





# Characterization of Flux-driven Josephson Parametric Amplifiers

Diploma Thesis

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August 2010

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# **Introduction and Motivation**

The interaction of light and matter is amongst the most thoroughly studied research topics in physics. The discovery of the photoelectric effect [1] has given rise to a new generation of experiments where light can no longer be treated classically. Today, all experiments where the quantum nature of light plays a decisive role fall into the realm of *quantum optics*. The most basic quantum optics experiments study the interaction of a single atom and a single photon. For several decades, reaching this regime has been a major research focus and has created the field of cavity quantum electrodynamics (QED) [2,3]. However, one limiting factor of all cavity QED experiments are small coupling strengths between the atom and the respective field stored in the cavity as the coupling strength is determined by the small dipole moment of the atom and the relatively large mode volume of the cavity.

Replacing the natural atom by a superconducting two-level system and the cavity by an on-chip transmission line resonator, the field of circuit quantum electrodynamics was born. With these superconducting circuits, the regime of strong coupling can be reached more easily [4, 5], as the coupling strength is several orders of magnitude larger than in cavity QED. In the field of circuit QED, many experiments known from the optical regime could be reproduced [6–9]. Furthermore, circuit QED also opened the door to new phenomena that are either difficult to observe in cavity QED [10] or do not have an optical analogon at all [11, 12].

But there is also a severe drawback of circuit QED. As the energy per photon is orders of magnitude smaller in the microwave (GHz) regime than in the optical regime, a single microwave photon has insufficient energy to trigger solid-state photo detectors such as avalanche diodes. Therefore, the microwave signals used in circuit QED have to be amplified before they can be detected. However, the predominantly used linear, phaseinsensitive amplifiers inevitably add noise to the signal that may be much larger than the signal itself.

The reason why these amplifiers add noise is due to a fundamental principle of quantum physics - the uncertainty principle. However, if an amplifier treats the quadratures of an input signal differently, the uncertainty principle allows for the noiseless amplification of one of the signal quadratures.

Proposals for the realization of phase-sensitive amplifiers involving the Josephson effect go back to the 1970s [13–16]. In 1988, B. Yurke *et al.* demonstrated the squeezing of 4.2 K thermal noise using a current-driven Josephson parametric amplifier (JPA) [17] with a noise temperature below the quantum limit of linear, phase-insensitive amplifiers. Two years later, experimental progress enabled R. Movshovich *et al.* to squeeze vacuum noise [18]. The flux-driven design of the Josephson parametric amplifier was introduced in 2008 by T. Yamamoto *et al.* [19] based on a theoretical proposal by T. Ojanen and J. Salo [20]. Up to this date, the use of JPAs was limited to squeeze thermal noise or vacuum noise. This changed in 2009 when a Josephson parametric amplifier was used by K. Lehnert *et al.* in a setup measuring the motion of a nanomechanical resonator [21]. Today, Josephson parametric amplifiers are of broad interest in the circuit QED community [19, 21-23].

In the course of this thesis, a flux-driven Josephson parametric amplifier, designed by T. Yamamoto and fabricated by I. Kunihiro, was studied. The working principle can be compared to a playground swing. By periodically leaning back and forth, the resonant frequency of the swing is varied periodically, increasing its deflection amplitude with time. The very same amplification principle applies to the flux-driven Josephson parametric amplifier. The oscillating system here is no mechanical pendulum, but its circuit analogon, a microwave resonator. The resonant frequency of such a resonator is determined by its capacitance and inductance, which is where the Josephson effect comes into play. A superconducting quantum interference device (SQUID) consisting of a superconducting loop intersected by two Josephson junctions terminates the center conductor of the resonator to ground, thus adding a flux-dependent nonlinear inductance to the resonator. Applying a periodically varying flux to the SQUID thus varies the resonant frequency periodically, amplifying the microwave signal coupled into the resonator.

This thesis is divided into three parts. In chapter 1, we shall lay the theoretical groundworks for a profound understanding of the flux-driven Josephson parametric amplifier. We will begin with the introduction of the coherent and squeezed states as two sets of basis states of the quantized electromagnetic field. After a short discussion of the principle of parametric amplification we will provide a detailed treatment of Josephson junctions and SQUIDs, the latter being one central building block of the flux-driven JPA. Subsequently, we shall have a look at the quantum mechanical treatment of Josephson parametric amplifiers. We will conclude the first chapter with a discussion of the noise properties of linear amplifiers.

A detailed description of the experimental setup is provided in chapter 2. We start with a short description of the samples and the corresponding sample holders before introducing the measurement setup. The latter involves a specialty, the use of mechanical microwave switches at Millikelvin temperatures. We describe our efforts to make switching at cryogenic temperatures possible.

In chapter 3, we present the experimental results obtained. In the course of this thesis, we provide a detailed characterization of two Josephson parametric amplifier samples. We have performed a thorough analysis of the central building block of our samples, the resonator. Both the phase-dependent and phase-independent gain of the JPAs were measured at different working points in order to find the parameter space were the amplifiers are working best. For the optimal working point, also the bandwidth was determined.

