv \frac{|W|}{Q_{12}} = \frac{Q_{12}+Q_{34}}{Q_{12}} = 1 + \frac{Q_{34}}{Q_{12}} = 1 - \frac{|Q_{34}|}{Q_{12}}$$

for Carnot process: $\Delta S_{12} + \Delta S_{34} = 0$

$(\Delta S_{12} > 0, \Delta S_{34} < 0)$

$$Q_{12} = T_1 \Delta S_{12}, \qquad Q_{34} = T_2 \Delta S_{34}$$

$$\Rightarrow \frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_2} = 0 \quad \Rightarrow \frac{Q_{34}}{Q_{12}} = -\frac{T_2}{T_1}$$



$$\eta \equiv \frac{|W|}{Q_{12}} = 1 + \frac{Q_{34}}{Q_{12}} = 1 - \frac{T_2}{T_1} = \frac{\Delta T}{T_{\mathrm{warm}}} \qquad \text{(Carnot)}$$

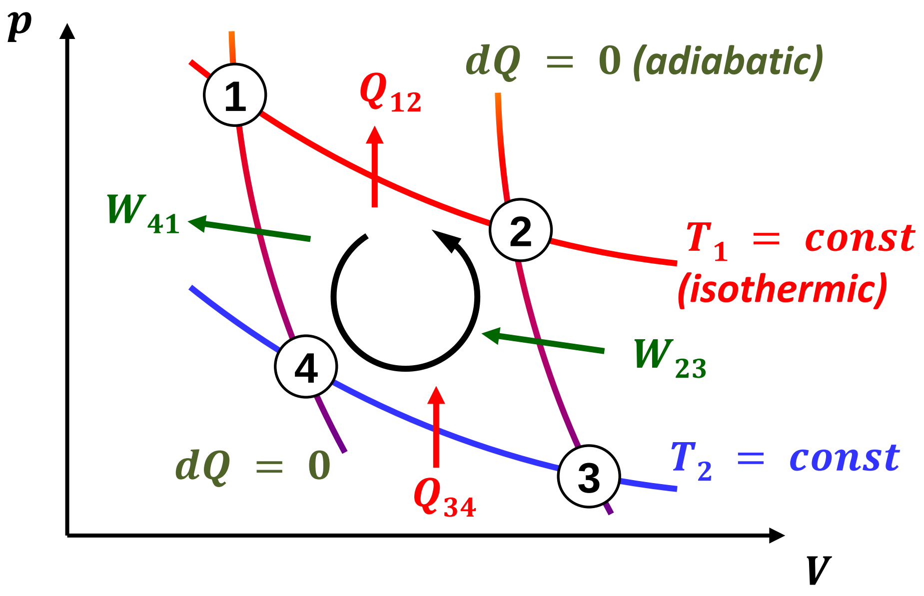
thermodynamic definition of temperature

- **Carnot process**: *technologically difficult* to realize

➔ better: ***gas circulation, compressor and expansion machine are spatially separated***

III.1.2 Expansion Machine

- **heat pumps (counterclockwise): heating and refrigerating machines**



- **heat pump:** ***heat is generated by mechanical work***
  - ***efficiency:***

$$h = \frac{\text{generated heat @ } T = T_1}{\text{performed work}} = \frac{Q_{12}}{|W|}$$

- **refrigerating machine:** ***removing heat (generating cold) by mechanical work***
  - ***efficiency:***

$$k = \frac{\text{removed heat @ } T = T_2}{\text{performed work}} = \frac{Q_{34}}{|W|}$$

- **ideal efficiency for reversible Carnot process:**

$$h_c = \frac{1}{\eta_c} = \frac{T_1}{T_1 - T_2} \geq 1$$

(increases with decreasing temperature difference $T_1 - T_2$)

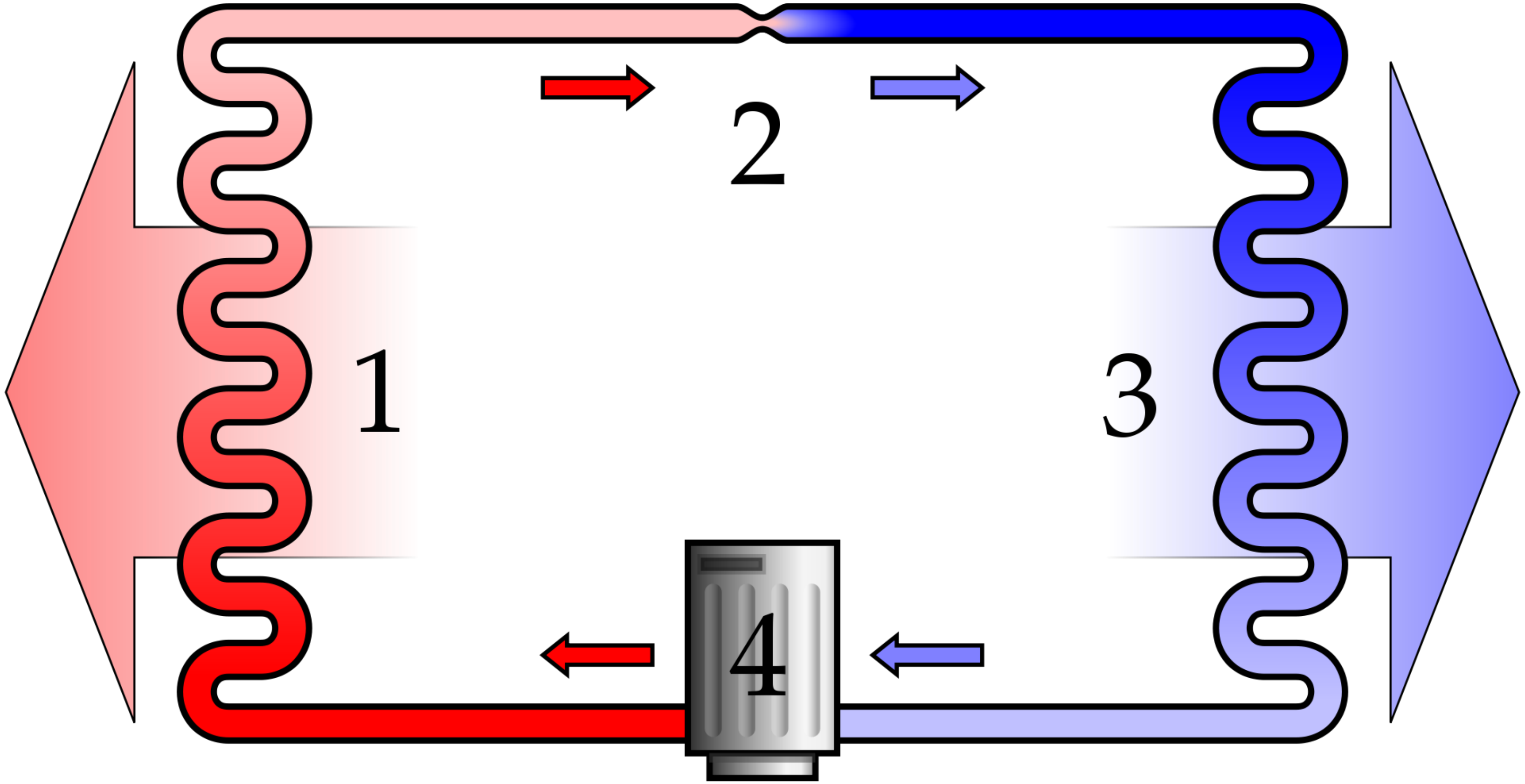
$$k_c = \frac{T_2}{T_1 - T_2} \geq 0$$

(decreases with increasing temperature difference $T_1 - T_2$)

III.1.2 Expansion Machine

- **application examples: heating and refrigerating machines**

e.g. heating of
swimming pool



e.g. air conditioning

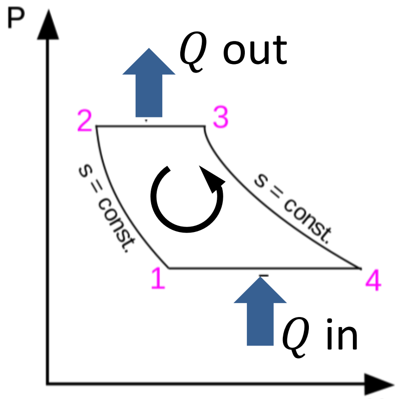
Wikimedia Commons

schematic diagram of a heat pump's vapor-compression refrigeration cycle:
1) condenser, 2) expansion valve, 3) evaporator, 4) compressor

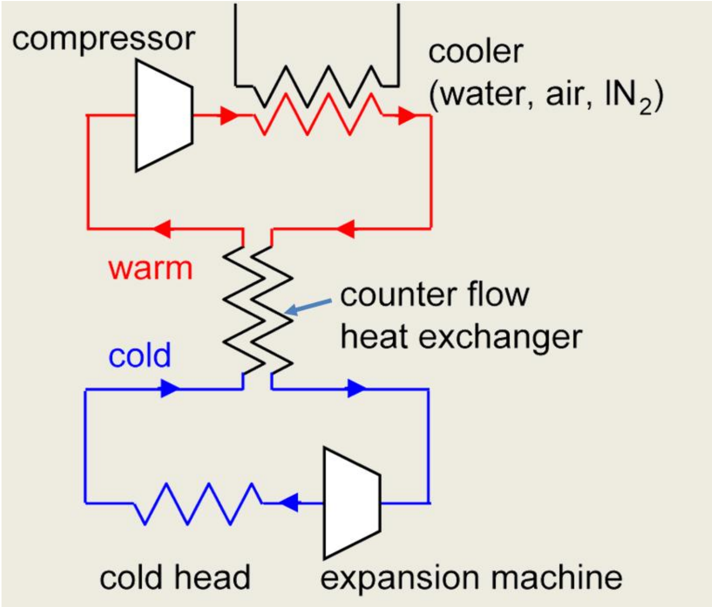
R. Gross © Walther-Meißner-Institut (2004 - 2024)

III.1.2 Expansion Machine

Brayton method ***(reverse Brayton cycle, isentropic compression and expansion)***



P-v Diagram



heat pump (conversion of mechanical work into heat, counter-clockwise circle)

$$C_V = \left.\frac{\mathrm{d}Q}{\mathrm{d}T}\right|_V = \left.\frac{\mathrm{d}U}{\mathrm{d}T}\right|_V$$
$$C_p = \left.\frac{\mathrm{d}Q}{\mathrm{d}T}\right|_p = \left.\frac{\mathrm{d}H}{\mathrm{d}T}\right|_p$$
$$H = U + pV$$

- entropy of a gas

we use: $\mathrm{d}S = \frac{\mathrm{d}Q}{T}$ and $pV = RT \;\Rightarrow\; V = \frac{RT}{p}$

$$\mathrm{d}Q = \mathrm{d}H - V\mathrm{d}p = C_p\mathrm{d}T - \frac{RT}{p}\mathrm{d}p$$

$$\mathrm{d}S = \frac{\mathrm{d}Q}{T} = \frac{C_p\mathrm{d}T - \frac{RT}{p}\mathrm{d}p}{T} = C_p\frac{\mathrm{d}T}{T} - R\frac{\mathrm{d}p}{p} \;\underbrace{\Rightarrow}_{\text{integration}}\; S_2 - S_1 = C_p\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

- compression and expansion are reversible: $S_2 - S_1 = 0$

$$0 = C_p\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = \ln\frac{T_2}{T_1} - \frac{R}{C_p}\ln\frac{p_2}{p_1} \;\Rightarrow\; \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{R/C_p}$$

we use: $C_p - C_V = R$ and $\frac{C_p}{C_V} = \kappa \;\Rightarrow$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-1/\kappa} = \left(\frac{p_2}{p_1}\right)^{(\kappa-1)/\kappa}$$

III.1.2 Expansion Machine

- medium: He gas

  Brayton method *(reverse Brayton cycle, isentropic compression and expansion)*

- e.g. **liquefaction of air:**
  - condensation on cold head
  - distillation in separation columns

➢ N_2	(77.4 K)	cooling
➢ Ar	(87.3 K)	inert gas
➢ O_2	(90.2 K)	welding

- **temperature reduction:**

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(\kappa-1)/\kappa}$$

$\kappa = C_p/C_V$ (= 5/3 for He)

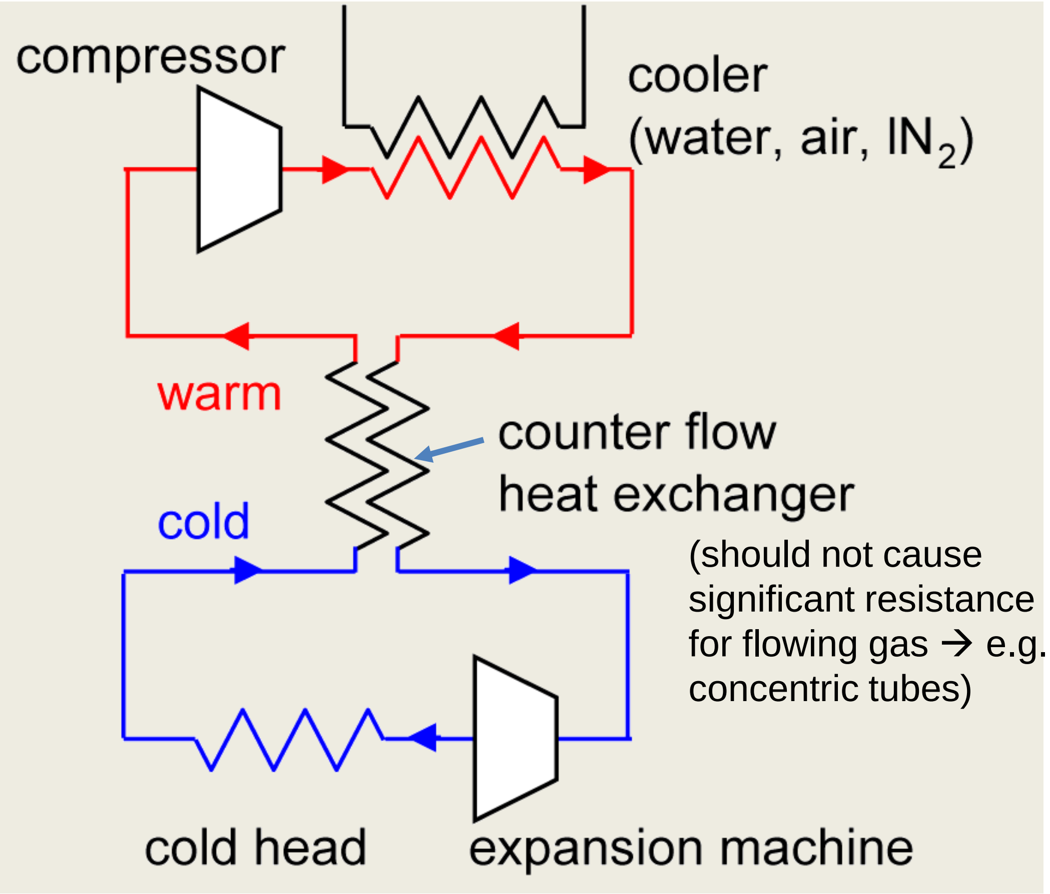
ideal, single-atomic gas: $\kappa = 1 + \left(\frac{2}{f}\right)$

- **efficiency:**

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \left(\frac{p_2}{p_1}\right)^{(\kappa-1)/\kappa}$$

➔ expansion from $p_1 = 100$ bar to $p_2 = 1$ bar results in ***T_2 = 50 K*** @ $T_1 = 300$ K

$T_2 = 8$ K can be reached in a two-stage cycle



(should not cause significant resistance for flowing gas → e.g. concentric tubes)

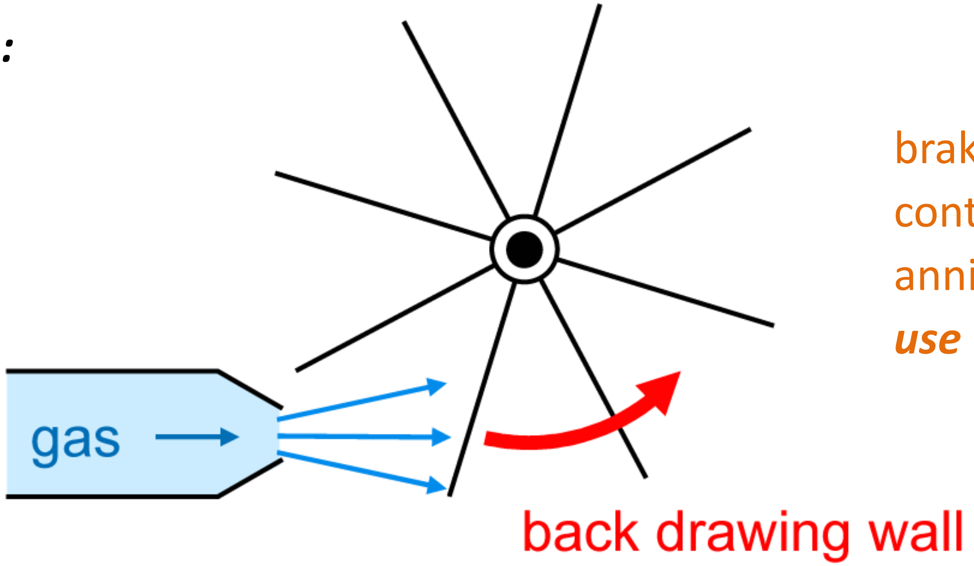
III.1.2 Expansion Machine

- **realizations of expansion machines:**

  - piston-cylinder machine similar to automobile engine
    crankshaft, camshaft, valve

  - cooling turbine $\Rightarrow$ ***commercially relevant***
    higher efficiency for larger throughput

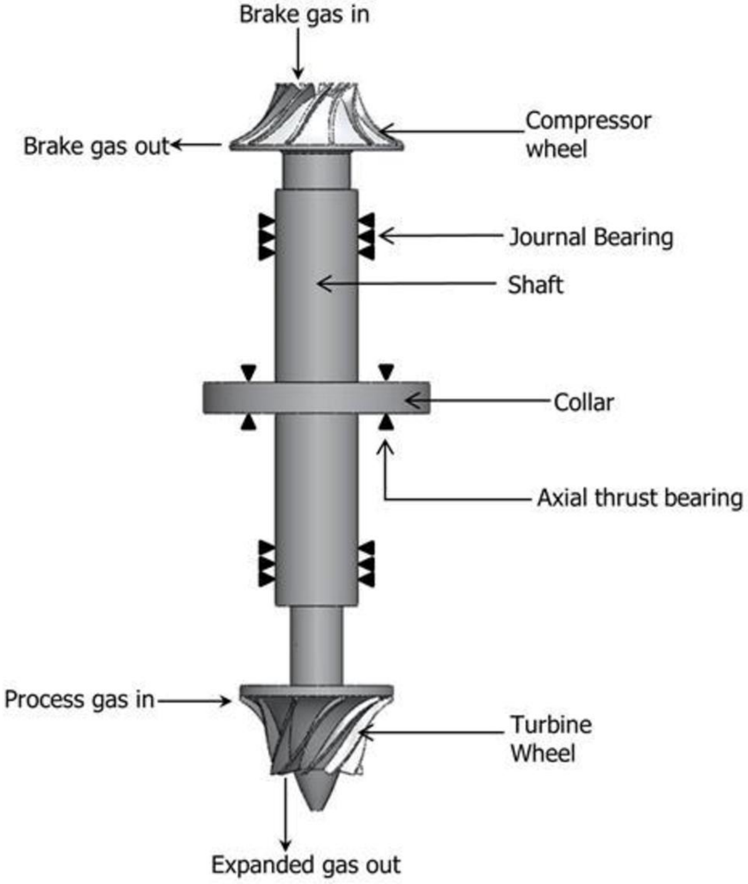
principle:

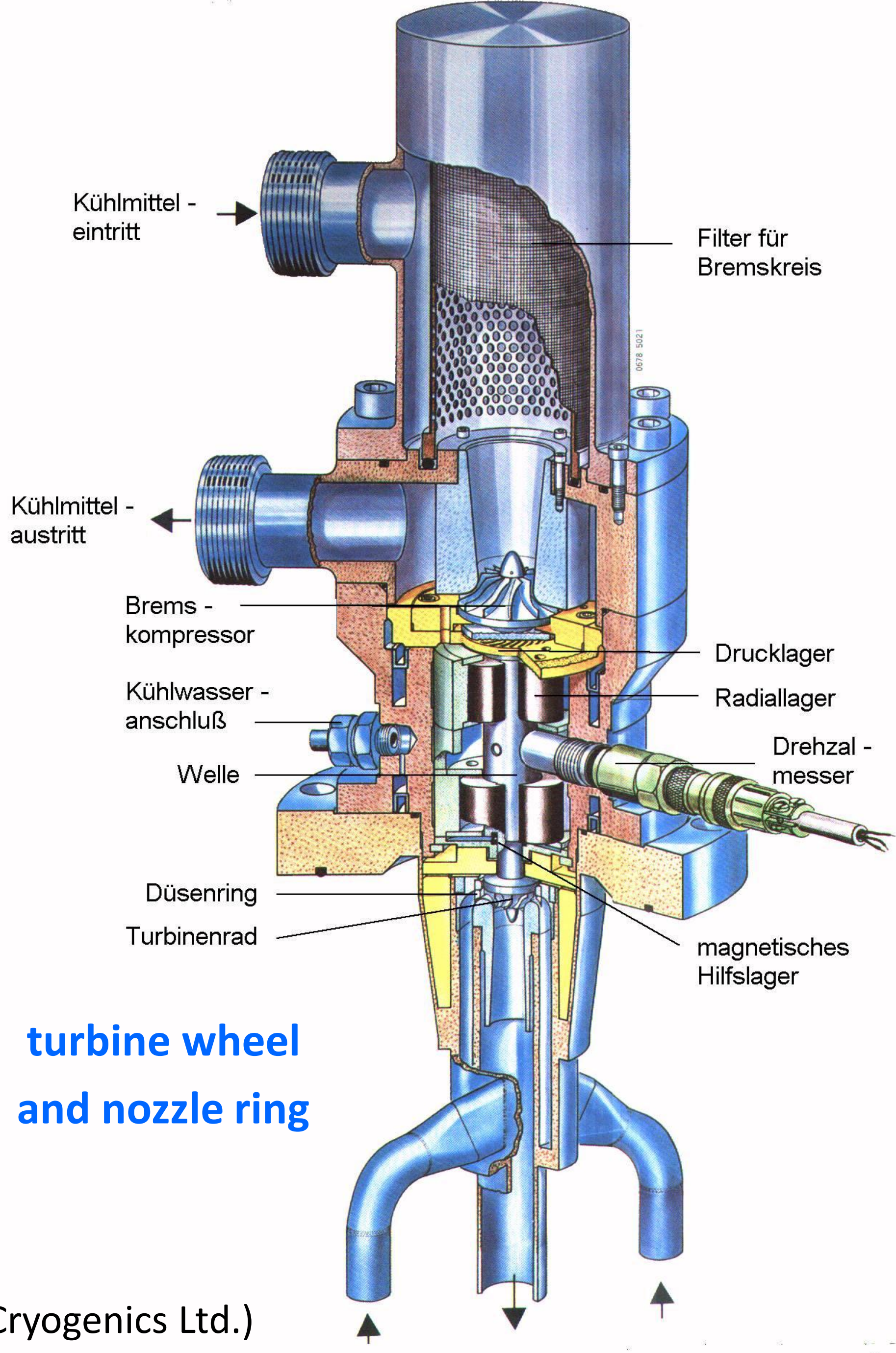


brake on turbine axis
controls rotational speed,
annihilates performed work,
use of gas bearings

III.1.2 Expansion Machine

- **turbine cooler (Sulzer machine)**





turbine wheel and nozzle ring

(Source: Linde Cryogenics Ltd.)

III.1.2 Expansion Machine

- **conclusions**

  - expansion machines are technologically simple
  - multi-stage arrangements for lower temperatures

    ⇒ ***almost down to 4.2 K***

  - but:

    ⇒ ***efficiency*** only acceptable for cooling turbines

  - no direct liquefaction of gas (mechanical problems)

    ⇒ ***liquefaction by Joule-Thomson stage***

  - for small-scale facilities:

    ⇒ ***regenerative machines*** better suited

Contents Part II: Quantum Transport in Nanostructures

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III.1 Generation of Low Temperatures

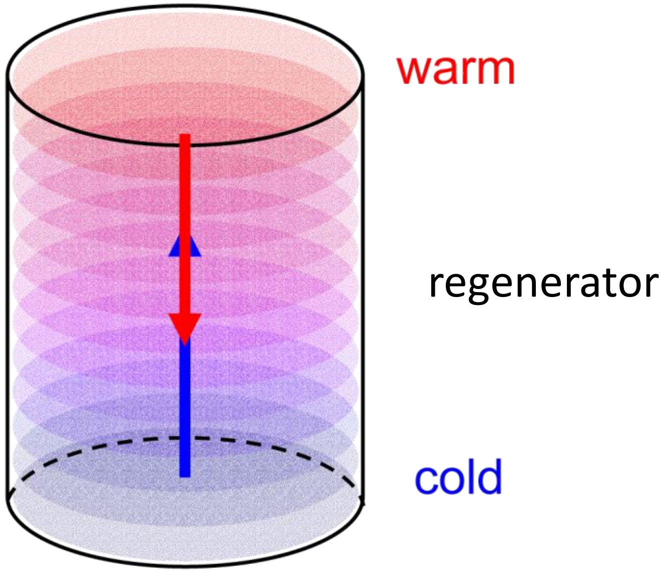
III.2 Thermometry

III.1.3 Regenerative Machines

- ***regenerator** replaces **heat exchanger***

  $\Rightarrow$ column with staple of fine metal meshes (Cu, Pb)

  → *low flow resistance*
  → *high heat capacity*
  → *low longitudinal heat conductivity*

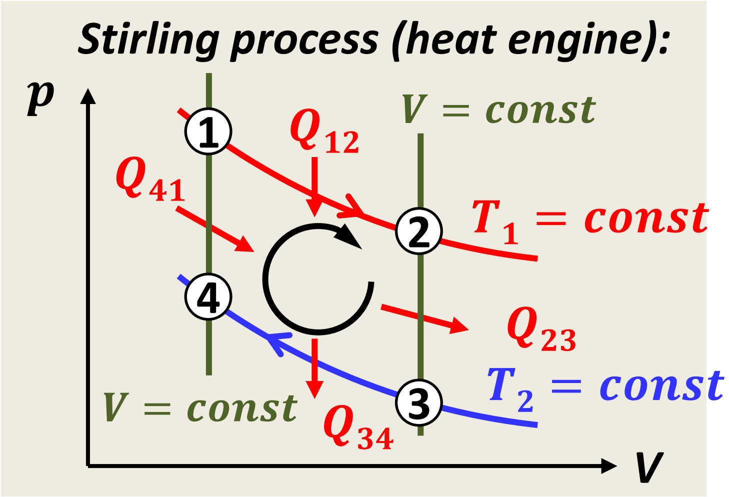


- alternating gas flow:

  cold gas upward $\Rightarrow$ ***cooling of meshes***
  warm gas downward $\Rightarrow$ ***cooling of gas***

- used in ***Stirling process***

Stirling process (heat engine):



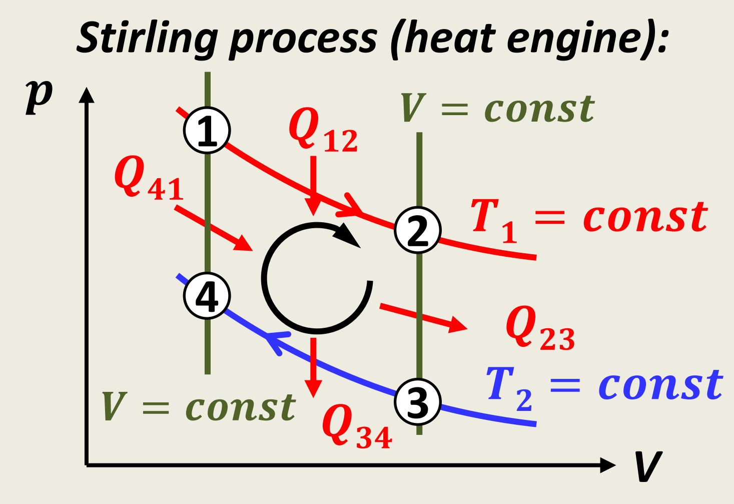
conversion of heat into mechanical work

$Q_{41} = Q_{23} = 0$ in ideal Carnot cycle (adiabatic process: $dQ = 0$)

heat removed in step 2 →3 has to be stored and provided in step 4 → 1

III.1.3 Regenerative Machines

- **Stirling machine**
  (clockwise, heat engine)



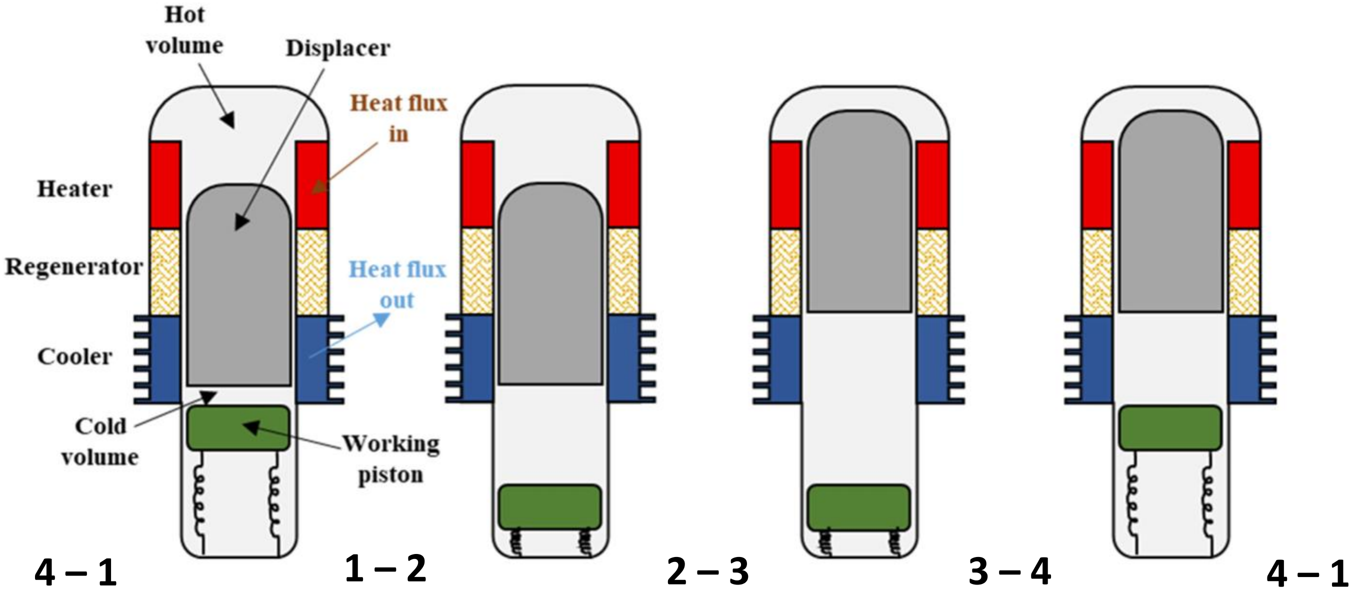
periodic expansion and compression of gas along two isotherms and two isochors

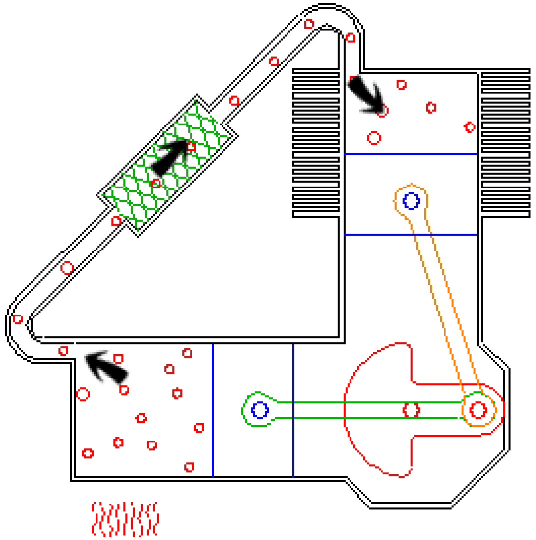
- *1→2: isothermal expansion, Q_{12} is added*
- *2→3: isochoric cooling, Q_{23} is removed*
- *3→4: isothermal compression, Q_{34} is removed*
- *4→1: isochoric warming, Q_{41} is added*

- *for isochoric steps there is no mechanical work*

$$Q_{23} = -Q_{41} = C_V \Delta T$$

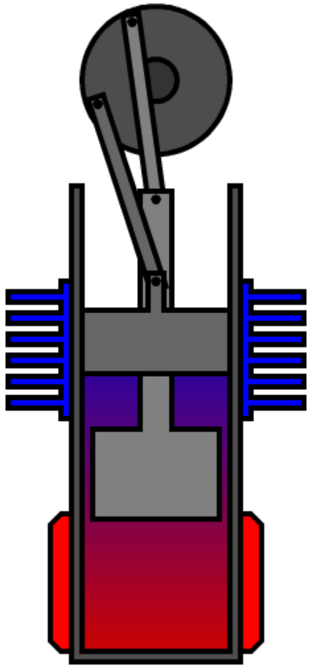
- *goal: intermediate storage of Q_{23} in regenerator to be able to add it again in step 4→1*
  → *use of two pistons with phase shift*



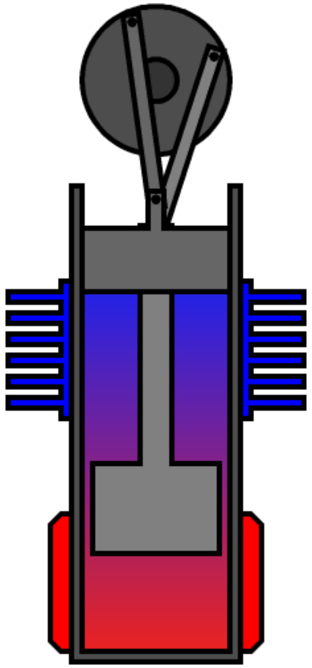


III.1.3 Regenerative Machines

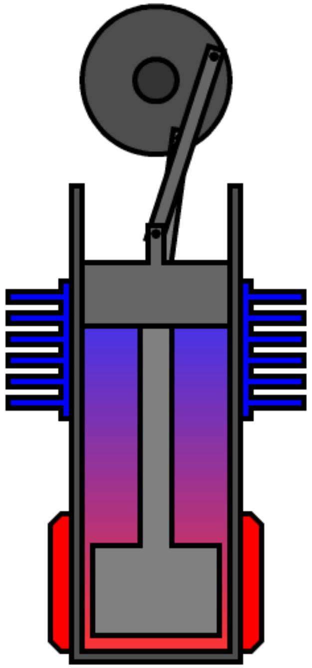
- **(beta) Stirling machine**



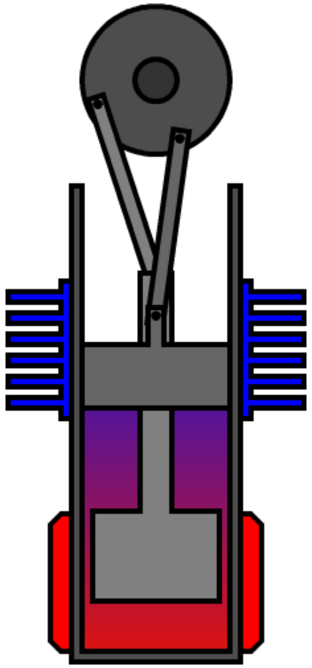
power piston (dark grey) has compressed the gas, the displacer piston (light grey) has moved so that most of the gas is adjacent to the hot heat exchanger



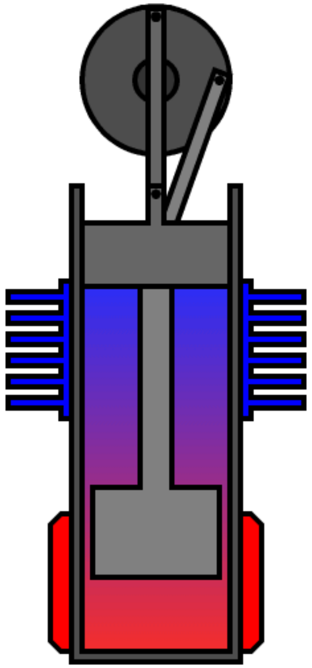
heated gas increases in pressure and pushes the power piston to the farthest limit of the power stroke.



displacer piston now moves, shunting the gas to the cold end of the cylinder.

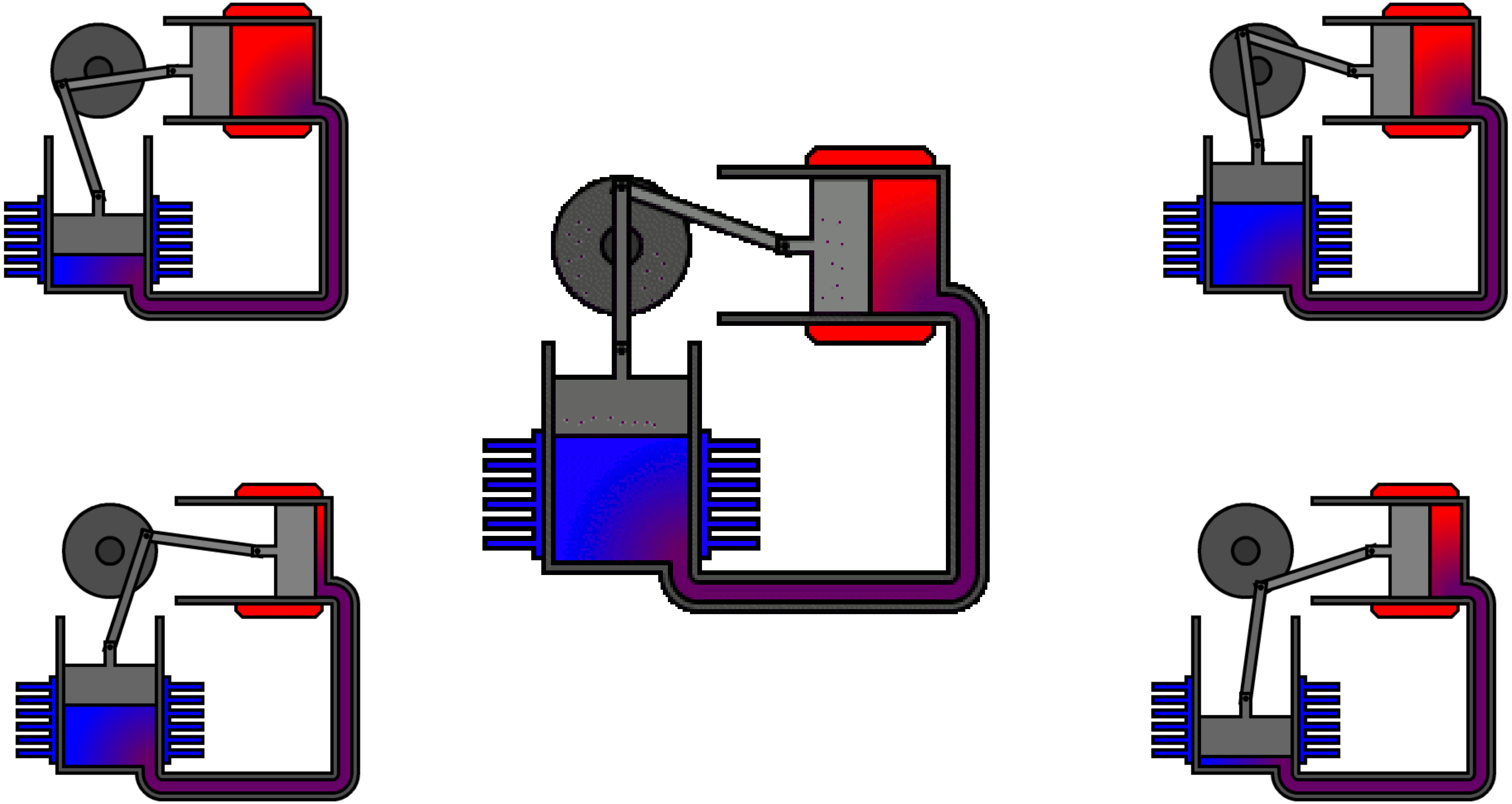


cooled gas is now compressed by the flywheel momentum. This takes less energy, since when it is cooled its pressure dropped.