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# 1 Theory of Josephson parametric amplifiers

The construction of low-noise or even noiseless amplifiers for the detection of weak quantum microwave signals is a fundamental challenge in the field of circuit QED. As mentioned in the introduction, noiseless amplification, however, is only possible if an amplifier treats its input signals differently with respect to phase. One promising representative of this class of amplifiers is the flux-driven Josephson parametric amplifier. In this chapter we shall therefore lay the foundations for a fundamental understanding of its functional principles. We will first establish the required quantum mechanical basis by introducing the coherent and squeezed states. After discussing the principle of parametric amplification on the basis of an example from classical mechanics, we provide a detailed description of the constituents of the flux-driven Josephson parametric amplifier. The central building block, the SQUID-terminated transmission line resonator, will be treated in detail before we provide a quantum mechanical description of the working principle of the JPA. We will conclude the theory section outlining the quantum mechanical limits of linear amplifiers due to the influence of vacuum fluctuations.

## 1.1 Minimum-uncertainty states

First of all, we introduce two sets of basis states of the quantized electromagnetic field. We will outline the class of coherent states as these come closest to a classical treatment of light. Squeezed states are vital for a profound understanding of Josephson parametric amplifiers as they establish the theoretical basis for its most important characteristic, namely the potential to amplify one signal quadrature without, in principle, adding any noise.

The quadratures  $X_1$  and  $X_2$  of a sinusoidal signal with amplitude  $A_0$ , frequency  $\omega$  and phase  $\varphi$  are defined by

$$X(t) = A_0 \cos(\omega t - \varphi)$$
  
=  $\underbrace{A_0 \cos(\varphi)}_{X_1} \cos(\omega t) + \underbrace{A_0 \sin(\varphi)}_{X_2} \sin(\omega t)$   
=  $X_1 \cos(\omega t) + X_2 \sin(\omega t)$   
=  $\operatorname{Re}\left[(X_1 + iX_2) e^{-i\omega t}\right],$  (1.1)

with the imaginary unit i.

In a coordinate system rotating with the angular frequency  $\omega$ , this sinusoidal signal is represented by the point  $(X_1, X_2)$ , called the *phase space representation* of X(t). We note that  $X_1$  and  $X_2$  are conjugate variables such as position and momentum. The derivation of coherent and squeezed states presented below mainly follows the book by Walls and Milburn [24].

#### 1.1.1 Coherent states

Coherent states play an important role in quantum electrodynamics. In the limit of large photon numbers, they represent the quantum mechanical state closest to a classical, sinusoidal electromagnetic wave.

Let us first consider the ground state, that is to say the state with lowest possible energy, of an arbitrary mode of the electromagnetic field as the vacuum state. In phase-space representation where the uncertainty of a state is depicted by an error contour in a plain defined by the real and imaginary part of the complex amplitude of an arbitrary quantum state, the vacuum state is represented by a circle centered at the origin, see Fig. 1.1. The fact that the uncertainty of this state is described by a circle can be understood considering that the vacuum exhibits no preferred direction with respect to phase and thus the orientation of  $X_1$  and  $X_2$  can be chosen arbitrarily. The radius of the circle can be interpreted as a measure of the vacuum energy [25].



Figure 1.1: The vacuum state in phase space representation.  $X_1$  and  $X_2$  are the real and imaginary parts of the complex amplitude of an arbitrary quantum state. The radius  $\Delta X_1 = \Delta X_2$  illustrates the uncertainties in the quadratures.

A coherent state  $|\alpha\rangle$  is now created by applying the unitary displacement operator,

$$\hat{D}(\alpha) = \exp\left(\alpha \hat{a}^{\dagger} - \alpha^* \hat{a}\right), \qquad (1.2)$$

on the vacuum, where  $\hat{a}^{\dagger}$  and  $\hat{a}$  are bosonic creation and annihilation operators and  $\alpha$  is an arbitrary, nonzero complex number,

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle. \tag{1.3}$$

In phase space representation, creating a coherent state corresponds to shifting the vacuum by  $\alpha$ , see Fig. 1.2.



Figure 1.2: A coherent state is generated by applying the displacement operator  $\hat{D}(\alpha)$  on the vacuum.

#### **1.1.2 Squeezed states**

A quantum mechanical state that has less uncertainty in one quadrature than in the other belongs to the class of the so-called *squeezed states*. We want to start by defining the class of states with minimum uncertainty. We therefore begin with a general bosonic annihilation operator of a single mode field,

$$\hat{a} = \frac{\hat{X}_1 + i\hat{X}_2}{2},\tag{1.4}$$

with Hermitian operators  $\hat{X}_1$  and  $\hat{X}_2$  representing the two quadratures of the single mode field with expectation values  $X_1$  and  $X_2$ . Expressed in creation and annihilation operators, the quadrature operators read

$$X_1 = \hat{a}^{\dagger} + \hat{a}, \tag{1.5a}$$

$$\hat{X}_2 = i \left( \hat{a}^{\dagger} - \hat{a} \right). \tag{1.5b}$$

From the commutation relations for the bosonic creation and annihilation operators we obtain the commutation relation for  $\hat{X}_1$  and  $\hat{X}_2$ ,

$$\left[\hat{X}_{1}, \hat{X}_{2}\right] = 2i.$$
 (1.6)