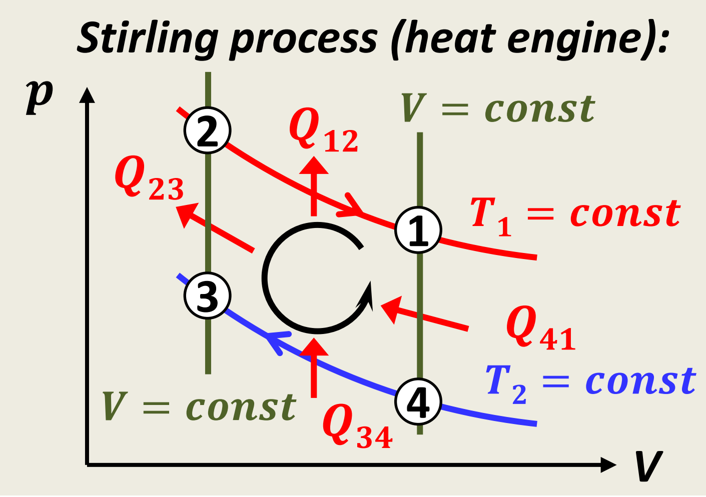
III.1.3 Regenerative Machines

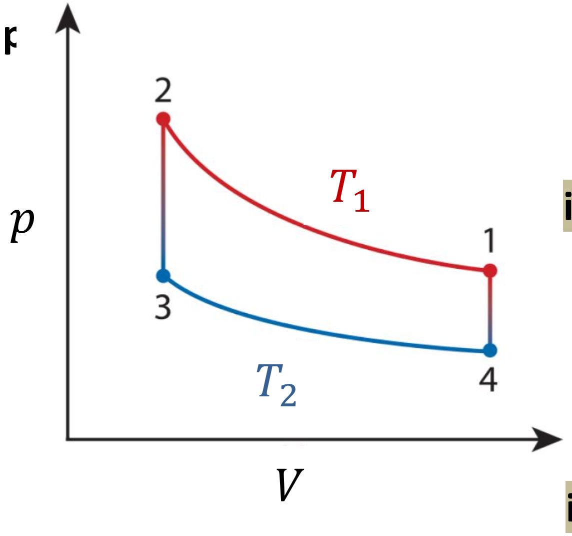
- **(alpha) Stirling machine**

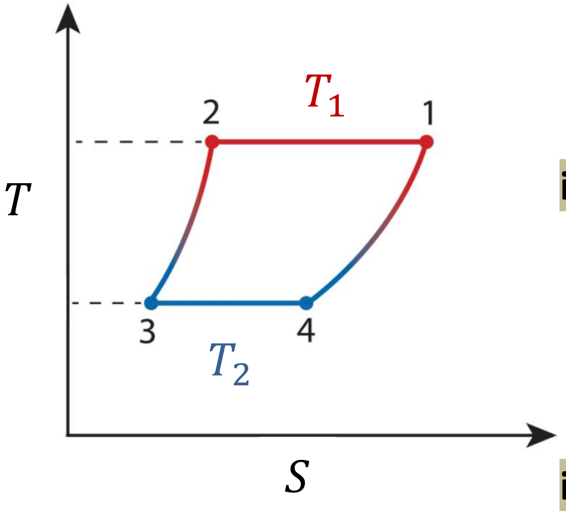


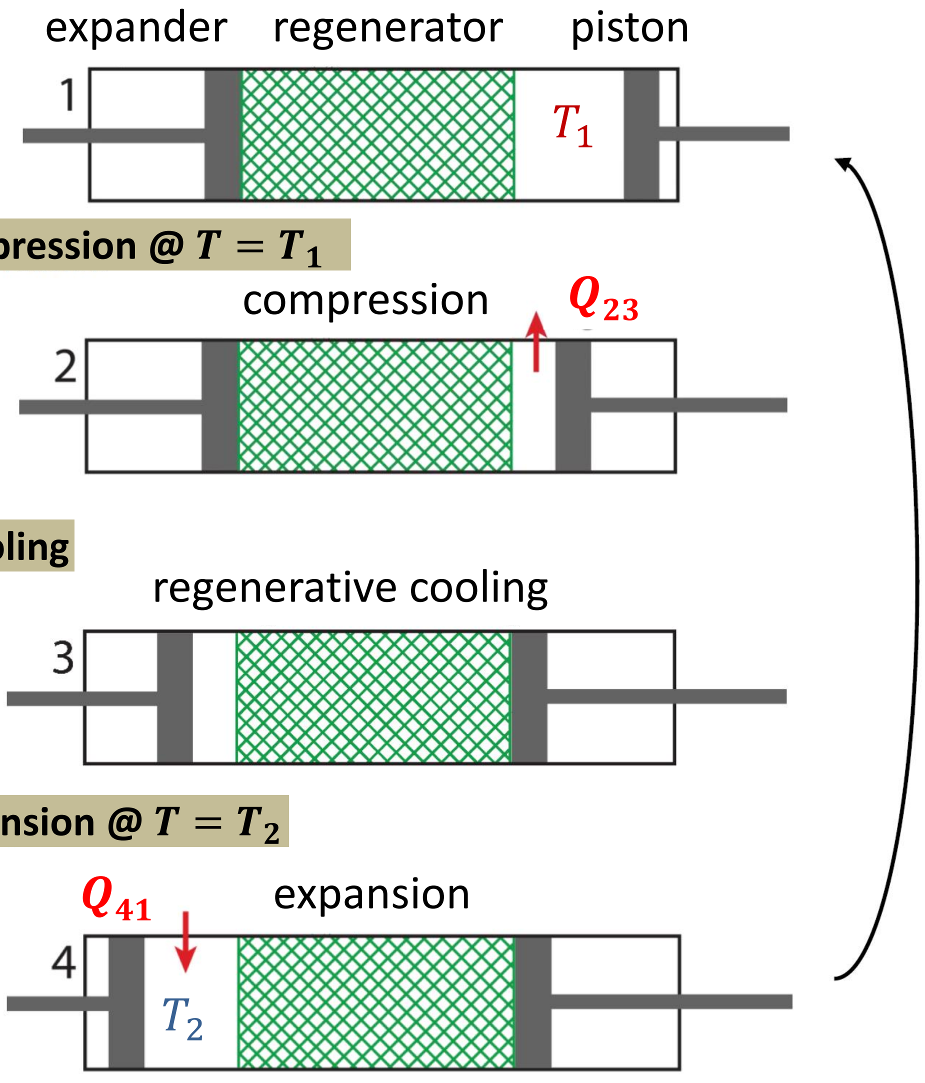
III.1.3 Regenerative Machines

- **Stirling machine**
  (counter-clockwise, heat pump









isothermal compression @ $T = T_1$

isochoric precooling

isothermal expansion @ $T = T_2$

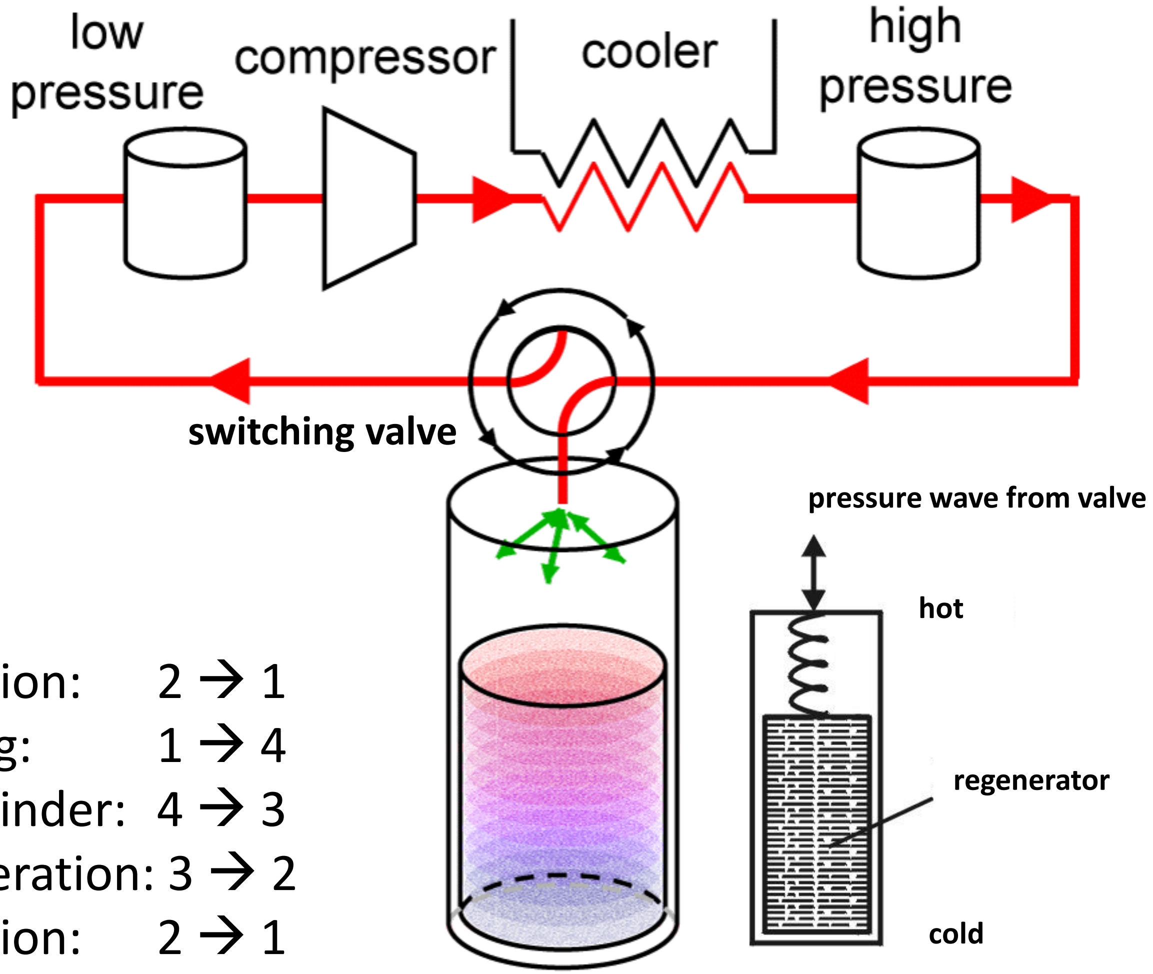
isochoric reheating

III.1.3 Regenerative Machines

- **Gifford-McMahon machine**

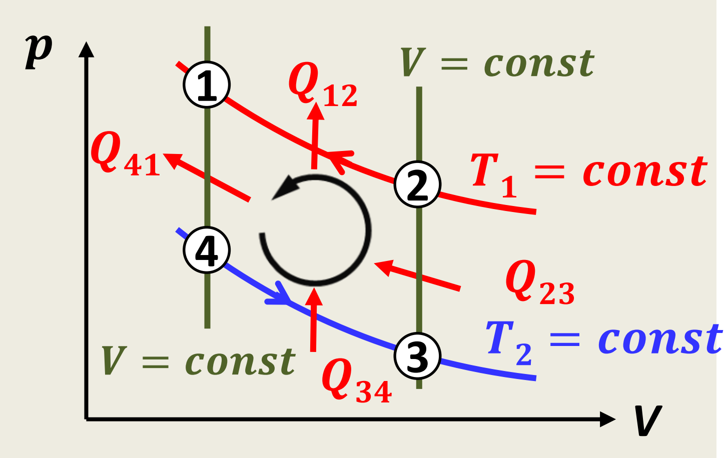
uses compressor with switching valve instead of piston

(counter-clockwise, heat pump)



heat pump

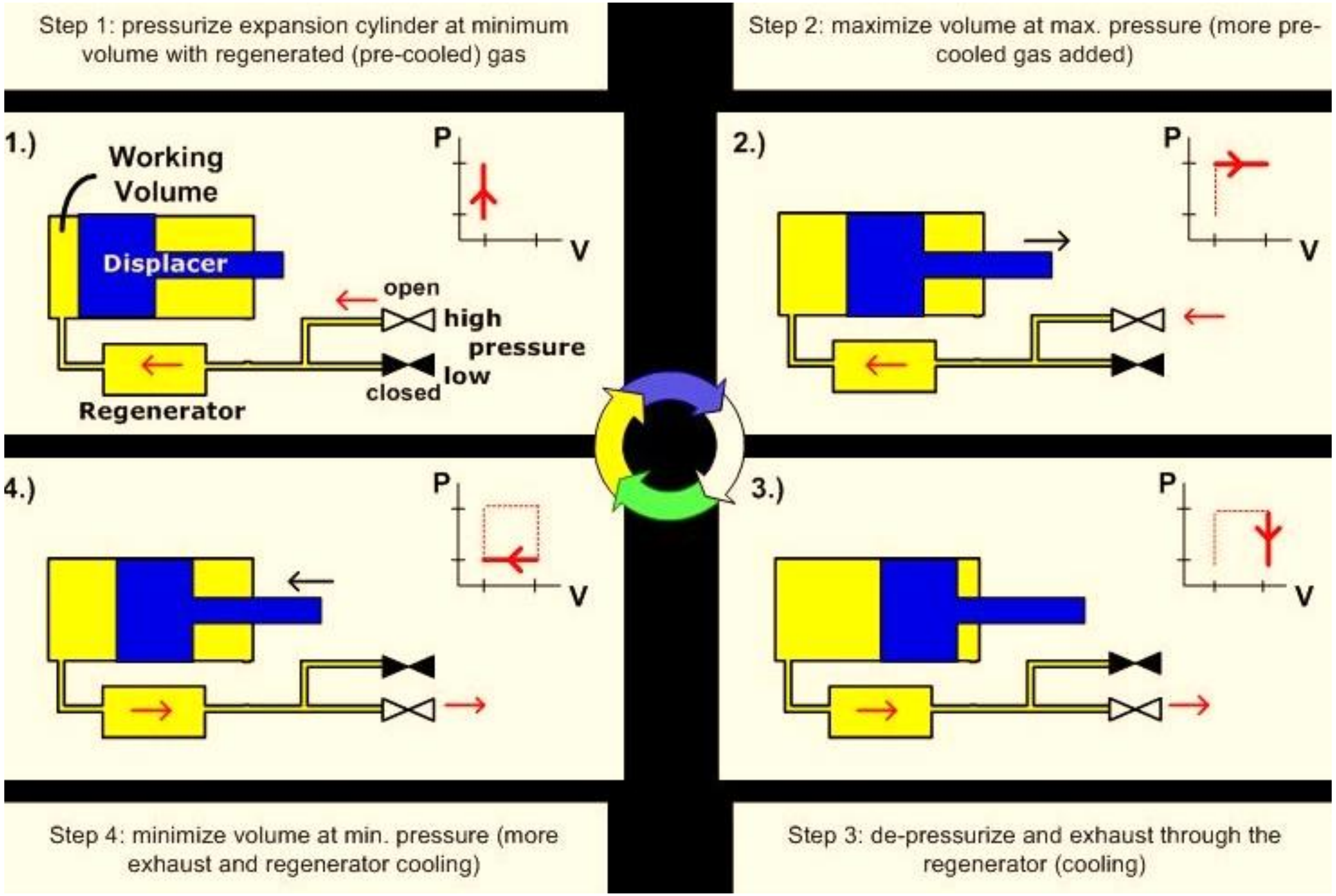
(conversion of mechanical work into heat)



1. warm compression: 2 → 1
2. isochoric cooling: 1 → 4
3. expansion in cylinder: 4 → 3
4. isochoric regeneration: 3 → 2
5. warm compression: 2 → 1

III.1.3 Regenerative Machines

- **Gifford-McMahon cycle**



Step 1: pressurize expansion cylinder at minimum volume with regenerated (pre-cooled) gas

Step 2: maximize volume at max. pressure (more pre-cooled gas added)

Step 3: de-pressurize and exhaust through the regenerator (cooling)

Step 4: minimize volume at min. pressure (more exhaust and regenerator cooling)

cryopump

two-stage cryocooler

III.1.3 Regenerative Machines

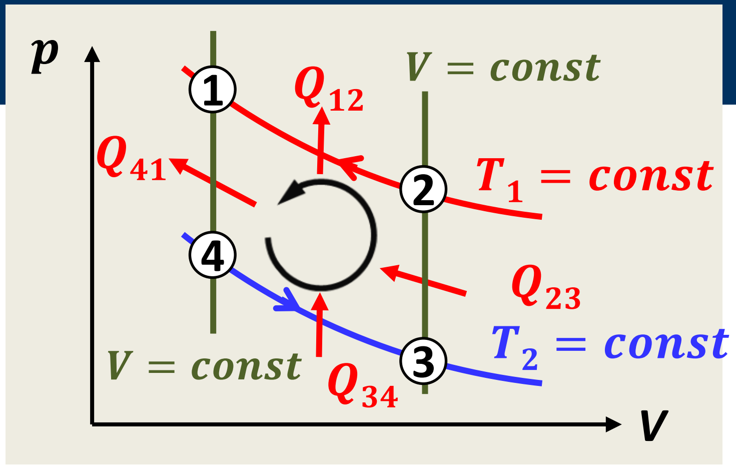
- **Pulse – Tube – Refrigerator**

  - the **pulse tube refrigerator** (PTR) or **pulse tube cryocooler** is based on the operation principle of Stirling coolers
  - PTR is made without moving parts in the low temperature part (in contrast with other cryocoolers, e.g. Stirling cryocooler and Gifford-McMahon cooler)
  - compact design possible → suitable for a wide variety of applications
  - minimum temperature about 2.5 K (with ^{4}He) and 1.3 K (with ^{3}He)

applications:

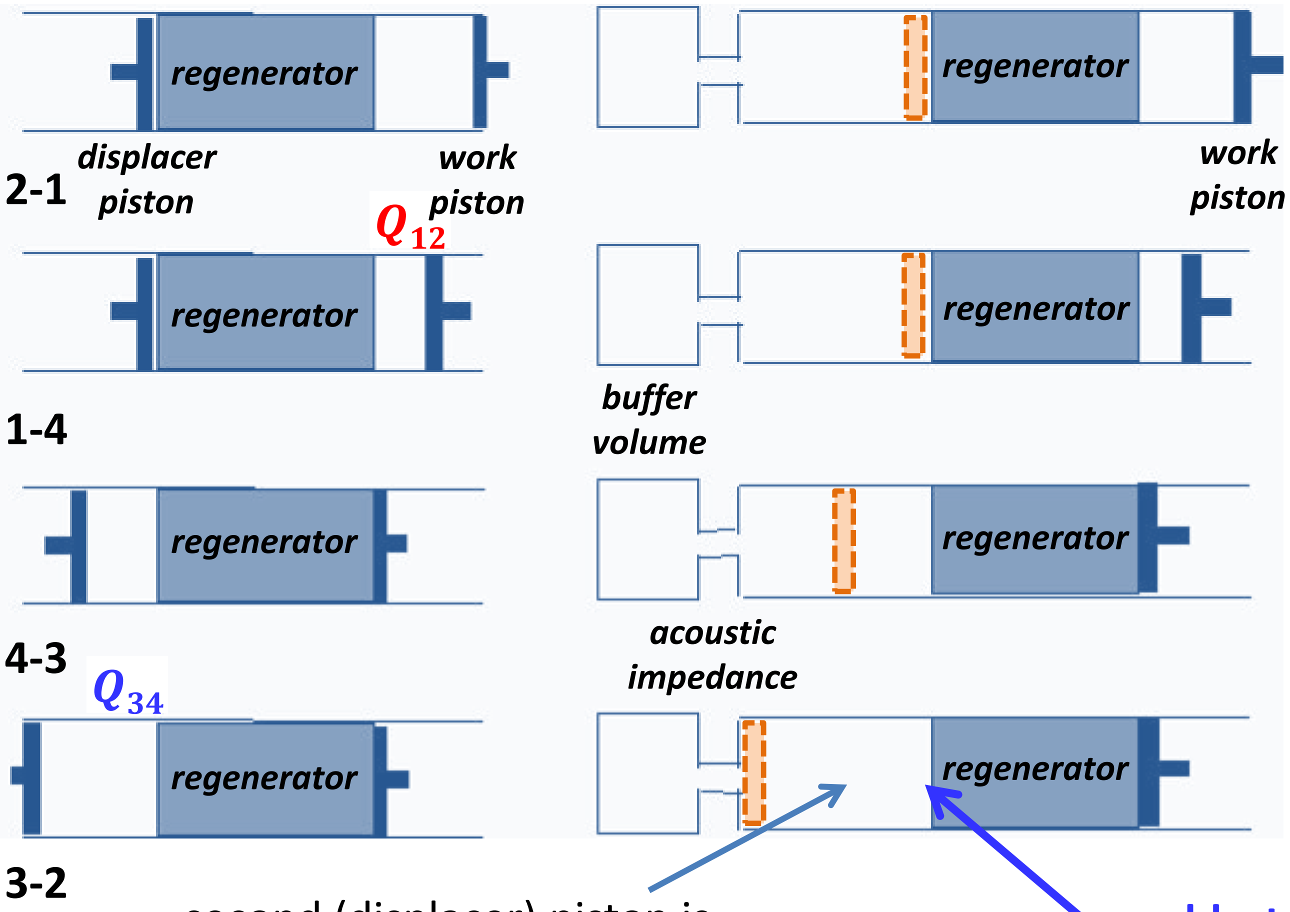
- industrial applications such as semiconductor fabrication (e.g. cryopumps)
- cooling of infrared sensors
- cooling of astronomical detectors (e.g. Atacama Cosmology Telescope or the QUBIC experiment – an interferometer for cosmology studies)
- **precoolers of dilution refrigerators**
  Kurt Uhlig (WMI), “Dry” dilution refrigerator with pulse-tube precooling, Cryogenics **44**, (2004), pp. 53–57
- suggested to be used to liquefy oxygen on Mars

III.1.3 Regenerative Machines



(counterclockwise, heat pump)

Stirling Cooler *Pulse – Tube – Refrigerator*



motion of gas volume element equivalent to motion of „displacer piston“

90° phase shift between motion of „displacer piston“ and „work piston“ realized by buffer volume

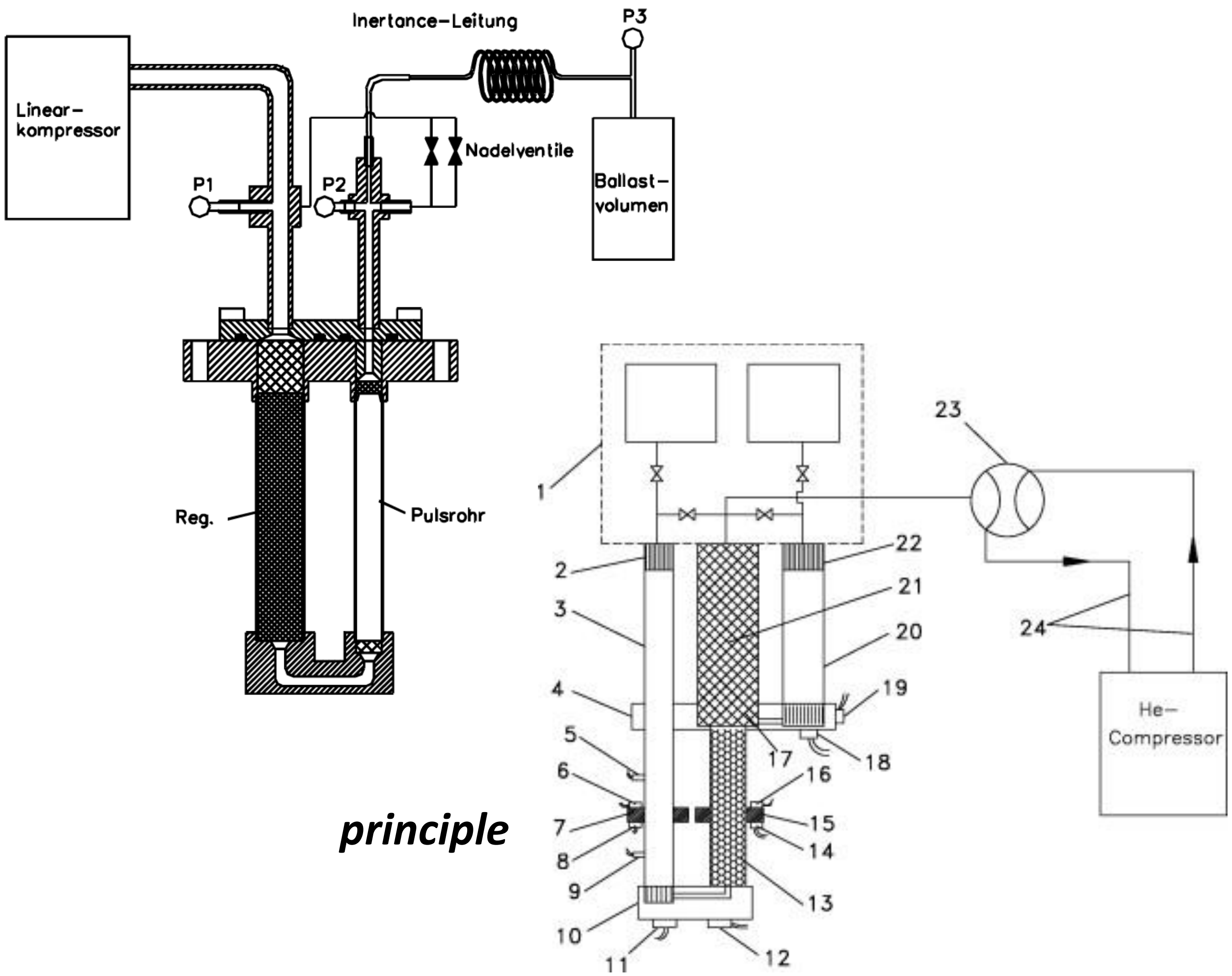
90° phase shift required for finite heat transport

second (displacer) piston is replaced by pulse tube (gas piston)

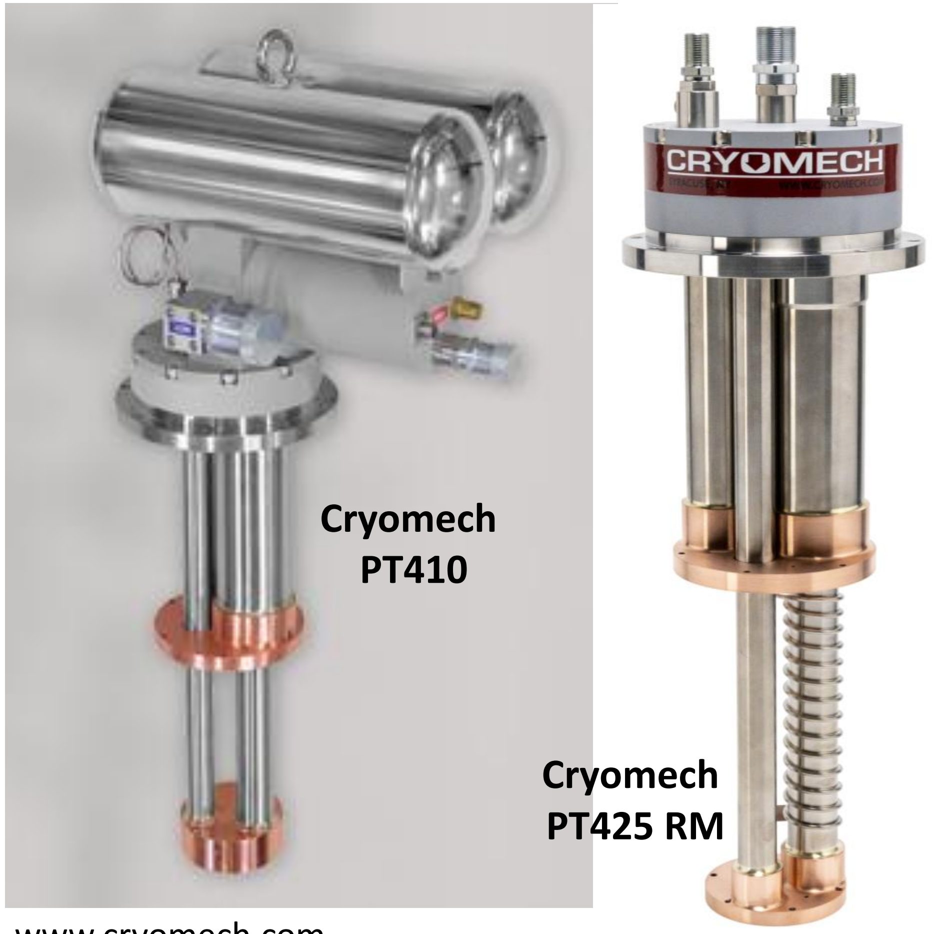
coldest spot between regenerator and pulse tube

III.1.3 Regenerative Machines

- **Pulse – Tube – Refrigerator (PTR)** (realizations)



principle

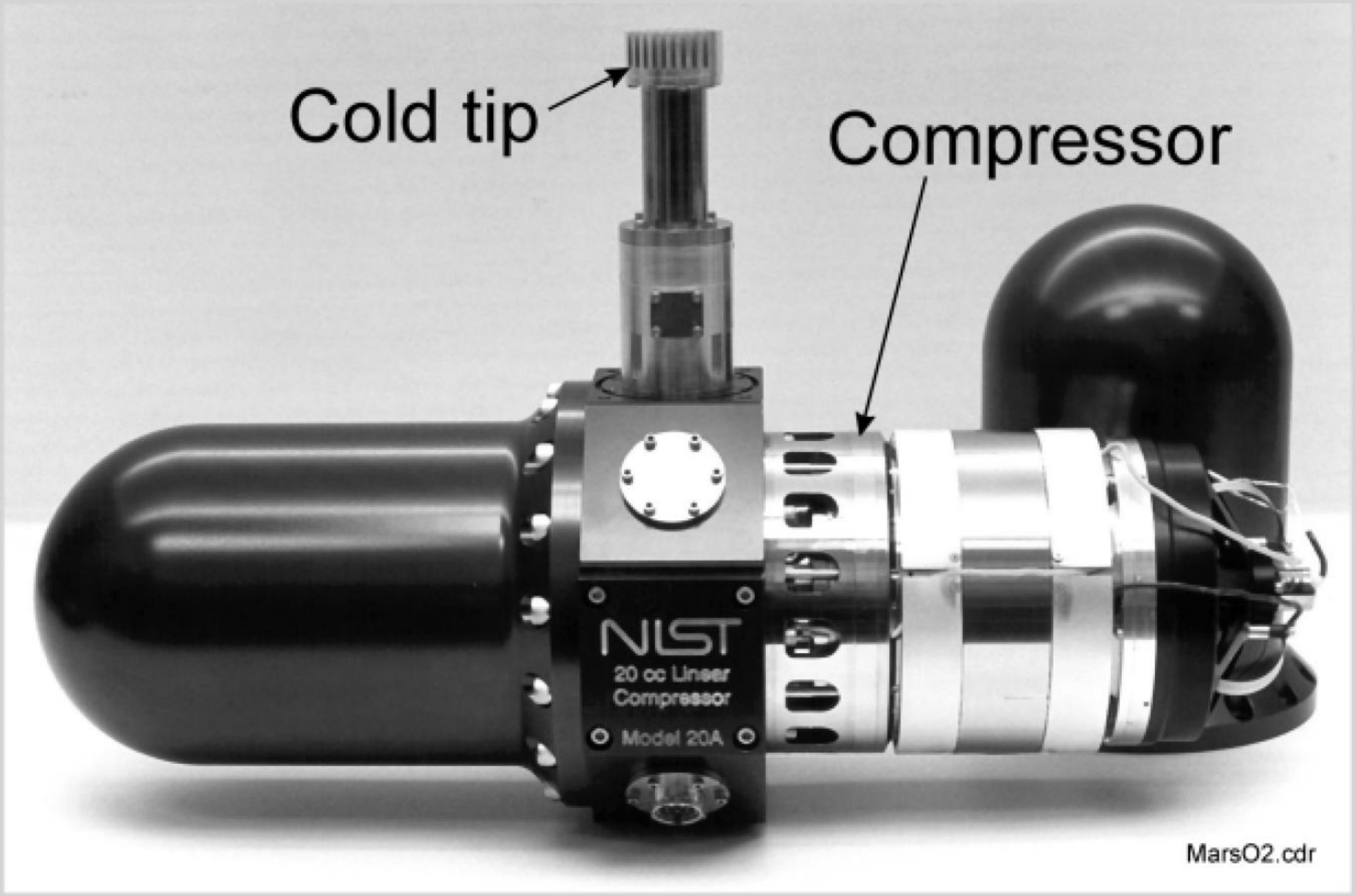


www.cryomech.com

commercially available
PTR with GM drive

1.0 W @ 4.2K, 35 W @ 45K, $T_{\min} = 2.5$ K
(2-stage PTR)

III.1.3 Regenerative Machines



pulse tube refrigerator for studies of liquefying oxygen on Mars (580 mm total length)

III.1.3 Regenerative Machines

- **Pulse – Tube – Refrigerator: summary**

  - presently very active development
  - no moving parts at low temperatures

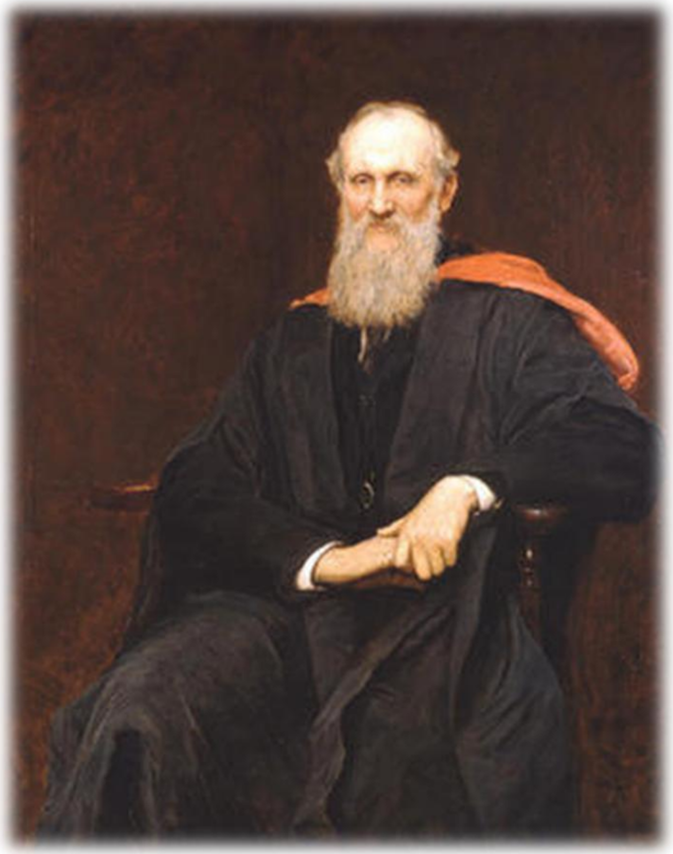
    $\Rightarrow$ ***long endurance***

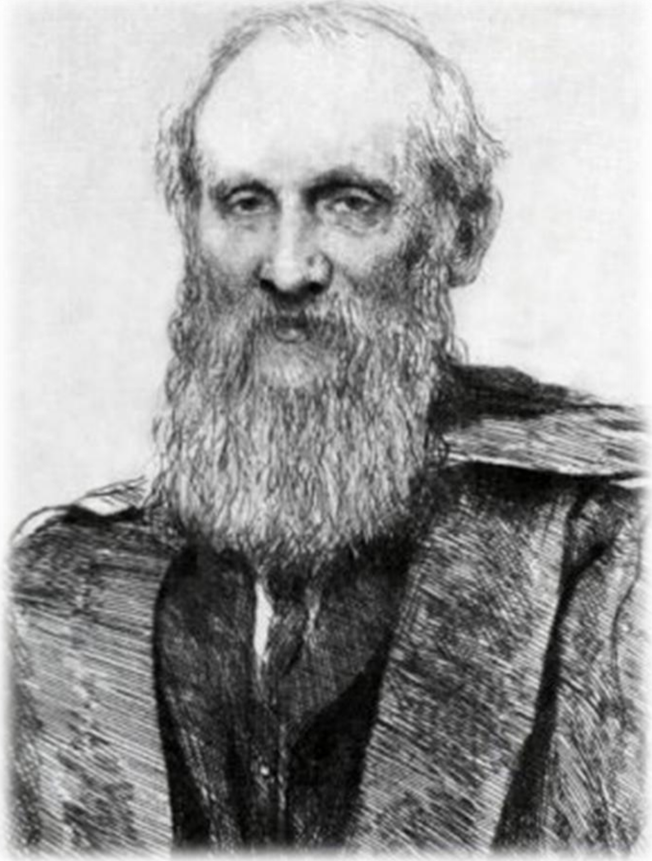
    mobile base stations and satellite applications

    (e.g. for superconductive microwave filters)
  - almost no vibrations
  - efficiency lower than for displacer system
  - only one simpler method:

    $\Rightarrow$ ***Joule-Thomson*** cooling

III.1.4 Joule-Thomson Cooling

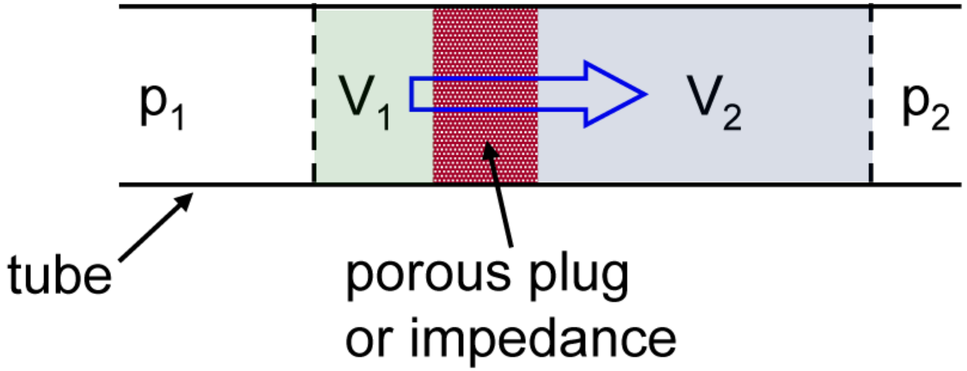




William Thomson (Lord Kelvin)
Born: 26 June 1824, Belfast, Northern Ireland
Died: 17 December 1907, Netherhall, Largs Ayrshire, Scotland

III.1.4 Joule-Thomson Cooling

- **Joule-Thompson process**
  - gas performs work against its own internal attractive forces
  - working medium/gas (V_1) flows through impedance and expands to V_2



$U_2 - U_1$ $\quad = \;$ 0 (adiabatic)

1st law of thermodynamics: $\Delta U = \Delta Q + \Delta W$

$$-\int_0^{V_2} p_2 dV_2 - \int_{V_1}^{0} p_1 dV_1 = -p_2V_2 + p_1V_1$$

$$\Rightarrow \boldsymbol{U_2 + p_2V_2 = U_1 + p_1V_1}$$

this means: process with **constant enthapy:** $H \equiv U + pV = const.$

- for **ideal gas**: $p_1V_1 = p_2V_2$ and hence $U_1 = U_2$, respectively $T_1 = T_2$ ➔ **no cooling !!**

III.1.4 Joule-Thomson Cooling

- **Joule-Thompson process: real gas**

transformation of gas into liquid on decreasing $\boldsymbol{T}$ and (or) increasing $\boldsymbol{p}$ due to
work against attractive interaction between the molecules

- ***weak long-range attraction:***
  tends to keep molecules closer together, same effect as additional compression of the gas

$$p_{\text{eff}} = p + \frac{a}{V^2}$$

a is a measure of the long-range attraction

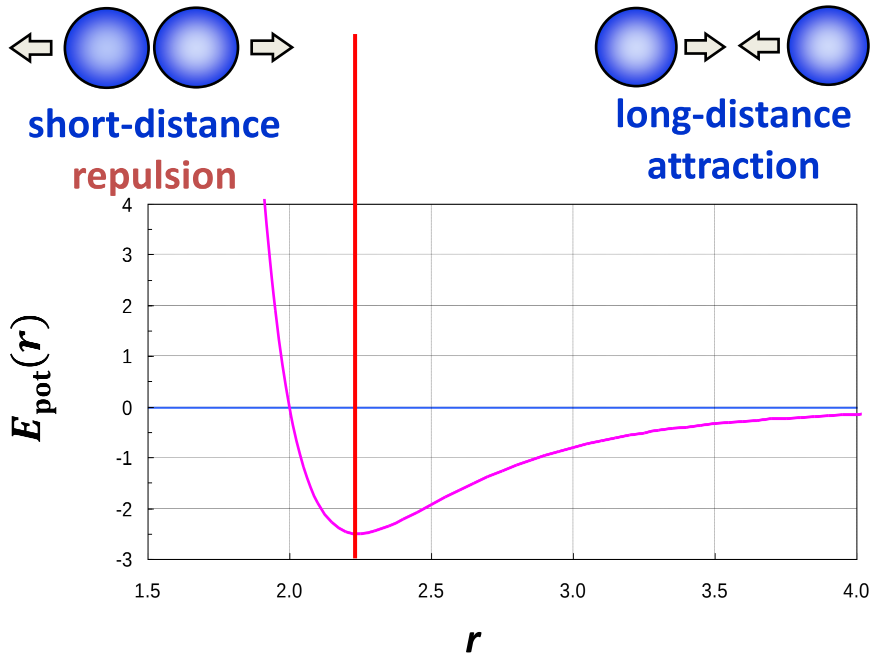
- ***strong short-range repulsion:***
  molecules are rigid:
  $p \rightarrow \infty$ as soon as the molecules "touch" each other.

$$V_{\text{eff}} = V - b$$

b ($\approx 4\pi\sigma^3/3$): "excluded volume" per particle

- **Van der Waals equation**:

$$\boxed{\left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT}$$

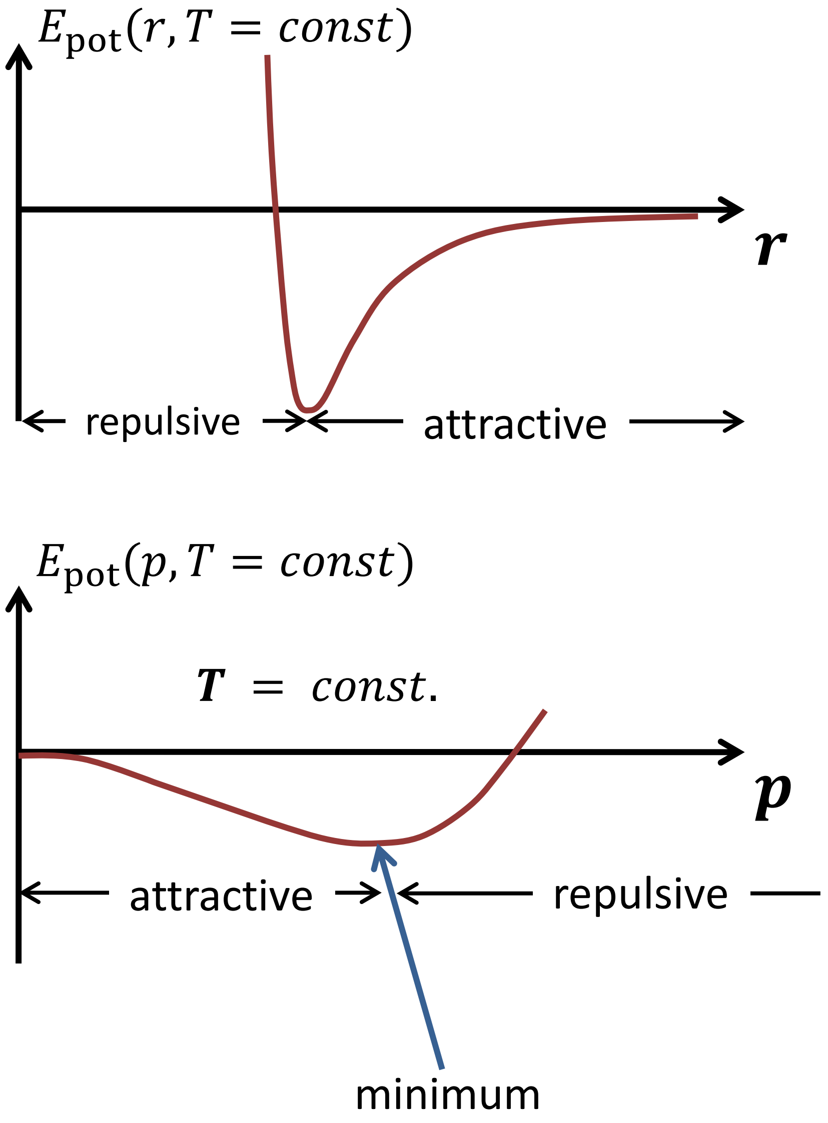


expansion (decrease of pressure):
low pressure:
attraction costs work
$\Rightarrow$ cooling of gas
high pressure:
repulsion provides work
$\Rightarrow$ heating of gas

III.1.4 Joule-Thomson Cooling

- **Joule-Thompson process: real gas**

interaction potential:



with $p \propto n \equiv \frac{N}{V} = \frac{1}{\langle r^3 \rangle}$

III.1.4 Joule-Thomson Cooling

- **Joule-Thompson process: more detailed analysis of isenthalpic expansion**

$$\Delta H = \left(\frac{\partial H}{\partial T}\right)_p \Delta T + \left(\frac{\partial H}{\partial p}\right)_T \Delta p = 0$$

with $\left(\frac{\partial H}{\partial T}\right)_p = C_p \quad \Rightarrow \; C_p \Delta T = -\left(\frac{\partial H}{\partial p}\right)_T \Delta p$ $\Rightarrow$ $-\frac{1}{C_p}\left(\frac{\partial H}{\partial p}\right)_T = \left(\frac{\Delta T}{\Delta p}\right)_H \equiv \mu_{\mathrm{JT}}$

Joule-Thomson coefficient

$\mu_{\mathrm{JT}} > 0$: cooling on expansion

$\mu_{\mathrm{JT}} < 0$: heating on expansion

with $\Delta H = T\Delta S + V\Delta p \quad \Rightarrow \quad \left(\frac{\partial H}{\partial p}\right)_T = T\left(\frac{\partial S}{\partial p}\right)_T + V$

and $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$

$$\Rightarrow \quad \mu_{\mathrm{JT}} = \left(\frac{\Delta T}{\Delta p}\right)_H = -\frac{1}{C_p}\left(\frac{\partial H}{\partial p}\right)_T = \frac{1}{C_p}\left(T\left(\frac{\partial V}{\partial T}\right)_p - V\right)$$

III.1.4 Joule-Thomson Cooling

- **Joule-Thompson process: more detailed analysis of isenthalpic expansion – ideal gas**

$$\mu_{\mathrm{JT}} = \left(\frac{\Delta T}{\Delta p}\right)_H = -\frac{1}{C_p}\left(\frac{\partial H}{\partial p}\right)_T = \frac{1}{C_p}\left(T\left(\frac{\partial V}{\partial T}\right)_p - V\right)$$

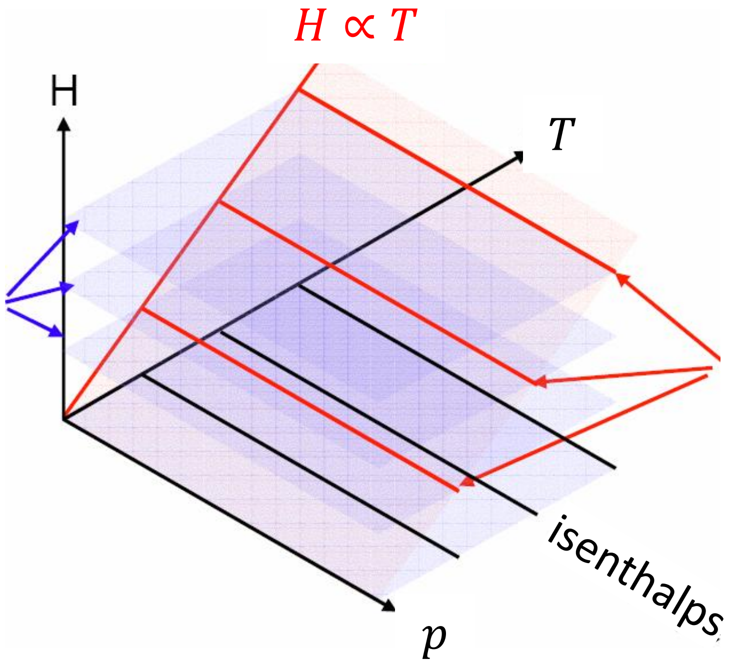
  - for ideal gas: $pV = RT \;\Rightarrow\; \left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{p} = \frac{V}{T} \;\Rightarrow\; \mu_{\mathrm{JT}} = 0$

$$H(T,p) = U + pV = \frac{3}{2}Nk_{\mathrm{B}}T + Nk_{\mathrm{B}}T = \frac{5}{2}Nk_{\mathrm{B}}T = const. \;\Rightarrow\; \mu_{\mathrm{JT}} \propto \left(\frac{\partial H}{\partial p}\right)_T = 0$$

equipartition theorem for monoatomic gas

ideal gas law

@ T = const



areas of $H = const.$

lines of intersection with $H = const.$

III.1.4 Joule-Thomson Cooling

- **Joule-Thompson process: more detailed analysis of isenthalpic expansion – real gas**