Next, we consider the universal uncertainty principle [26] for arbitrary Hermitian operators  $\hat{A}$  and  $\hat{B}$ ,

$$\Delta A \cdot \Delta B \ge \frac{1}{2} \left| \left\langle \Psi \left| \left[ \hat{A}, \hat{B} \right] \right| \Psi \right\rangle \right|, \qquad (1.7)$$

in which  $\Psi$  is an arbitrary normalized function and the uncertainty  $\Delta A$  is defined as the root mean square of the deviation of A from its mean, i.e.  $(\Delta A)^2 = \langle \Psi | (\hat{A} - A_0)^2 | \Psi \rangle$  where the mean value of A is defined by  $A_0 = \langle \Psi | \hat{A} | \Psi \rangle$ . The uncertainty  $\Delta B$  is defined in the same way. We then get the uncertainty principle for  $X_1$  and  $X_2$ ,

$$\Delta X_1 \cdot \Delta X_2 \ge 1. \tag{1.8}$$

As an example let us consider a harmonic oscillator with frequency  $\omega_0$  and mass m. The corresponding annihilation operator reads

$$\hat{a} = \frac{1}{2} \left( \sqrt{\frac{2m\omega_0}{\hbar}} \hat{q} - i\sqrt{\frac{2}{m\omega_0\hbar}} \hat{p} \right)$$
(1.9)

with the position operator

$$\hat{q} = \sqrt{\frac{\hbar}{2m\omega_0}} \left( \hat{a} + \hat{a}^{\dagger} \right) \tag{1.10}$$

and the momentum operator

$$\hat{p} = i\sqrt{\frac{\hbar m\omega_0}{2}} \left(\hat{a} - \hat{a}^{\dagger}\right).$$
(1.11)

For this special case, the uncertainty principle (1.8) turns into the well-known Heisenberg uncertainty principle

$$\Delta p \cdot \Delta q \ge \frac{\hbar}{2}.\tag{1.12}$$

A family of minimum-uncertainty states is now defined by considering the equal sign in (1.8):

$$\Delta X_1 \cdot \Delta X_2 = 1 \tag{1.13}$$

Among these minimum-uncertainty states, a special case are the coherent states discussed in Section 1.1.1, where the uncertainty is distributed equally on both quadratures,

$$\Delta X_1 = \Delta X_2 = 1. \tag{1.14}$$

A more general class of states are the *squeezed states*, where the uncertainty is reduced in one quadrature and increased in the other<sup>1</sup>, see Fig. 1.3,

$$\Delta X_1 < \Delta X_2 \text{ or } \Delta X_1 > \Delta X_2. \tag{1.15}$$

From the quantum electrodynamics' point of view, squeezed states are generated by applying the squeezing operator  $\hat{S}(\epsilon)$ ,

$$\hat{S}(\epsilon) = \exp\left(\frac{1}{2}\epsilon^*\hat{a}^2 - \frac{1}{2}\epsilon\hat{a}^{\dagger 2}\right),\tag{1.16}$$

where  $\epsilon = re^{2i\varphi}$  defines the squeezing factor  $r = |\epsilon|$ , a measure for the attenuation and amplification of the uncertainty in the respective quadratures. The angle  $\varphi$  accounts for the fact that squeezing is not necessarily carried along the axis  $X_1$  and  $X_2$ , see Fig. 1.3. A general squeezed state  $|\alpha, \epsilon\rangle$  is generated from the vacuum by first squeezing the vacuum and then displacing it, see Fig. 1.4,

$$|\alpha, \epsilon\rangle = \hat{D}(\alpha)\,\hat{S}(\epsilon)\,|0\rangle\,. \tag{1.17}$$

## 1.2 The principle of parametric amplification

In classical physics, the concept of parametric amplification can be understood considering a pendulum whose length can be varied periodically by means of a drive, see Fig. 1.5.

<sup>&</sup>lt;sup>1</sup>In literature, a squeezed state that also is a state of minimum uncertainty, i.e.  $\Delta X_1 < \Delta X_2$  and  $\Delta X_1 \cdot \Delta X_2 = 1$ , is sometimes referred to as an *ideal* squeezed state [27].



Figure 1.3: A squeezed state in phase space representation with the squeezing factor r.  $Y_1 + iY_2 = (X_1 + iX_2) \cdot e^{-i\varphi}$  is the rotated complex amplitude.



Figure 1.4: A squeezed state is generated by first squeezing the vacuum and subsequent displacement.

If the length l of the pendulum is constant in time, the classical equation of motion describing the time-dependence of the small angle deflection  $\varphi$  of the pendulum is given by

$$\ddot{\varphi} + \frac{g}{l}\varphi = 0, \tag{1.18}$$

where g is the acceleration of gravity. The eigenfrequency  $\omega_0$  of the pendulum reads

$$\omega_0 = \sqrt{\frac{g}{l}}.\tag{1.19}$$

Parametric amplification of the oscillation amplitude is obtained by setting the drive (cf. Fig. 1.5) to twice the eigenfrequency and thus periodically varying the pendulum  $length^2$ 

$$l \to l(t) = l + \Delta l \cdot \cos\left(2\omega_0 t\right). \tag{1.20}$$

The differential equation obtained by substituting the length (1.20) into (1.18) can be solved numerically. The solution for the initial conditions  $\dot{\varphi}(0) = 0$ ,  $\varphi(0) = 1$  is shown in

<sup>&</sup>lt;sup>2</sup>An everyday example, the pumping of a playground swing, can also be modeled by a parametrically driven harmonic oscillator. The case where one stands on a swing and periodically leans back and forth is straightforward to understand [28]. In the seated case, however, it is only a good approximation [29].