  – for real gas: $\left(p+\frac{a}{V_m^2}\right)(V_m-b)=RT \Longrightarrow H(T,p)=U+pV=\frac{5}{2}Nk_\mathrm{B}T+\Delta U(p,T)\neq const.$

  – ***at low densities:*** we can use ***approximation*** $p \gg \frac{a}{V^2}, V \gg b$ and obtain

$$pV+\frac{a}{V}-pb=RT, \qquad \left|\frac{\partial}{\partial T}(\ldots)_p \right. \Longrightarrow p\left(\frac{\partial V}{\partial T}\right)_p-\frac{a}{V^2}\left(\frac{\partial V}{\partial T}\right)_p=R \Rightarrow \left(\frac{\partial V}{\partial T}\right)_p=\frac{R}{p-\frac{a}{V^2}}$$

insert into

$$\mu_\mathrm{JT}=\left(\frac{\Delta T}{\Delta p}\right)_H=-\frac{1}{C_p}\left(\frac{\partial H}{\partial p}\right)_T=\frac{1}{C_p}\left(T\left(\frac{\partial V}{\partial T}\right)_p-V\right)$$

$$\Longrightarrow \quad \mu_\mathrm{JT}=\left(\frac{\Delta T}{\Delta p}\right)_H\approx\frac{1}{C_P}\left(2\frac{a}{RT}-b\right)$$

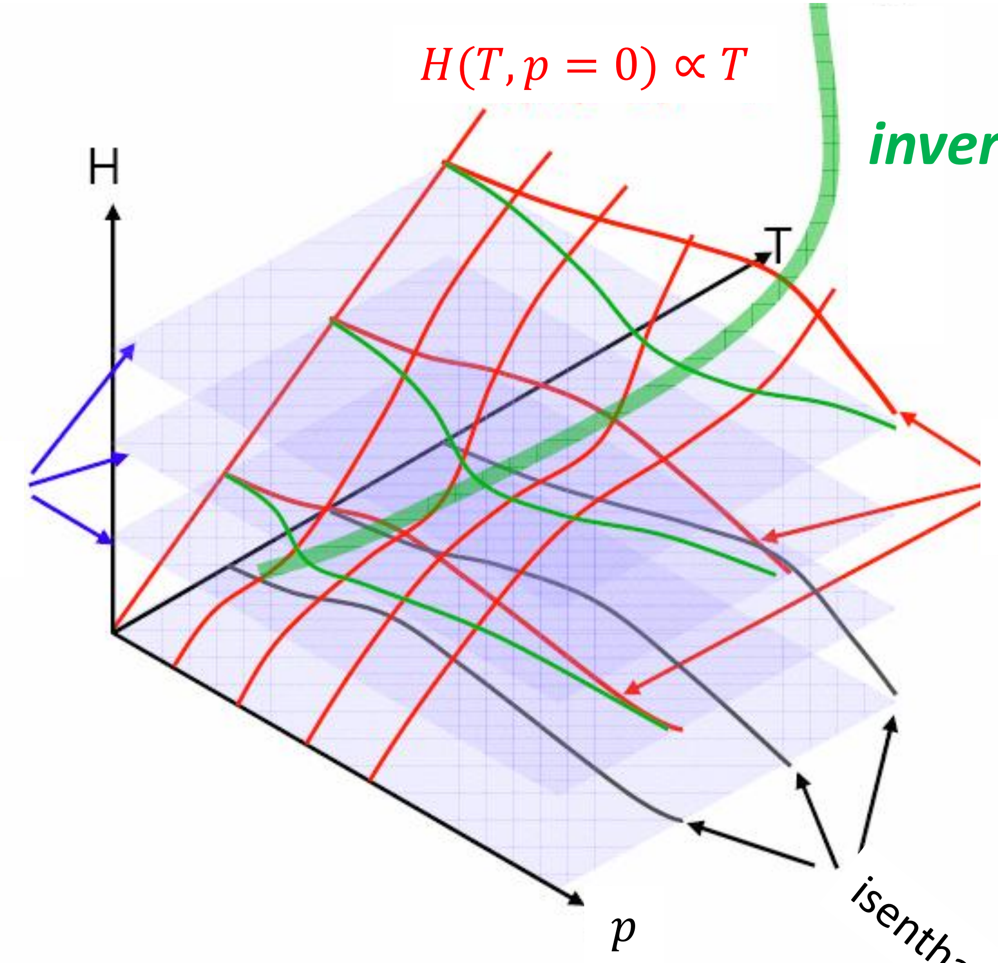
$\mu_\mathrm{JT}>0$ for $T<2a/bR \Rightarrow$ cooling on expansion

$\mu_\mathrm{JT}<0$ for $T>2a/bR \Rightarrow$ heating on expansion

inversion temperature: $T_\mathrm{inv}=\frac{2a}{bR}$

III.1.4 Joule-Thomson Cooling

- **Joule-Thompson process: more detailed analysis of isenthalpic expansion – real gas**



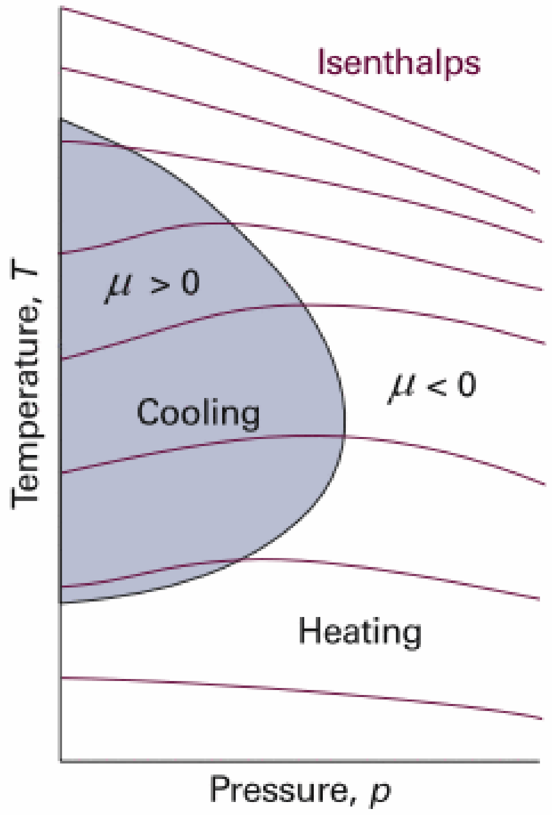
$H(T, p = 0) \propto T$

inversion curve

areas of
$H = const.$

lines of intersection with
$H = const.$

isenthalps



$$H(T, p) = U + pV = \frac{5}{2} N k_B T + \Delta U(p, T) \neq const.$$

III.1.4 Joule-Thomson Cooling

- **Joule-Thompson process: more detailed analysis of isenthalpic expansion – real gas**

  – without "low density" approximation

$$\mu_{\mathrm{JT}} = \frac{(2a/RT)(1-b/V^2)-b}{C_p[1-(2a/VRT)(1-b/V)^2]}$$

low density ***approximation*** $p \gg \frac{a}{V^2}, V \gg b$:

$$\mu_{\mathrm{JT}} \approx \frac{1}{C_P}\left(2\frac{a}{RT}-b\right)$$

inversion curve: points where $\mu_{\mathrm{JT}} = 0$:

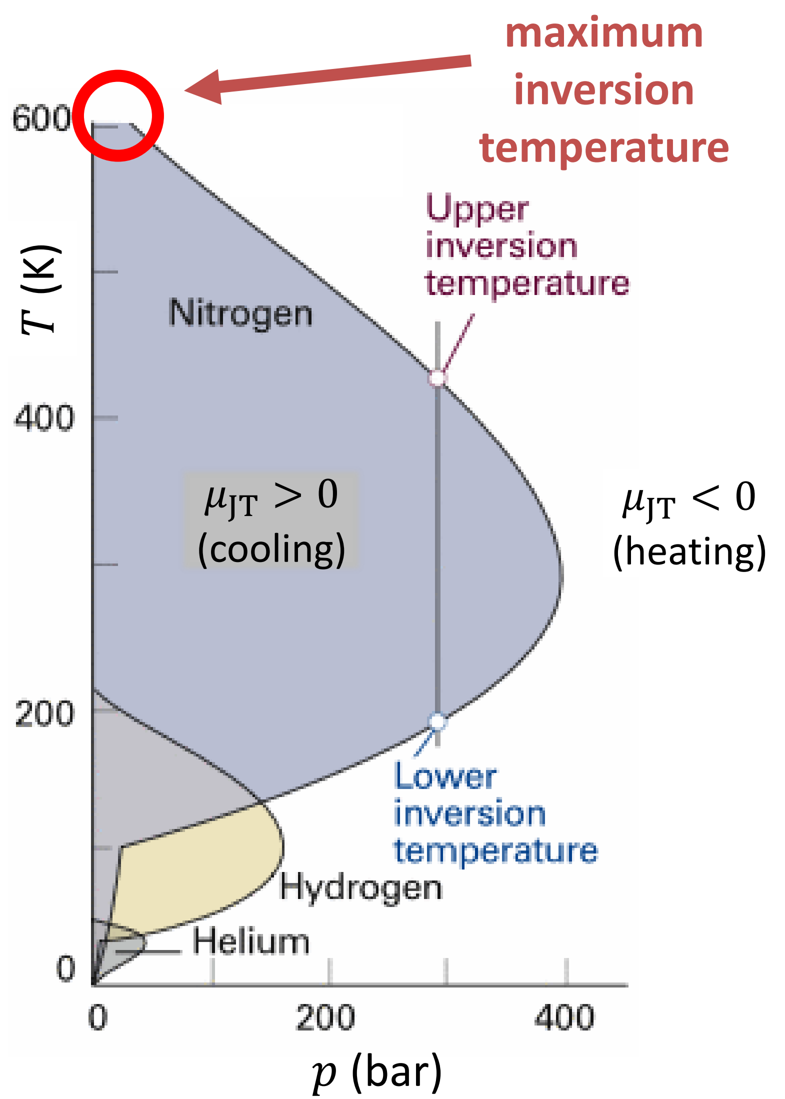
for vdW gas: $(2a/RT)(1-b/V)2 \;=\; b$

inversion temperature:

$$T_{\mathrm{inv}} = \frac{2a}{bR}\left(1-\frac{b}{V}\right)^2$$

equation of state gives $T_{\mathrm{inv}}(p,T)$

maximum inversion temperature: $\boldsymbol{T_{\mathrm{inv}} = \frac{2a}{bR}}$

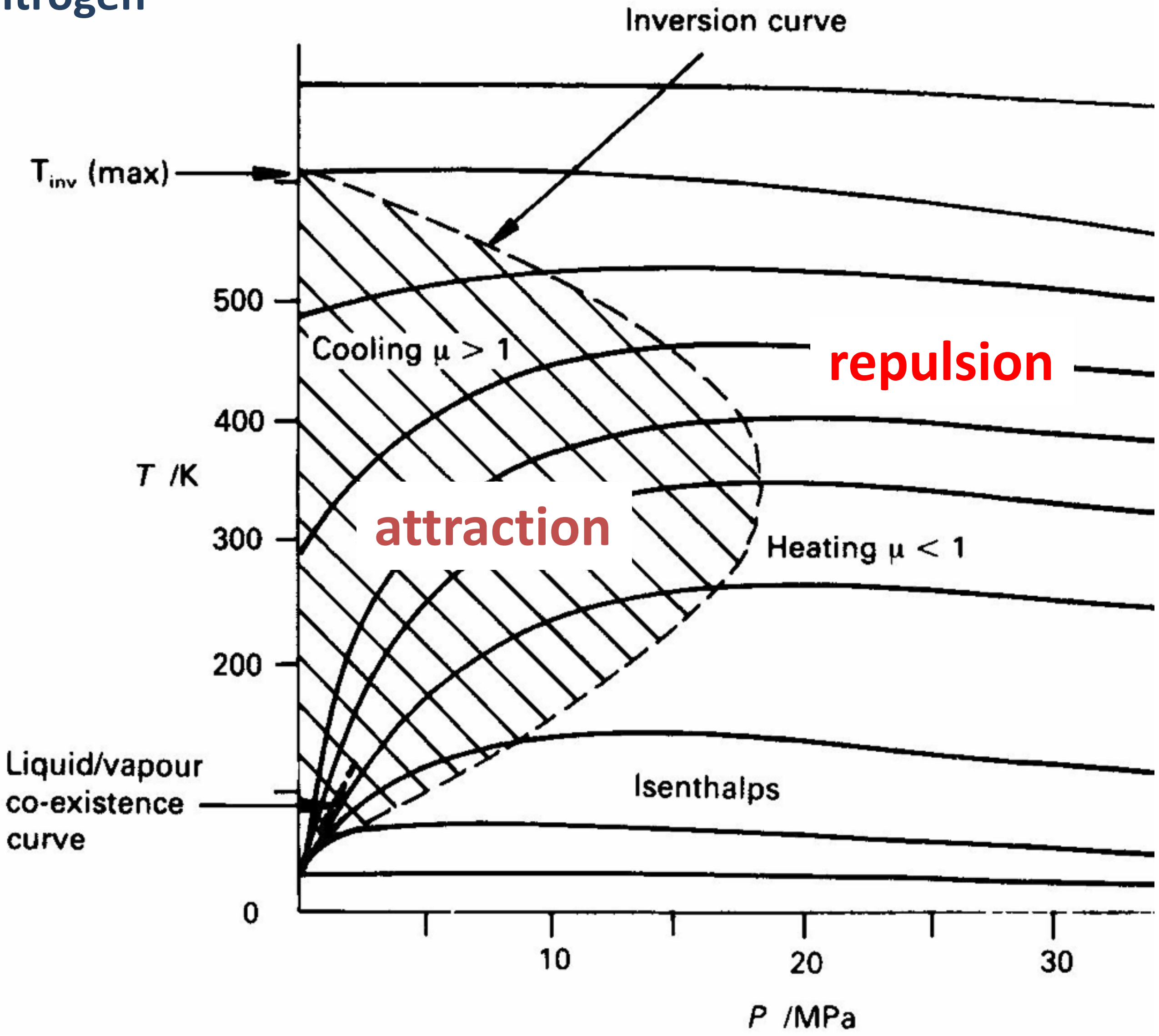


III.1.4 Joule-Thomson Cooling

- **Joule-Thompson coefficient for nitrogen**

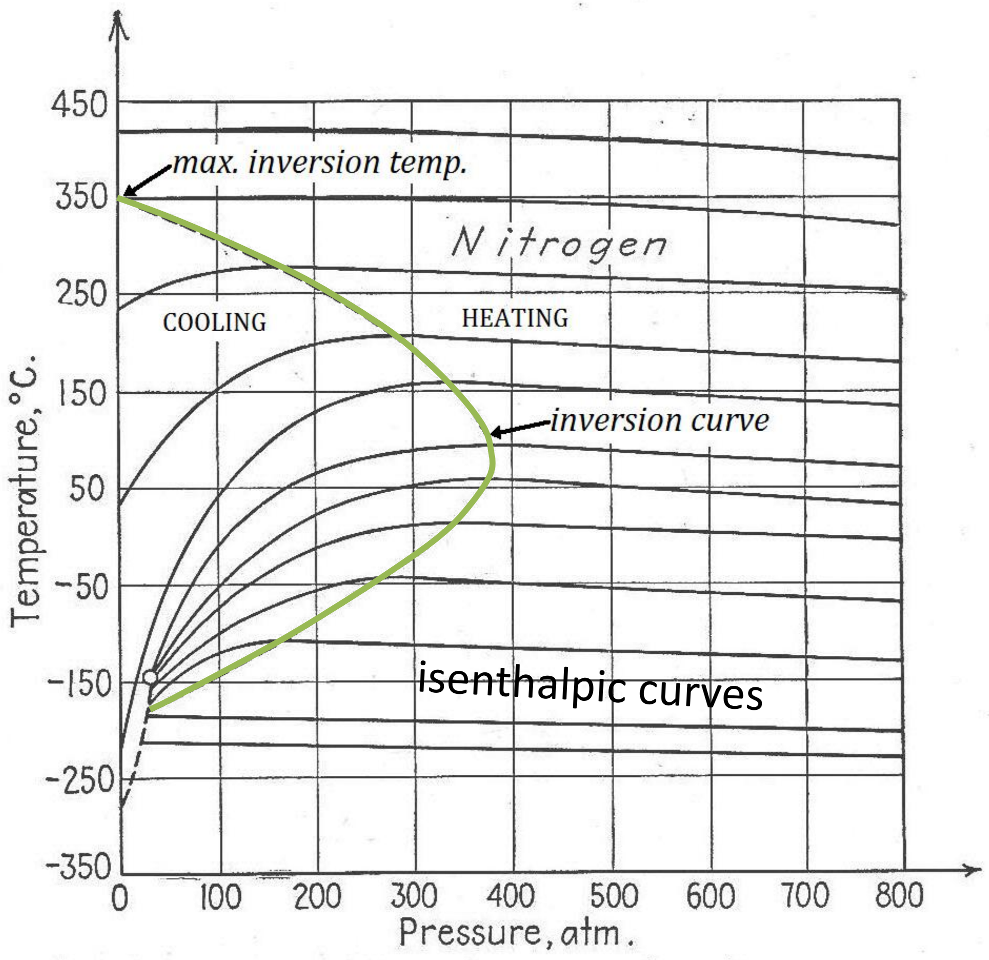
$$\mu_{\mathrm{JT}} \propto \left(\frac{\partial T}{\partial p}\right)_H$$

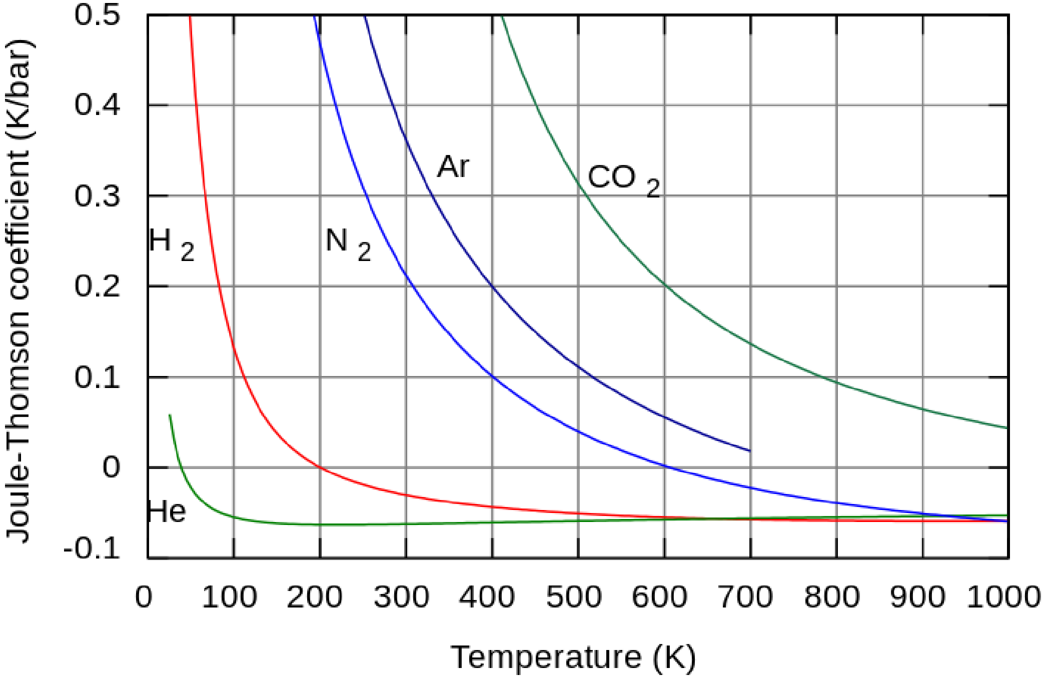
slope of isenthalps



III.1.4 Joule-Thomson Cooling

- **Joule-Thompson coefficient for nitrogen**





Joule-Thomson coefficients for various gases at atmospheric pressure

III.1.4 Joule-Thomson Cooling

- **Joule-Thompson cooling: inversion temperature**

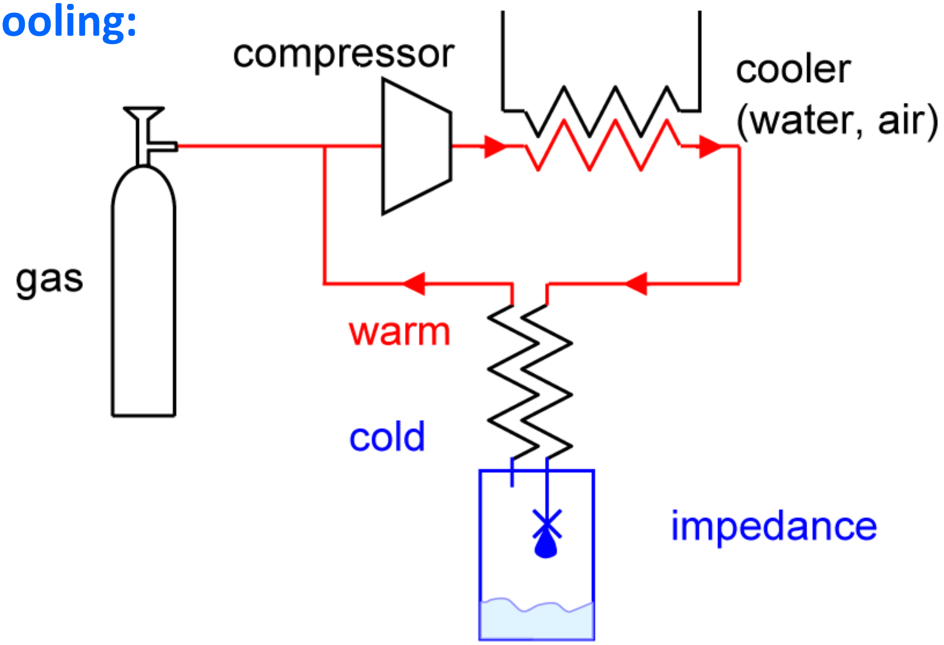
gas	**maximum inversion temperature [K]**
Helium-3	(23)
Helium-4	45
Hydrogen	205
Neon	250
Nitrogen	621
Air	603
Carbon monoxide	652
Argon	794
Oxygen	761
Methane	939
Carbon dioxide	1500
Ammonia	1994

vdW gas can be liquefied only for $T < T_{\mathrm{inv}}$!!!

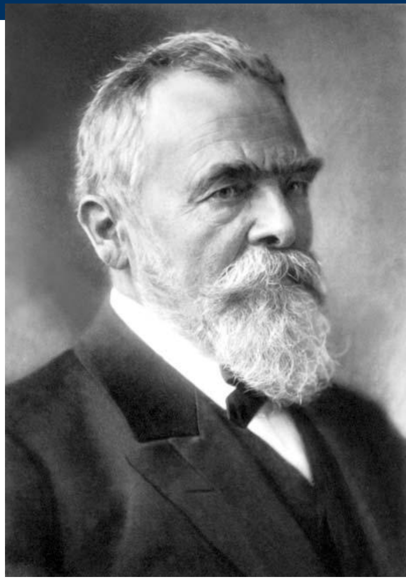
III.1.4 Joule-Thomson Cooling

- **“Linde process”**

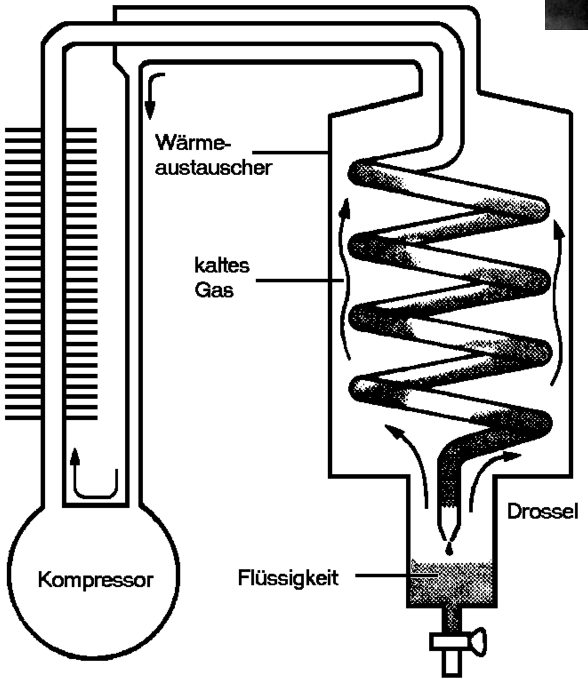
closed cycle cooling:



- gas is cooled by JT-expansion until liquid drops out the impedance
- patent application by Carl von Linde on May 12, 1903 (liquefaction of oxygen)

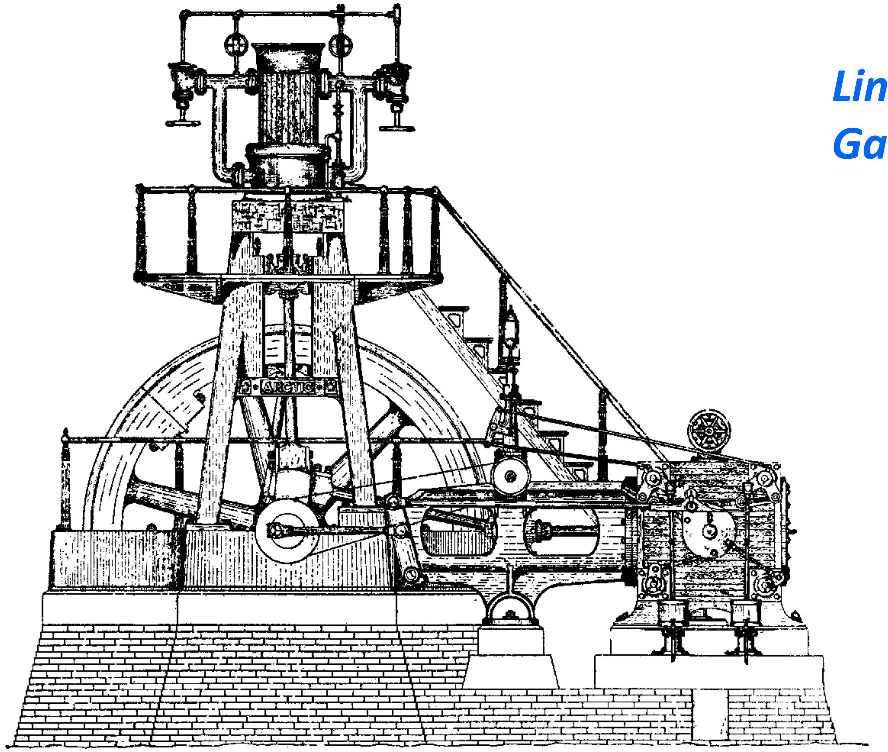


Carl von Linde
(1842 – 1934)

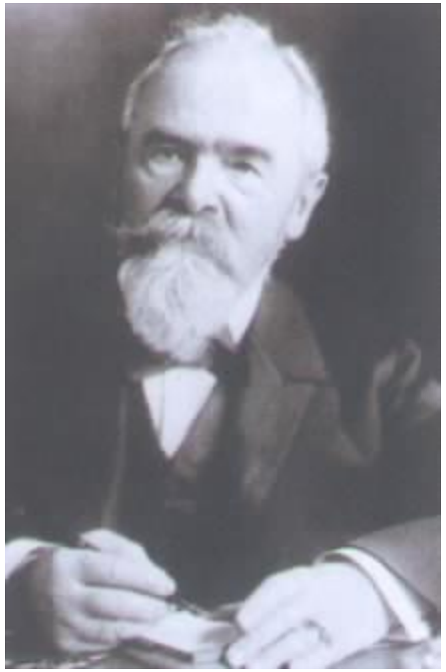


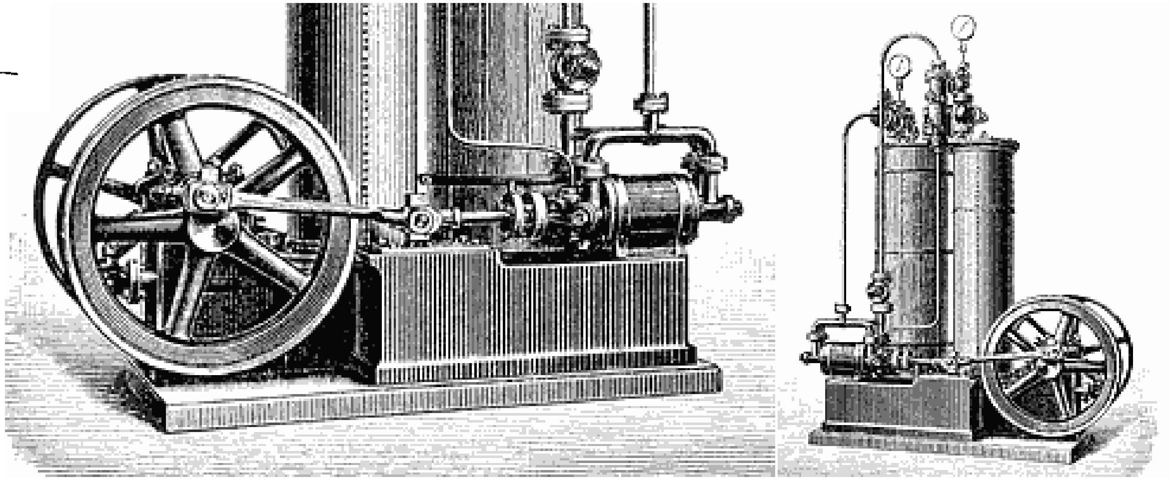
(Source: PTB Braunschweig)

III.1.4 Joule-Thomson Cooling



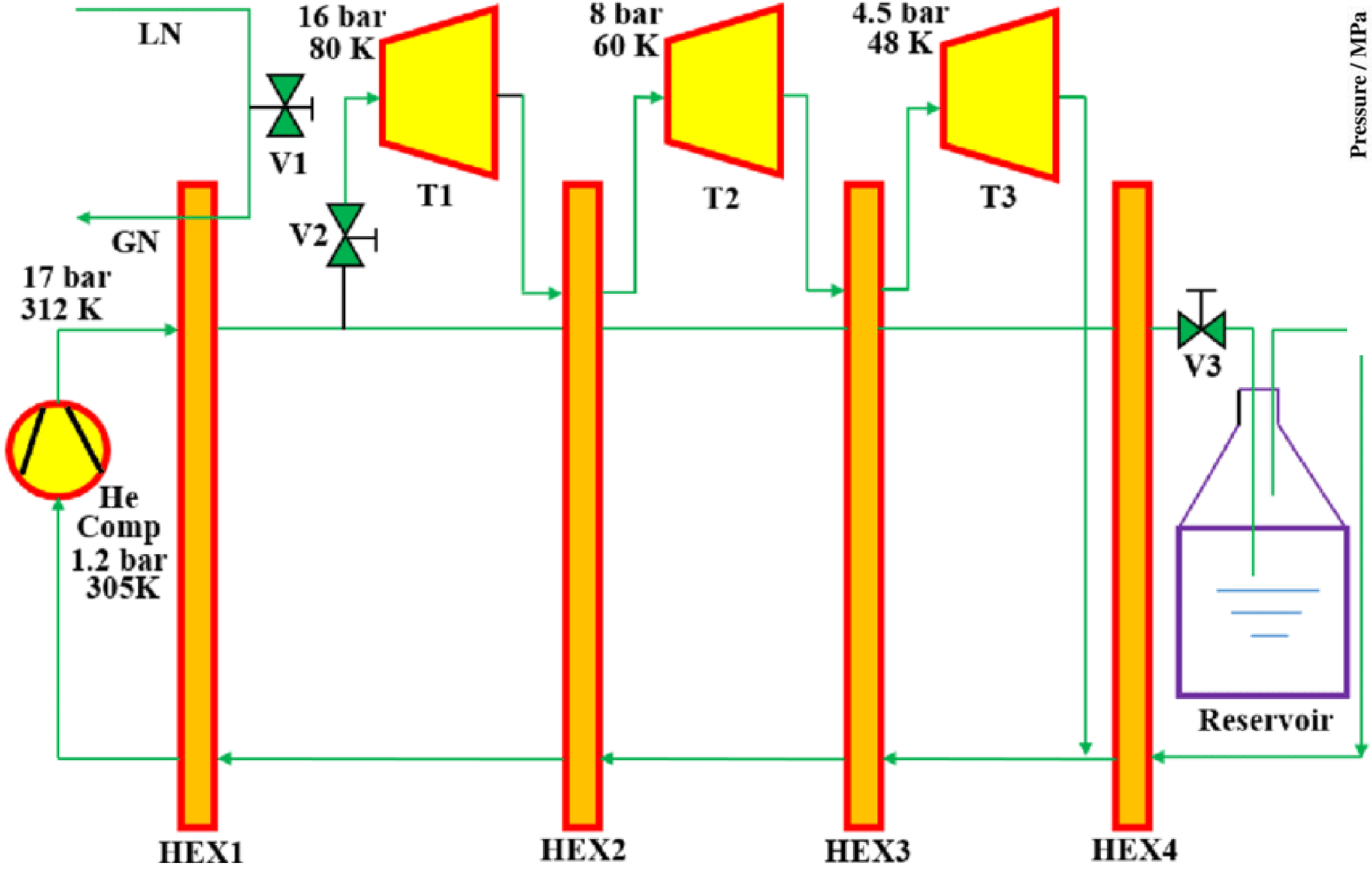
Lindesche
Gasverflüssigungsanlage (1895)



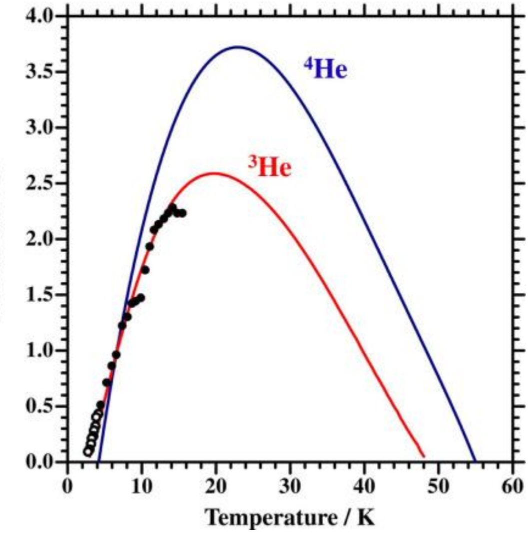


III.1.4 Joule-Thomson Cooling

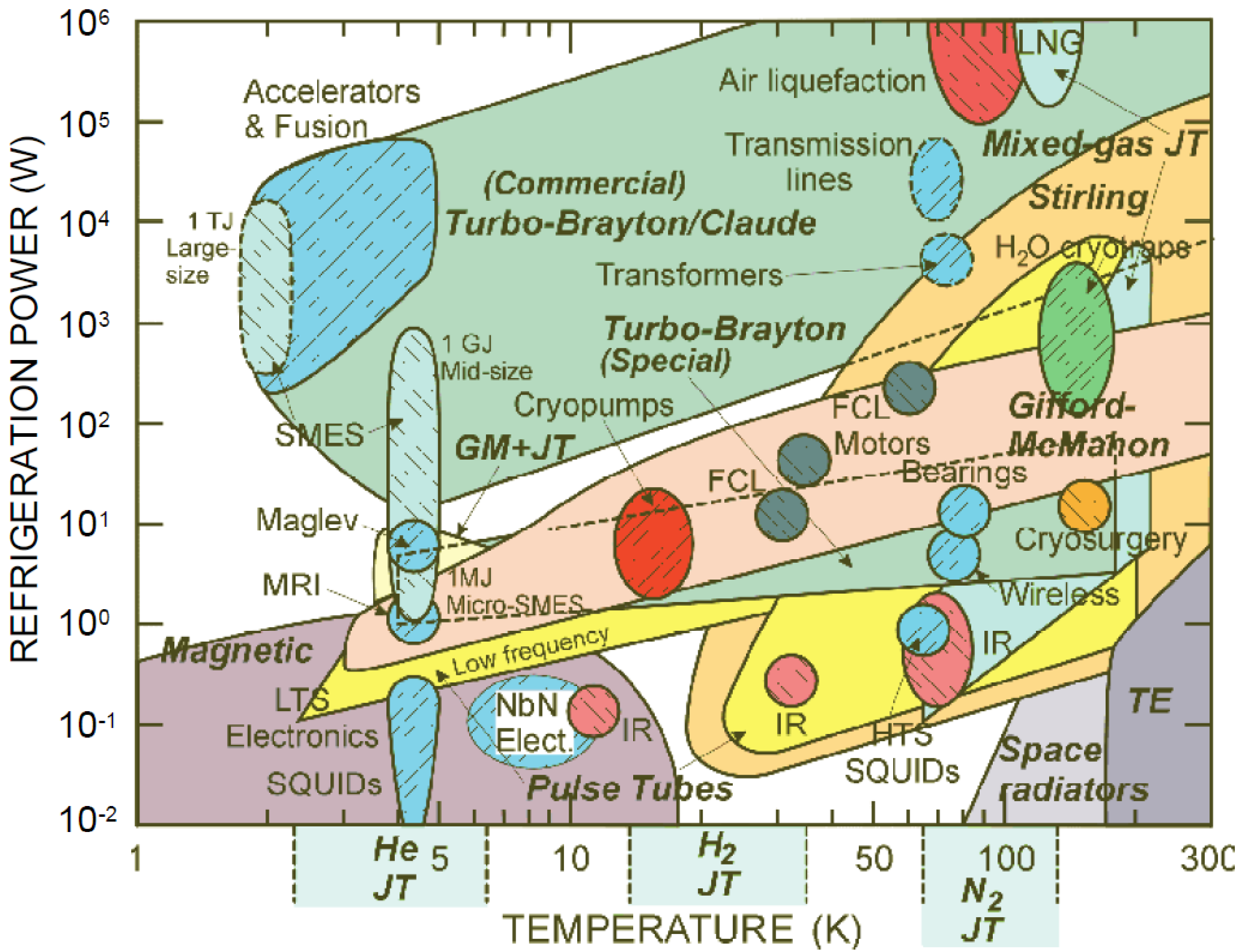
- **Helium liquefaction by Joule-Thompson cooling**



schematics of a Helium liquefier

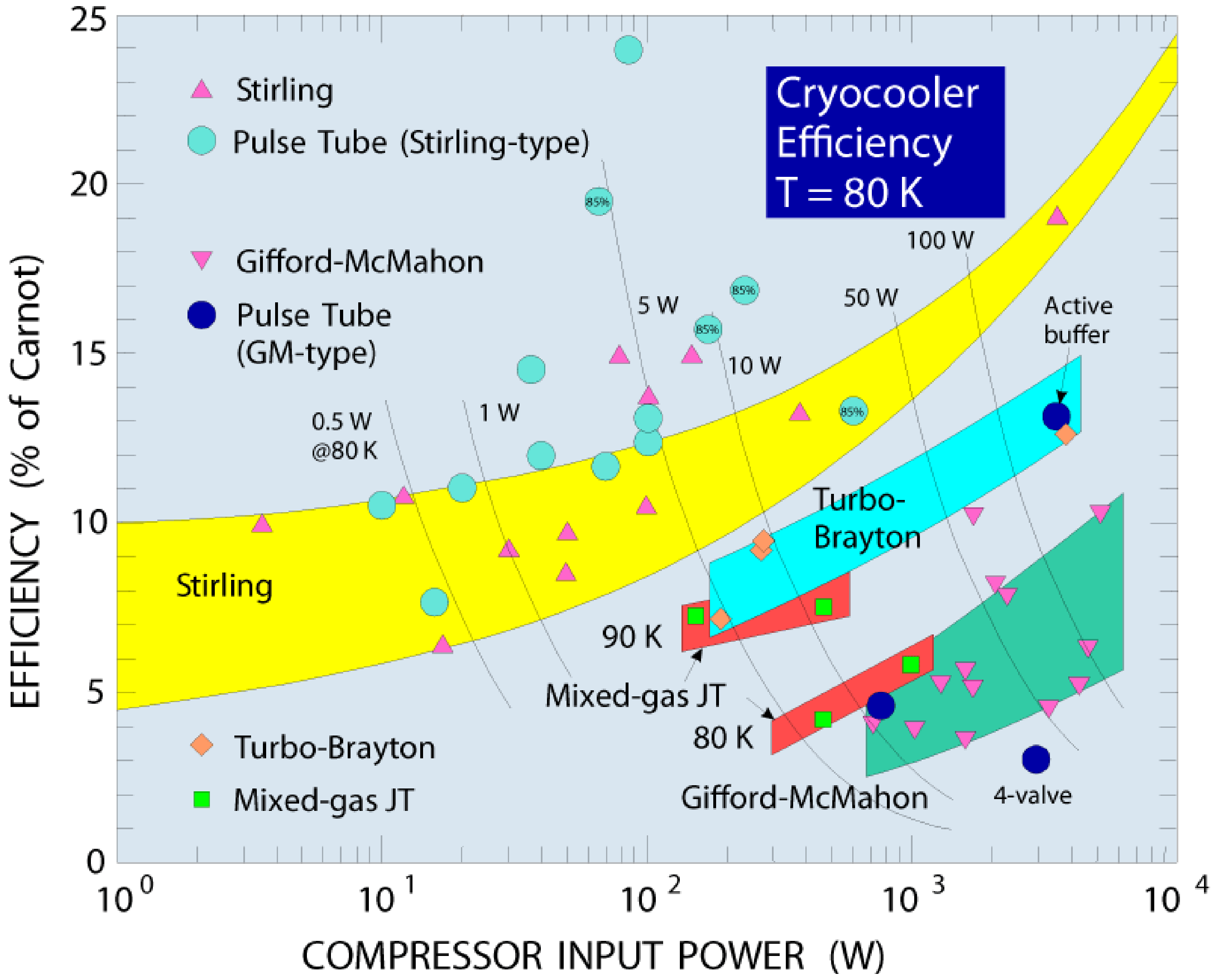


III.1.5 Summary



application of cooling techniques in plane of refrigeration power versus temperature

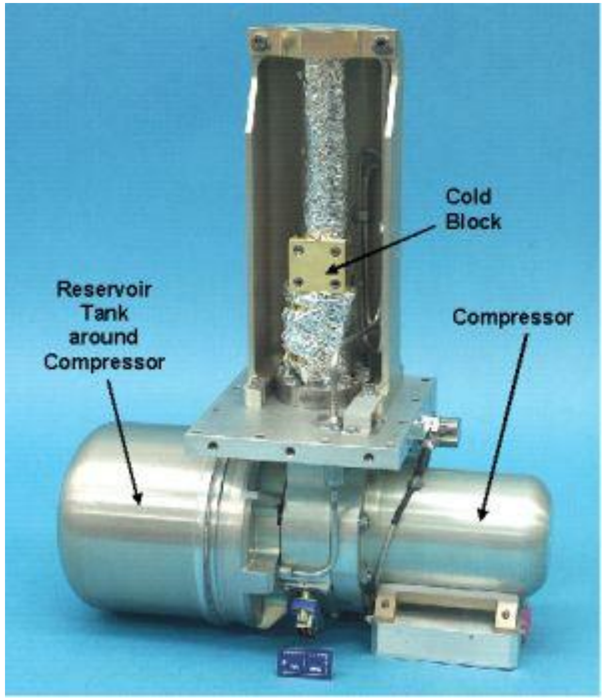
III.1.5 Summary



efficiency of cryocoolers

III.1.5 Summary

Northrop Grumman's HEC cryocooler



Sumitomo Heavy Industries

specification for cryocooler:

- 1 Watt of cooling @ 80 K, rejecting heat at 300 K
- 10 year life
- 230 K to 340 K survival temperature
- survival of launch vibration (non-operating)
- low exported vibration
- high efficiency
- no maintenance possible ➔ oil-free

Stirling cycle miniature cryocooler:

- lightweight cooler, ideal for cooling of sensors and other electronics when low power consumption is important
- mean time before failure of 24,000 hours
- cooling capacity of 1 W @ 80 K
- power consumption of only 55 W.

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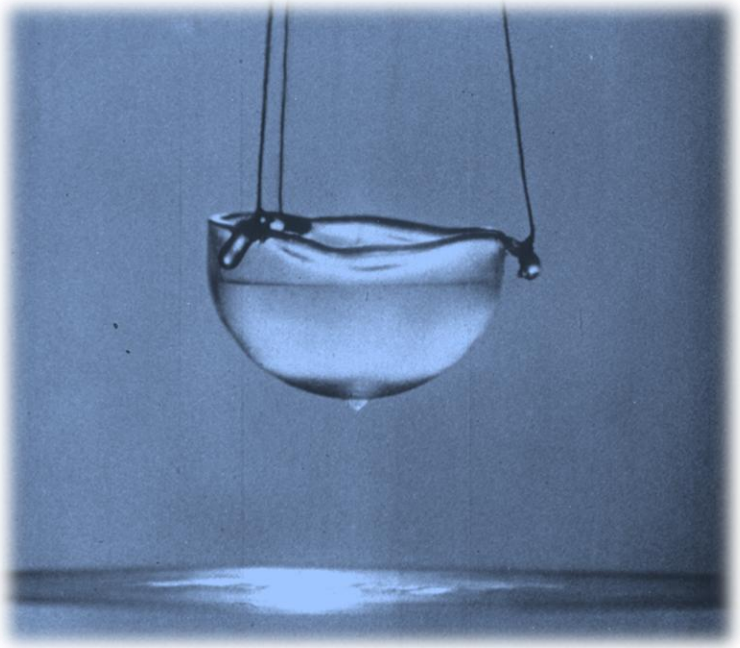

Lecture No. 12

R. Gross
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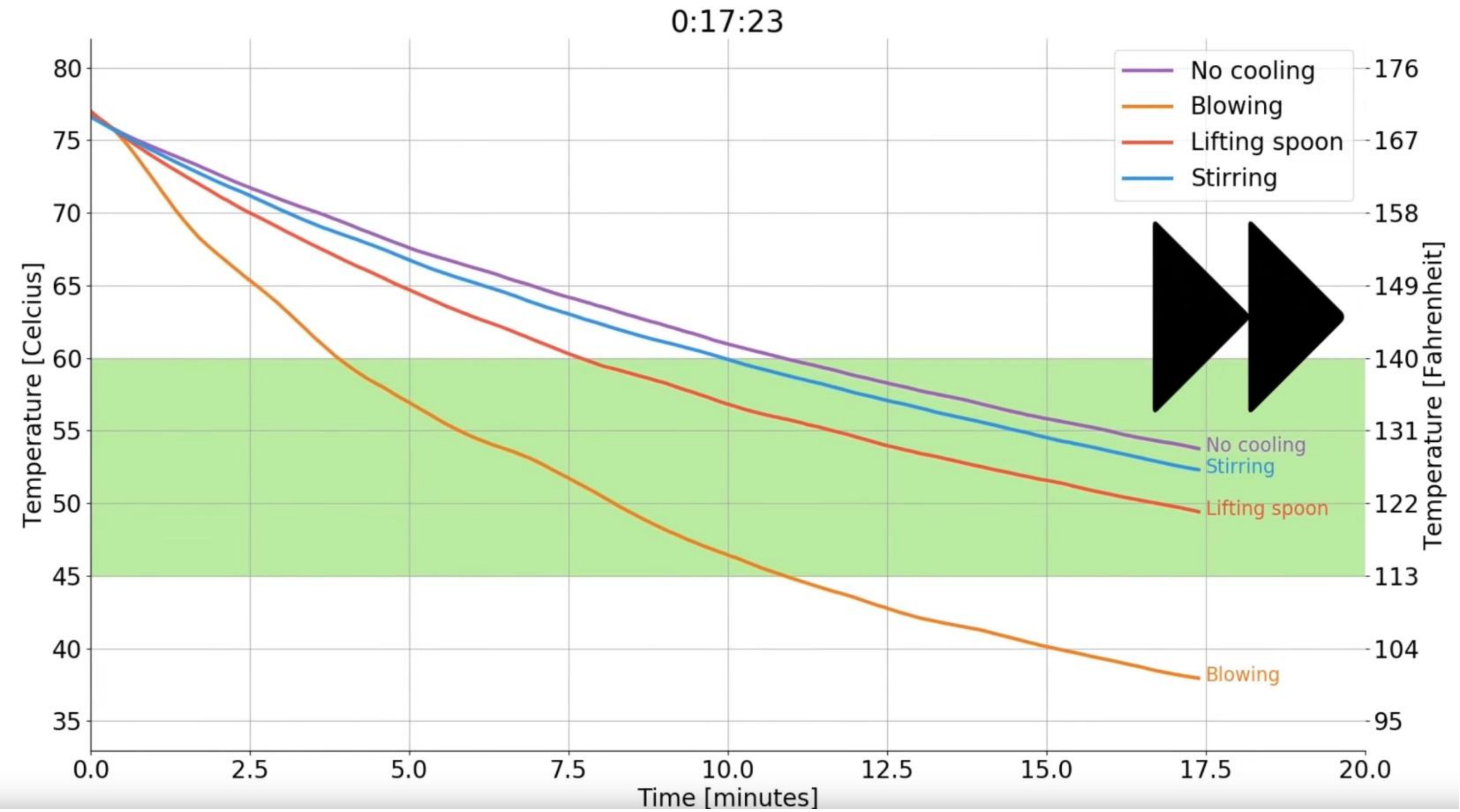
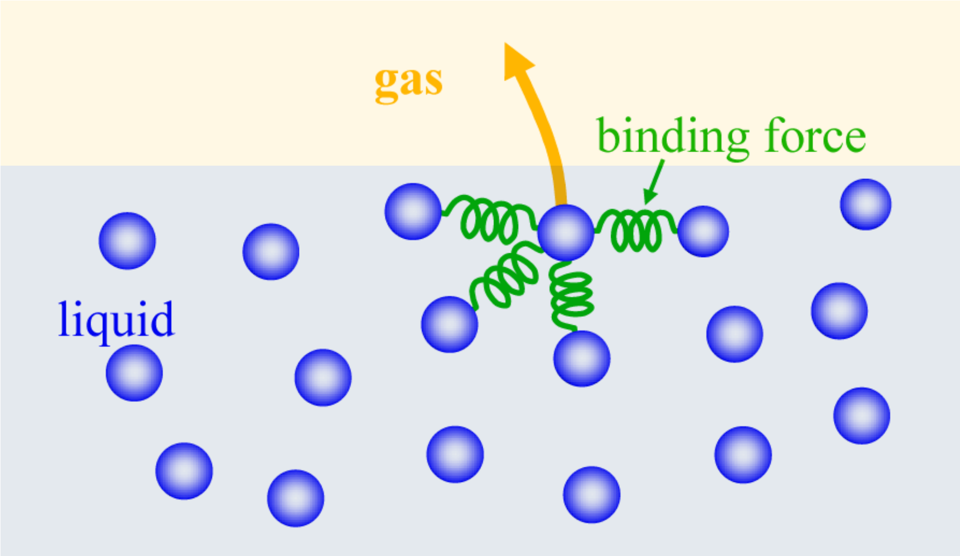
III.1.6 Evaporation Cooling



III.1.6 Evaporation Cooling

- **physics of evaporation cooling**
  - everyday experience: sweating, wind direction, cooling of coffee, …

    ⇒ **moisten finger, evaporation cooling**
  - microscopically:



  - evaporation: ***work required to overcome binding forces***

    ⇒ only the fastest molecules will do it

    ⇒ high-energy particles are lost

    ⇒ liquid cools down

III.1.6 Evaporation Cooling

- **physics of evaporation cooling**

  - limit of evaporation cooling: ***$k_\mathrm{B}T$ becomes too small compared to ΔH_vap*** when lowering T (heat of evaporation)
  - ΔH_vap should be small to reach large cooling power at low temperatures
  - numbers: about 1 K can be reached with liquid ^{4}He, about 0.3 K with ^{3}He

  - boiling point can be calculated by using the Clausius-Clapeyron equation, if heat of vaporization and the vapor pressure of the liquid at a certain temperature are known

III.1.6 Evaporation Cooling

- **physics of evaporation cooling: Clausius-Clapeyron equation**

$$\frac{\mathrm{d}p}{\mathrm{d}T}=\frac{S_{\mathrm{gas}}-S_{\mathrm{liquid}}}{V_{\mathrm{gas}}-V_{\mathrm{liquid}}}=\frac{\Delta H_{\mathrm{vap}}}{\left(V_{\mathrm{gas}}-V_{\mathrm{liquid}}\right)T}\simeq\frac{\Delta H_{\mathrm{vap}}}{V_{\mathrm{gas}}\,T}$$

ΔH_{vap}: molar latent heat [J/mole]
≈ 90 J/mole for ^{4}He

  - approximate expression using $pV = RT$ (ideal gas):

$$\frac{\mathrm{d}p}{p}=\frac{\Delta H_{\mathrm{vap}}}{RT^2}\,dT$$

  - integration yields (assuming that ΔH_{vap} is constant over the considered T range):

$$\ln\left(\frac{p}{p_0}\right)=\frac{\Delta H_{\mathrm{vap}}}{R}\left(\frac{1}{T_0}-\frac{1}{T}\right)$$

  - normal boiling temperature:

pressure above liquid

$$T_{\mathrm{B}}(p)\;=\left(\frac{1}{T_0}-\frac{R}{\Delta H_{\mathrm{vap}}}\ln\left(\frac{p}{p_0}\right)\right)^{-1}$$

boiling temperature at p_0

boiling point corresponds to the temperature at which the vapor pressure of the liquid equals the surrounding environmental pressure

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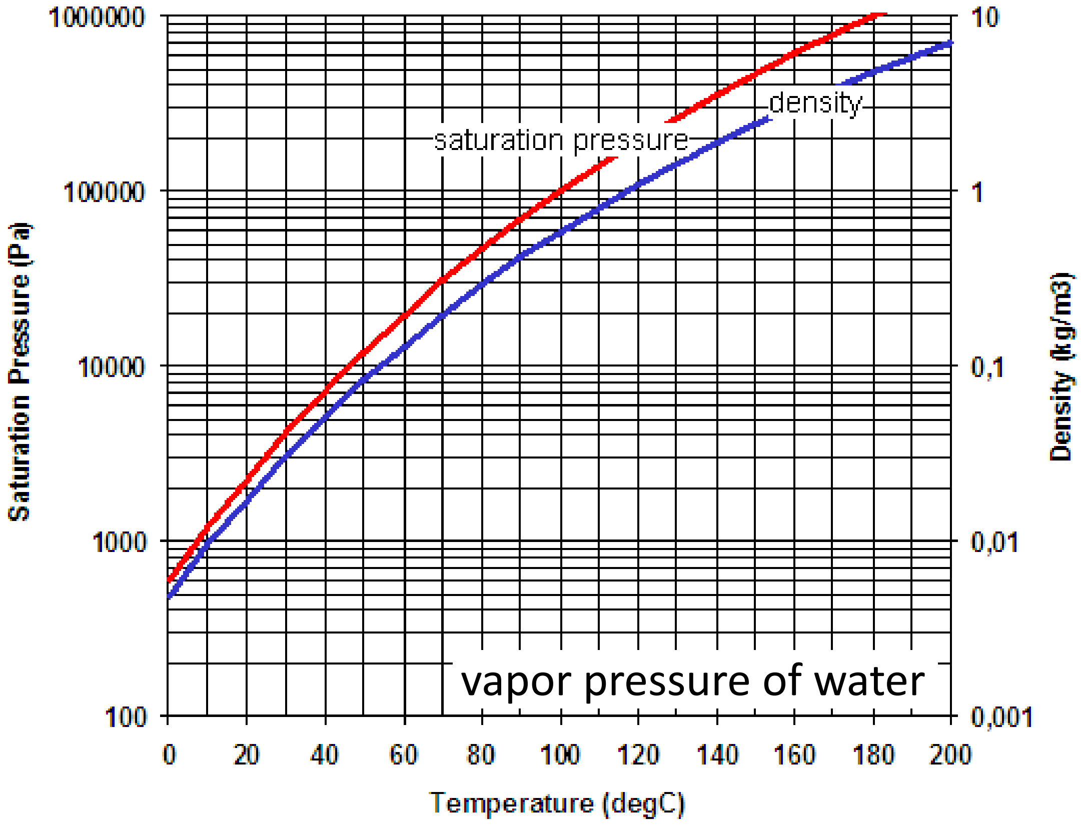
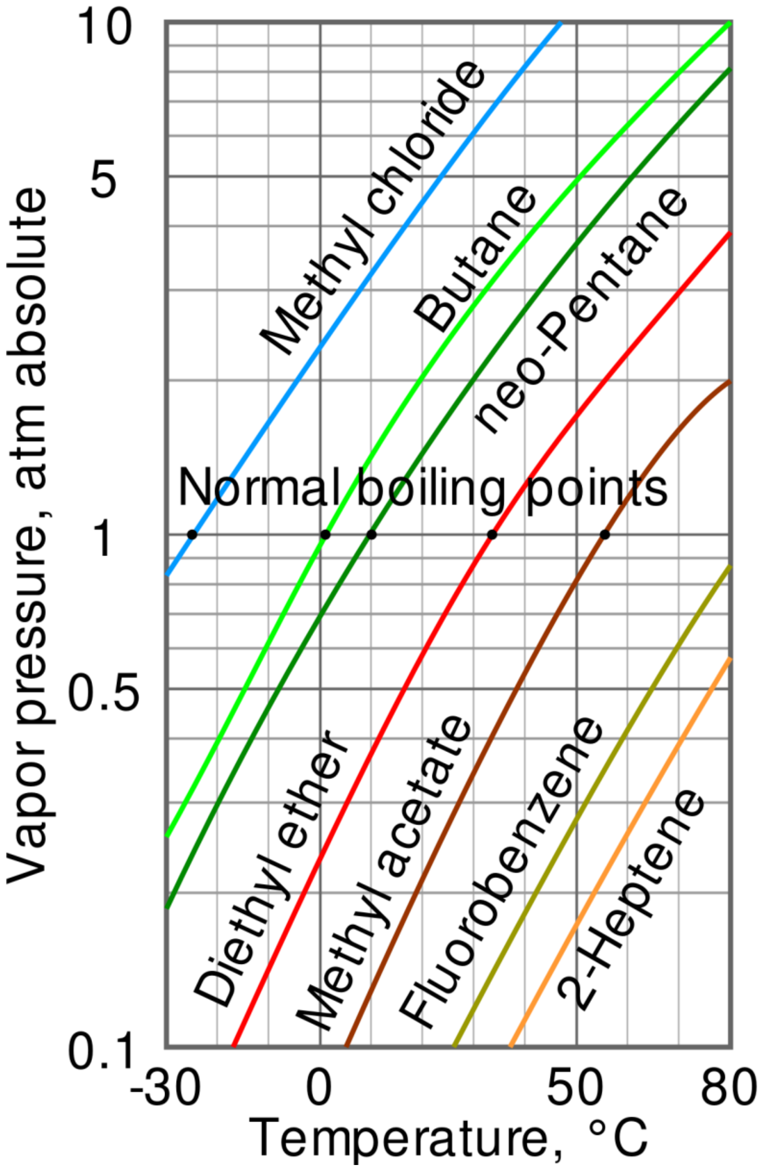
III.1.6 Evaporation Cooling

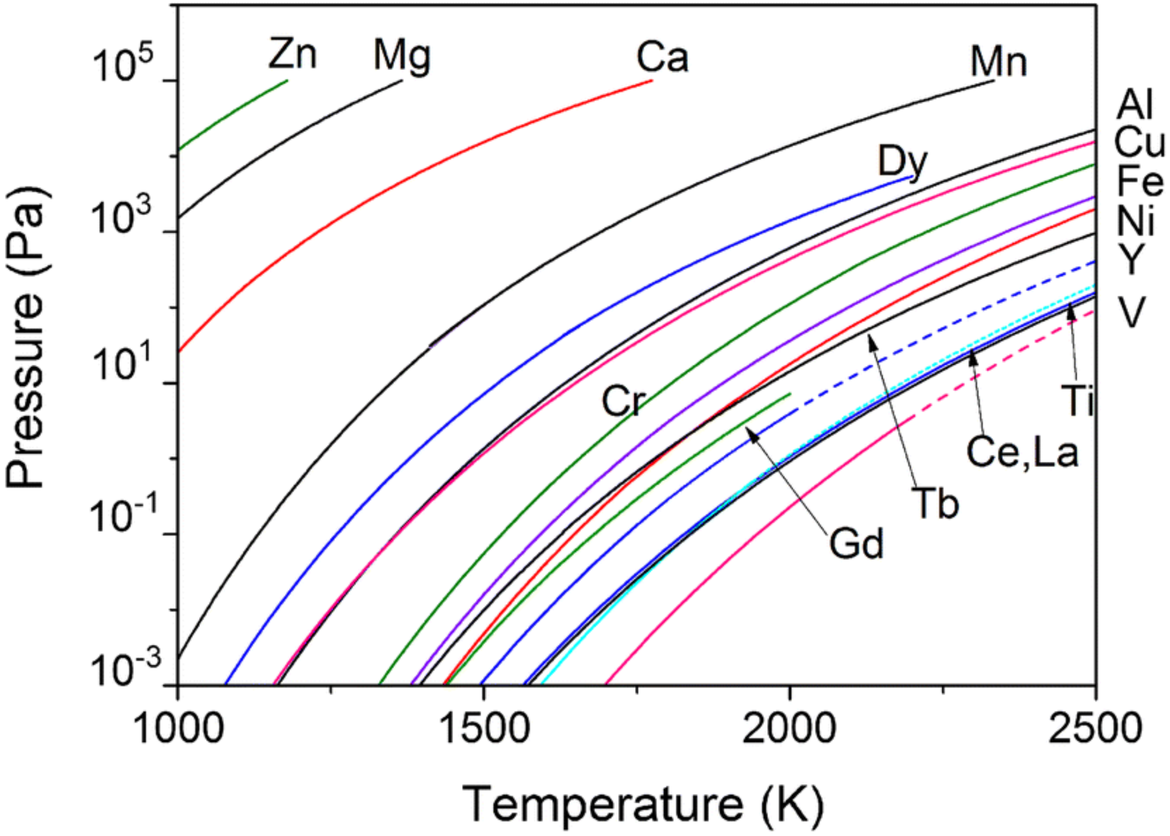
- **physics of evaporation cooling: vapor pressure**

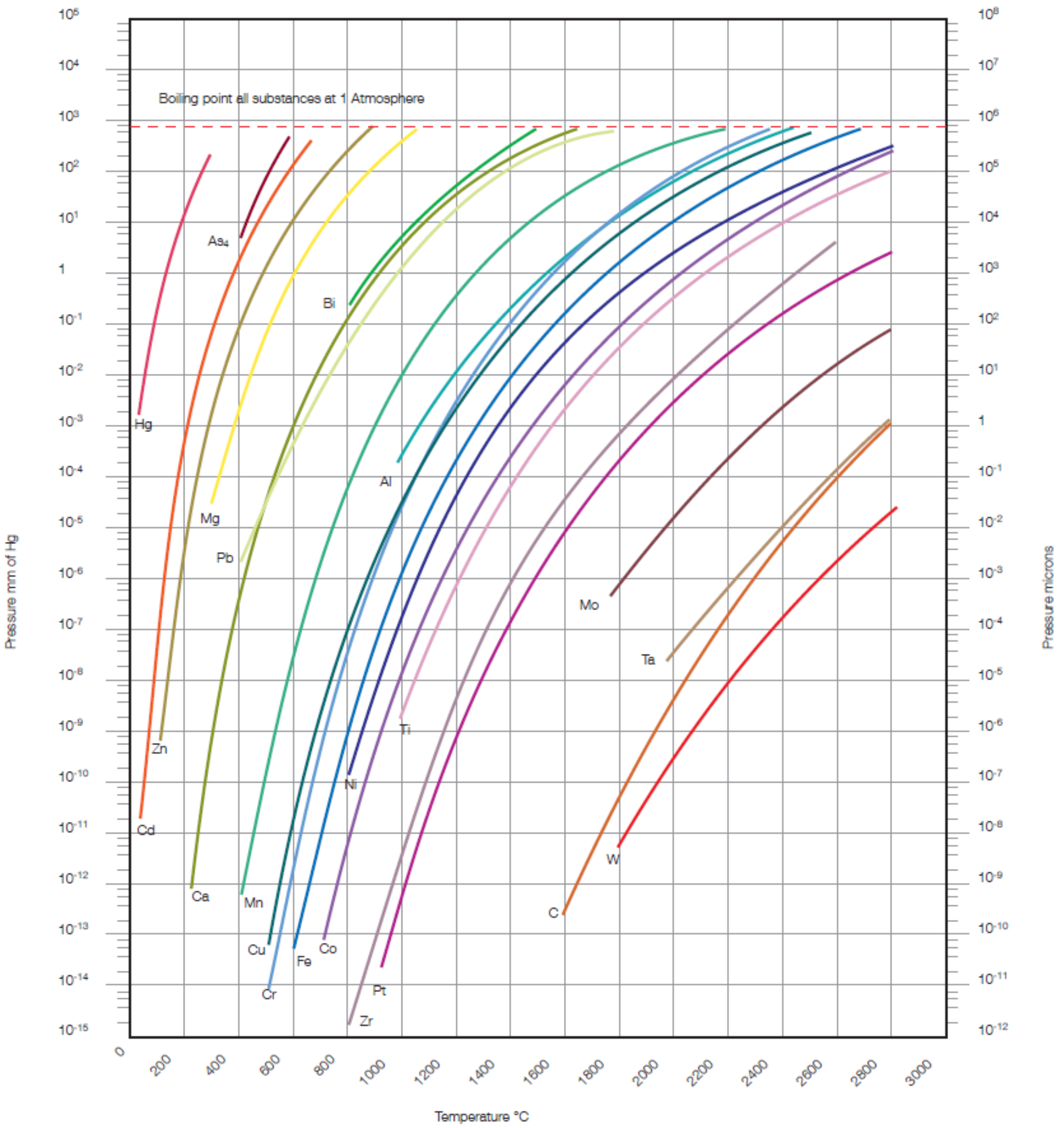




III.1.6 Evaporation Cooling

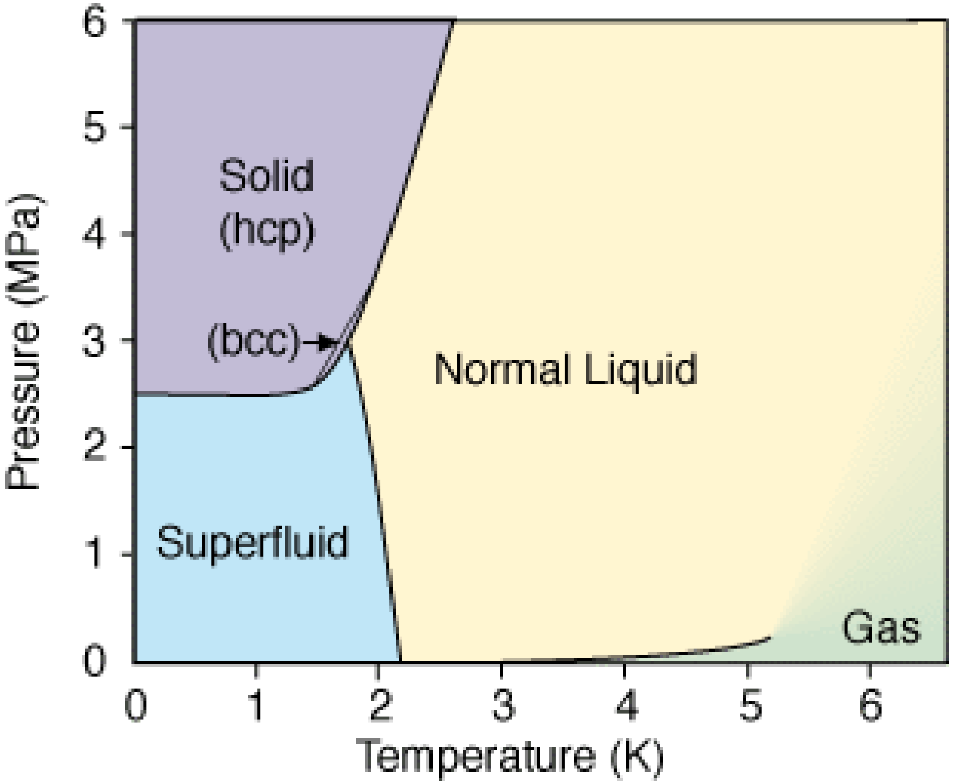
- **physics of evaporation cooling:**
  vapor pressure of some elements

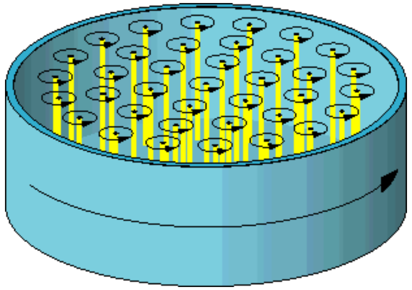




III.1.6 Evaporation Cooling

- **liquid ^{4}He (cf. chapter I)**
  - boson
  - liquid down to 0 K (@ 1 atm)
  - superfluid ^{4}Helium at 2.17 K
    - Bose condensation: macroscopic number of atoms in ground state
    - very low viscosity
    - very high heat conduction
    - strange thermomechanical effects
    - creeping on vertical surfaces
    - vortex core with radius 0.8 Å @ 0.6K
    - explained by a two-fluid model
  - density 125 kg/m^3





III.1.6 Evaporation Cooling

- **liquid helium cryostats**

  – LHe has small latent heat

    ➔ good thermal insulation by vacuum is required

    ➔ LHe container of poor thermal conductivity → glass or stainless steel

    ➔ thermal radiation shield at liquid Nitrogen temperature to reduce black-body radiation

➢ bath cryostat - sample is immersed in the LHe

➢ gas flow cryostat - sample is located in cold He gas

III.1.6 Evaporation Cooling

- **liquid helium container**

narrow neck to minimize

- ***heating by radiation***
- ***heating by thermal conduction***

typical losses

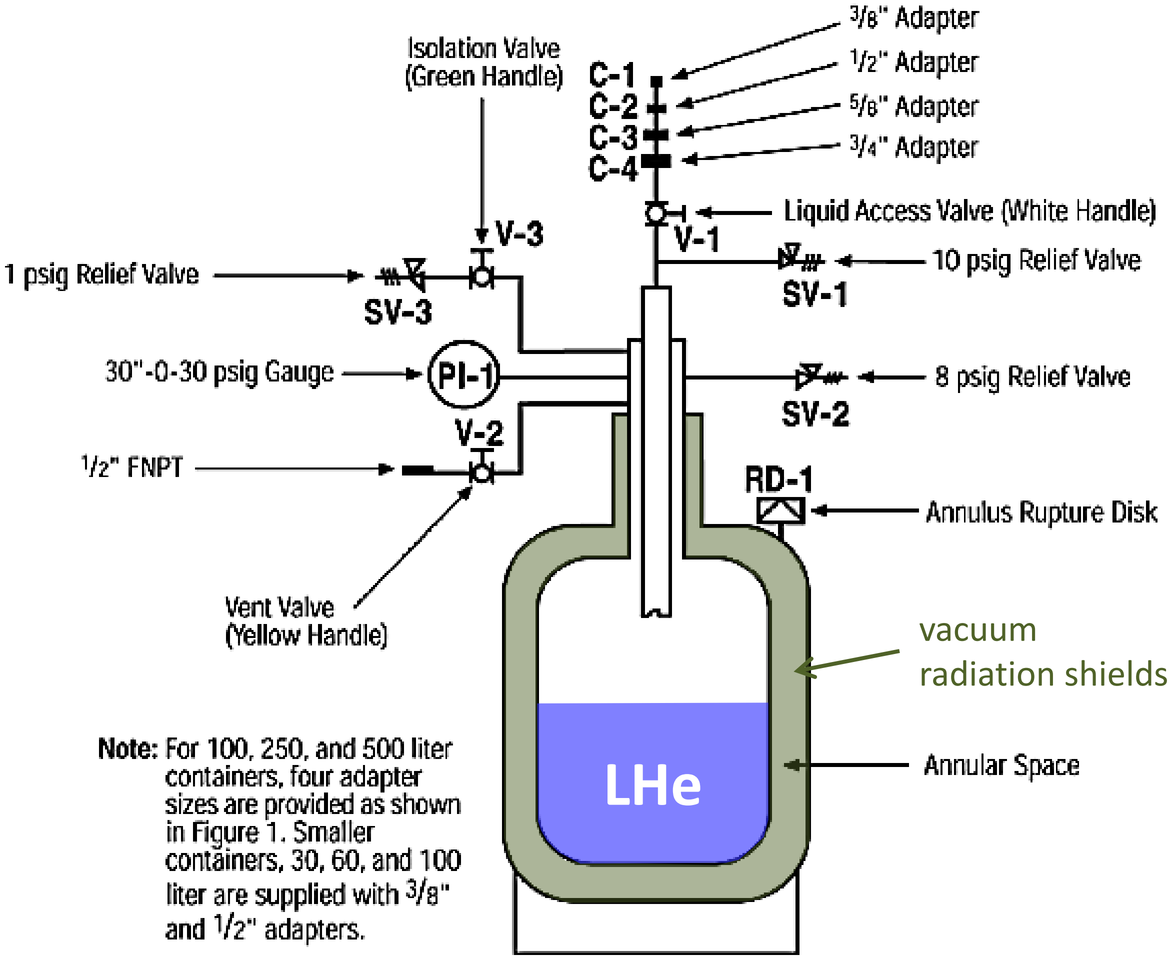
- ***1 liter of LHe / day***

STRATOS® 100 SL

STRATOS® 250 SL

STRATOS® 380 SL

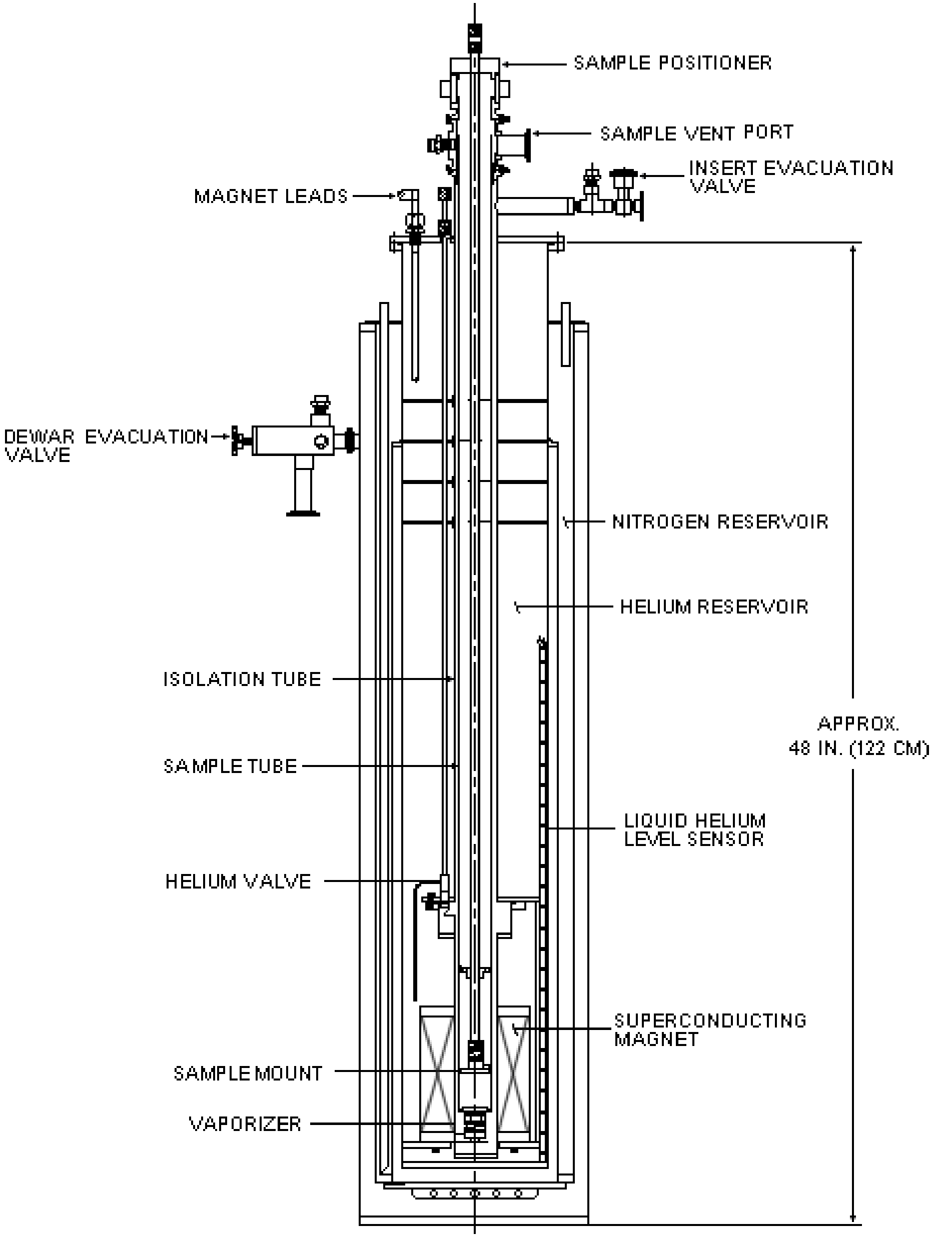
STRATOS® 1000 SL



Note: For 100, 250, and 500 liter containers, four adapter sizes are provided as shown in Figure 1. Smaller containers, 30, 60, and 100 liter are supplied with 3/8" and 1/2" adapters.

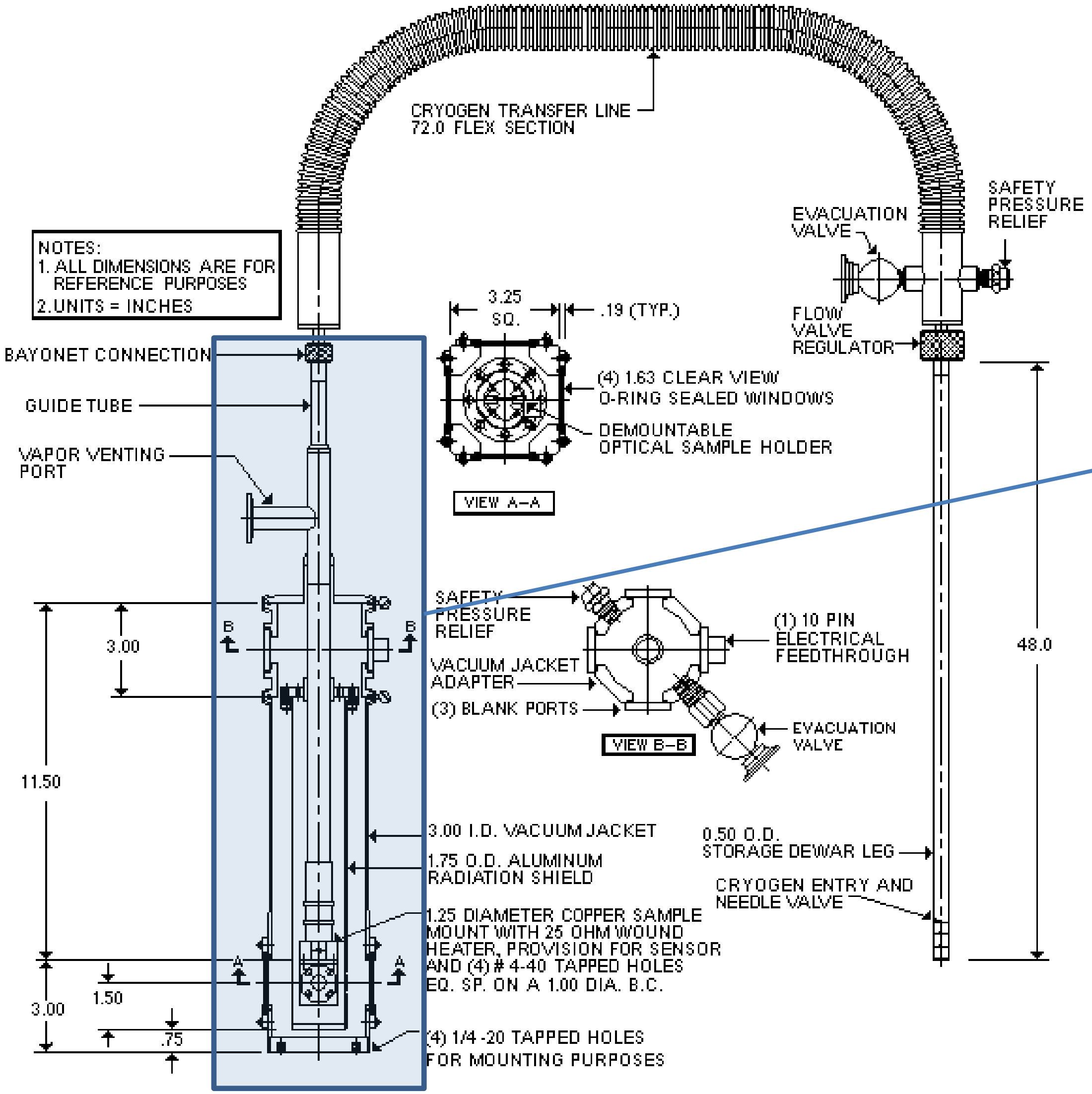
III.1.6 Evaporation Cooling

- **liquid helium bath cryostat**



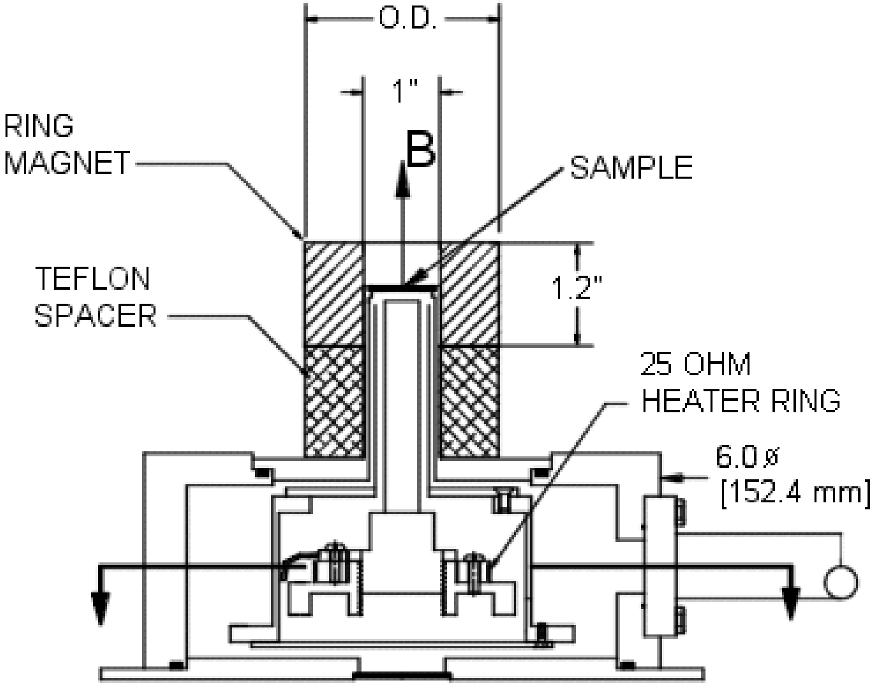
III.1.6 Evaporation Cooling

- **liquid helium gas flow cryostat (1)**



III.1.6 Evaporation Cooling

- **liquid helium gas flow cryostat (2)**



Possible Permanent Magnet Layout with ST-500 Microscopy Cryostat

- no liquid Nitrogen required
- radiation shield cooled by cold helium return gas

III.1.6 Evaporation Cooling

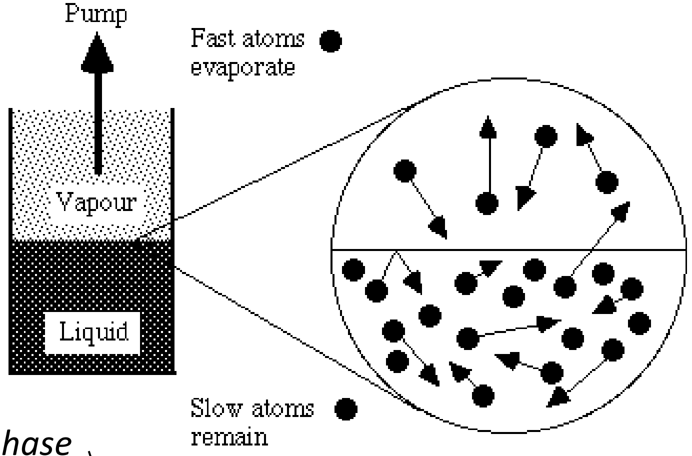
- **liquid helium temperatures below 4.2 K**
  - reduce the vapour pressure over bath of ^{4}Helium
  - temperatures down to about 1 K achievable at a typical pumping power of 10 m^3/h

Clausius-Clapeyron equation:

$$\frac{dp}{dT} = \frac{\Delta H_{\mathrm{vap}}}{(V_{\mathrm{gas}} - V_{\mathrm{liquid}})T} \simeq \frac{\Delta H_{\mathrm{vap}}}{V_{\mathrm{gas}} T} \Rightarrow \begin{array}{l} T_B(p) = \left(\frac{1}{T_0} - \frac{R\ln(p/p_0)}{\Delta H_{\mathrm{vap}}}\right)^{-1} \\ \ln\frac{p}{p_0} = \frac{\Delta H_{\mathrm{vap}}}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right) \end{array}$$

ΔH_{vap}: molar latent heat [J/mole]

$\simeq N_A \cdot E_{\mathrm{binding}} \simeq 90$ J/mole for ^{4}He

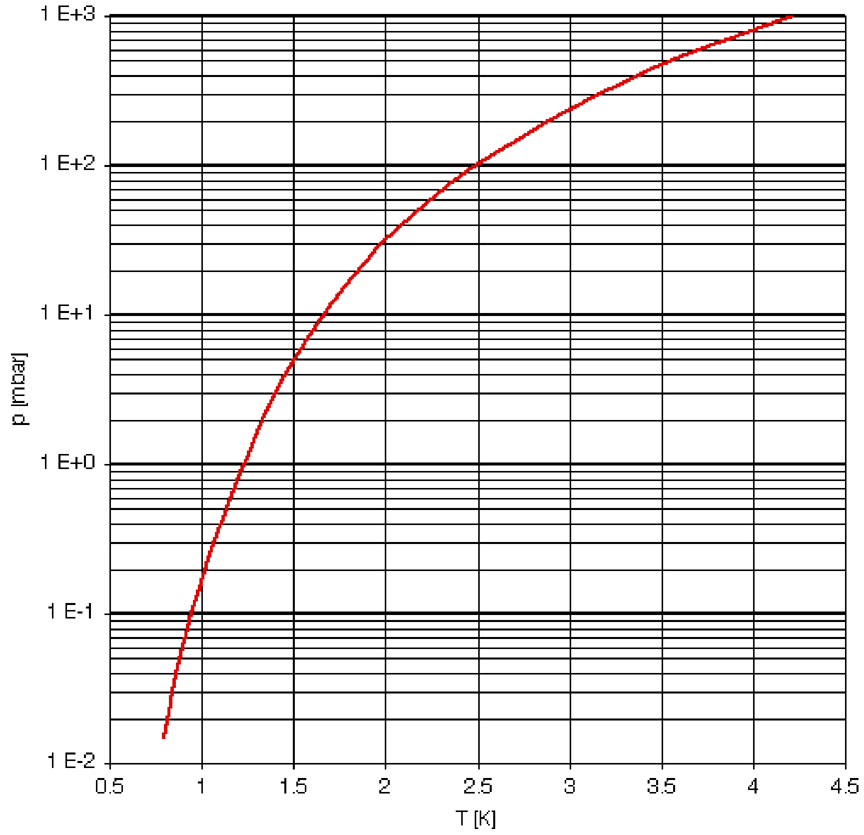


rate of atoms going to gas phase

cooling power:

$$\dot{Q} = \dot{n}_{\mathrm{gas}} \Delta H_{\mathrm{vap}} \propto p \propto \exp\left(-\frac{\Delta H_{\mathrm{vap}}}{RT}\right)$$

→ up to 10 mW cooling power @ 1.2K and 10 m^3/h



III.1.6 Evaporation Cooling

- **liquid ^{3}He (cf. chapter I)**
  - fermion
  - superfluid at 2.5mK

    formation of weakly bound fermions: Cooper pairs
  - density 59 kg/m^3
  - higher vapour pressure than ^{4}He due to smaller latent heat:

    $\Delta H_{\mathrm{vap}} = 40$ J/mole → **cooling power ≈ 80 mW @ 1.2K and 10 m^3/h** pumping speed
  - 0.3 K by pumping ^{3}He vapour

    some cm^3

    0.1 mW cooling power @ 0.3K

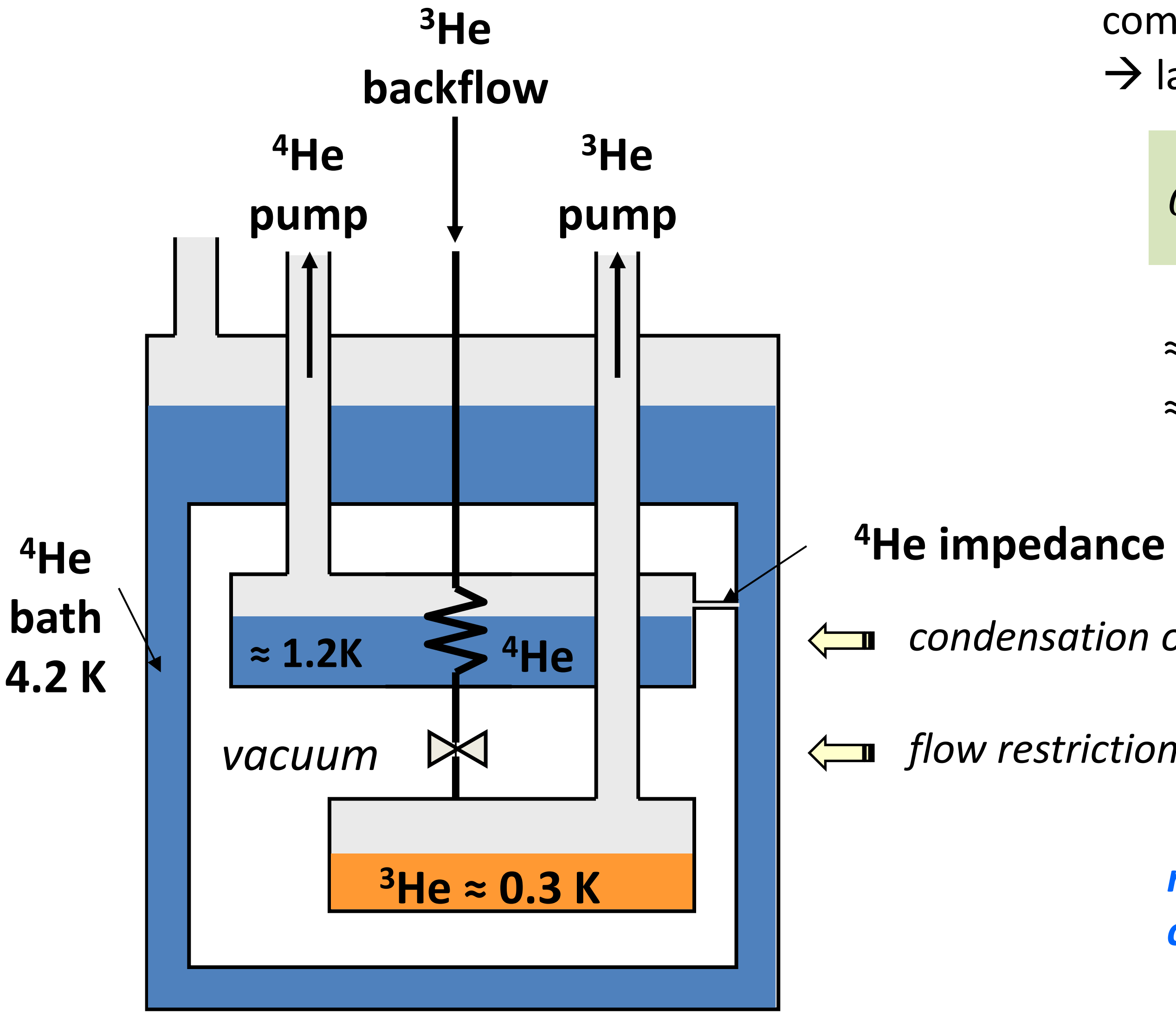
  - ^{3}He obtained by nuclear reactions

    extremely expensive

    1 liter of ^{3}He gas costs more than US $5.000 (2022)

III.1.6 Evaporation Cooling

- **liquid ^{3}He cryostat**



latent heat of ^{3}He: $\Delta H_{\text{vap}} = 40$ J/mole as compared to 90 J/mole for ^{4}He
→ larger cooling power

$$\dot{Q} = \dot{n}_{\text{gas}} \Delta H_{\text{vap}} \propto p \propto \exp\left(-\frac{\Delta H_{\text{vap}}}{RT}\right)$$

≈ 80 mW @ 1.2 K for ^{3}He as compared to
≈ 10 mW @ 1.2 K for ^{4}He

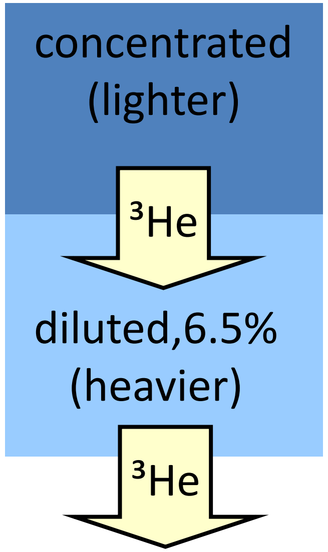
⇦ *condensation of backflowing ^{3}He gas*

⇦ *flow restriction for condensed ^{3}He*

minimum temperature ≈ 300 mK
cooling power ≈ 0.1 mW

III.1.7 ^{3}He/^{4}He Dilution Cooling

- **^{3}He/^{4}He dilution refrigeration (cf chapter I.6)**
  - operation principle:
    - → remove ^{3}He atoms from the dilute phase below $T_{\mathrm{tri}} = 0.87$ K
    - → transport of ^{3}He atoms across phase boundary to maintain equilibrium concentration
    - → corresponds to evaporation of ^{3}He from concentrated phase
    - → ***cooling effect as the latent heat of evaporation is removed***



concentrated (lighter)

diluted,6.5% (heavier)

- for Fermi liquid:

  → $\boldsymbol{C}_{\mathrm{V,concentrated}} < \boldsymbol{C}_{\mathrm{V,diluted}}\ (\boldsymbol{x_3 = 0.065})$ $\quad (C_V \propto T/T_{\mathrm{F}},\ \ T_{\mathrm{F}} \propto n_{3\mathrm{He}}^{2/3})$

- with $U = Q = \int_0^T C_V(T')\,dT' = \frac{\gamma}{2}\,T^2$ we therefore obtain ($C_V = \gamma T, \gamma =$ Sommerfeld coefficient):

$$\boldsymbol{U}_{\mathrm{concentrated}}(\boldsymbol{T}) < \boldsymbol{U}_{\mathrm{diluted}}(\boldsymbol{T})$$

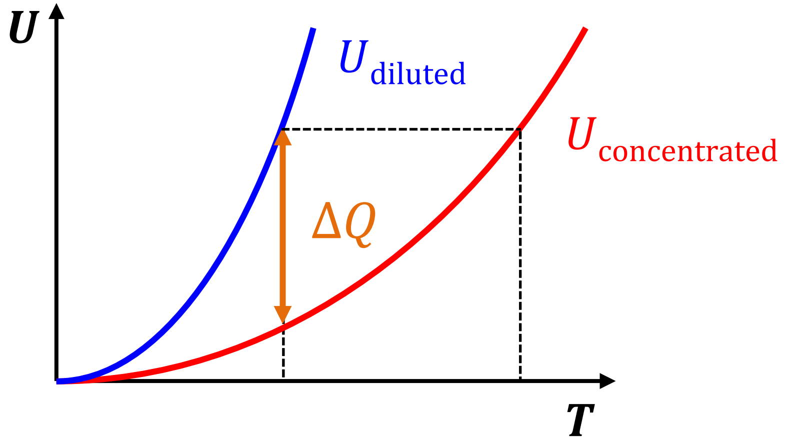
- on transition across phase boundary:

$$\mathrm{d}G = 0 = \mathrm{d}U - T\mathrm{d}S \ \Rightarrow\ \mathrm{d}U = T\mathrm{d}S = \mathrm{d}Q$$

→ removal of heat $\boldsymbol{\Delta Q = T\Delta S = T\,[S_{\mathrm{dil}}(T) - S_{\mathrm{con}}(T)]}$

➔ ***cooling effect***

→ ***^{3}He/^{4}He dilution refrigerator***



III.1.7 ^{3}He/^{4}He Dilution Cooling

- ^{3}He/^{4}He dilution refrigeration: cooling power **(cf chapter I.6)**
  - assumption: *one mole of ^{3}He crosses boundary between diluted and concentrated phase*
  - removed heat: $\Delta Q = T\Delta S = T\,[S_\mathrm{dil}(T) - S_\mathrm{con}(T)]$
  - cooling power: $\Delta\dot{Q} = \left(\frac{\Delta Q}{n_\mathrm{3He}}\right)\cdot \dot{n}_\mathrm{3He} \Rightarrow$ $\Delta\dot{Q} = \dot{n}_\mathrm{3He} T\,[s_\mathrm{dil}(T) - s_\mathrm{con}(T)]$ $\quad s_\mathrm{dil} = \frac{S_\mathrm{dil}}{n_\mathrm{3He}}, s_\mathrm{con} = \frac{S_\mathrm{con}}{n_\mathrm{3He}}$
  - since there is no volume change: $\mathrm{d}Q = \mathrm{d}U = T\mathrm{d}S = C_V \mathrm{d}T, \quad U(T) = \int_0^T C_V \mathrm{d}T', \quad S(T) = \int_0^T \frac{C_V}{T'}\mathrm{d}T'$

➢ with $C_\mathrm{mol} = \frac{\pi^2 N_A k_\mathrm{B}^2 T}{2\varepsilon_\mathrm{F}}$ and $\varepsilon_\mathrm{F} = \frac{\hbar^2}{2m_\mathrm{3He}^*}\left(\frac{3\pi^2 N_\mathrm{3He}}{V}\right)^{2/3} = \frac{\hbar^2}{2m_\mathrm{3He}^*}\left(\frac{3\pi^2 N_A}{V_\mathrm{mol}}\right)^{2/3}$ *(standard expressions for Fermi liquid)*

we obtain the entropy

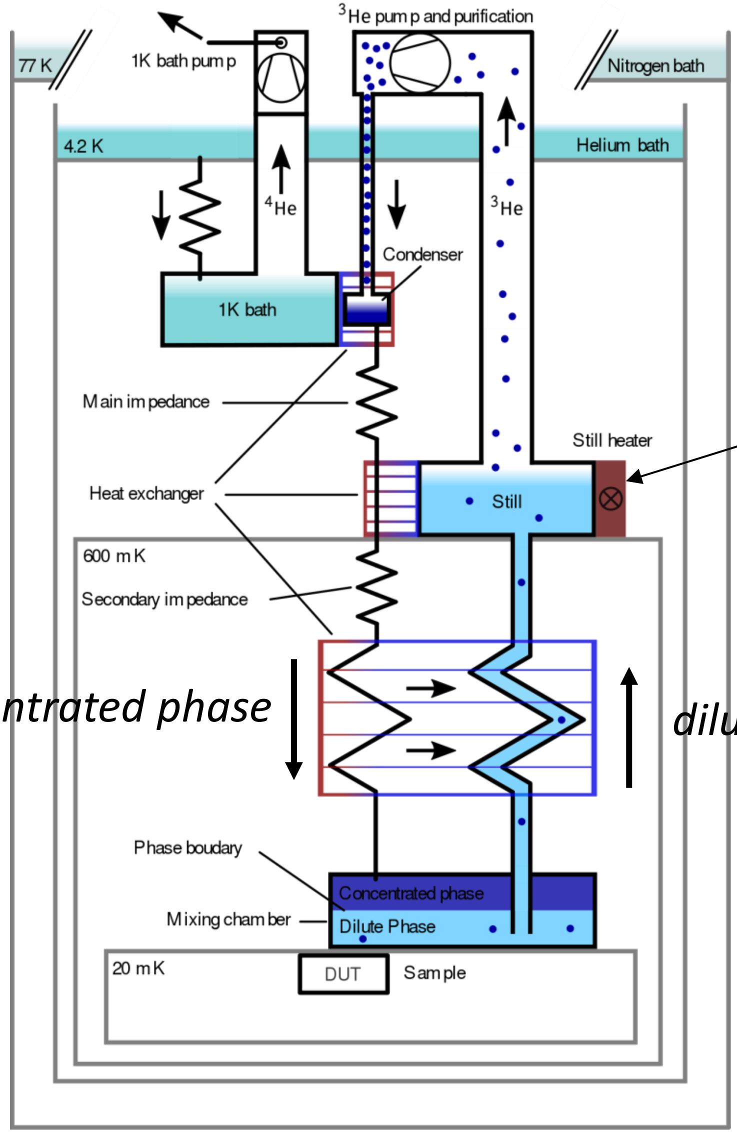
$$S_\mathrm{mol} = \frac{\pi^2 N_A k_\mathrm{B}^2 m_\mathrm{3He}^*}{\hbar^2}\left(\frac{V}{3\pi^2 N_\mathrm{3He}}\right)^{2/3} T \propto \frac{T}{n_\mathrm{3He}}$$

⇨ $S_\mathrm{con} < S_\mathrm{dil}$ (as $n_\mathrm{3,con} > n_\mathrm{3,dil}$)

⇨ $\dot{Q}(T) = 84\,\dot{n}_\mathrm{3He} T^2 [\mathrm{W}]$ (cooling power ∝ ^{3}He throughput $\dot{n}_3$ [mol/s])

III.1.7 ^{3}He/^{4}He Dilution Cooling

- **^{3}He/^{4}He dilution refrigeration: large cooling power $\dot{Q}(T) = 84\,\dot{n}_{3\mathrm{He}}T^2$[W] requires large ^{3}He throughput**



still
(≈ 06 - 0.7 K)

concentrated phase

mixing chamber
(≈ 10 - 20 mK)

heater to allow for effective pumping of ^{3}He

dilute phase

concentrated phase
dilute phase

pumping of ^{3}He generates osmotic pressure

→ ^{3}He flows from mixing chamber to still
→ only possible if ^{3}He atoms cross phase boundary
→ cooling

minimum temperature ≈ 1.5 mK

III.1.7 ^{3}He/^{4}He Dilution Cooling

- **^{3}He/^{4}He dilution refrigeration**

  – example: desired cooling power: 10^{-5} W
  still temperature: 0.7 K
  mixing chamber temperature: 10 mK

  – what is the required pumping speed ??

$$\dot{n}_3 = \frac{10^{-5}}{84\,(10^{-2})^2} = 0.0012\text{ mole / s}$$

$$\dot{Q} = 84\,\dot{n}_3\,T^2$$

we assume that ^{3}He is an ideal gas (R = 8.31 J / mole K)

$$\dot{V} = \dot{n}_3 RT/p$$

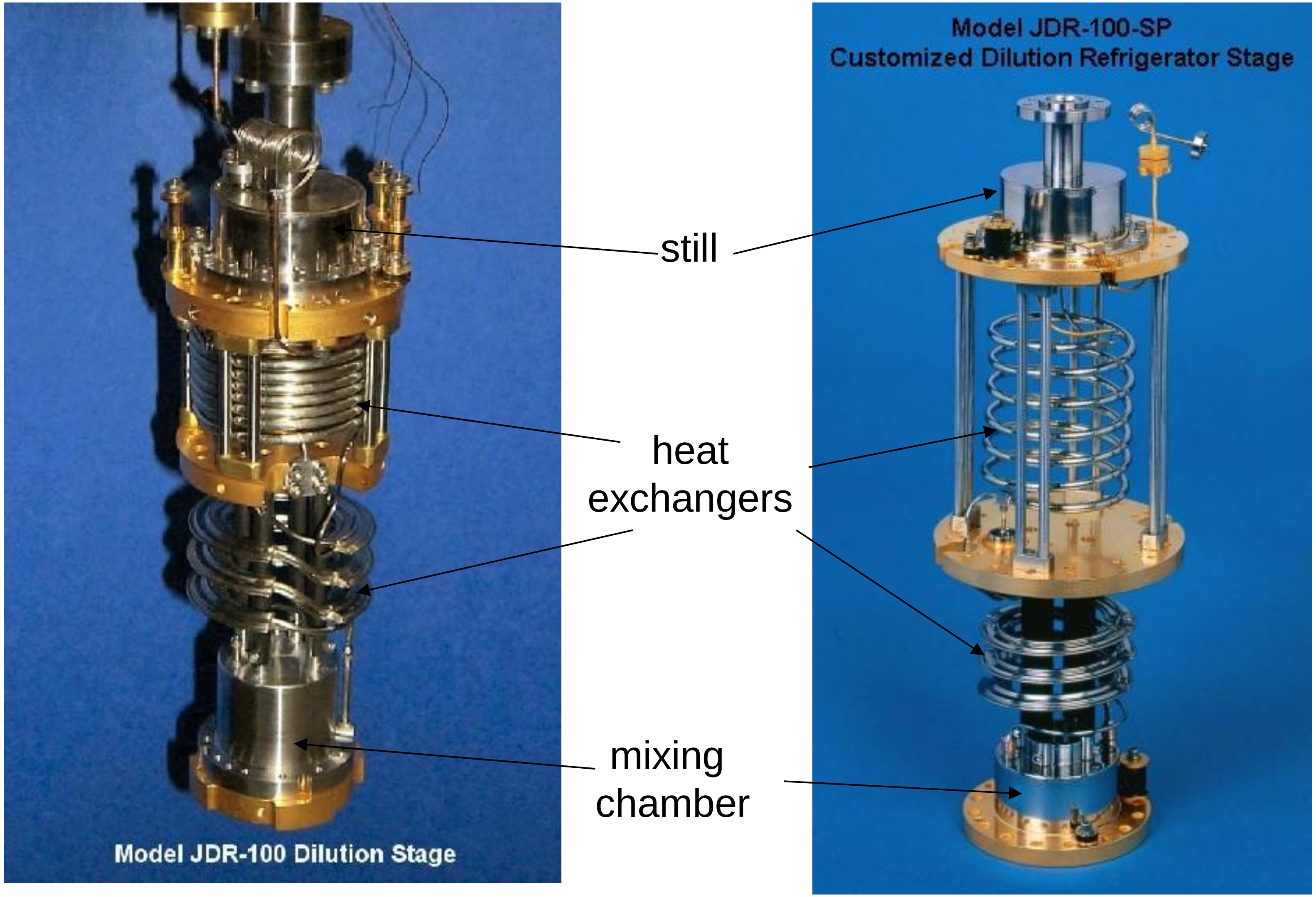
$$R = 8.31\text{ J/mole K}$$

  – numbers: vapor pressure of ^{3}He at 0.7 K: 0.0828 mbar = 8.28 Pa
  → @ 300 K we obtain:

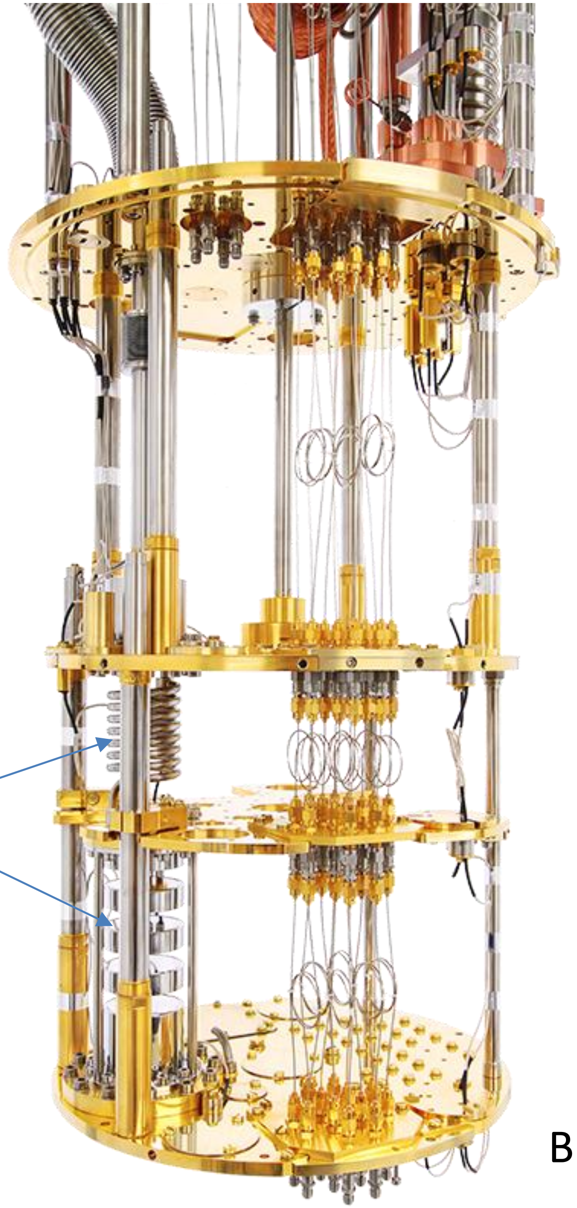
$$\dot{V} = 0.0012 \cdot 8.31 \cdot 300/8.28 = 0.363\text{ m}^3/\text{s} \approx 360\text{ l/s}$$

→ **large ^{3}He pump is required**

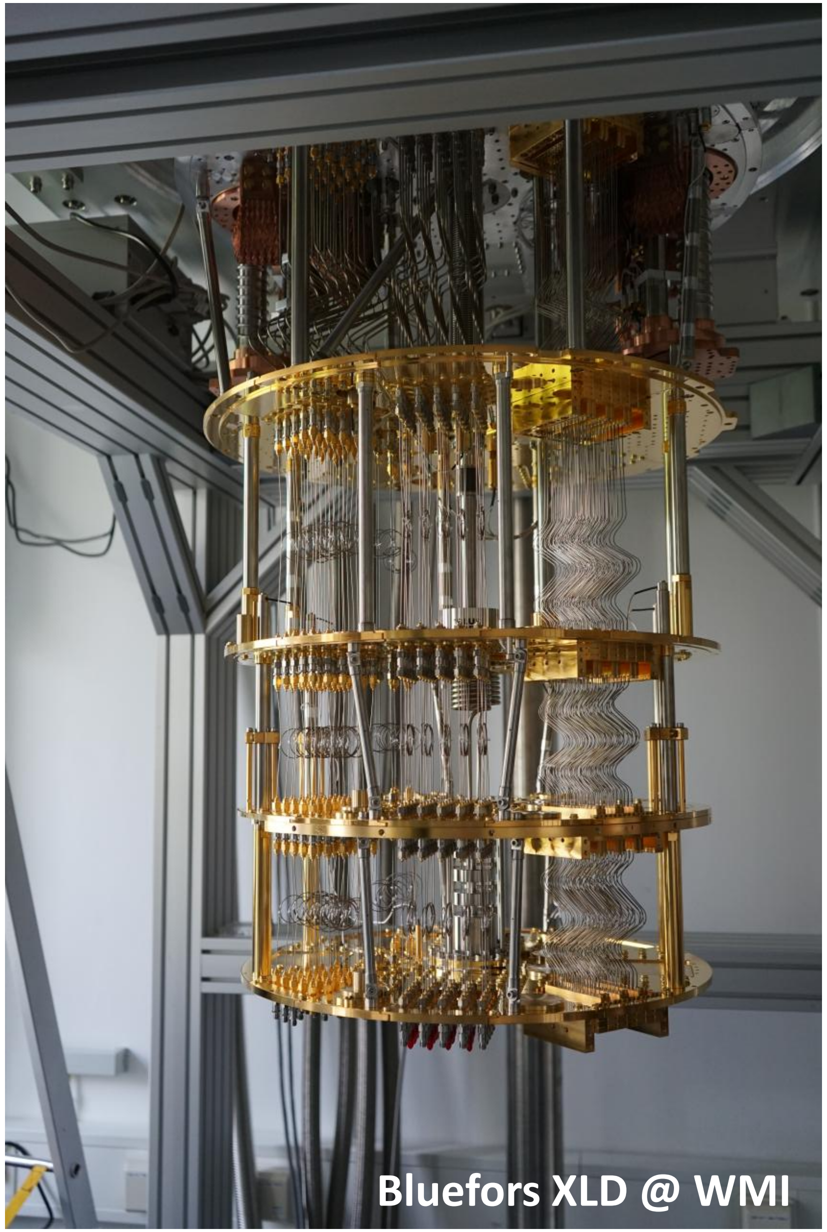
III.1.7 ^{3}He/^{4}He Dilution Cooling



III.1.7 ^{3}He/^{4}He Dilution Cooling

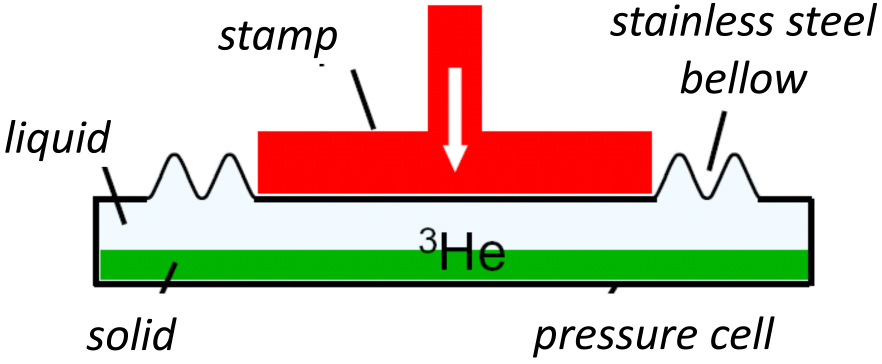


Bluefors



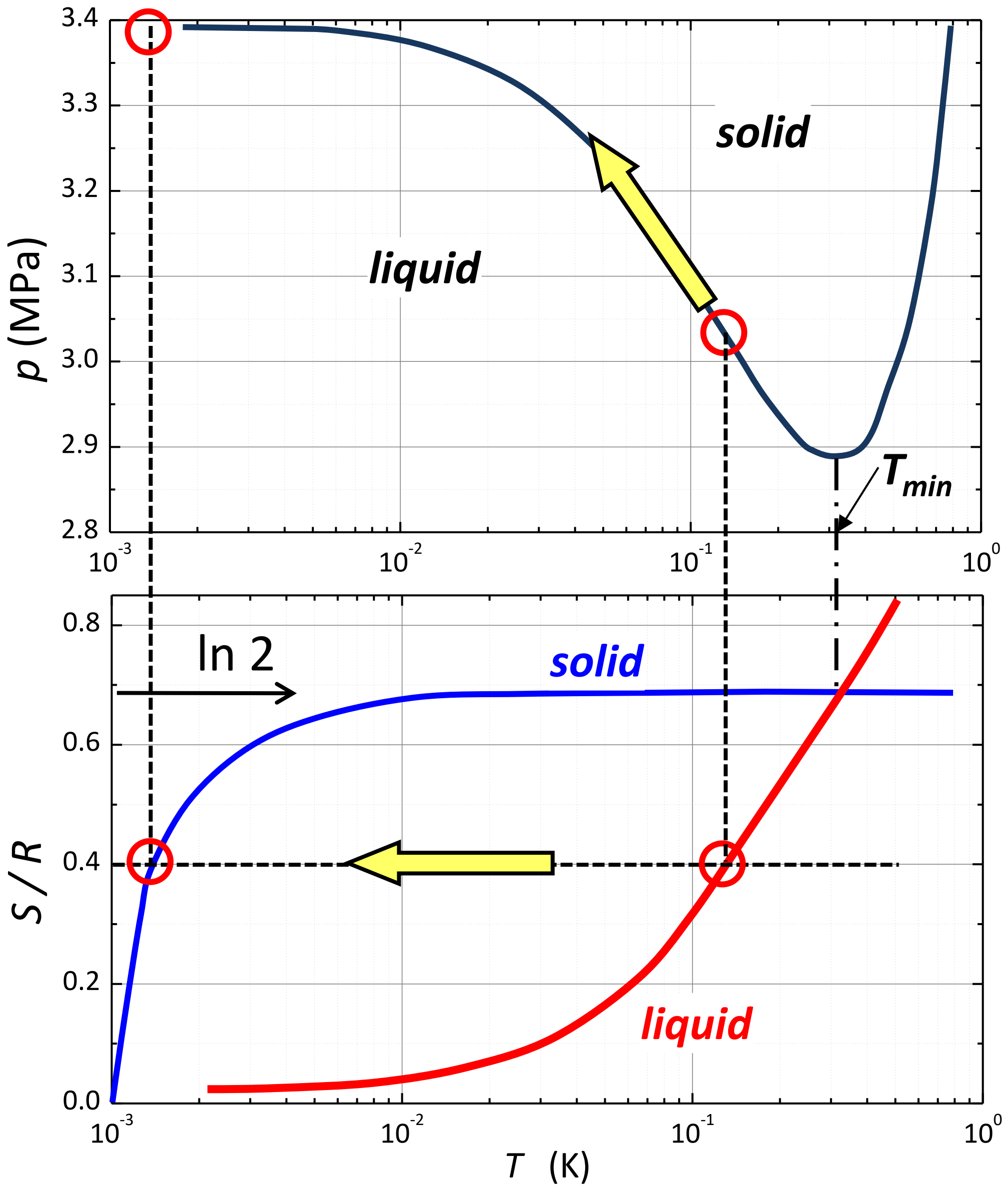
III.1.8 Pomeranchuk Cooling

- **Pomerantchuk cooling (cf. chapter I.5.2)**



  - precooling to $T < T_{\text{min}}$
  - adiabatic compression
    ➔ *solidification and cooling*
  - lowest T: ≈ 1.5 mK

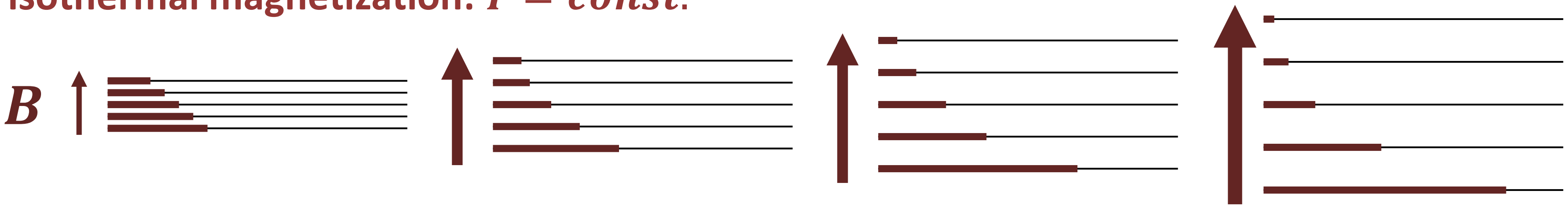
    ➔ *limitation due to antiparallel spin ordering in solid ^{3}He*



III.1.9 Adiabatic Demagnetization Cooling

- **principle: combination of isothermal magnetization and adiabatic demagnetization**

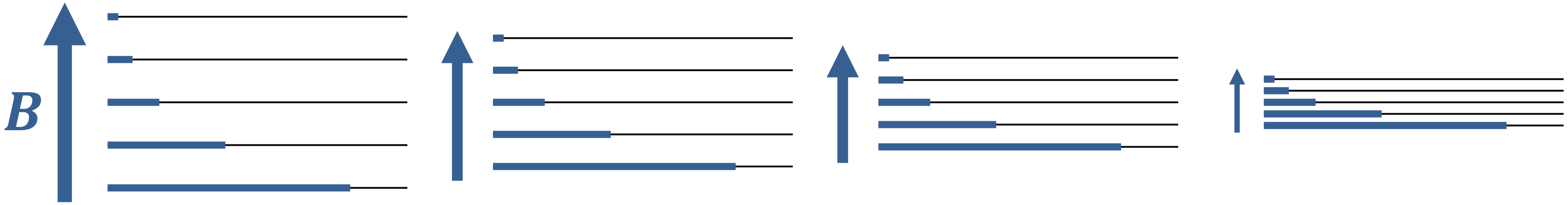
  - **isothermal magnetization: $T = const.$**



$$\frac{\Delta Q}{\Delta S} = T = const$$

➔ switch on B_{ext} at $T = const$, magnetic work is done, generated heat ΔQ is dumped into heat sink
➔ reduction of entropy if $T = const.$

  - **isenthalpic or adiabatic demagnetization: $S = const.$**

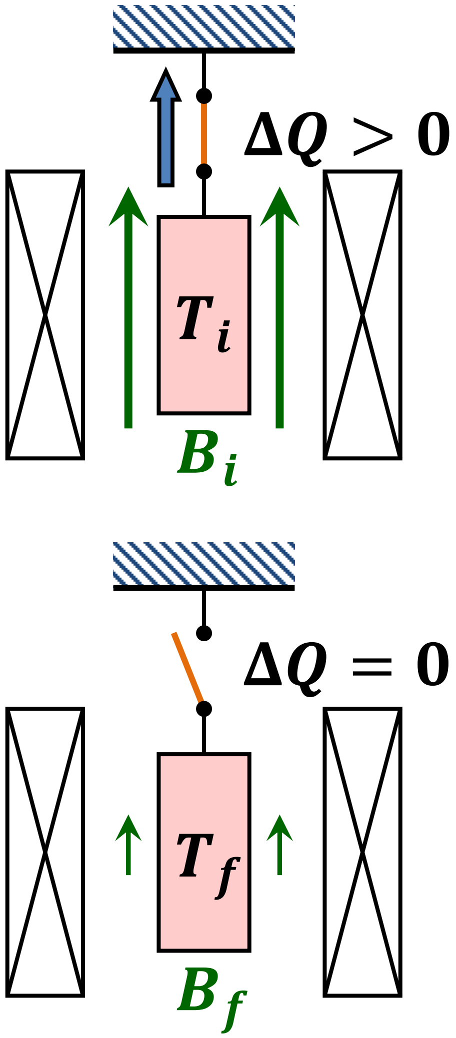


$$\frac{\Delta Q}{\Delta T} = S = const$$

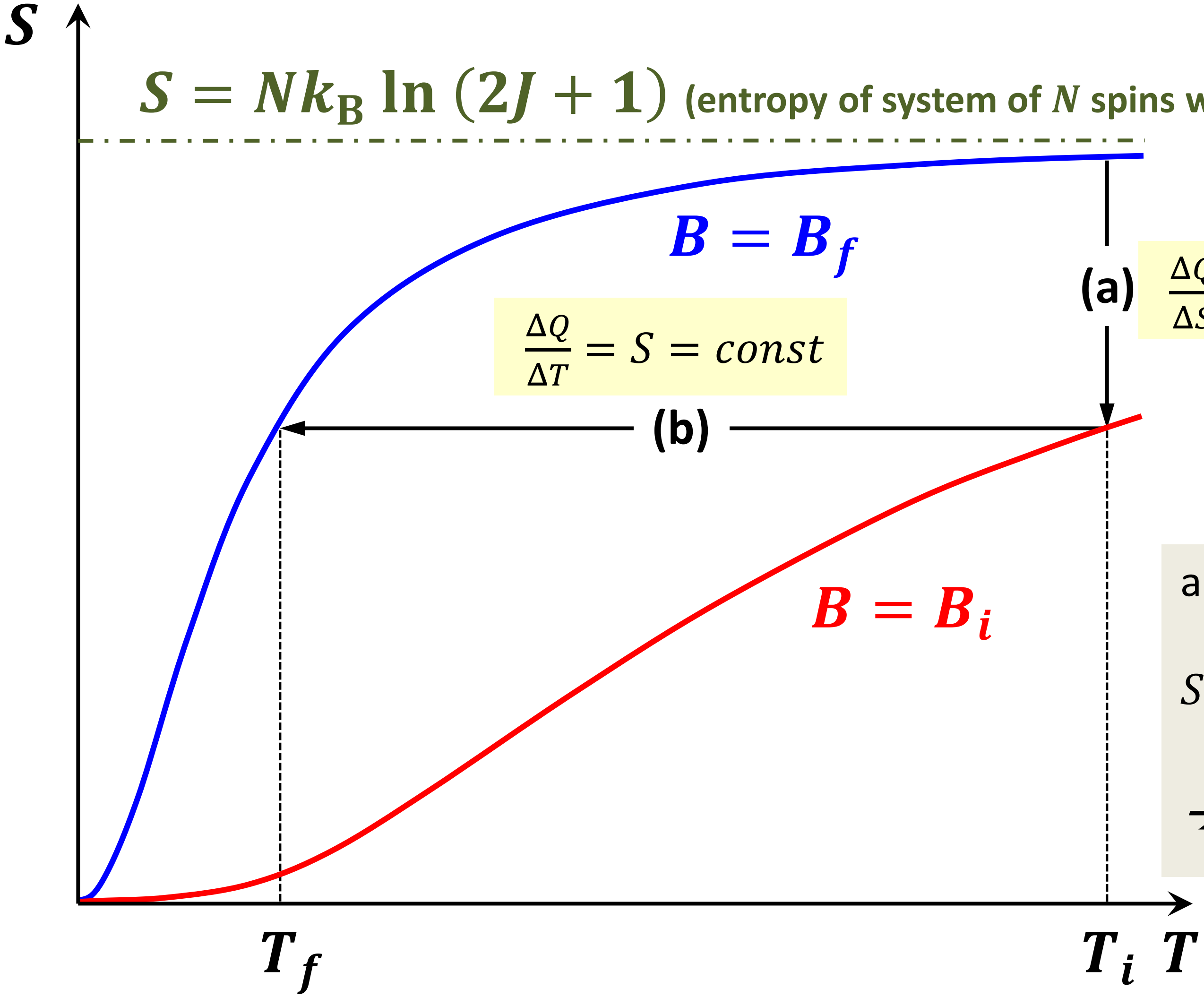
➔ switching off B_{ext} is increasing entropy S of the magnetic system
➔ to keep $S = const$, T has to be reduced and, as $\frac{\Delta Q}{\Delta T} = S = const$, heat ΔQ is removed from spin system

III.1.9 Adiabatic Demagnetization Cooling

- adiabatic demagnetization



$S = N k_\mathrm{B} \ln\left(2J+1\right)$ (entropy of system of N spins with total spin J)



$\frac{\Delta Q}{\Delta S} = T = const$

$\frac{\Delta Q}{\Delta T} = S = const$

adiabatic demagnetization:

$S\left(\frac{B_i}{T_i}\right) = S\left(\frac{B_f}{T_f}\right)$

$\rightarrow T_f = T_i \cdot \frac{B_f}{B_i}$

III.1.9 Adiabatic Demagnetization Cooling

- **thermodynamics of adiabatic demagnetization**
  - which amount of heat ΔQ_{spin} can be absorbed by the spin systems ?

$$\Delta Q_{\text{spin}}(B=0) = \int_{T_f}^{T_i} C_{\text{spin}} dT = \int_{T_f}^{T_i} T \left(\frac{\partial S_{\text{spin}}}{\partial T}\right)_B dT$$

(cooling capacity)

  - entropy of spin system with spin quantum number J for $g\mu_{\text{B}}B \ll k_{\text{B}}T$:

$$S = Nk_{\text{B}} \left\{ \ln(2J+1) - \frac{g^2 J(J+1)\mu_{\text{B}}^2}{6k_{\text{B}}^2} \quad \frac{B^2 + B_{\text{int}}^2}{T^2} \right\}$$

finite B_{int} due to finite spin-spin interaction

  - isenthalpic demagnetization ($S = const.$):

$$\frac{B_i^2 + B_{\text{int}}^2}{T_i^2} = \frac{B_f^2 + B_{\text{int}}^2}{T_f^2}$$

  - final temperature

$$\boxed{T_f = T_i \sqrt{\frac{B_f^2 + B_{\text{int}}^2}{B_i^2 + B_{\text{int}}^2}}}$$

remaining internal field due to finite magnetic interactions (should be as small as possible)

III.1.9 Adiabatic Demagnetization Cooling

- **adiabatic demagnetization**

paramagnetic salts:

e.g MAS = $MnSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$

→ cooling of electron spins

→ material with large entropy *S/R* but large B_{int}

→ lowest temperatures $T_f \approx 100$ mK

→ large cooling capacity

e.g CMN = $2Ce(NO_3)_3 \cdot 2Mg(NO_3)_2 \cdot 24H_2O$

→ cooling of electron spins

→ material with small entropy *S/R* but small B_{int}

→ lowest temperatures $T_f \approx 2$ mK

→ small cooling capacity

nuclear demagnetization:

e.g ^{63}Cu ($L = \frac{3}{2}$) or ^{65}Cu ($L = \frac{3}{2}$)

($B_{\mathrm{int}} \approx 0.3$ mT, $T_i \approx$ 10 mK, $B_i \approx 3$ T)

→ cooling of nuclear spins

→ $T_f\ (B_f = 0) \approx$ 1 μK

→ problem: transfer of spin temperature to lattice

long spin-lattice relaxation time

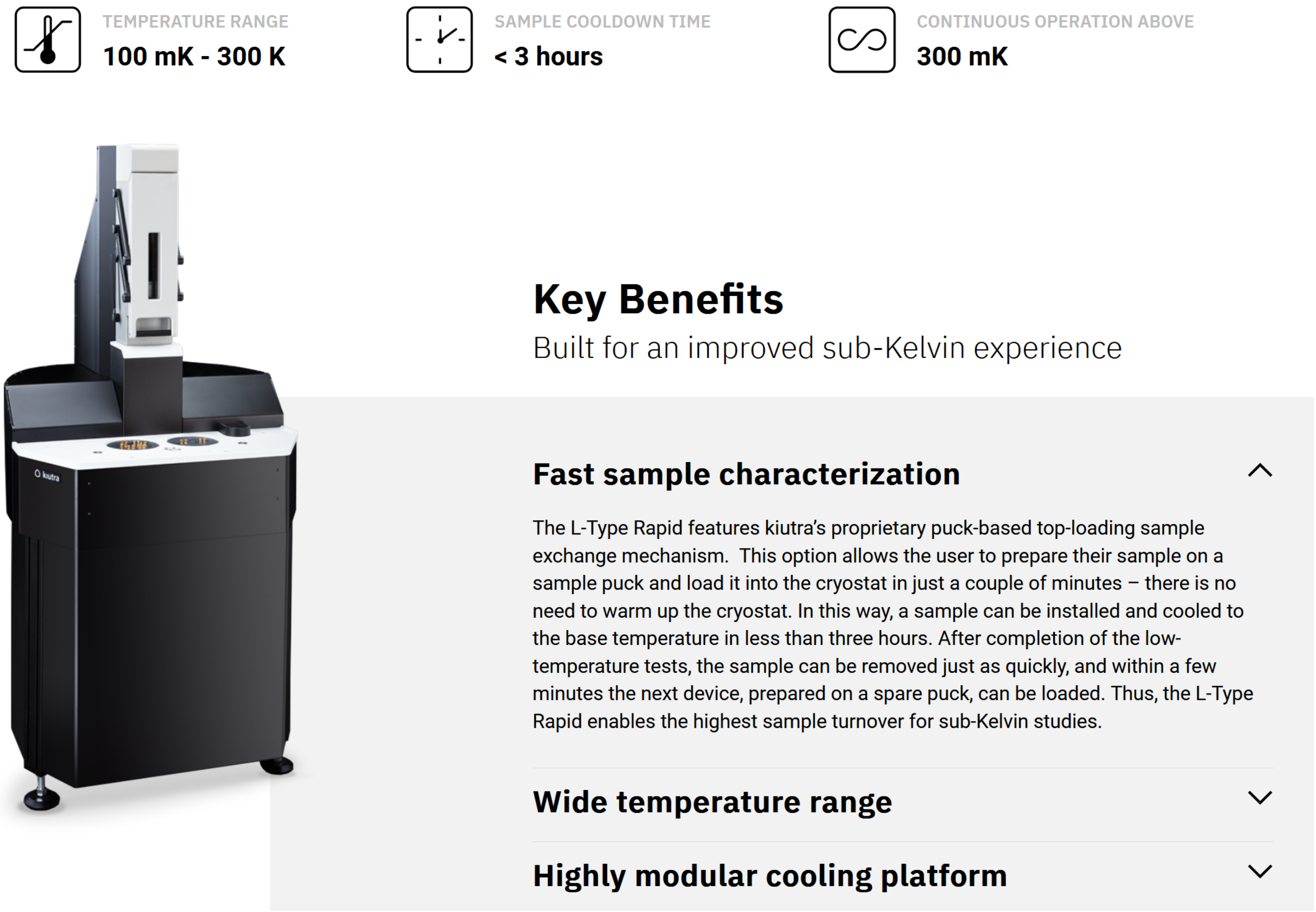
other materials: ^{141}PrNi$_5$ ($L = \frac{5}{2}$), ^{195}Pt ($L = \frac{1}{2}$)

III.1.9 Adiabatic Demagnetization Cooling

Cryogen-free Two Stage Adiabatic Demagnetization Refrigerator from Janis

A cryogen-free two stage adiabatic demagnetization refrigerator using a 4 K pulse tube cryocooler. Gallium Gadolinium Garnet (GGG) and Ferric Ammonium Alum (FAA) paramagnetic pills were used for the first and second stage of the ADR, with Kevlar string supports for each stage. The FAA stage reaches a base temperature below 50 mK, and remains below 100 mK for more than two days.