Figure 1.5: A pendulum whose length can be varied periodically.

Fig. 1.6. It can be seen that the work done by the drive to periodically vary the length of the pendulum increases the angular deflection of the pendulum in time. For clarity, we have neglected damping terms in our calculation that would limit the deflection.

From these classical considerations, we shall now pass on to a quantum mechanical treatment of parametric amplification. To derive the Hamiltonian of a quantum mechanical driven harmonic oscillator (cf. Appendix A), we again start with the unperturbed equation of motion

$$\frac{d^2q}{dt^2} + \omega_0^2 q = 0 \tag{1.21}$$

and introduce a modulation of the eigenfrequency  $\omega_0 \to \omega_0 [1 + \delta \cos(\alpha \omega_0 t)]$ , in which  $\delta$  is the amplitude and  $\alpha \omega_0$  is the frequency of the modulation. Neglecting  $\delta^2$ -terms, this leads to

$$\frac{d^2q}{dt^2} + \omega_0^2 \left[1 + 2\delta \cos\left(\alpha\omega_0 t\right)\right] q^2 = 0.$$
(1.22)

The Hamiltonian corresponding to this equation of motion is

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + \frac{m}{2}\omega_0^2 \left[1 + 2\delta\cos\left(\alpha\omega_0 t\right)\right]\hat{q}^2.$$
(1.23)

Introducing annihilation and creation operators (1.10, 1.11) leads to the Hamiltonian of a parametrically pumped harmonic oscillator,

$$\hat{\mathcal{H}} = \hbar\omega_0 \left[ \hat{a}^{\dagger} \hat{a} + \frac{1}{2} + 2\delta \cos\left(\alpha\omega_0 t\right) \left( \hat{a} + \hat{a}^{\dagger} \right)^2 \right].$$
(1.24)

The vacuum component of  $\hat{\mathcal{H}}$ , i.e. the term  $\frac{1}{2}\hbar\omega_0$ , is usually omitted as it merely represents a constant energy offset. In the following sections we will treat the flux-driven Josephson parametric amplifier, a device that is described by this Hamiltonian.



Figure 1.6: Parametrically pumping a pendulum increases its deflection with time. In a real setup, however, friction limits the maximum amplitude. We also note that for large deflections, the small-angle approximation used in (1.18) no longer holds.

## 1.3 The Josephson effect

The Josephson effect, predicted by Brian D. Josephson in 1962, is observed for two weakly coupled superconductors. Among other possibilities, such a coupling can be achieved by isolating the two superconductors by means of an insulating layer, see Fig. 1.7. Analogical to a normal metal - insulator - metal junction, where electrons can tunnel through the barrier if it is sufficiently thin, Cooper pairs can tunnel through the insulator between the two superconductors. The supercurrent in the two superconductors is described by the macroscopic wave functions  $\Psi_1$  and  $\Psi_2$ ,

$$\Psi_1 = \sqrt{n_1^*} e^{i\theta_1},\tag{1.25a}$$

$$\Psi_2 = \sqrt{n_2^*} e^{i\theta_2},\tag{1.25b}$$

in which  $\sqrt{n_1^*}$  and  $\sqrt{n_2^*}$  are the Cooper pair densities and  $\theta_1$  and  $\theta_2$  are the phases of the macroscopic wave functions of the two superconductors. For thin insulating barriers, the two macroscopic wave functions overlap. Similar to the hydrogen molecule, where molecular binding is a result of the coupling energy that arises from the overlap of the electronic wave functions of the constituting atoms, the two weakly coupled superconductors can also be regarded as a molecule. The corresponding coupling energy is called the *Josephson coupling energy* and will be derived below.



Figure 1.7: A Josephson junction where two superconductors are connected via an insulating layer.

#### 1.3.1 The Josephson equations

The Josephson equations establish a relation between the phase differences of the macroscopic wave functions of the superconductors and the supercurrent across the insulating barrier between them. For a detailed derivation we refer to Ref. [30].

In the presence of a magnetic field, the gauge-invariant phase difference  $\gamma$  is defined as [31]

$$\gamma(\vec{r},t) = \theta_2(\vec{r},t) - \theta_1(\vec{r},t) - \frac{2\pi}{\Phi_0} \int_{1}^{2} \vec{A}(\vec{r},t) \cdot d\vec{l}, \qquad (1.26)$$

where  $\Phi_0 = \frac{h}{2e}$  is the magnetic flux quantum and  $\vec{A}$  is a magnetic vector potential. The integration path is along the direction of current, for an SIS<sup>3</sup>-type junction the path is across the barrier from superconductor 1 to superconductor 2. The first Josephson equation, also known as the current-phase relation, describes the Josephson current across the barrier as a function of the gauge-invariant phase difference,

$$I_s(\gamma) = I_c \sin \gamma, \tag{1.27}$$

in which  $I_c$  is the maximum or critical Josephson current. In the absence of scalar and vector potentials, the first Josephson equation simplifies to

$$I_s(\gamma) = I_c \sin\left(\theta_2 - \theta_1\right) \tag{1.28}$$

and denotes a supercurrent varying sinusoidally with the phase difference  $\theta_2 - \theta_1$  across the junction.