III.1.9 Adiabatic Demagnetization Cooling



TEMPERATURE RANGE
100 mK - 300 K

SAMPLE COOLDOWN TIME
< 3 hours

CONTINUOUS OPERATION ABOVE
300 mK

Key Benefits

Built for an improved sub-Kelvin experience

Fast sample characterization

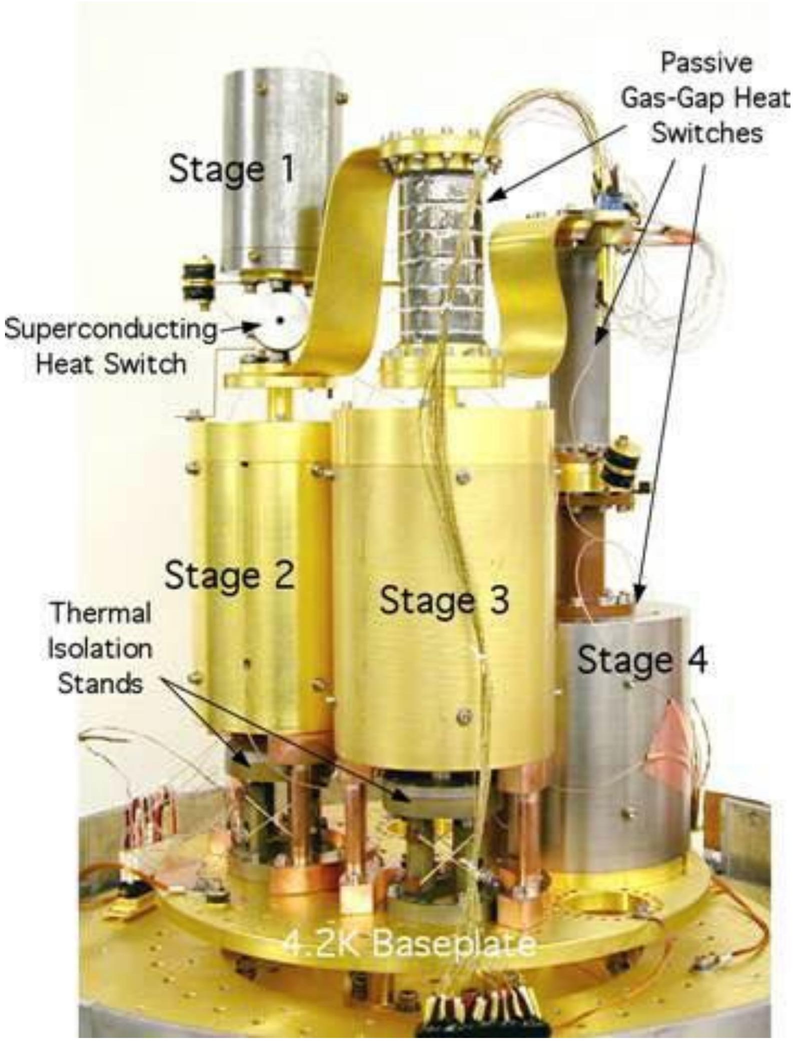
The L-Type Rapid features kiutra's proprietary puck-based top-loading sample exchange mechanism. This option allows the user to prepare their sample on a sample puck and load it into the cryostat in just a couple of minutes – there is no need to warm up the cryostat. In this way, a sample can be installed and cooled to the base temperature in less than three hours. After completion of the low-temperature tests, the sample can be removed just as quickly, and within a few minutes the next device, prepared on a spare puck, can be loaded. Thus, the L-Type Rapid enables the highest sample turnover for sub-Kelvin studies.

Wide temperature range

Highly modular cooling platform

kiutra GmbH
Flößergasse 2
81369 Munich, Germany

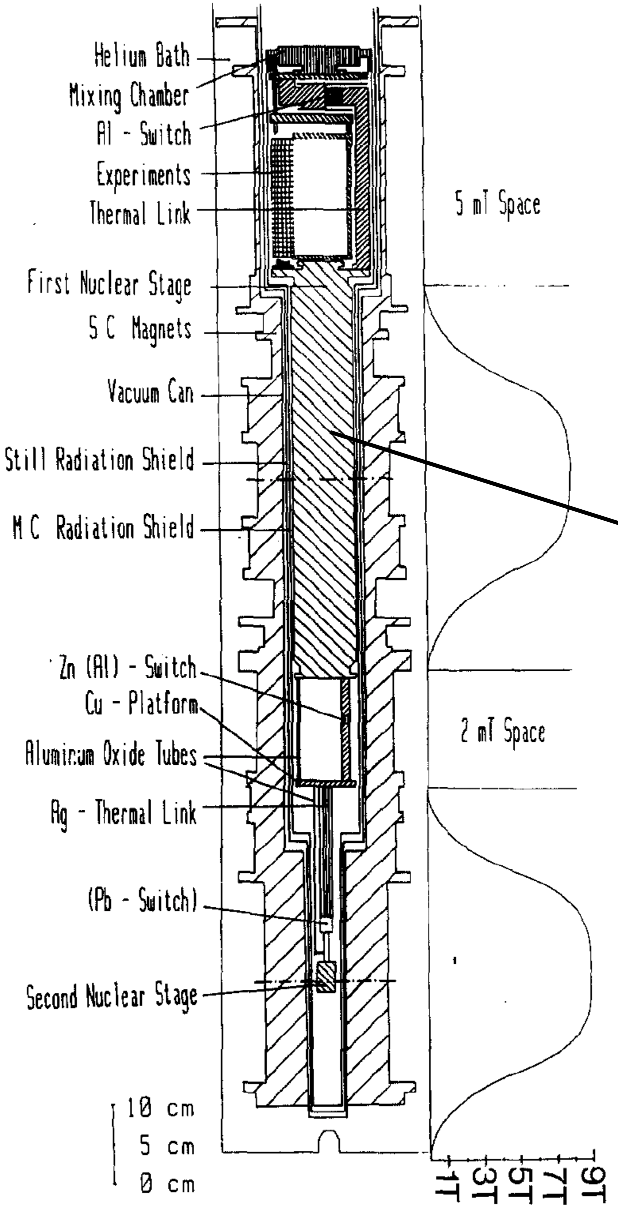
III.1.9 Adiabatic Demagnetization Cooling



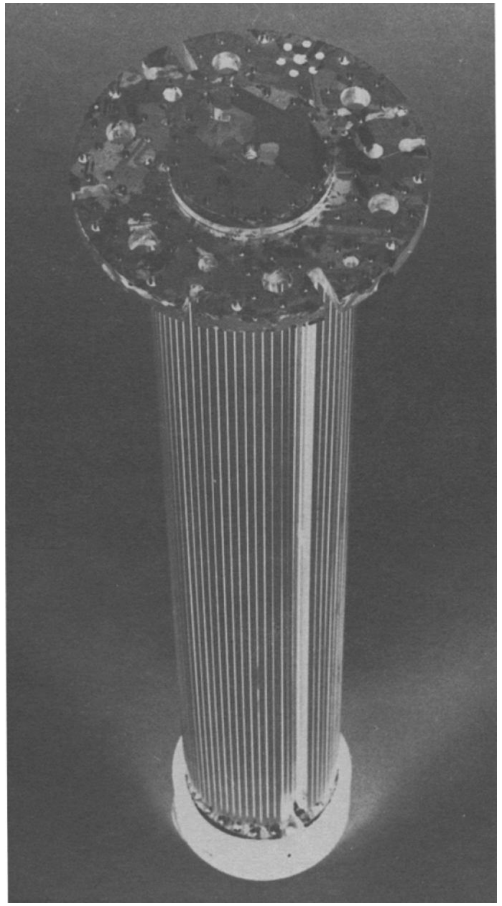
Continuous Adiabatic Demagnetization Refrigerator (CADR) under development at NASA´s Goddard Space Flight Center

- CADR to cool from below 5K to ≈ 35 mK
- advantage: no stored cryogens
  → maximizing the lifetime/mass ratio for the instrument

III.1.9 Adiabatic Demagnetization Cooling

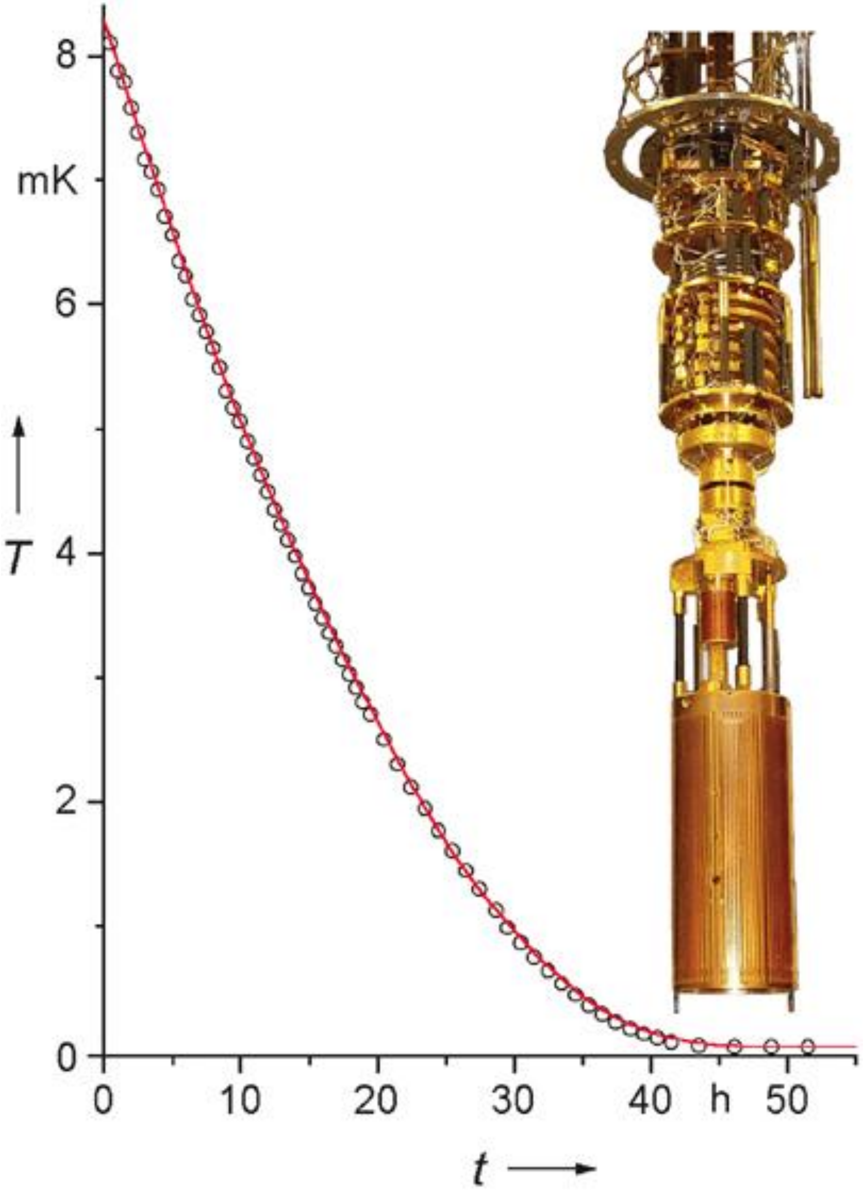


Cu demagnetization stage
(length: 525 mm, diameter: 78 mm)



R. Gloss et al.,
J. Low Temp. Phys. 73, 101 (1988)

III.1.9 Adiabatic Demagnetization Cooling



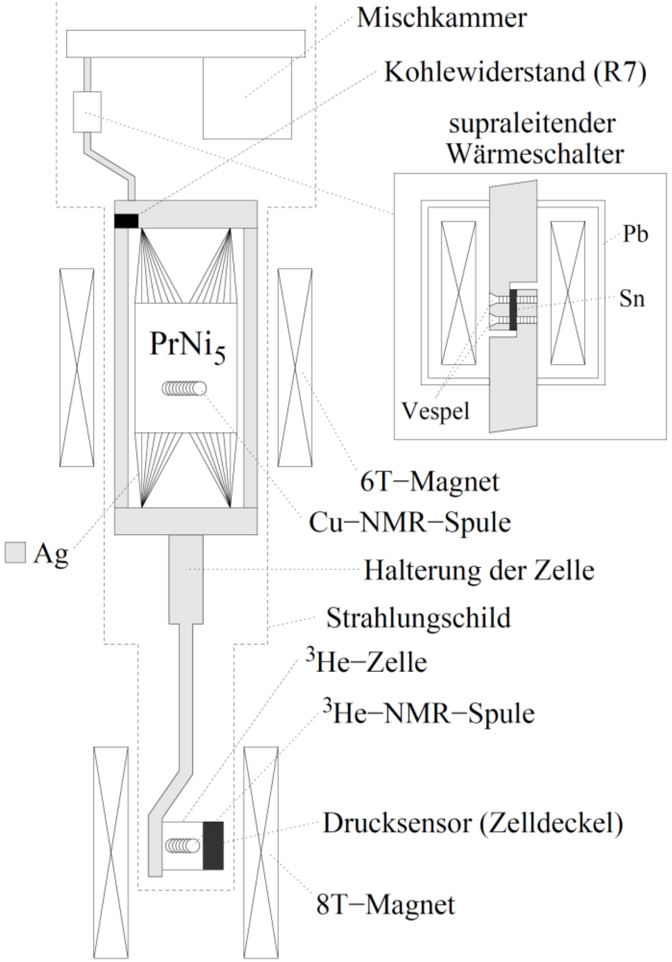
µK facility of PTB Berlin:

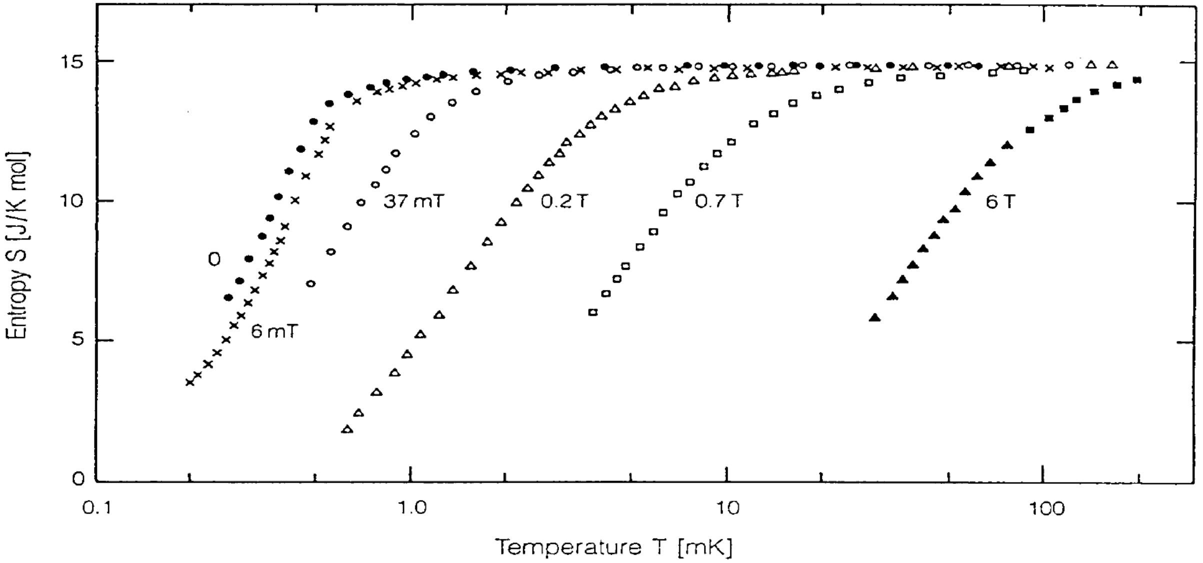
lattice temperatures measured on the 105-mol-copper stage of the Berlin microkelvin facility with Pt-NMR. The achieved minimal temperature was 23.3 µK. The red line depicts the calculated course of temperature for the thermodynamically optimized demagnetization function.

heat leak: below 1.5 nW.

III.1.9 Adiabatic Demagnetization Cooling

„Bayerische Millimühle 2“ @ WMI



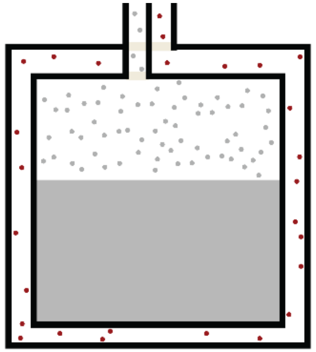


III.1.9 Adiabatic Demagnetization Cooling

- **heat links in nuclear adiabatic demagnetization systems**

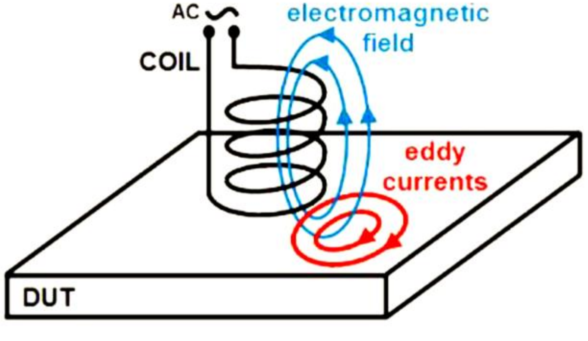
external heat leaks

- direct thermal conduction (e.g. wires)
- thermal conduction by convection
- thermal radiation



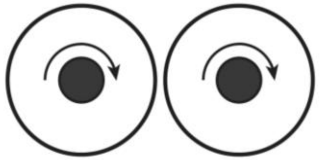
dissipation by eddy currents

- vibrations inside a stati B-field
- time-varying B-fields

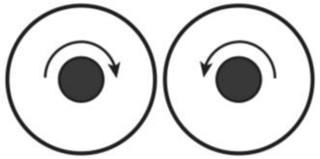


temporally decaying heat leaks

- Otho-to-para conversion of hydrogen
- hydrogen bubbles (e.g. in metals)



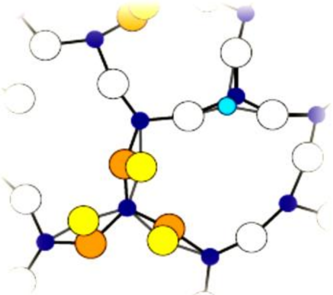
ortho hydrogen (parallel spin)

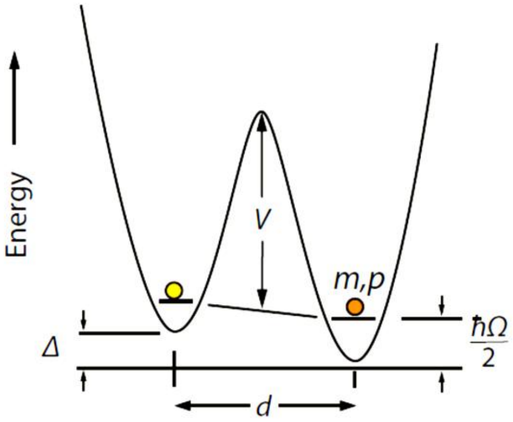


para hydrogen (anti-parallel spin)

internal sources of dissipation

- tunneling systems
- two-level fluctuators





Contents Part II: Quantum Transport in Nanostructures

Contents:

III.1 Generation of Low Temperatures

➡ III.2 Thermometry

III.2 Thermometry

III.2.1 Thermometry - Introduction

- **temperature and temperature scales**
  - ***temperature of a system in thermodynamic equilibrium:***
    defined as the relation between the amount of heat δQ incident on the system during an infinitesimal quasi-static transformation, and the variation δS of its entropy during this transformation:

$$\boxed{T = \frac{\delta Q}{\delta S}}$$

➔ for reversible Carnot process ($dS = 0$):

$$\boxed{0 = \oint \frac{\delta Q}{T}}$$

  - ***Lord Kelvin*** (1854): there is an absolute zero of temperature scale

➔ $T_0 = 0\,K = -273.15°\mathrm{C}$ $1\,\mathrm{K} = 1°\mathrm{C}$ see http://www.its-90.com

Kelvin scale   *Celsius scale (1742)*

III.2.1 Thermometry – Introduction

- **William Thomson (Lord Kelvin)**

Born	26 June 1824, Belfast, Co. Antrim, Ireland
Died	17 December 1907 (aged 83)[1], Largs, Ayrshire, Scotland [1]
Residence	Cambridge, England Glasgow, Scotland
Nationality	United Kingdom of Great Britain and Ireland
Institutions	University of Glasgow
Alma mater	Glasgow University, Peterhouse, Cambridge


Lord Kelvin by Hubert von Herkomer
http://br.geocities.com/saladefisica3/fotos/kelvin.gif

William Thomson (Lord Kelvin)

a variety of physical phenomena and concepts with which Thomson is associated are named Kelvin:

- ***Kelvin material***
- ***Kelvin water dropper***
- ***Kelvin wave***
- ***Kelvin-Helmholtz instability***
- ***Kelvin-Helmholtz mechanism***
- ***Kelvin-Helmholtz luminosity***
- ***The SI unit of temperature, Kelvin***
- ***Kelvin transform in potential theory***
- ***Kelvin's circulation theorem***
- ***Kelvin-bridge (also known as Thomson-bridge)***

III.2.1 Thermometry – Introduction

- **temperature scales**

  – **SI temperature scale**

    the SI temperature scale is the Kelvin scale. It defines the triple point of water as the numerical value of 273.16, i.e., 273.16 K. The unit of temperature in this scale is the Kelvin (K).

  – **Celsius scale:**

    the Celsius scale has units of °C (degrees Celsius) with the size of the unit equal to 1 Kelvin.

$$T(°\mathrm{C}) = T(\mathrm{K}) - 273.15$$

  – **agreement of bureaus of standards:**

    → ***ITS-90 temperature scale*** **for *T* > 0.65 K** *(Comité International des Poids et Messures 1990)*

    the ITS-90 is defined by 17 fixed points and 4 defining instruments. It spans a temperature range from 0.65 K to 10 000 K. For cryogenic purposes the three defining instruments are helium vapor pressure thermometry, gas thermometry, and platinum resistance thermometry.

  –

    → ***PLTS-2000 temperature scale*** **for lower *T*** *(Provisonal Low Temperature Scale, melting curve of ^{3}He)*

    the PLTS-2000 is defined by a polynomial, relating the melting pressure of ^{3}He to temperature from the range 0.9 mK to 1 K. The pressure to temperature relationship is based on primary thermometers such as Johnson noise and nuclear orientation.

III.2.1 Thermometry – Introduction

- **temperature scales**

  - **new definition of the Kelvin**

    The Kelvin, symbol K, is a unit of measurement for temperature

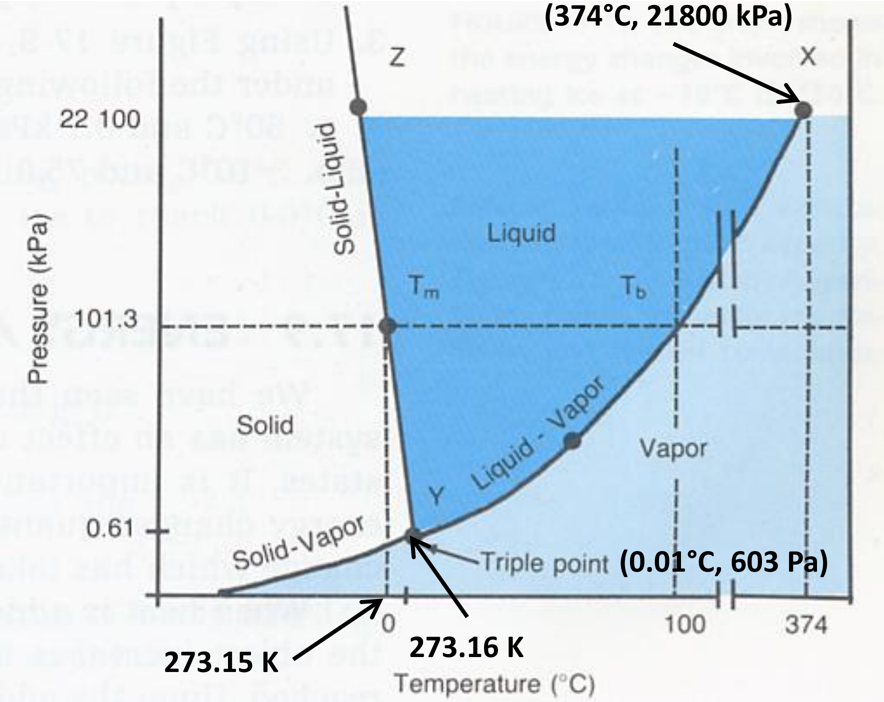
    The Kelvin scale is an absolute scale, which is defined such that 0 K is absolute zero and a change of thermodynamic temperature T by 1 Kelvin corresponds to a change of thermal energy $k_\mathrm{B}T$ by 1.380649×10^{-23} J.

    The Boltzmann constant $k_\mathrm{B} = 1.380649 \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$ was exactly defined in the 2019 redefinition of the SI base units such that the triple point of water is $T = \ 273.16 \pm 0.0001$ K

III.2.1 Thermometry – Introduction

- **The Water Triple Point**

The triple point of water is the most important defining thermometric fixed point used in the calibration of thermometers to the **International Temperature Scale of 1990 (ITS-90)**.
It is the sole realizable defining fixed point common to the **Kelvin Thermodynamic Temperature Scale (KTTS)** and the ITS-90; the assigned value on these scales is **273.16 K (0.01°C)**



III.2.1 Thermometry – Introduction

- **defining Fixed Points of the ITS-90**

Number	Temperature		Substance [a]	State [b]	Wr (T_{90})
	T_{90}/K	T_{90}/°C			
1	3 to 5	-270.15 to -268.15	He	V	
2	13.8033	-259.3467	e-H_2	T	0.001 190 07
3	~17	~-256.15	e-H_2 (or He)	V (or G)	
4	~20.3	-252.85	e-H_2 (or He)	V (or G)	
5	24.5561	-248.5939	Ne	T	0.008 449 74
6	54.3584	-218.7916	O_2	T	0.091 718 04
7	83.8058	-189.3442	Ar	T	0.215 859 75
8	234.3156	-38.8344	Hg	T	0.844 142 11
9	273.16	0.01	H_20	T	1.000 000 00
10	302.9146	29.7646	Ga	M	1.118 138 89
11	429.7485	156.5985	In	F	1.609 801 85
12	505.078	231.928	Sn	F	1.892 797 68
13	692.677	419.527	Zn	F	2.568 917 30
14	933.473	660.323	Al	F	3.376 008 60
15	1234.93	961.78	Ag	F	4.286 420 53
16	1337.33	1064.18	Au	F	
17	1357.77	1084.62	Cu	F	

[a] All substances except ^{3}He are of natural isotopic composition, e-H_2 is hydrogen at the equilibrium concentration of the ortho- and para-molecular forms.

[b] V: vapour pressure point; T: Triple Point (temperature at which the solid, liquid and vapour phases are in equilibrium); G: gas thermometer point; M,F melting point, freezing point (temperature, at a pressure of 101 325 Pa, at which the solid and liquid phases are in equilibrium)

see http://www.its-90.com

III.2.1 Thermometry – Introduction

- **temperature measurement**

  - definition of temperature via reversible Carnot process is not well suited for establishing useful measuring methods

  - in practice: use of ***fixpoints*** and ***interpolation polynoms***

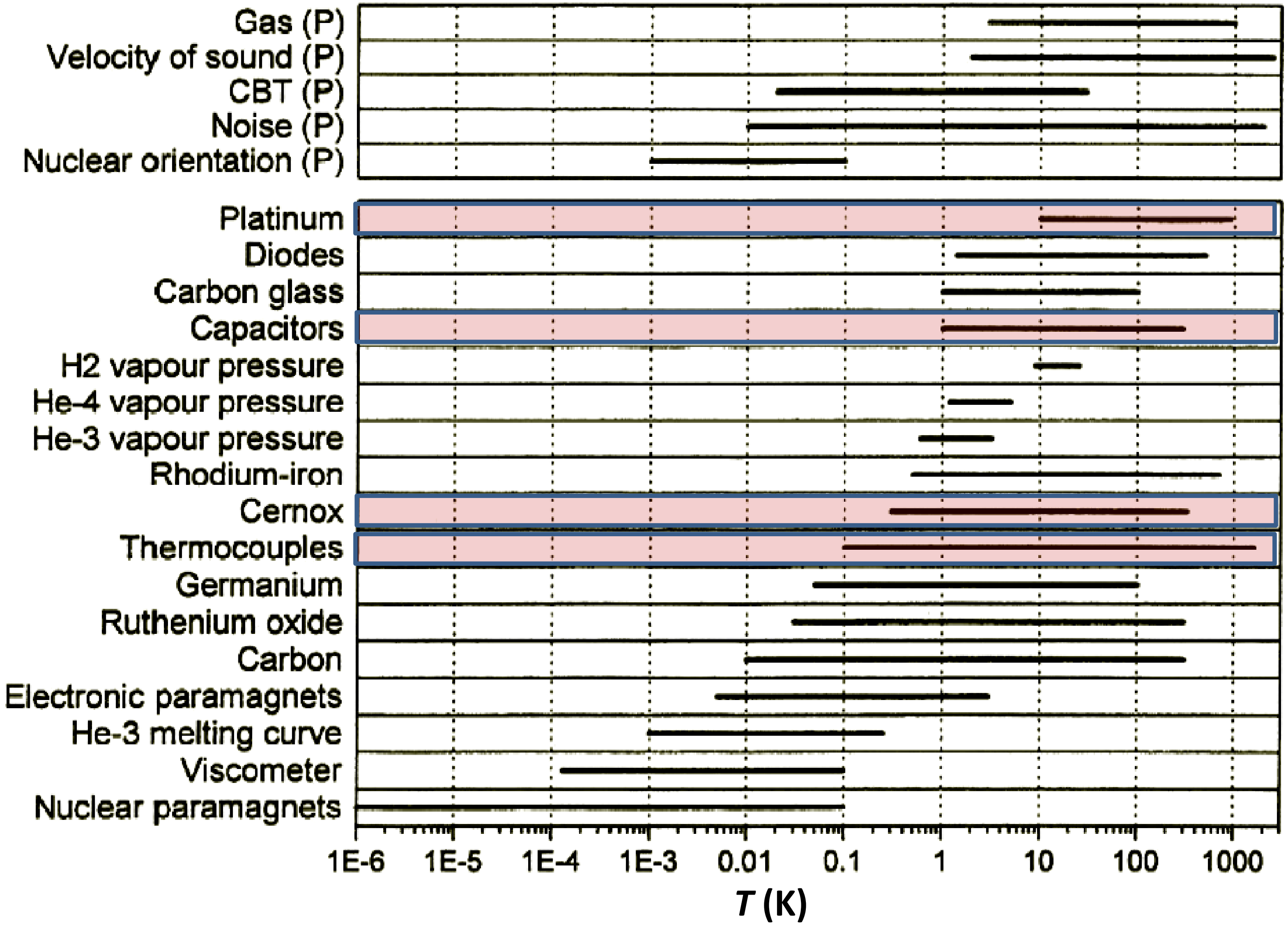
  - ***primary thermometers:***
    → measured quantity is related directly to temperature *(in a theoretically predictably way)*
    → no calibration is required

  - ***secondary thermometers:***
    → measured quantity varies with temperature in a reproducible way
    → must be calibrated using a primary thermometer

  - requirements for temperature measurement:
    - → good thermal contact between thermometer and sample
    - → low self-heating
    - → fast response to temperature changes

III.2.1 Thermometry – Introduction

- **typical temperature range of some thermometers**



III.2.1 Thermometry – Introduction

- most common thermometers for 1K < *T* < 300 K
  - gas thermometer: $p = p(T)$
    - Helium gas ≈ ideal gas down to 10K:
  - vapour pressure thermometer: $T_{\text{liquid}} = f(p_{\text{vapor}})$
    - pressure of 10 Pa corresponds to 0.4 K for ^{3}He
  - thermocouples: $V_{\text{th}} = V_{\text{th}}(T)$
  - resistance thermometry: $R = R(T)$
    - 1K - 300K
    - semiconductors (e.g. Ge doped with Arsenic has 100-500 Ω/K @ 4.2K, self-heating around 10 µA)
    - p-n junction diode (problem with high bias current → self heating)
  - capacitance thermometry: $C = C(T)$
    - based on temperature change of dielectric properties
    - virtually no magnetic field-induced errors
  - noise thermometer: $S = S(T)$
    - Johnson noise in resistor: $S_V = 4k_{\text{B}}TR$
    - like gas thermometer, but with electrons
    - with SQUID measurements: 0.1% @ 1K

III.2.1 Thermometry – Introduction

- most common thermometers for $T < 1$ K

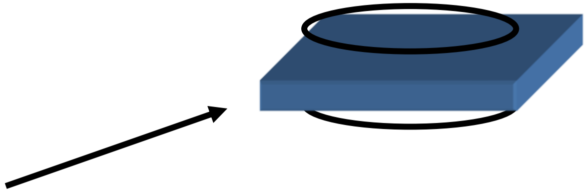
1 mK $\leq T \leq$ 1 K:

- ***magnetic suceptibility thermometer***

Curie's law: $\chi = \mu_0 \frac{M}{B} = \frac{C}{T}$

M: magnetization
B: applied magnetic field
C: Curie constant

mutual inductance between two coils: $m = m_0 \, f \, \chi$



→ Cerium magnesium nitrate (CMN) useful from 1 K – 10 mK
→ low temperature limit set by magnetic ordering at ≈ 1 mK

- ***resistance thermometers***

T < 1 mK:

- ***nuclear Magnetic Resonance (NMR) thermometer***

→ temperature dependence of spin relaxation
→ platinum ideal choice for NMR thermometry

Contents Part II: Quantum Transport in Nanostructures

Contents:

III.2.2 Primary Thermometers

- **gas thermometers**

  - ideal gas would be a perfect thermometer:

$$pV = nRT$$

➔ *measure pressure at constant volume*

  - for real gases life is more complicated ➔ deviations from ideal behavior

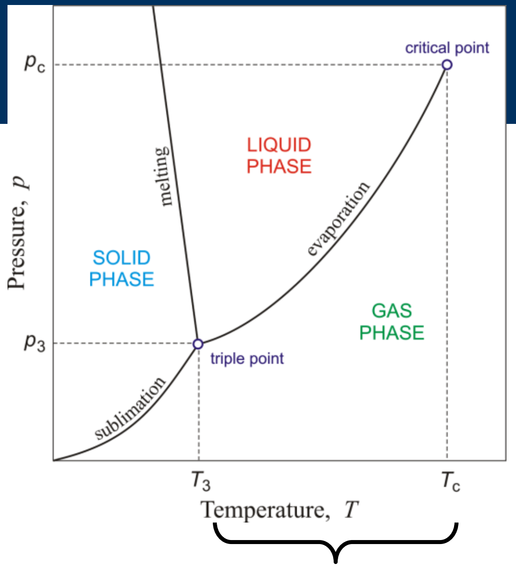
$$pV = n\{RT + b(T)p + c(T)T^2 + d(T)T^3 + \cdots\}$$

virial coefficients (tabulated ITS-90 values)

  - systematic errors:
    - → dead volumes
    - → thermal expansion of cell, elastic deformation of cell
    - → adsorption and desorption from walls

➔ *mainly used in calibration laboratories !*

III.2.2 Primary Thermometers



limited T range

- **vapour pressure thermometry**
  - Clausius-Clapeyron equation: $$\frac{dp}{dT} = \frac{L(T)}{(V_{\mathrm{gas}} - V_{\mathrm{liquid}})T} \approx \frac{\Delta H_{\mathrm{vap}}(T)}{V_{\mathrm{gas}}T}$$
  - for ideal gas ($pV = RT$): $$R \ln p + const. = \int \frac{\Delta H_{\mathrm{vap}}(T)}{T^2} dT$$
  - if $\Delta H_{\mathrm{vap}}(T)$ is known → ***determine T of liquid (e.g. He) via measurement of He pressure above liquid***
  - in practice, a set of secondary vapour pressure scales is used:

    ITS-90: $$\boxed{T = \sum_i A_i \left\{\frac{(\ln p + B)}{C}\right\}^i}$$ *(in principle no primary thermometer !!)*
  - with ^{3}He and ^{4}He: ITS defined down to 0.65 K

III.2.2 Primary Thermometers

- **helium vapor pressure thermometer**

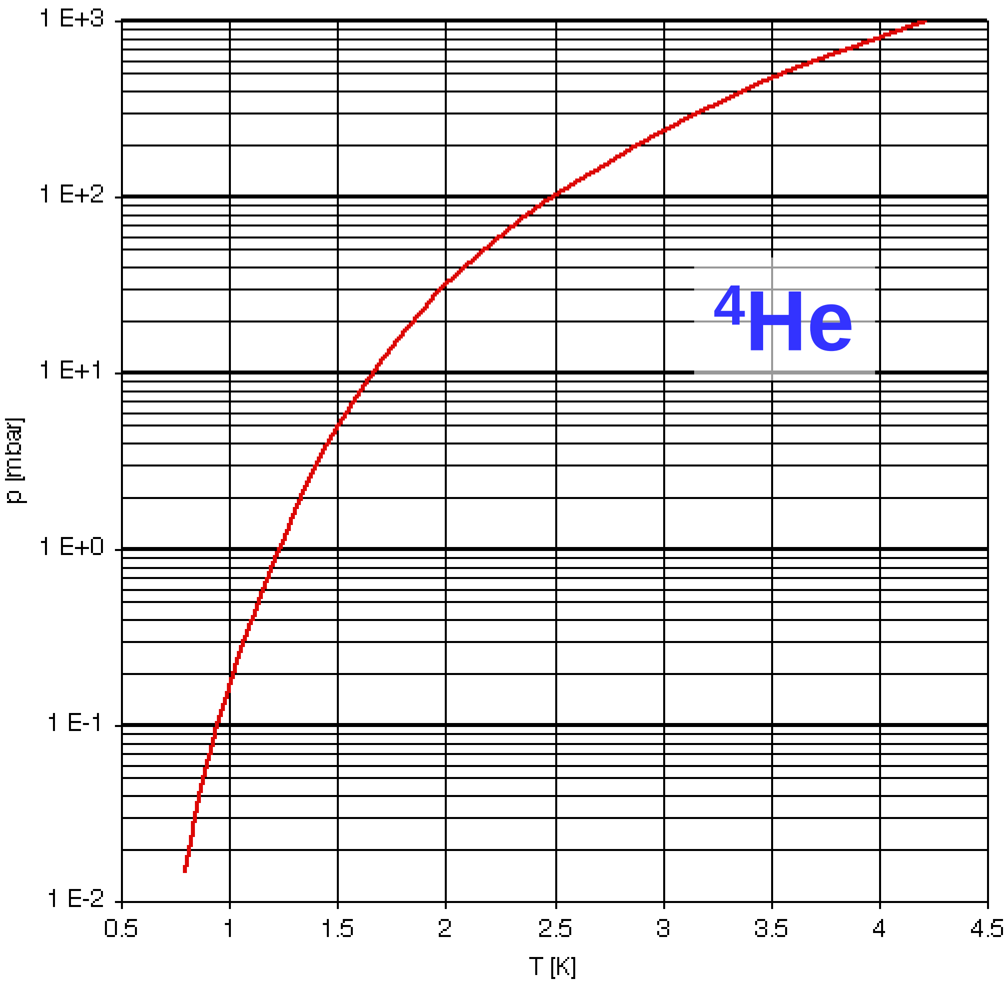
values of the constants for the helium vapour-pressure, and the temperature range for which each equation, identified by its set of constants, is valid (see http://www.its-90.com).