The second Josephson equation reads

$$\frac{\partial \gamma}{\partial t} = \frac{2\pi}{\Phi_0} \int_{1}^{2} \vec{E} \left( \vec{r}, t \right) \cdot d\vec{l}, \qquad (1.29)$$

where  $\int_{1}^{2} \vec{E}(\vec{r},t) \cdot d\vec{l}$  corresponds to a voltage drop across the junction. Therefore, the second Josephson equation is also called the voltage-phase relation. In the case of a

<sup>&</sup>lt;sup>3</sup>superconductor-insulator-superconductor

constant voltage V applied to the Josephson junction, this equation simplifies to

$$\frac{\partial \gamma}{\partial t} = \frac{2\pi}{\Phi_0} V. \tag{1.30}$$

Integrating this equation over time yields

$$\gamma(t) = \gamma_0 + \frac{2\pi}{\Phi_0} V \cdot t. \tag{1.31}$$

Inserting this result in the first Josephson equation (1.27), we obtain that the Josephson current  $I_s(t) = I_c \sin \gamma(t)$  oscillates at the Josephson frequency

$$\frac{\nu}{V} = \frac{\omega}{2\pi V} = \frac{1}{\Phi_0} = 483.5979 \frac{\text{MHz}}{\mu \text{V}}.$$
(1.32)

The Josephson constant  $K_{J-90} := \frac{1}{\Phi_0}$  is used to define the Volt since 1990 [32].

As mentioned above, a Josephson junction can be considered as a molecule with a finite binding energy due to the overlap of the macroscopic wave functions. In order to derive the energy stored in the junction, we integrate the power from time t = 0 to time  $t = t_0$ ,

$$E_J = \int_{0}^{t_0} I_s V \, dt. \tag{1.33}$$

Substituting the two Josephson equations 1.27 and 1.30 into 1.33 we obtain

$$E_J = \int_{0}^{t_0} \left( I_c \sin \tilde{\gamma} \right) \left( \frac{\Phi_0}{2\pi} \frac{d\tilde{\gamma}}{dt} \right) dt.$$
 (1.34)

With the phase difference  $\gamma(0) = 0$  and  $\gamma(t_0) = \gamma$  this integral rewrites to

$$E_J = \frac{\Phi_0 I_c}{2\pi} \int_0^\gamma \sin \tilde{\gamma} \, d\tilde{\gamma}. \tag{1.35}$$

Integration gives an expression for the energy stored in the junction, the *Josephson* coupling energy,

$$E_J = \frac{\Phi_0 I_c}{2\pi} \left( 1 - \cos \gamma \right) = E_{J0} \left( 1 - \cos \gamma \right).$$
 (1.36)

Another notable property of the Josephson junction is that it behaves as a nonlinear inductance. To demonstrate this, we consider the time derivative of the current-phase relation (1.27),

$$\frac{dI_s}{dt} = I_c \cos \gamma \frac{d\gamma}{dt}.$$
(1.37)

Using the voltage-phase relation yields

$$\frac{dI_s}{dt} = I_c \cos\gamma \frac{2\pi}{\Phi_0} V. \tag{1.38}$$

Comparing this result with the law of induction,  $V = L \cdot \frac{dI}{dt}$ , we can define the Josephson inductance  $L_s$ ,

$$L_s = \frac{\Phi_0}{2\pi I_c \cos\gamma} = L_c \frac{1}{\cos\gamma} \tag{1.39}$$

with the critical Josephson inductance

$$L_c = \frac{\Phi_0}{2\pi I_c}.\tag{1.40}$$

#### 1.3.2 The dc SQUID

Derivating the Josephson equations, we have seen that the macroscopic wave function manifests itself in temporal interference resulting in the Josephson ac currents. Now we shall consider a device where spatial interference can be observed in analogy to the double slit experiment. Figure 1.8 shows a schematic illustration of a *Superconducting Quantum Interference Device*, SQUID. It consists of a superconducting loop that is intersected by two Josephson junctions. The ring is penetrated by a magnetic field perpendicular to its surface normal. A transport current I flows across the loop. The Josephson junctions however will provide a limit for the maximum current  $I_{s,max}$  that can be sent across the loop. For a detailed analysis of the SQUID, we refer to Ref. [33]. In the course of this work however we will restrict our discussion of the SQUID to the aspects relevant for understanding the functionality of the Josephson parametric amplifier.



Figure 1.8: A dc SQUID consists of a superconducting loop that is intersected by two Josephson junctions. A current I, divided into  $I_1$  and  $I_2$  along the loop arms, is sent through the SQUID. A magnetic field B is applied to the SQUID. Flux quantization requires the generation of a circular current J.

If a magnetic field is applied to a superconducting loop without Josephson junctions, the total magnetic flux in the loop will be an integer multiple of the flux quantum  $\Phi_0$ . This effect, known as flux quantization, was postulated by F. London in 1950 [34] and experimentally proven by R. Doll and M. Näbauer in Munich [35] and B. S. Deaver and W. M. Fairbanks in Stanford [36]. As the superconducting loop can be exposed to arbitrary magnetic fields, a ring current J in the superconducting loop is generated such that its contribution to the total flux rounds the external flux to the nearest integer multiple of  $\Phi_0$  i.e.,

$$\Phi_{\rm tot} = \Phi_{\rm ext} + LJ = n \cdot \Phi_0, \tag{1.41}$$

where L is the self inductance of the loop,  $\Phi_{\text{ext}}$  is the external magnetic flux and n is an integer.

Using the denotations from Fig. 1.8, we shall now provide a derivation for the maximum current through the SQUID depending on of the external magnetic flux [32]. The current I through the SQUID is divided into the currents  $I_1$  and  $I_2$  flowing along the SQUID arms. Current conservation demands

$$I = I_1 + I_2. (1.42)$$

As we assume the loop arms and the two Josephson junctions to be identical, the current I will be distributed on both arms. Taking the ring current into account, we can write down expressions for  $I_1$  and  $I_2$ ,

$$I_1 = \frac{I}{2} + J,$$
 (1.43a)

$$I_2 = \frac{I}{2} - J.$$
 (1.43b)

With the current-phase relation (1.27) and assuming that the critical currents  $I_c$  of both Josephson junctions are identical, we get

$$\frac{I}{2} + J = I_c \sin \gamma_1, \qquad (1.44a)$$

$$\frac{I}{2} - J = I_c \sin \gamma_2, \qquad (1.44b)$$

in which  $\gamma_1$  and  $\gamma_2$  are the gauge-invariant phase differences across the two Josephson junctions. It can now be shown [32] that integrating the phase gradient  $\nabla \gamma$  over the loop (cf. Fig. 1.8) yields an expression for the phase difference

$$\gamma_2 - \gamma_1 = \frac{2\pi}{\Phi_0} \Phi_{\text{tot}} = \frac{2\pi}{\Phi_0} (\Phi_{\text{ext}} + LJ).$$
 (1.45)

We now assume that we can neglect the contribution of LJ to the magnetic flux. It is evident that the ring current J cannot be larger than the critical current  $I_c$  of the Josephson junctions. Therefore, the flux LJ has to be smaller than  $LI_c$ . We now demand that the flux LJ is small compared to half a flux quantum, that is

$$\beta_c := \frac{2LI_c}{\Phi_0} \ll 1,\tag{1.46}$$

where  $\beta_c$  is called the screening parameter. If we neglect the flux contribution of the ring current, we get  $\Phi_{\text{tot}} = \Phi_{\text{ext}}$ . Combining (1.44) and (1.45) allows to eliminate  $\gamma_2$ , yielding

$$I = I_c \left[ \sin \gamma_1 + \sin \left( 2\pi \frac{\Phi_{\text{ext}}}{\Phi_0} + \gamma_1 \right) \right].$$
 (1.47)

Defining the variable  $\chi := \gamma_1 + \pi \frac{\Phi_{\text{ext}}}{\Phi_0}$  and applying trigonometric identities allows to rewrite (1.47) to

$$I = 2I_c \cdot \sin \chi \cdot \cos \left( \pi \frac{\Phi_{\text{ext}}}{\Phi_0} \right).$$
 (1.48)

At a given flux  $\Phi_{\text{ext}}$  and current I,  $\chi$  will adjust itself to satisfy (1.48). The current I can only grow as long as  $-1 < \sin \chi < 1$ . For larger currents I, (1.48) can no longer be satisfied and a voltage will drop across the SQUID. We have therefore found an expression for the maximum supercurrent across the SQUID,

$$I_{s,\max} = 2I_c \left| \cos \left( \pi \frac{\Phi_{\text{ext}}}{\Phi_0} \right) \right|. \tag{1.49}$$

The reasoning used to derive the inductance of a single Josephson junction (1.37 - 1.40) can also be used to derive the flux-dependent inductance of a SQUID [37].

$$L_{c,\text{SQUID}}(\Phi_{\text{ext}}) = \frac{\hbar}{2eI_{s,\text{max}}} = \frac{\Phi_0}{4\pi I_c \left|\cos\left(\pi \frac{\Phi_a}{\Phi_0}\right)\right|}$$
(1.50)

## 1.4 The layout of the flux-driven Josephson parametric amplifier

In this section we shall give an overview of our Josephson parametric amplifier, designed and manufactured by T. Yamamoto and K. Inomata at NEC in the group of Y. Nakamura [19]. We will get to know the constituents and their role for the operation of the amplifier. A circuit diagram of the sample is shown in Fig. 1.9. We provide a detailed analysis of the main constituents, namely the transmission line resonator with the coupling capacitor, the dc SQUID and the pump line.

#### 1.4.1 Lumped-element model for the transmission line resonator

The central element of our Josephson parametric amplifier is a transmission line resonator with capacitance C, inductance L, resistance R and the coupling capacitance  $C_c$ . Describing the resonator using a lumped-element approach [38] yields a resonant frequency of

$$\omega_0 = \frac{1}{\sqrt{LC}}.\tag{1.51}$$

The calculations presented below will provide expressions for the internal and external quality factor. With these expressions, suitable design parameters for the resonator can be chosen in order to adjust the quality factors to the desired values.



Figure 1.9: Sample layout (top figure) and circuit diagram (bottom figure) of a flux driven Josephson parametric amplifier. A magnetic flux  $\Phi$  is applied to the SQUID. The output signal of the Josephson parametric amplifier is detected at the same port where the input signal is applied.