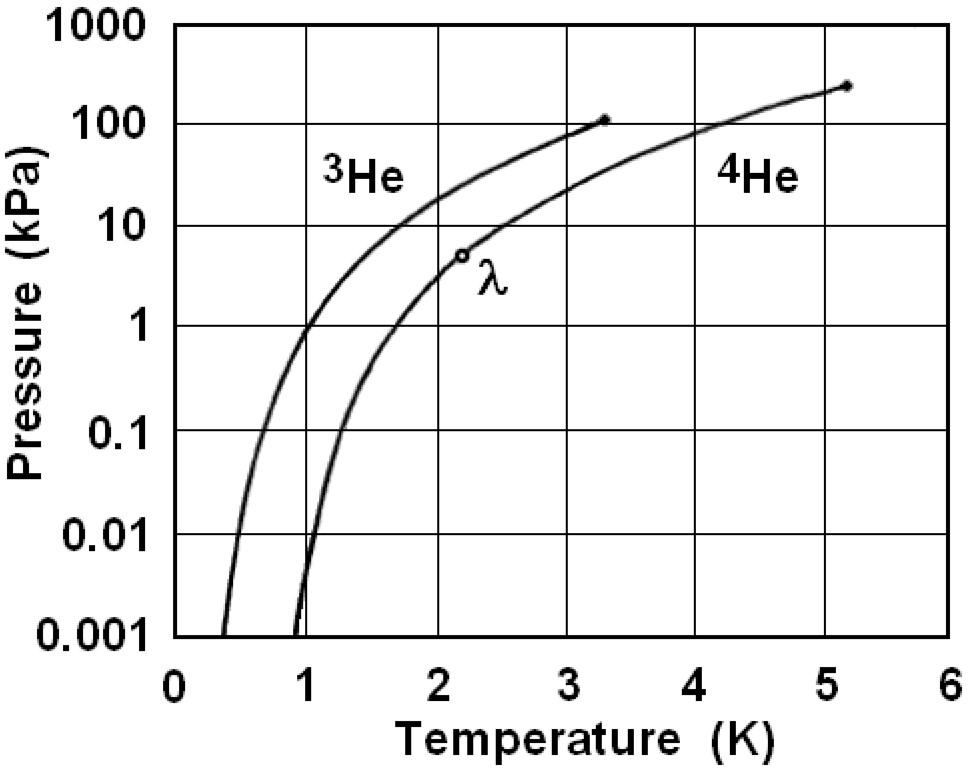
	^{3}He **0.65 K to 3.2 K**	**^{4}He** **1.25 K to 2.1768 K**	**^{4}He** **2.1768 K to 5.0 K**
A_0	1.053 447	1.392 408	3.146 631
A_1	0.980 106	0.527 153	1.357 655
A_2	0.676 380	0.166 756	0.413 923
A_3	0.372 692	0.050 988	0.091 159
A_4	0.151 656	0.026 514	0.016 349
A_5	-0.002 263	0.001 975	0.001 826
A_6	0.006 596	-0.017 976	-0.004 325
A_7	0.088 966	0.005 409	-0.004 973
A_8	-0.004 770	0.013 259	0
A_9	-0.054 943	0	0
B	7.3	5.6	10.3
C	4.3	2.9	1.9

$$T = \sum_i A_i \left\{ \frac{(\ln p + B)}{C} \right\}^i$$

III.2.2 Primary Thermometers

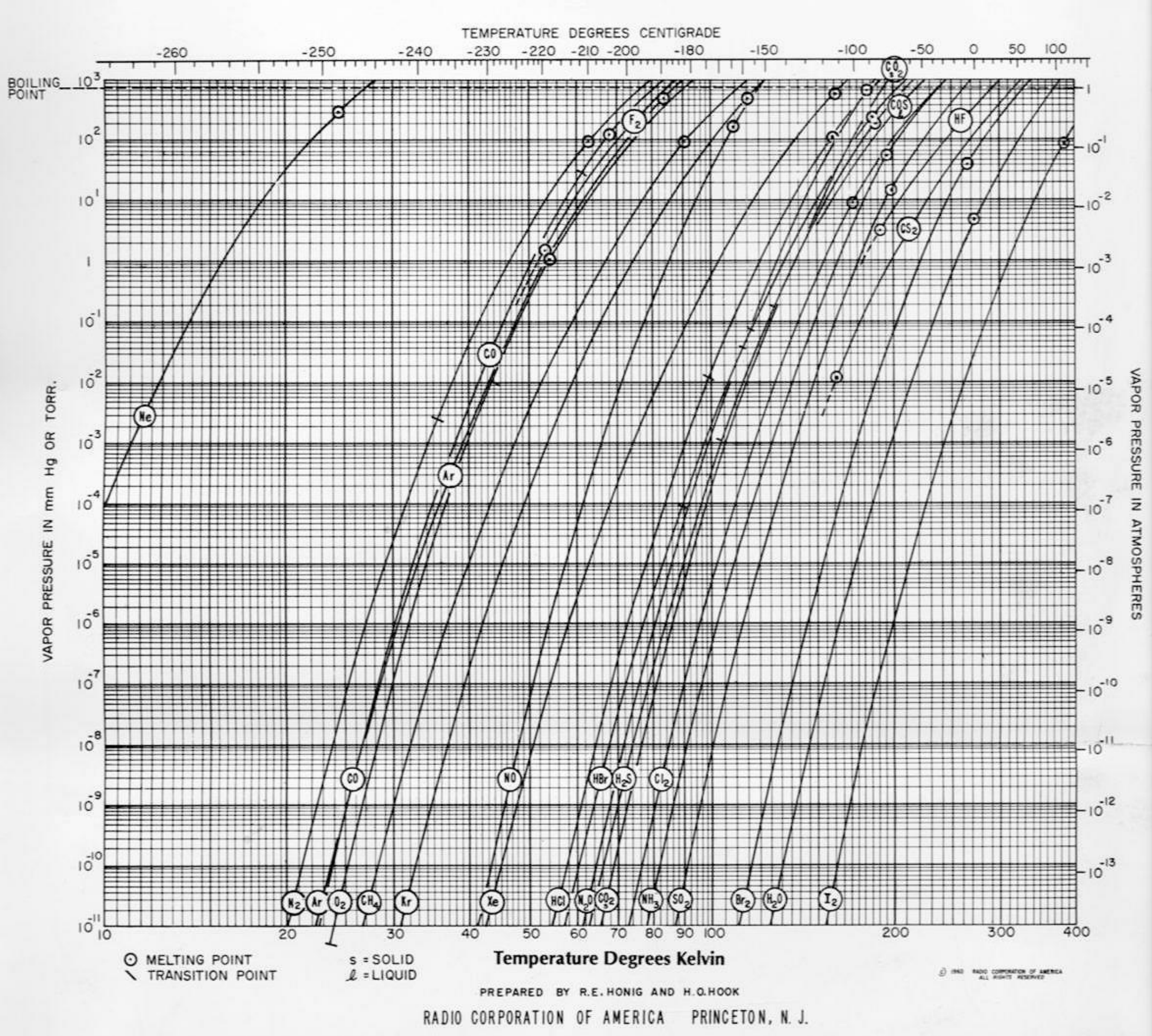
- **helium vapor pressure thermometer**





III.2.2 Primary Thermometers

- **vapor pressure thermometry**

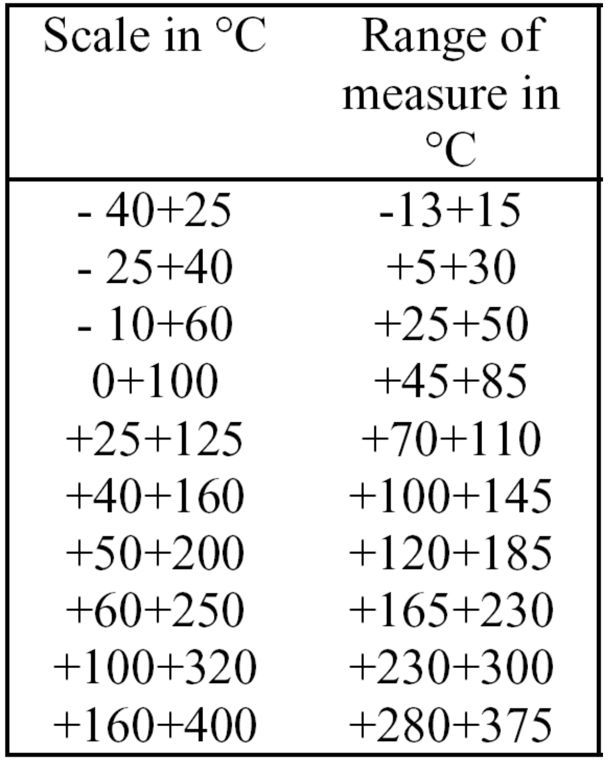


www.mallister.com/graphics/vapor3.jpg

III.2.2 Primary Thermometers

- **vapor pressure thermometry**

www.bm-industries.com



Scale in °C	Range of measure in °C
- 40+25	-13+15
- 25+40	+5+30
- 10+60	+25+50
0+100	+45+85
+25+125	+70+110
+40+160	+100+145
+50+200	+120+185
+60+250	+165+230
+100+320	+230+300
+160+400	+280+375

III.2.2 Primary Thermometers

- **^{3}He melting curve thermometry**

  – use of melting curve of ^{3}He to define PLTS-2000 temperature scale ***down to 0.9 mK***

  – polynom for melting curve:

$$p = \sum_{i=-3}^{9} \alpha_i T^i$$

→ coefficients given by **PLTS-2000**
→ also use of 4 fix points

minimum of melting curve,
transition temperatures to A and B phase and
afm order of nuclear spins in solid ^{3}He

III.2.2 Primary Thermometers

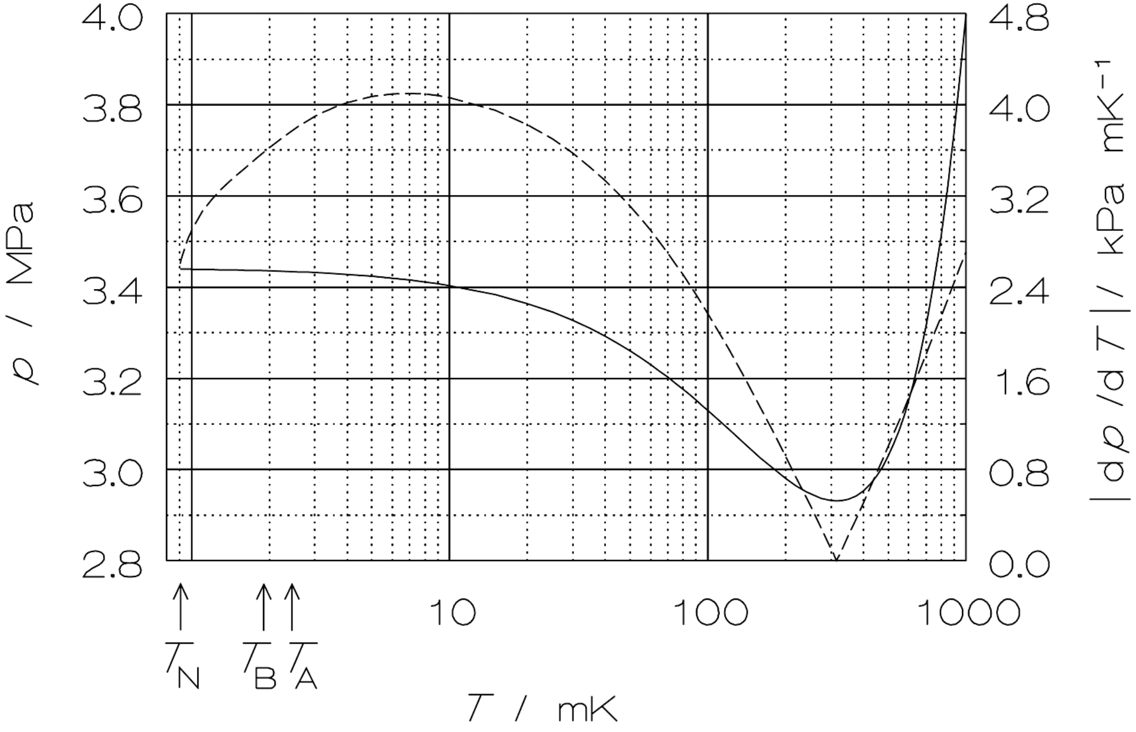
- **^{3}He melting curve thermometry**

$$p/\mathrm{MPa} \;=\; \sum_{i=-3}^{+9} a_i (T_{2000}/\mathrm{K})^i$$

$$a_{-3} = -1.385\,544\,2 \cdot 10^{-12}$$
$$a_{-2} = 4.555\,702\,6 \cdot 10^{-9}$$
$$a_{-1} = -6.443\,086\,9 \cdot 10^{-6}$$
$$a_{0} = 3.446\,743\,4 \cdot 10^{0}$$
$$a_{1} = -4.417\,643\,8 \cdot 10^{0}$$
$$a_{2} = 1.541\,743\,7 \cdot 10^{1}$$
$$a_{3} = -3.578\,985\,3 \cdot 10^{1}$$
$$a_{4} = 7.149\,912\,5 \cdot 10^{1}$$
$$a_{5} = -1.041\,437\,9 \cdot 10^{2}$$
$$a_{6} = 1.051\,853\,8 \cdot 10^{2}$$
$$a_{7} = -6.944\,376\,7 \cdot 10^{1}$$
$$a_{8} = 2.683\,308\,7 \cdot 10^{1}$$
$$a_{9} = -4.587\,570\,9 \cdot 10^{0}$$

source: R.L. Rusby et al. (2001)

melting curve of ^{3}He



Point	p/MPa	T_{2000}/mK
minimum	2.93113	315.24
A	3.43407	2.444
A-B	3.43609	1.896
Néel	3.43934	0.902

III.2.2 Primary Thermometers

- **noise thermometry**

  - **Nyquist theorem**: $S_V = 4 k_\mathrm{B} T R$

    voltage noise power spectral density (V²/Hz)

    temperature

    resistance

    valid only in the low frequency limit $f \ll k_\mathrm{B}T / h$ (≈ 20 GHz @ 1K)

  - temperature determined by measurement of $\boldsymbol{S_V}$ and $\boldsymbol{R}$

  - ***example***: $R = 10\ \mathrm{k}\Omega$, $T = 1$ K, band width $\Delta f = 105$ Hz

    → $(S_V\,\Delta f)^{1/2} \approx 2 \cdot 10^{-7}$ V → sensitive amplifier required → SQUID preamplifier

    → $P \approx 10^{-18}$ W

III.2.2 Primary Thermometers

- **superconducting fix point thermometers**
  - based on the precise measurement of the transition temperatures of superconductors
  - available from NIST at Boulder

ITS-90

Element	TC90/ K
Cd	0.5200 ± 0.0030
Zn	0.8500 ± 0.0030
Al	1.1810 ± 0.0025
In	3.4145 ± 0.0025
Pb	7.1997 ± 0.0025
Nb	9.2880 ± 0.0025

NIST fixpoint device

$^*T^C$ (K)	Substance	Width (mK)	Reproducibility (1 standard deviation) (mK)
0.0160	°W	0.7	± 0.2
0.0230	°Be	0.2	0.1
0.0990	°Ir	0.8	0.1
0.1605	°AuAl2	0.3	0.1
0.2065	°AuIn2	0.4	0.15
0.5190	+Cd	0.5-8.0	0.3
0.8510	+Zn	2.5-10	0.4
1.1796	+Al	1.5-4.0	0.4
3.4145	+In	0.5-2.5	0.2
7.1996	+Pb	0.6-2.0	0.2

°SRM 768 [Schooley and Soulen (1982)]

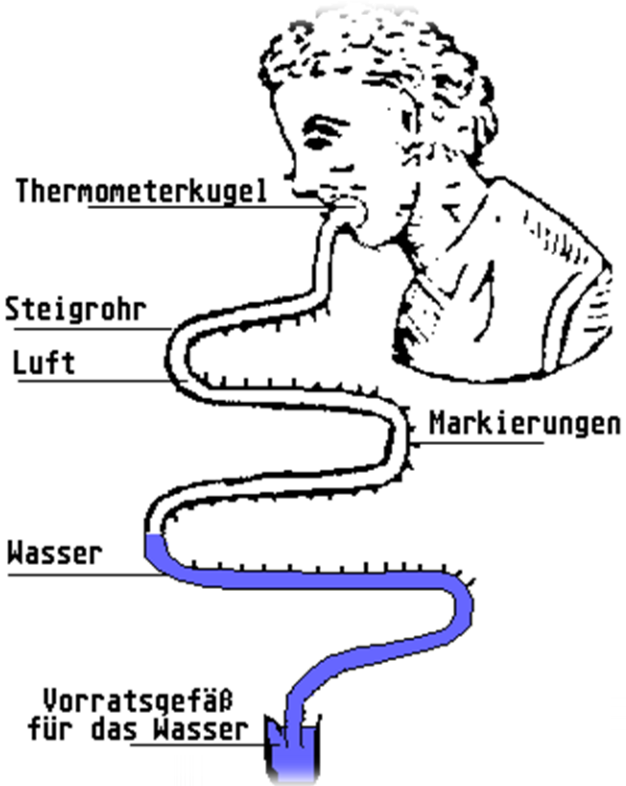
+SRM 767 [Schooley et al. (1980)] * T_{90} above 0.5 K.

Contents Part II: Quantum Transport in Nanostructures

Contents:

III.2.3 Secondary Thermometers

- there are many ….



III.2.3 Secondary Thermometers

- **resistance thermometers**

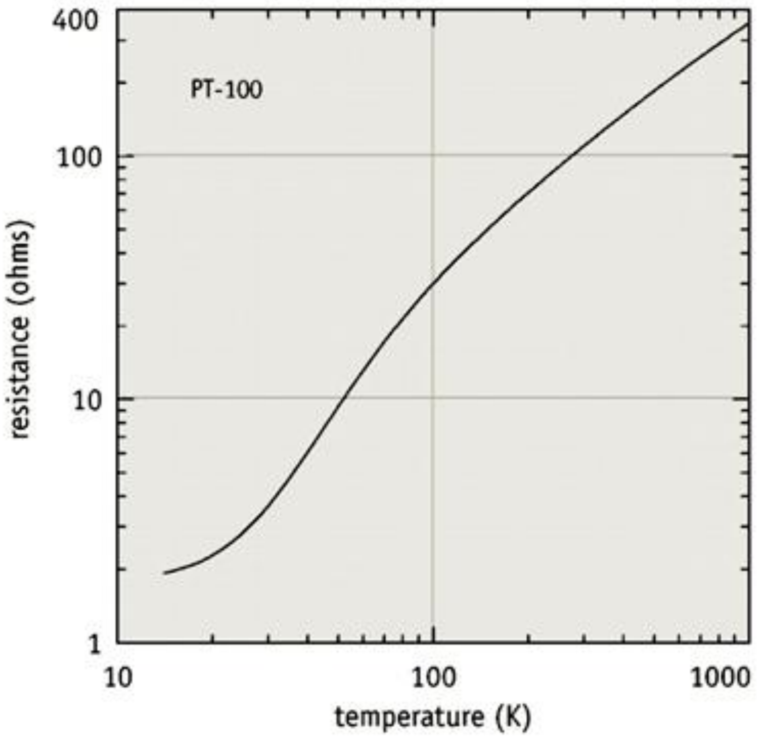
  - required: well established relation between resistance and temperature, sufficiently large dR/dT
  - advantage: resistance easy to measure → resistance thermometry very popular
  - fact: temperature variation of resistance may have very different physical origin
  - commonly used:

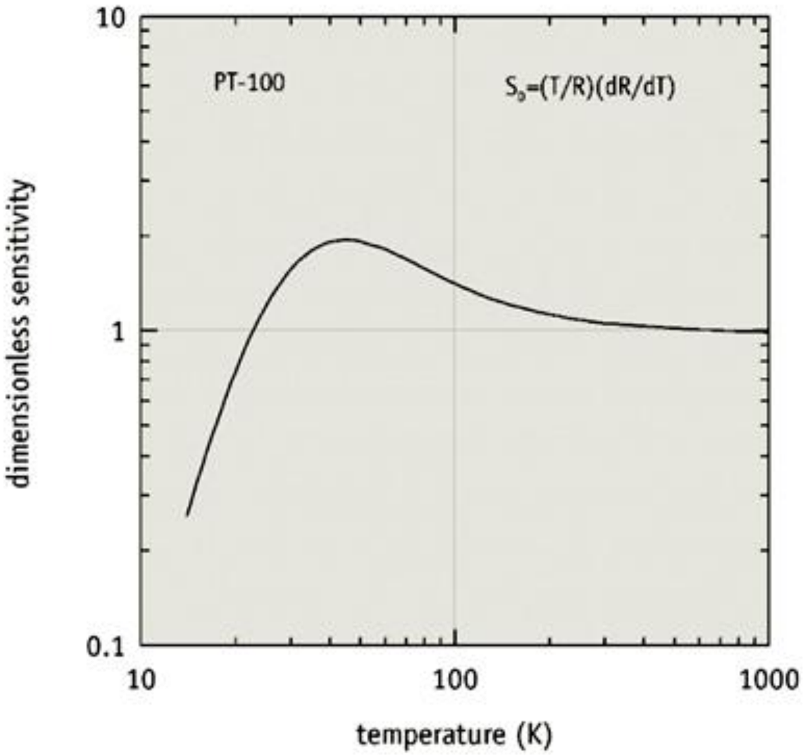
    → Pt resistors (PT-100, PT-1000)
    → RhFe resistors
    → carbon resistors (Speer, Allen-Bradley)
    → carbon glass resistors
    → Ge resistors
    → RuO_2 resistors

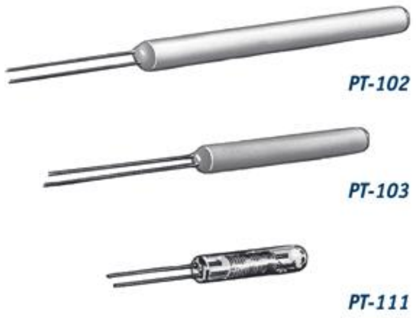
III.2.3 Secondary Thermometers

- **Platinum resistors**

The platinum resistance thermometer (PRT) is very widely used below 500 °C as a thermometric sensor. There is a wide range of quality of PRT available, from the standard instrument (SPRT) of the ITS-90 to some industrial types (IPRT) that are accurate only to within a few tenths of a kelvin or, perhaps, even a kelvin or more. The major difference of the industrial type of fabrication from the standard type is not just the purity of platinum, but also the less strain-free mounting of the film or wire which is embedded (partially or totally) in a cement (glass or refractory). Furthermore, in most cases, the thermometer body is not hermetically sealed.







Source: Lake Shore Cryotronics, Inc.

III.2.3 Secondary Thermometers

- **Platinum resistors**

  - *ITS-90 requirement for Pt resistance thermometer (PRT)*

    temperatures are determined in terms of the ratio of the resistance $R\,(T_{90})$ at a temperature T_{90} and the resistance R(273.16 K) at the triple point of water:

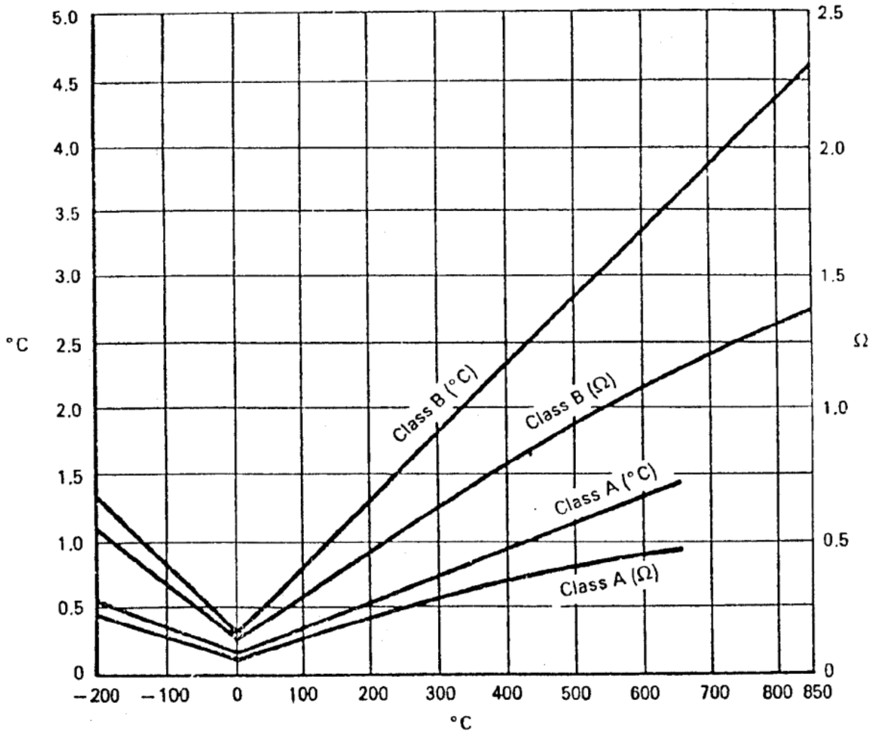
    $$W(T_{90}) \;=\; R(T_{90})/R(0.01\,°\mathrm{C}) \quad \Rightarrow \quad \begin{aligned} &\boldsymbol{W(29.7646\,°C) \;\geq\; 1.118\,07} \\ &\boldsymbol{W(-38.8344\,°C) \;\leq\; 0.844\,235} \end{aligned}$$

  - *industrial PRT*

    for $0 < T < 100$°C

    $R = R_0 \cdot (1 + a \cdot T)$

    $a = 3.85 \cdot 10^{-3}$ / K



allowed errors in °C:

Class A: $dT \;=\; \pm(0.15\,°\mathrm{C} \;+\; 0{,}002 \;\cdot\; T)$

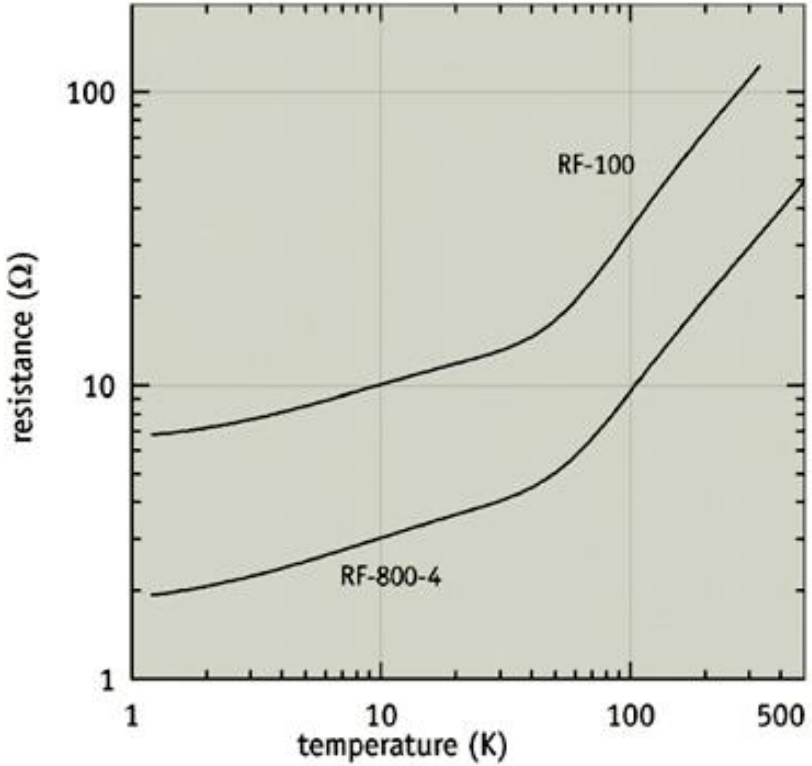
Class B: $dT \;=\; \pm(0.30\,°\mathrm{C} \;+\; 0{,}005 \;\cdot\; T)$

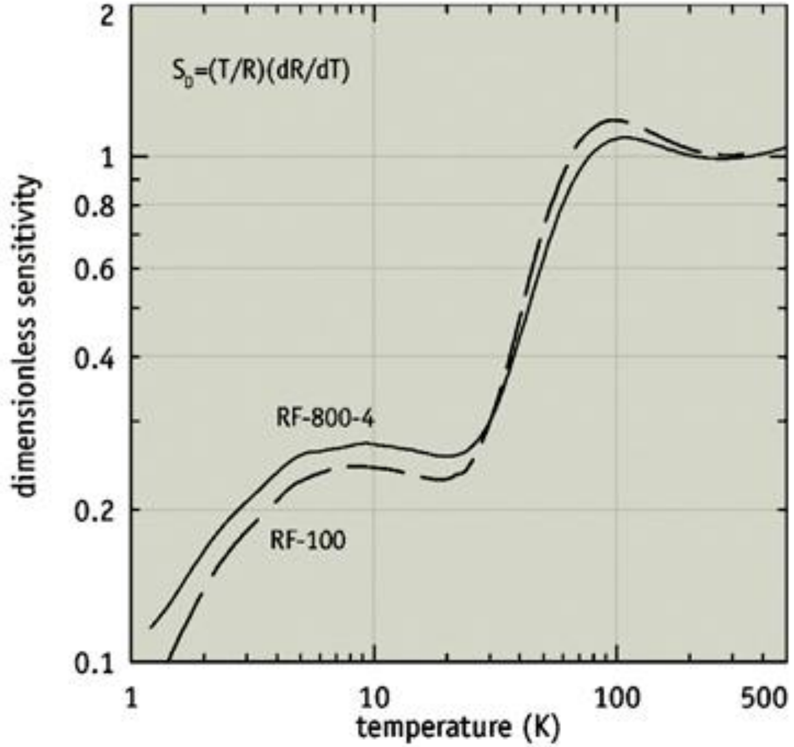
1/3 Class B: $dT \;=\; \pm 1/3 \;\cdot\; (0.30\,°\mathrm{C} \;+\; 0.005 \;\cdot\; T)$

III.2.3 Secondary Thermometers

- **RhFe resistor thermometer**

Rhodium with 0.5% Fe





Source: Lake Shore Cryotronics, Inc.

R. Gross © Walther-Meißner-Institut (2004 - 2024) - additional topic

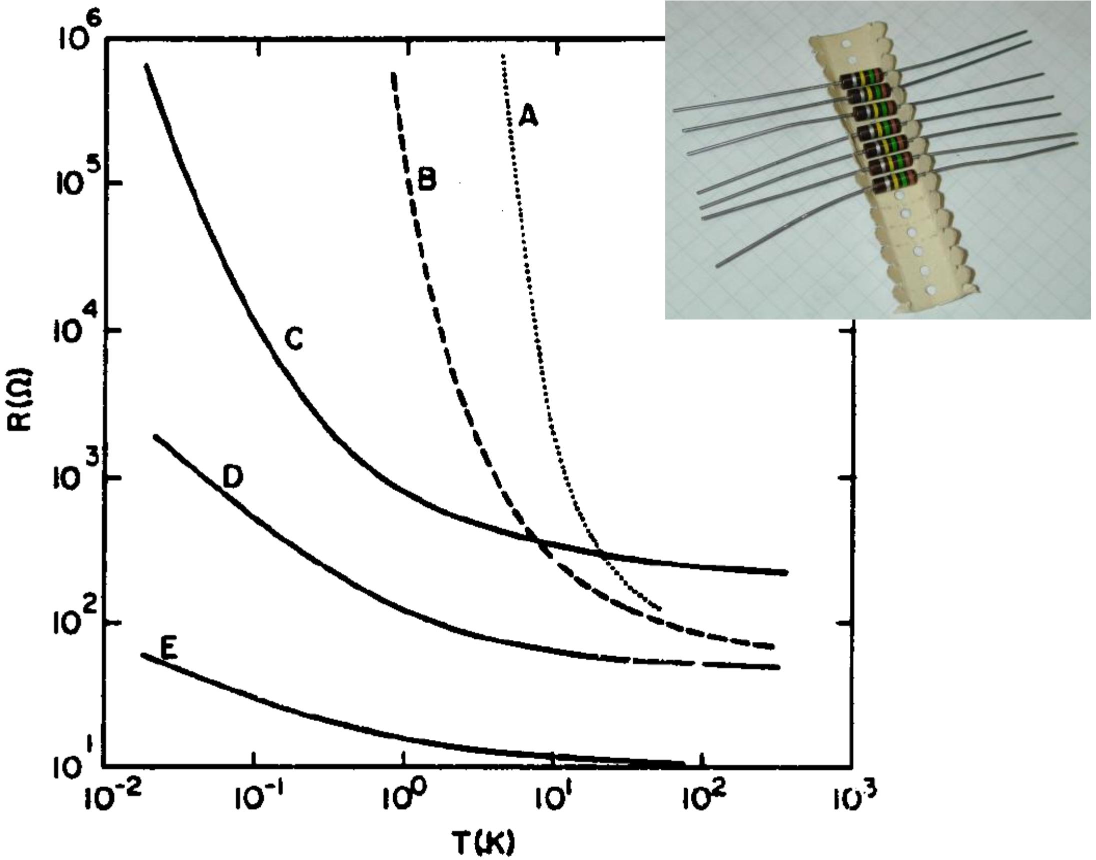
III.2.3 Secondary Thermometers

- **carbon resistors**

commercial carbon-composition resistors manufactured by Allen-Bradley were introduced as temperature sensors by Clement and Quinnell in 1952. The carbon composition resistor is a small cylinder consisting of graphite with a binder encased in an outer phenolic shell.

The carbon resistors used as thermometers are generally characterized by their room temperature resistance and their wattage [see Rubin (1980)], and have come largely from the following manufacturers:

- *Allen-Bradley*
- *Airco Speer (usually referred to simply as Speer)*
- *Ohmite*
- *Matsushita,*
- *CryoCal*

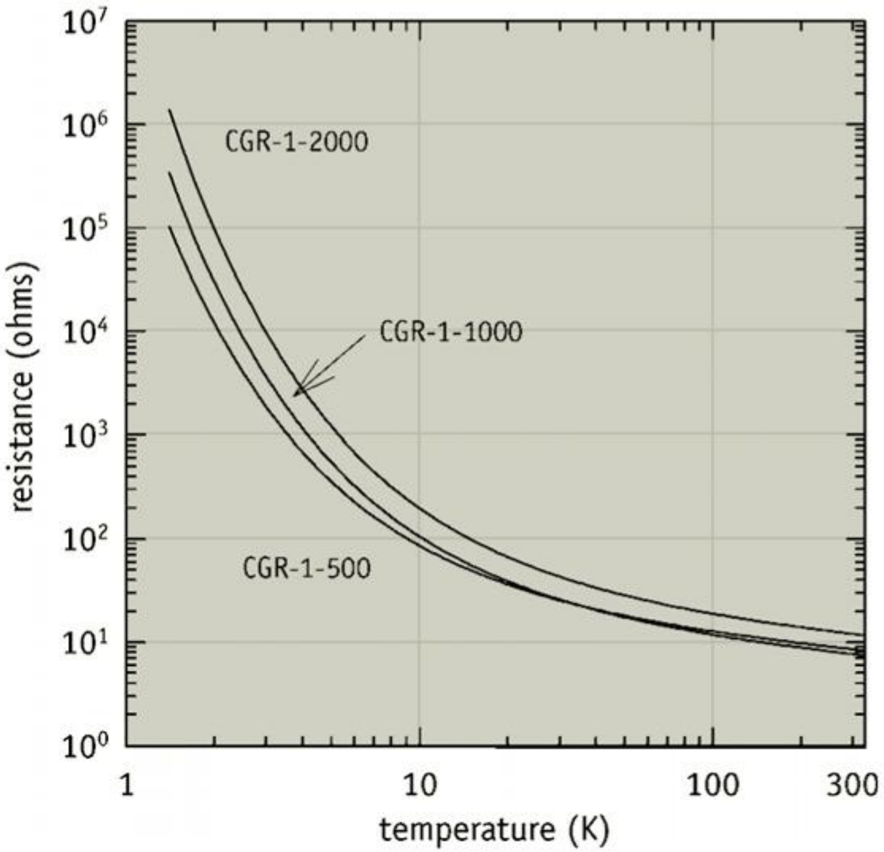


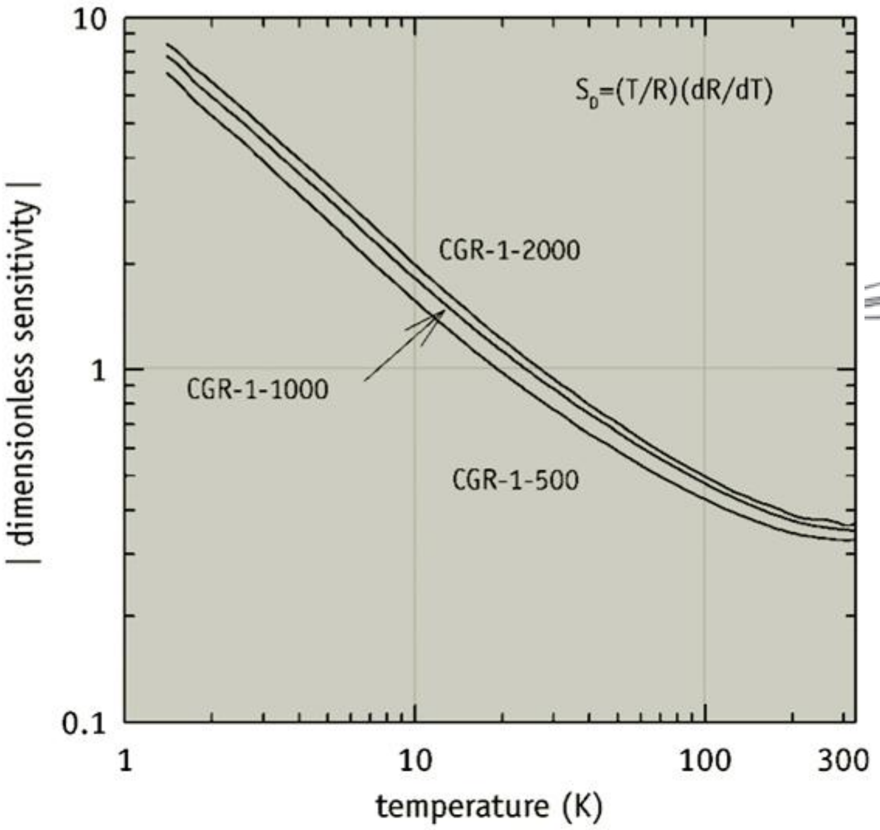
A: thermistor,
B: 68 Ω Allen-Bradley,
C: 220 Ω Speer (grade 1002),
D: 51 Ω Speer (grade 1002),
E: 10 Ω Speer (grade 1002)

III.2.3 Secondary Thermometers

- **carbon glass resistors**

A porous glass is prepared by removing the boron-rich phase from a borosilicate alkaline glass to leave a material having the appearance of silicate spheres of about 30 nm diameter, randomly distributed and separated by 3 to 4 nm pores. The spaces are then partially filled with high-purity carbon to form amorphous fibres ➔ better stability than carbon resistors.





Source: Lake Shore Cryotronics, Inc.

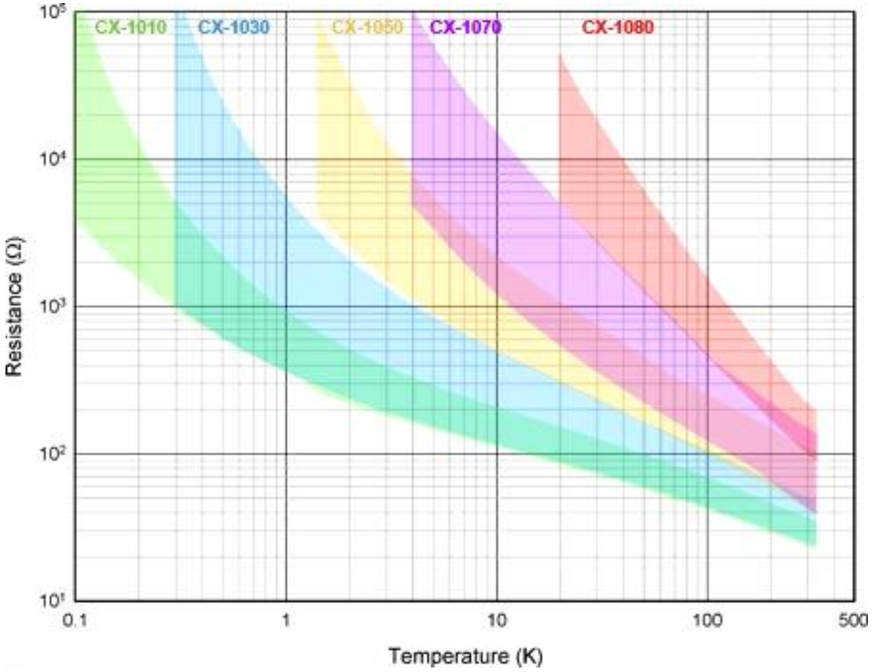
III.2.3 Secondary Thermometers

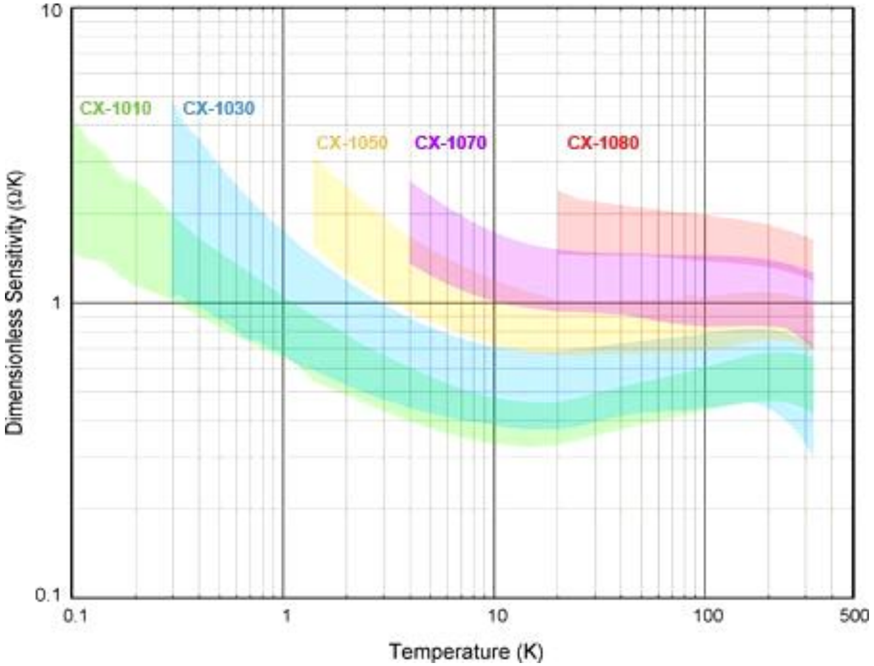
- **Cernox resistors**

Patents:
#5,363,084, Nov. 1994, "*Film Resistors Having Trimmable Electrodes*"

#5,367,285, Nov. 1994, "Cernox™", "*Metal Oxy-nitride Resistance Films and Methods of Making the Same,*"

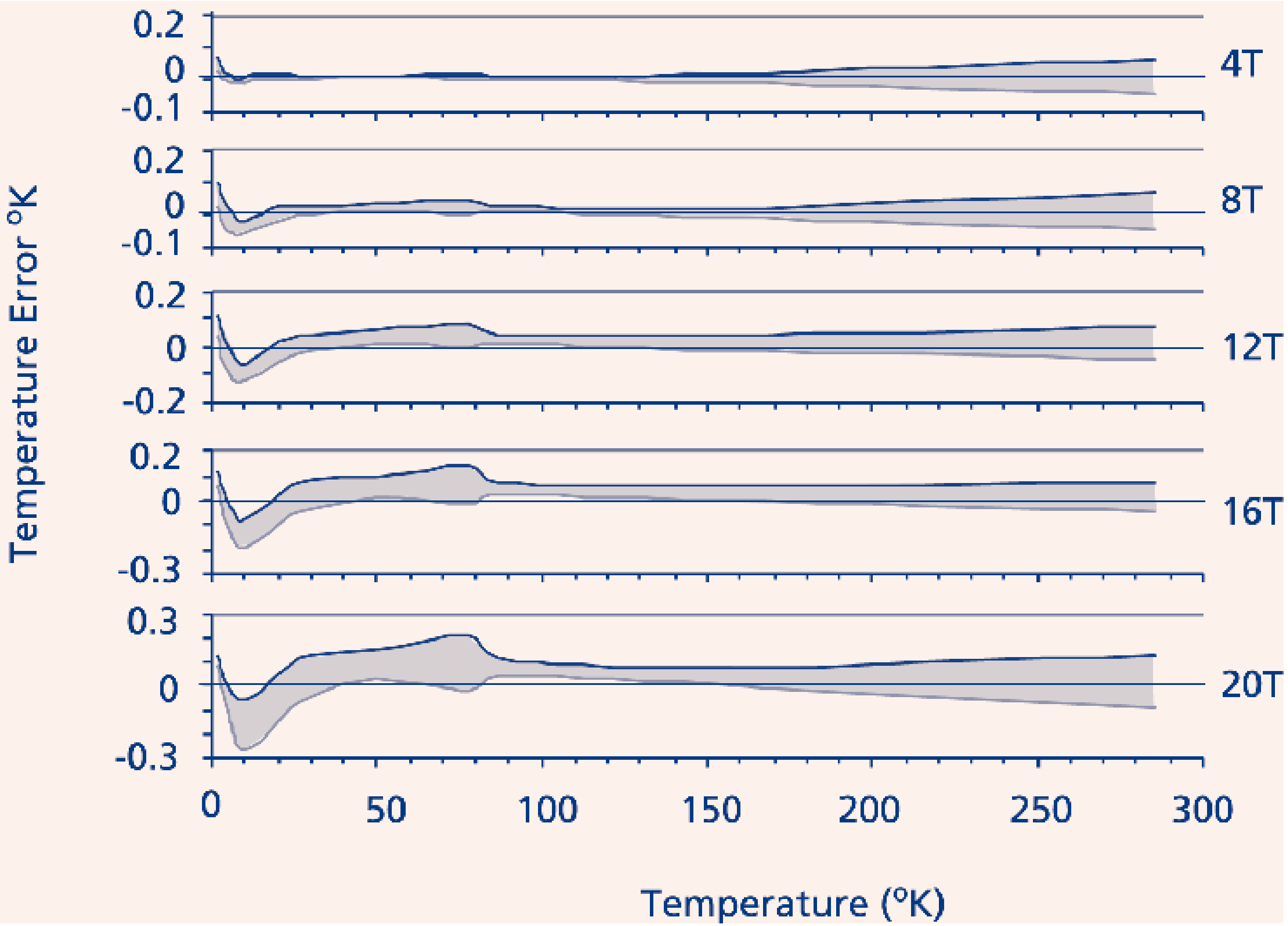
➔ ***small magnetic field coefficient***: $\Delta T/T$ typically smaller than 0.5% @ 19 T





III.2.3 Secondary Thermometers

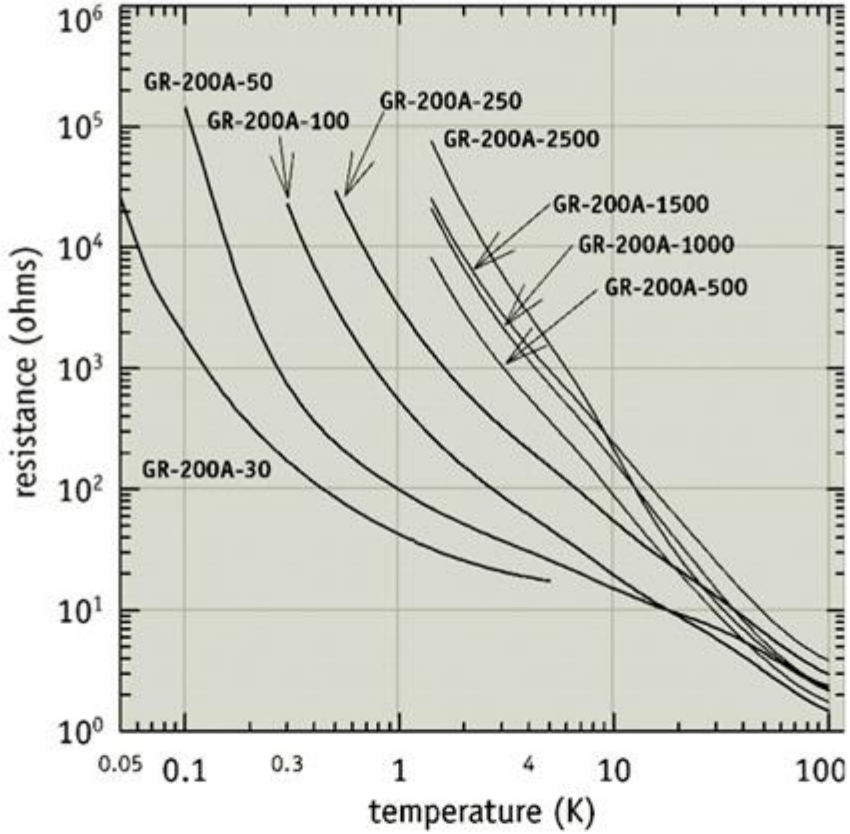
- **Cernox resistors**

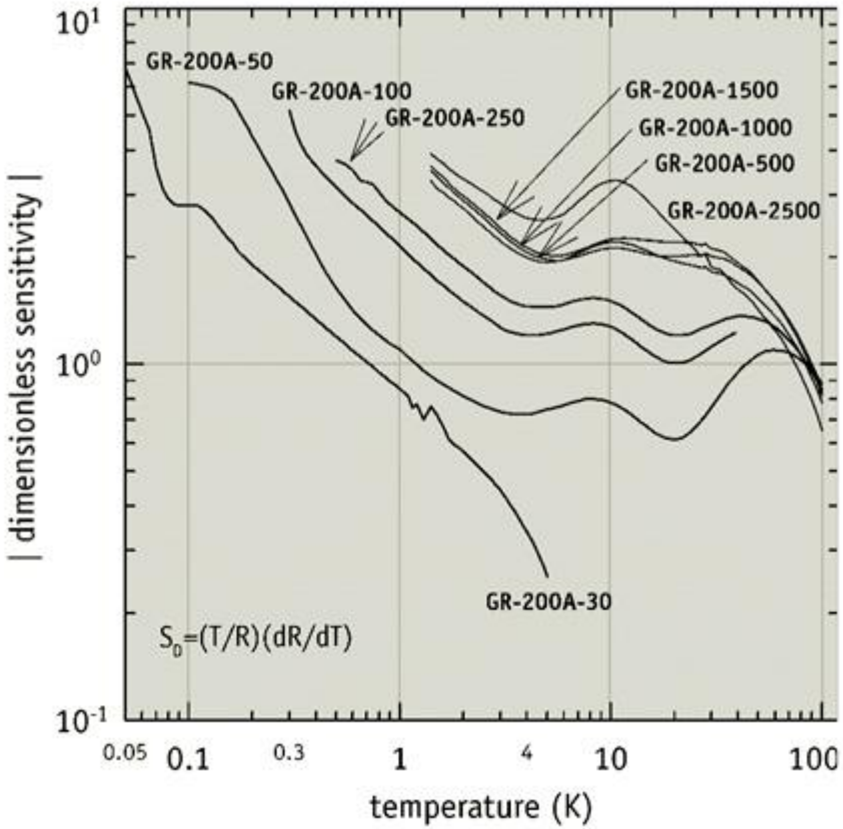


Low temperature thermometry in high magnetic fields VII. Cernox sensors to 32 T, B.L. Brandt *et al, Rev. Sci. Instrum., vol 70, No 1, 1999, pp 104-110.*

III.2.3 Secondary Thermometers

- **Germanium resistors**



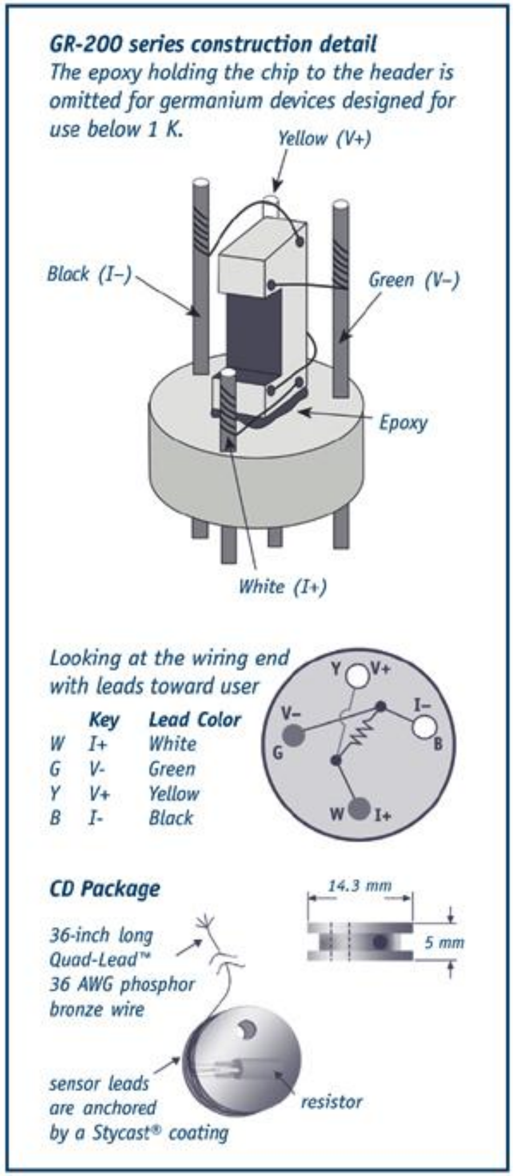


Source: Lake Shore Cryotronics, Inc.