The input impedance  $Z_{in}$  of such an *RLC*-resonator with coupling capacitance  $C_c$  (cf. Fig. 1.10) is given by

$$Z_{\rm in} = \frac{1}{i\omega C_c} + \frac{1}{\frac{1}{R} + \frac{1}{i\omega L} + i\omega C}$$
  
$$= \frac{1}{i\omega C_c} + \frac{1}{\frac{1}{R} + i\omega C \left(1 - \frac{\omega_0^2}{\omega^2}\right)}$$
  
$$\approx \frac{1}{i\omega C_c} + \frac{1}{\frac{1}{R} + 2i \left(\omega - \omega_0\right) C},$$
 (1.52)

for  $\omega \approx \omega_0$ . The expression for  $Z_{\rm in}$  can be divided into real and imaginary parts,

$$Z_{\rm in} = \frac{R}{1 + 4C^2 R^2 (\omega - \omega_0)^2} - i \left[ \frac{1}{\omega C_c} + \frac{2(\omega - \omega_0) C R^2}{1 + 4C^2 R^2 (\omega - \omega_0)^2} \right].$$
 (1.53)

At resonant frequency, the imaginary part of the impedance equals zero [39]. As a consequence of the coupling capacitance, this *loaded resonant frequency*  $\omega_L$  will differ from  $\omega_0$  and is given by

$$\omega_L = \frac{CR\omega_0 \left(4C + C_c\right) \pm \sqrt{C \left(R^2 \omega_0^2 C_c^2 C - 4C - 2C_c\right)}}{2CR \left(2C + C_c\right)}.$$
(1.54)

With the approximation  $R^2 \omega_0^2 C_c^2 C \gg 4C, 2C_c$ , the two solutions reduce to

$$\omega_{L,1} = \omega_0, \tag{1.55a}$$

$$\omega_{L,2} = \frac{\omega_0}{1 + \frac{C_c}{2C}}.\tag{1.55b}$$

The only physically relevant solution is  $\omega_{L,2}$ . The other solution  $\omega_{L,1}$  implies a very large real part of the impedance [39] resulting in a large impedance mismatch between both sides of the coupling capacitance, thus preventing a signal from being coupled into the resonator via the coupling capacitance. In what follows we will refer to the relevant solution of (1.54) as  $\omega_L$ .

In the case of the Josephson parametric amplifier, however, the resonator is not connected to ground directly, but via a SQUID with the flux-dependent inductance (1.50). Hence, the total inductance of the resonator reads

$$L_{\rm tot} = L + L_{c,\rm SQUID}.$$
 (1.56)

Substituting L by  $L_{\text{tot}}$  in (1.51) yields the flux-dependent resonant frequency

$$\omega_r(\Phi_{\text{ext}}) = \frac{\omega_0}{\sqrt{1 + \frac{L_{c,\text{SQUID}}(\Phi_{\text{ext}})}{L}}},\tag{1.57}$$

where  $\omega_0$  is the resonant frequency and L is the inductance of the quarter-wavelength transmission line resonator without the SQUID.

Another characteristic quantity of a resonator is its quality factor Q with the general definition [40]

$$Q = \omega_L \cdot \frac{\text{time-average energy stored in the system}}{\text{energy loss per second in the system}},$$
(1.58)

in which  $\omega_L$  is the resonant frequency of the resonator. The bandwidth  $\Delta \omega_{\text{FWHM}}$  of the resonator is found to be the inverse quality factor [41],

$$\Delta\omega_{\rm FWHM} = \frac{\omega_0}{Q},\tag{1.59}$$

where  $\Delta \omega_{\text{FWHM}}$  is the full width at half maximum (FWHM) of the resonance peak of the resonator, see Fig. 1.11. We will now consider the different loss channels of the resonator.

First we consider a resonator that is not coupled to the environment. Loss therefore is only determined by the imperfections of the resonator. The resonator is operated at a finite temperature, so quasi-particle excitations give rise to a loss mechanism. Furthermore, high-frequency electromagnetic fields can penetrate the surface of a superconductor up to a certain depth, a mechanism comparable to the skin effect in normal metals. Due to the presence of quasi-particle excitations, the resistance of the superconductor becomes finite [42]. The resistance taking all these loss mechanism into account is called the



Figure 1.10: The circuit of the coupled transmission line resonator (left) can be transformed into its so-called Norton equivalent circuit (right) [45]. The resonator part of the circuit is marked by the blue area. I represents the microwave source that delivers the input signal to the resonator.  $Z_{in}$  denotes the input impedance seen by the source. Expressions for the quantities  $L, R, C, R^*$ and  $C^*$  are provided in text.

surface resistance  $R_s$ . Its dependence on the temperature T, the frequency  $\omega$  and the residual resistance  $R_{\rm res}$  is given by [32]

$$R_s \propto \frac{\omega^2}{T} e^{-\frac{\Delta_0}{k_B T}} + R_{\rm res}, \qquad (1.60)$$

in which  $\Delta_0$  is the energy gap of the superconductor and  $k_B$  is the Boltzmann constant. As we can see, the surface resistance scales with  $\omega^2$  in contrast to the skin effect of a normal metal where the surface resistance is proportional to  $\sqrt{\omega}$  [43].