III.2.3 Secondary Thermometers

- **Germanium resistors**

Source: Lake Shore Cryotronics, Inc.


GR-200 series construction detail

The epoxy holding the chip to the header is omitted for germanium devices designed for use below 1 K.

Looking at the wiring end with leads toward user

	Key	Lead Color
W	I+	White
G	V-	Green
Y	V+	Yellow
B	I-	Black

CD Package

36-inch long Quad-Lead™ 36 AWG phosphor bronze wire

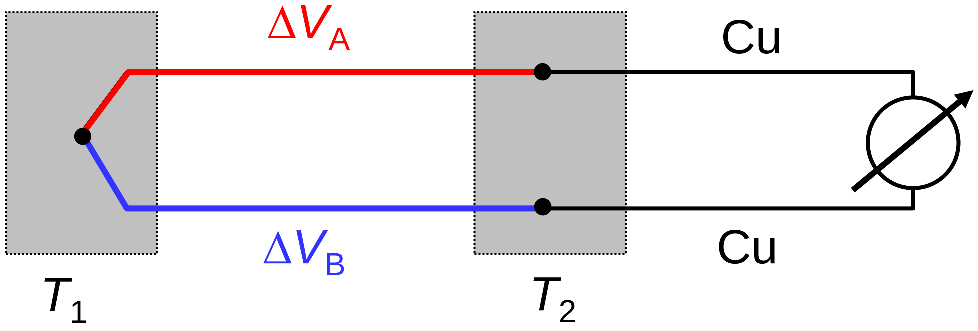
sensor leads are anchored by a Stycast® coating

III.2.3 Secondary Thermometers

- **Thermocouples**

Thermocouples are pairs of dissimilar metal wires joined at least at one end, which generate a net thermoelectric voltage between the the open pair according to the size of the temperature difference between the ends, the relative Seebeck coefficient of the wire pair and the uniformity of the wire-pair relative Seebeck coefficient.

- based on Seebeck effect: $\Delta V = S \cdot \Delta T$ S = thermopower
- measurement of difference of thermovoltages of two different materials



well-known reference temperature

III.2.3 Secondary Thermometers

- Thermocouples
  - **Chromel-Gold/Iron (0.07%)**
    consists of a Gold (Au)-0.07 at % Iron (Fe) as the negative thermoelement and a Ni-Cr alloy (Chromel) as the positive thermoelement. This thermocouple is more widely used because of ist relatively high thermoelectric sensitivity (>15 µV/K above 10 K).
  - **Type E (Chromel (Ni-Cr-alloy)** / **Constantan (Cu-Ni-alloy))**
    has the highest sensitivity among the three standard thermocouple types typically used at low temperatures (types E, K, and T). The best choice for temperatures down to 40 K.
  - **Type K (Chromel (Ni-Cr-alloy) / Alumel (Ni-Al-alloy))**
    recommended for continuous use in inert atmospheres. Has a sensitivity of 4.1 mV/K at 20 K (about ½ of Type E).
  - **Type T (Copper / Constantan)**

many more !!

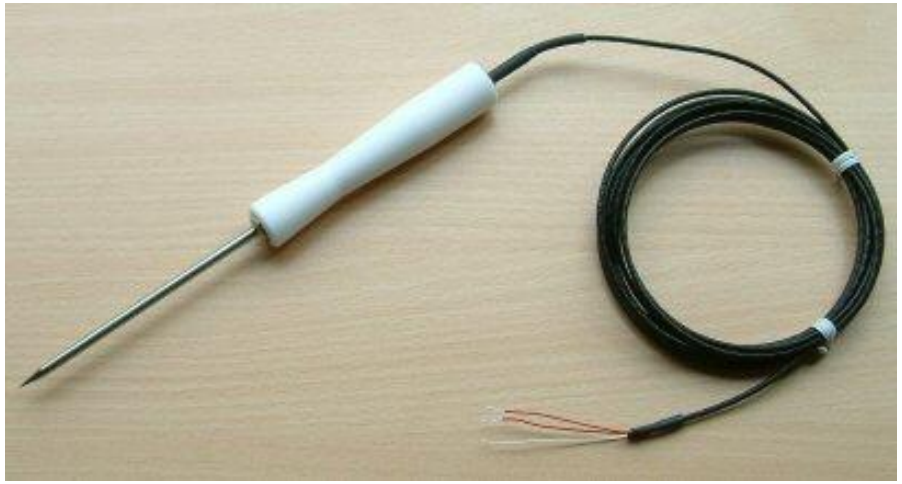
III.2.3 Secondary Thermometers

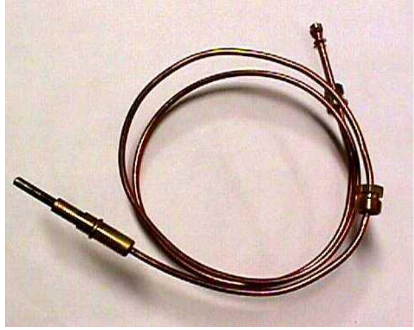
- **Thermocouples**

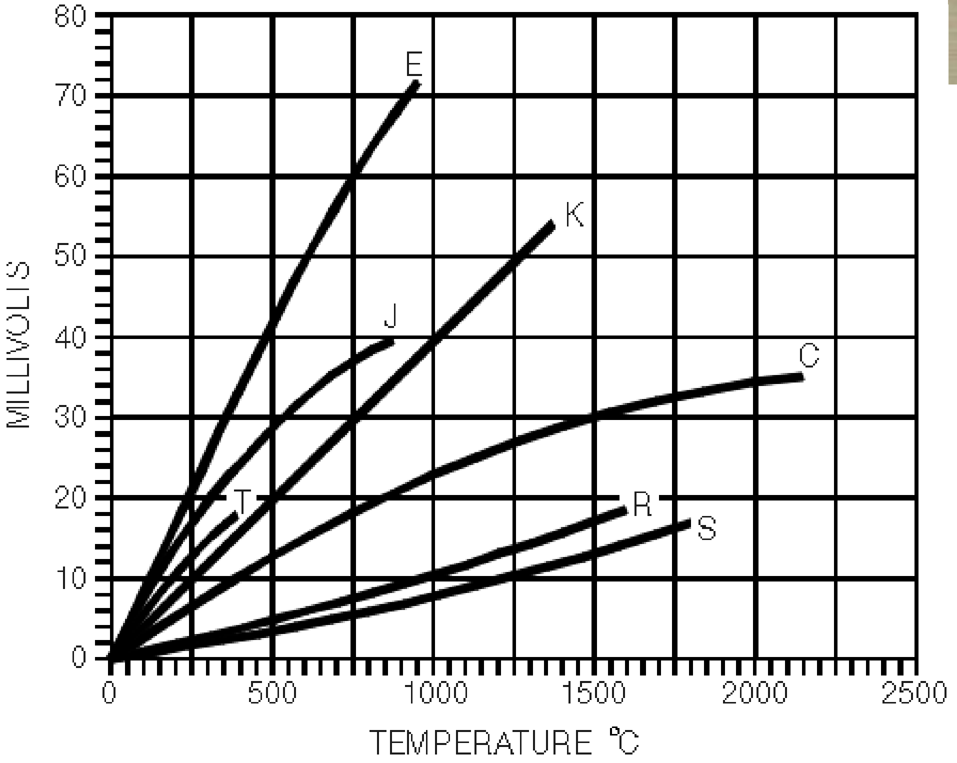
Thermocouple Type	Names of Materials	Useful Application Range
B	Platinum30% Rhodium (+) Platinum 6% Rhodium (-)	2500 -3100F 1370-1700°C
C	W5Re Tungsten 5% Rhenium (+) W26Re Tungsten 26% Rhenium (-)	3000-4200F 1650-2315°C
E	Chromel (+) Constantan (-)	200-1650F 95-900°C
J	Iron (+) Constantan (-)	200-1400F 95-760°C
K	Chromel (+) Alumel (-)	200-2300F 95-1260°C
N	Nicrosil (+) Nisil (-)	1200-2300F 650-1260°C
R	Platinum 13% Rhodium (+) Platinum (-)	1600-2640F 870-1450C
S	Platinum 10% Rhodium (+) Platinum (-)	1800-2640F 980-1450°C
T	Copper (+) Constantan (-)	-330-660F -200-350°C

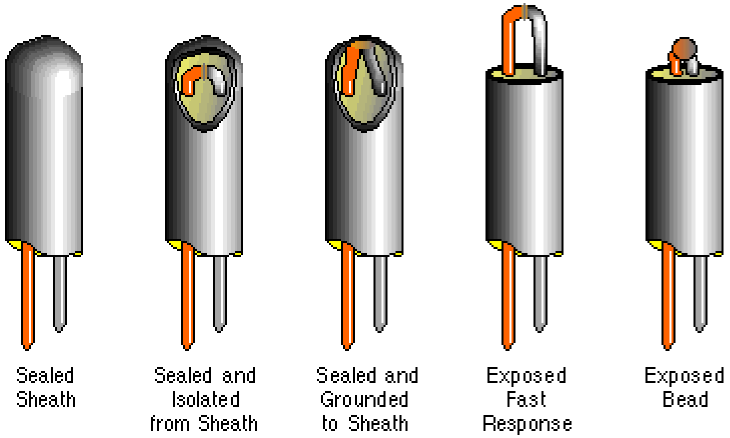
III.2.3 Secondary Thermometers

- **Thermocouples**









Thermocouple Sheath Options

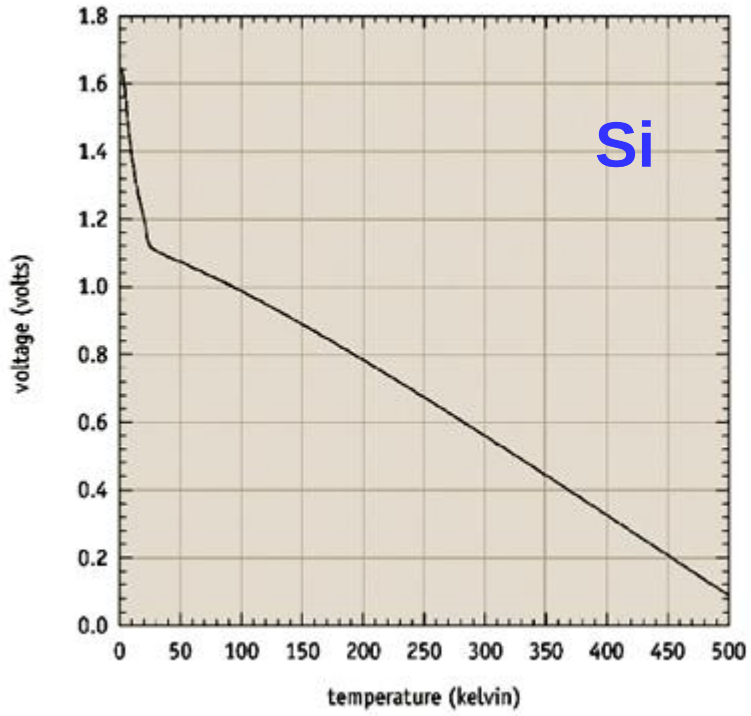
III.2.3 Secondary Thermometers

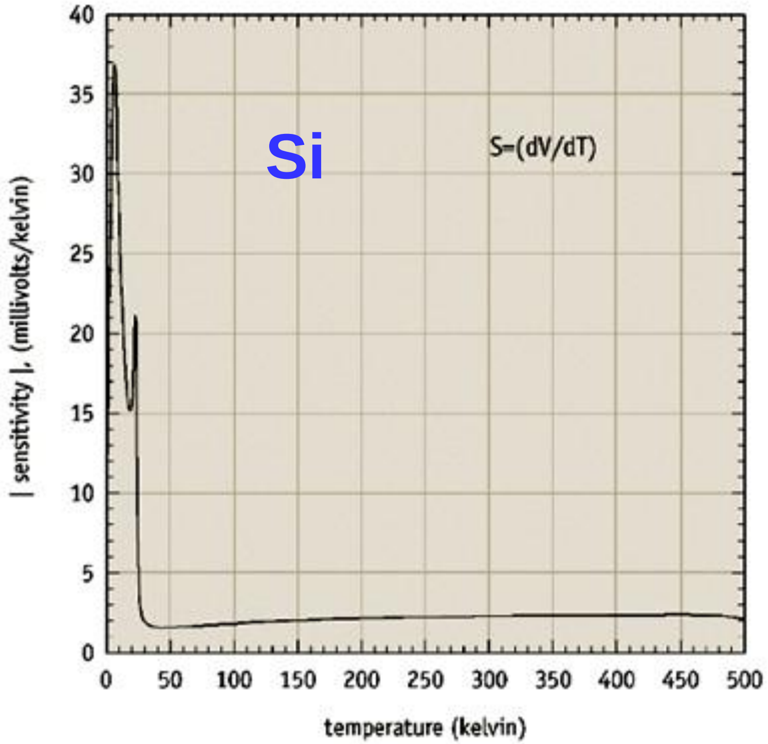
- **Diode Thermometers**

The temperature-indicating parameter is the forward-biased junction voltage, which decreases approximately linearly with increasing temperature when the current is kept constant, since $I \quad \exp(eV/k_\mathrm{B}T)$

→ The typical I-V characteristic is such as to make the internal impedance of the device very high (easily greater than 100 kΩ) at small currents; or else - using a larger current - one encounters unacceptably high power dissipation at low temperatures.

→ There is a transition region in the conduction mechanism around 20 K that makes fitting a V-T characteristic over the whole temperature range difficult for GaAs and impossible for Si



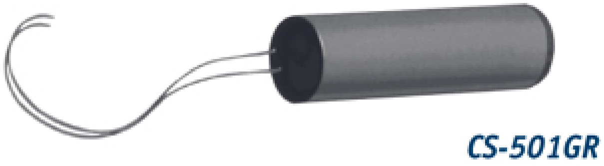


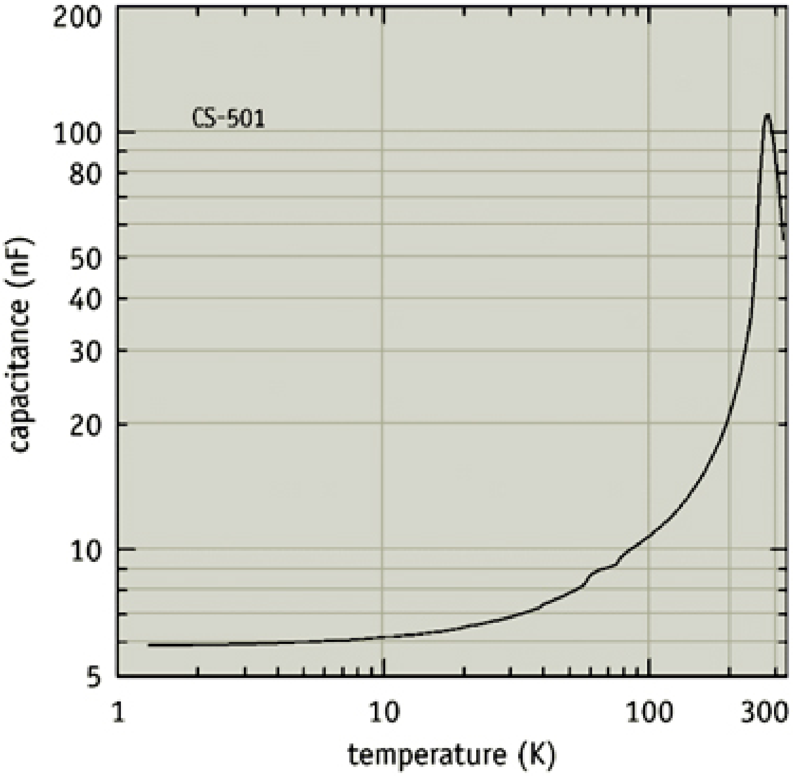
Source: Lake Shore Cryotronics, Inc.

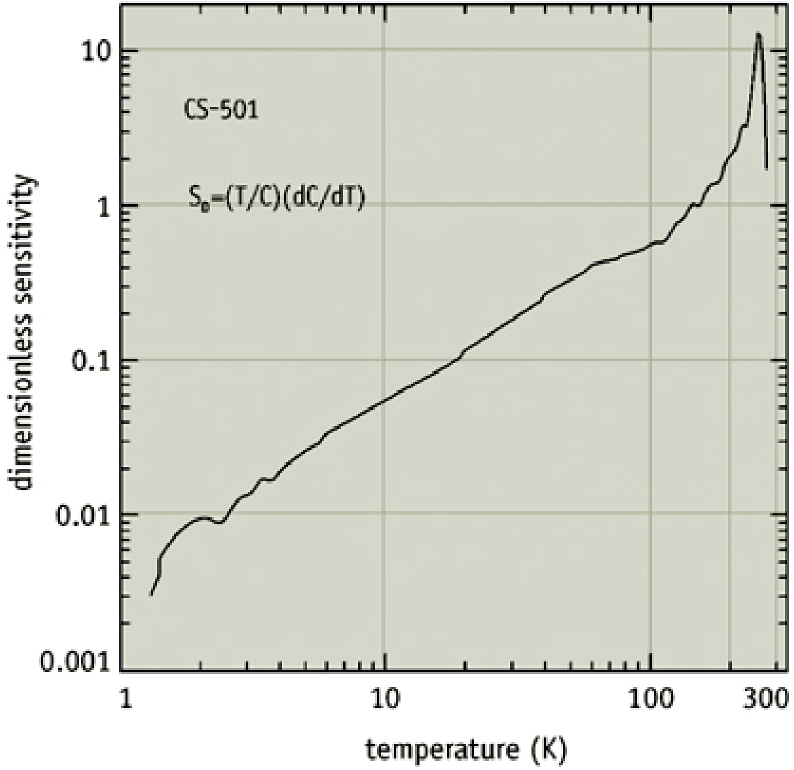
III.2.3 Secondary Thermometers

- **Capacitive Thermometers**
  - based on the well defined relation between the dielectric constant and temperature
  - temperature is determined via a capacitance measurement
  - advantage: virtually no magnetic field dependence

- **Capacitive Thermometers**



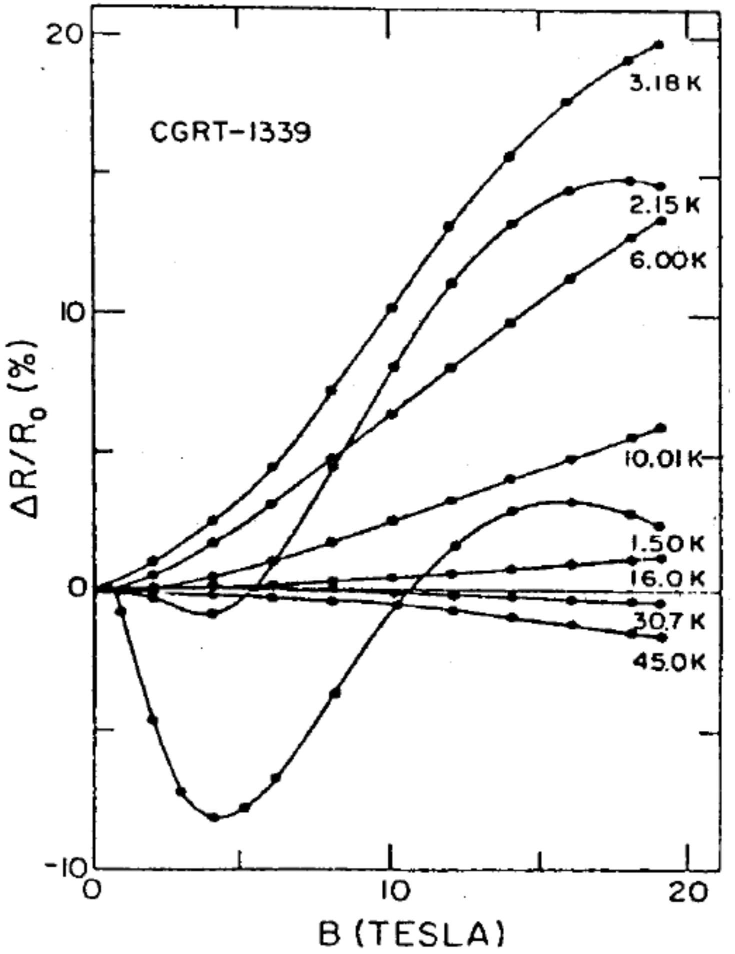




Source: Lake Shore Cryotronics, Inc.

III.2.3 Secondary Thermometers

- **magnetic field effects**

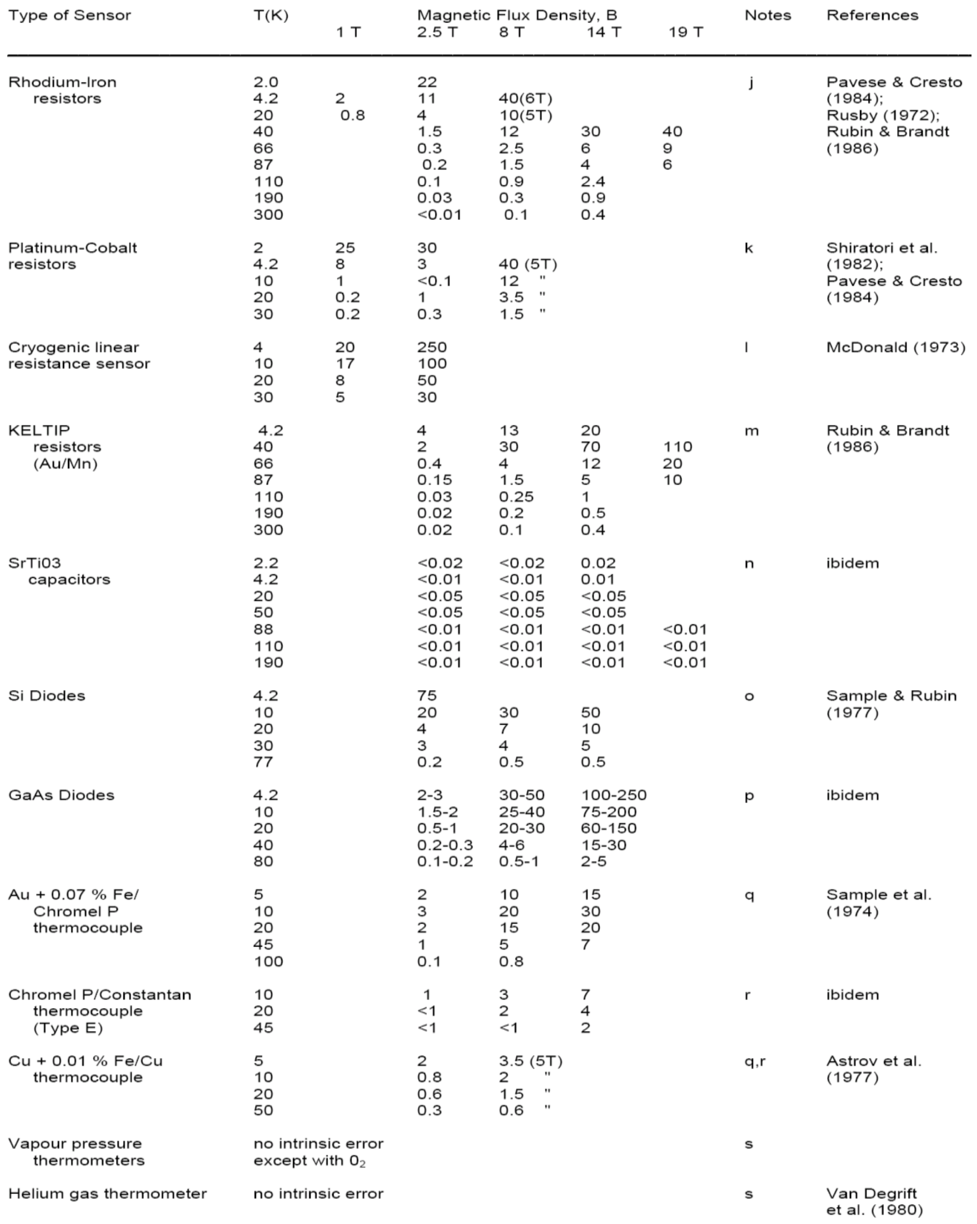


Magnitude of relative temperature error |ΔT|/T (%) for values of B

Type of Sensor	T(K)	Magnetic Flux Density, B					Notes	References
		1 T	2.5 T	8 T	14 T	19 T		
Carbon radio resistors							a	Sample and Rubin (1977)
Allen-Bradley (2.7, 3.9, 5.6, 10 C)	0.5		2-4	5-13	7-20			
	1.0		2-4	6-15	9-25			
	2.5		1-5	6-18	10-30			
	4.2		1-5	5-20	10-35			
Allen-Bradley (47, 100, 220 Ω)	4.2		<1	5	10		a	ibidem
	10		<1	3	5			
	20		<1	1	2			
Speer, Grade 1002 (100, 220, 470 C)	0.5		0-2	0-1	0-6		b	ibidem
	1.0		1-2	2-4	3-9			
	2.5		3-5	1-4	7-14			
	4.2		4-9	2-5	4-13			
Matsushita (68, 200, 510 C)	1.5		1-2	10-15			c	ibidem
	2.1		1	10-15				
	4.2		2-3	4-8				
KVM carbon composite resistors	2.4		3	8 (5T)			d	Astrov et al. (1977)
	4.2		1.5	5 "				
	10		0.4	1.4 "				
	20		0.1	0.4 "				
	80		<0.01	<0.01 "				
Carbon-Glass Resistors	2.2		0.1	1.5	3	4	e	Rubin and Brandt (1986)
	4.2		0.5	2	5	7		
	10		0.2	1.1	3	4		
	20		<0.01	0.02	0.03	0.13		
	45		0.07	0.5	1.3	2		
	88		0.06	0.5	1.3	2		
	190		0.04	0.3	1.0	1.7		
	310		<0.01	0.2	0.6	1.1		
Thermistors	4.2		<0.05	1	3		f	Sample and Rubin (1977)
	10		<0.05	0.3	1			
	20		<0.05	0.1	0.5			
	40		<0.05	0.1	0.5			
	60		<0.05	0.1	0.3			
Germanium Resistors	2.0		8-10	60			g	ibidem
	4.2		5-20	30-55	60-70			
	10		4-15	25-60	60-75			
	20		3-20	15-35	50-80			
	70		3-10	15-30	25-50			
Germanium Resistors TSG-2	4.2		30	120			g	Astrov et al. (1977)
	20		2.5	6				
Specially doped Ge resistors KG	4.2		<0.2.	0.5 (6T)			h	ibidem; Matacotta et al. (1984)
	10		<0.2	<0.5 "				
	20		<0.5	2-3 "				
	30		<0.5	5 "				
	80		0.15	0.5 "				
Platinum Resistors	10		100|	250|	|	|	i	Pavese & Cresto (1984); Neuringer et al. (1971); Rubin & Brandt (1986)
	20		2-8|20	25|100	250|	|		
	40		0.5|<1	3|5	6|10	9|		
	66		0.1|<0.5	0.8| 2	2| 5	4|		
	87		0.04|<0.5	0.4| 1	1| 2	2|		
	110		0.02|	0.2|	0.6|	1|		
	190		<0.01|	0.06|	0.2|	0.3|		
	300		<0.01|	0.02|	0.07|	0.13|		
			(a) (b)	(a) (b)	(a) (b)	(a) (b)		

III.2.3 Secondary Thermometers

- **magnetic field effects**



Type of Sensor	T(K)		Magnetic Flux Density, B				Notes	References
		1 T	2.5 T	8 T	14 T	19 T		
Rhodium-Iron resistors	2.0		22				j	Pavese & Cresto (1984); Rusby (1972); Rubin & Brandt (1986)
	4.2	2	11	40(6T)				
	20	0.8	4	10(5T)				
	40		1.5	12	30	40		
	66		0.3	2.5	6	9		
	87		0.2	1.5	4	6		
	110		0.1	0.9	2.4			
	190		0.03	0.3	0.9			
	300		<0.01	0.1	0.4			
Platinum-Cobalt resistors	2	25	30				k	Shiratori et al. (1982); Pavese & Cresto (1984)
	4.2	8	3	40 (5T)				
	10	1	<0.1	12 "				
	20	0.2	1	3.5 "				
	30	0.2	0.3	1.5 "				
Cryogenic linear resistance sensor	4	20	250				l	McDonald (1973)
	10	17	100					
	20	8	50					
	30	5	30					
KELTIP resistors (Au/Mn)	4.2		4	13	20		m	Rubin & Brandt (1986)
	40		2	30	70	110		
	66		0.4	4	12	20		
	87		0.15	1.5	5	10		
	110		0.03	0.25	1			
	190		0.02	0.2	0.5			
	300		0.02	0.1	0.4			
SrTi03 capacitors	2.2		<0.02	<0.02	0.02		n	ibidem
	4.2		<0.01	<0.01	0.01			
	20		<0.05	<0.05	<0.05			
	50		<0.05	<0.05	<0.05			
	88		<0.01	<0.01	<0.01	<0.01		
	110		<0.01	<0.01	<0.01	<0.01		
	190		<0.01	<0.01	<0.01	<0.01		
Si Diodes	4.2		75				o	Sample & Rubin (1977)
	10		20	30	50			
	20		4	7	10			
	30		3	4	5			
	77		0.2	0.5	0.5			
GaAs Diodes	4.2		2-3	30-50	100-250		p	ibidem
	10		1.5-2	25-40	75-200			
	20		0.5-1	20-30	60-150			
	40		0.2-0.3	4-6	15-30			
	80		0.1-0.2	0.5-1	2-5			
Au + 0.07 % Fe/ Chromel P thermocouple	5		2	10	15		q	Sample et al. (1974)
	10		3	20	30			
	20		2	15	20			
	45		1	5	7			
	100		0.1	0.8				
Chromel P/Constantan thermocouple (Type E)	10		1	3	7		r	ibidem
	20		<1	2	4			
	45		<1	<1	2			
Cu + 0.01 % Fe/Cu thermocouple	5		2	3.5 (5T)			q,r	Astrov et al. (1977)
	10		0.8	2 "				
	20		0.6	1.5 "				
	50		0.3	0.6 "				
Vapour pressure thermometers	no intrinsic error except with 0_2						s	
Helium gas thermometer	no intrinsic error						s	Van Degrift et al. (1980)

III.2.3 Secondary Thermometers

Table 1.1: Summary of Some Properties of Most-Commonly-Used Thermometers

Thermometer	Usual Temperature Range	Thermometric Quantity	Typical Uncertainty
Germanium	1 K to 100 K	elec. resist.*	ΔT / T < 2x10-4
Rhodium-Iron	0.5 K to 30 K	elec. resist.*	0.3 mK
Platinum-Cobalt (industrial type)	2 K to 20 K	elec. resist.*	10 mK
Carbon	0.5 K to 30 K	elec. resist.*	ΔT / T < 5x10-3
Carbon-glass	0.5 K to 100 K	elec. resist.*	ΔT / T < 1 x1 0-3
Diode	4 K to 300 K	junction voltage	-50 mK
Vapour-pressure	various subranges between 0.5 K and 100 K	pressure	-1 mK
Mercury-in-glass	-50 °C to 250 °C	thermal expansion of mercury	0.1 K
Thermocouples	4 K to 2500 °C	thermoelectromotive force	
Type S	-50 °C to 1600 °C		0.3 K < 1000 °C 1 K > 1000 °C
Type R	-50 °C to 1600 °C		0.3 K < 1000°C 1 K > 1000 °C
Type B	300 °C to 1800 °C		0.5 K to 2 K
Type T	-200 °C to 350 °C		0.1 K
Type E	-200 °C to 870 °C		0.1 K < 300 K 1 K > 300 K
Type J	0 °C to 760 °C		0.5 K < 300 °C 2 K > 300 °C
Type K	-200 °C to 1260 °C		0.1 K < 200 °C 1 K 200-1000 °C 3 K > 1000 °C
Type N	0 °C to 1300 °C		0.1 K < 200 °C 0.5 K 200-1000 °C 3 K > 1000 °C
W/Re	1000 °C to 2400 °C		3-10 K

III.2.3 Secondary Thermometers

Thermometer	Usual Temperature Range	Thermometric Quantity	Typical Uncertainty
Thermistor	-80 °C to 250 °C	elec. resist.*	0.1 K (much better if use confined to very small temperature range)
Platinum: SPRT	14 K to 630 °C	elec. resist.*	0.5 mK
IPRT	20 K to 600 °C		50 mK
Radiation	100 °C to 3000 °C	spectral radiance of source	1 K < 1000 °C 5 K > 1000 °C

III.2.3 Secondary Thermometers

- **Magnetic Susceptibility Thermometers**

  – Curie-Weiss law: $\chi = \dfrac{C}{T - TC}$

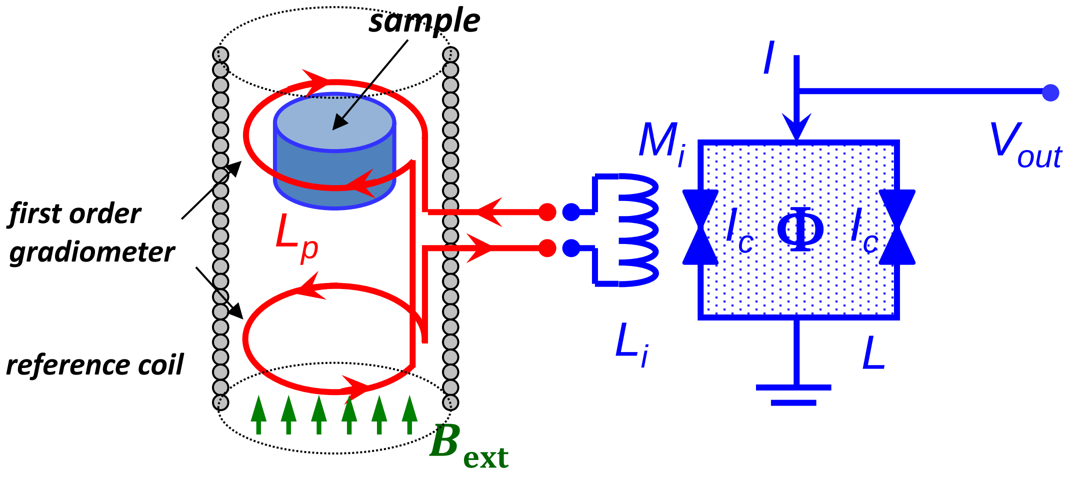
  ➔ measure χ as a function of T, calibaration requires determination of C and T_C

  – experimental techniques:

  → ***inductance bridge***: compare inductance of coil containing the magnetic material to empty reference coil

  → ***SQUID magnetometer***:

  resolution:
  → $\Delta T/T \approx 10 - 10$ @ 1 K
  → increases ≈ $1/T$



T.C.P. Chui et al., Phys. Rev. Lett. **69**, 3005 (1992)

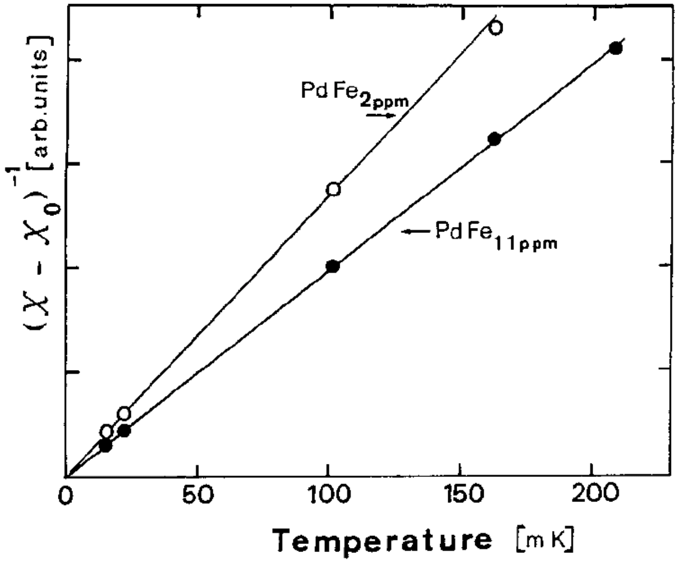
III.2.3 Secondary Thermometers

- **Magnetic Susceptibility Thermometers**

  – materials (use of electronic magnetic susceptibility):

→ paramagnetic salts: e.g. Ce-Mg-nitrate (CMN)
++ very low ordering temperature: $T_C \sim 2$ mK
++ large Curie constant
- - long time constant at low T $(\sim 100\ \mathrm{s})$
- - cannot be used in vacuum (instable)

→ nonmagnetic metals: e.g. PdFe or AuEr



- usable down to about 0.3 mK
- response time: 1 s @ 10 mK

M. Jutzler, B. Schröder, K. Gloos, F. Pobell,
Z. Phys. B64, 115 (1986)

III.2.3 Secondary Thermometers

- **Magnetic Susceptibility Thermometers**

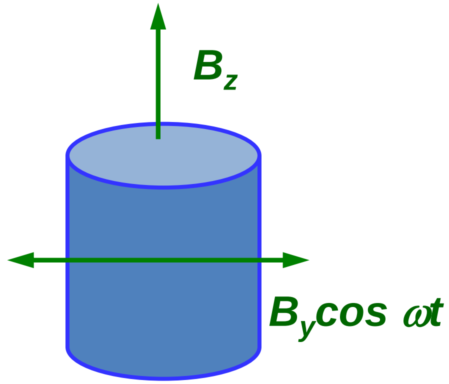
  – materials (use of nuclear susceptibility):

    → use of nonmagnetic metals: e.g. Cu

    - - nuclear moments much smaller → sensitive SQUID magnetometer
    - - perturbing magnetic impurities → very pure materials
    (e.g. 1 ppm Fe in Cu gives similar signal as all Cu nuclear spins)
    ++ can be used below 1 mK

III.2.3 Secondary Thermometers

- **Nuclear Magnetic Resonance Thermometers**
  - are not based on static orientation of nuclear moments as in susceptibility thermometers
  - have selectivity to specific nuclear moments (less sensitive to magnetic impurities)
  - usable at temperatures below 1 mK
  - materials: predominantly ^{195}Pt
  - experimental techniques: stationary and pulsed NMR



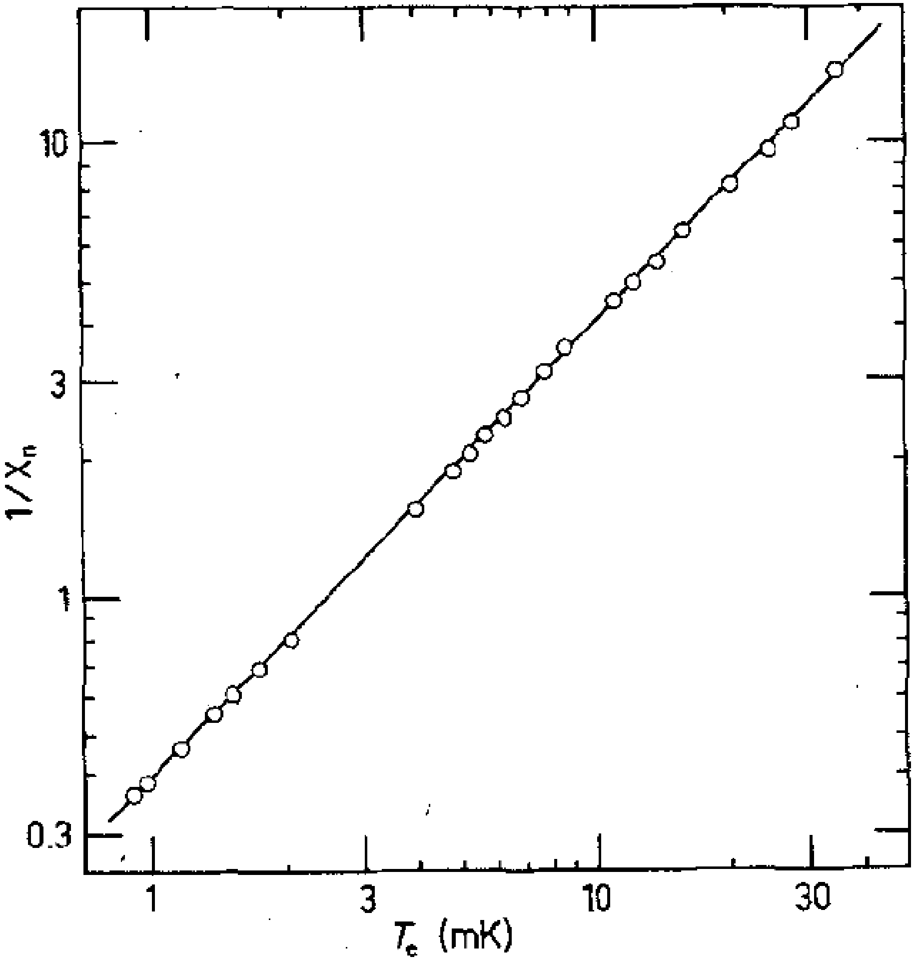
→ measures resonance absorption of high frequency signal
→ allows to determine induced $M_y \propto M_0(T) \propto 1/TN$ at resonance
→ requires small $B_y \approx 1\ \mu T$ to avoid saturation effects

→ measures decay of induced $M_y(t)$ after 90° pulse ⍰
→ amplitude $dM_y(t = 0)$ or integral of decay curve $\propto M_0(T) \quad 1/TN$

➔ ***general problem***: one measures temperature T_N of nuclear spins
→ are they in thermal equilibrium with lattice ??

III.2.3 Secondary Thermometers

- **Nuclear Magnetic Resonance Thermometers**



comparison of the Curie and Korringa temperatures of a Pt sample:

the inverse nuclear susceptibility $1/\chi_N$ *(arb. units)* is plotted against the electron temperature

$$T_e = \left(\frac{1}{t_1}\right) \cdot 29.9 \times 10^{-3}\ \mathrm{K\ sec.}$$

A.I. Ahonen et al.,
J. Low Temp. Phys. **25**, 421 (1976)