In a resonator, losses do not only occur in the conductor [44]. In the dielectric, two-level systems can be excited. Regardless of material losses, energy is also lost by radiation into free space [39]. The quality factor taking all these loss mechanisms of the conductor and the dielectric into account is called the *internal quality factor*  $Q_{int}$  given by [38]

$$Q_{\rm int} = \omega_L R C_{\rm eff}|_{\omega = \omega_L}, \qquad (1.61)$$

with the characteristic parameters

$$C_{\text{eff}}(\omega) = C + C^*(\omega), \qquad (1.62)$$

$$C^*(\omega) = \frac{C_c}{1 + \omega^2 C_c^2 Z_0^2},\tag{1.63}$$

$$Z_0 = \sqrt{\frac{L}{C}},\tag{1.64}$$

in which  $Z_0$  is the load impedance, see Fig. 1.10.

Now we consider a perfect cavity that is coupled to the environment, in our case via the coupling capacitance  $C_c$ , see Fig. 1.9. The corresponding quality factor is hence called

the external quality factor  $Q_{\text{ext}}$  and its dependence on the coupling capacitance is given by

$$Q_{\text{ext}} = \omega_L R^* C_{\text{eff}}|_{\omega = \omega_L} \tag{1.65}$$

in which  $R^*$  is defined as

$$R^*(\omega) = \frac{1 + \omega^2 C_c^2 Z_0^2}{\omega^2 C_c^2 Z_0}.$$
(1.66)

The overall quality factor of the cavity, also called the *loaded quality factor*, taking all loss channels into account is given by [46]

$$\frac{1}{Q} = \frac{1}{Q_{\text{int}}} + \frac{1}{Q_{\text{ext}}}.$$
 (1.67)

Another very important quantity is the reflection coefficient  $\Gamma$ . If an input signal  $A_{in}$  is applied to the resonator, the detected output signal will be  $\Gamma \cdot A_{in}$ . The theoretical value for the complex quantity  $\Gamma$  is given by (cf. Appendix A)

$$\Gamma = \frac{\left(\omega - \omega_L\right)^2 + i\kappa_2\left(\omega - \omega_L\right) + \frac{\kappa_1^2 - \kappa_2^2}{4}}{\left(\omega - \omega_L + i\frac{\kappa_1 + \kappa_2}{2}\right)^2}$$
(1.68)

with the coupling constants

$$\kappa_1 = \frac{\omega_L}{Q_{\text{ext}}},\tag{1.69}$$

$$\kappa_2 = \frac{\omega_L}{Q_{\text{int}}}.$$
(1.70)

Magnitude and phase of the complex reflection coefficient  $\Gamma$  are shown in Fig. 1.11. If the external quality factor is greater than the internal quality factor, the resonator is *undercoupled*. Otherwise, for  $Q_{\text{ext}} < Q_{\text{int}}$ , the resonator is *overcoupled*.

In Fig. 1.12, we compare the magnitude and phase of  $\Gamma$  for different values of  $Q_{\text{int}}$  for a fixed external quality factor. It can be seen that the phase shows only marginal dependence on  $Q_{\text{int}}$ , whereas the magnitude shows a strong dependence. The latter can be understood considering that, as discussed above, the internal quality factor is a measure for imperfections of the resonator. The smaller  $Q_{\text{int}}$  is, the more signal is lost inside the resonator. Thus, the detected output signal level is smaller for decreasing  $Q_{\text{int}}$ .

The lumped-element approach presented here provides a relatively simple model for the calculation of the resonator parameters. However, the real physical device is a transmission line resonator with length l and distributed capacitance  $C_l$  and inductance  $L_l$  per unit length. The modes of the electric field in such a resonator depend on the boundary conditions. Considering a resonator with one end open and the other directly connected to ground, one has the case of a quarter-wavelength resonator with resonant frequency

$$\widetilde{\omega}_0 = \frac{1}{4l\sqrt{L_lC_l}}.$$
(1.71)



Figure 1.11: Magnitude (top) and phase (bottom) of the reflection coefficient  $\Gamma$  (cf. Eq. 1.68) as a function of frequency. The calculation was performed with  $Q_{\text{ext}} = 100$ ,  $Q_{\text{int}} = 600$ ,  $\omega_L = 6.0$  GHz for the overcoupled case and  $Q_{\text{ext}} = 1000$ ,  $Q_{\text{int}} = 600$ ,  $\omega_L = 6.0$  GHz for the undercoupled case. The black arrows in the top plot denote the full width at half maximum (FWHM).

#### 1.4.2 The pump line

The pump line, the second building block of the Josephson parametric amplifier, is inductively coupled to the SQUID, see Fig. 1.9. By applying a microwave signal at frequency  $\omega_{\text{pump}}$  to the pump port, the SQUID loop can be interspersed with a periodically varying magnetic flux. This flux contribution adds to the external magnetic flux which is constant in time and used to set the resonant frequency of the transmission line resonator in terms of (1.57). In our experiments, this external flux was generated by a coil mounted in the vicinity of the sample.



Figure 1.12: Magnitude (top) and phase (bottom) of the reflection coefficient  $\Gamma$  (cf. Eq. 1.68) as a function of frequency for different values of  $Q_{\text{int}}$ . The resonator parameters  $Q_{\text{ext}} = 300$  and  $\omega_L = 6.0$  GHz are the same for all three curves. The magnitude strongly depends on the internal quality factor whereas the phase shows only little dependence.

As now the magnetic flux penetrating the SQUID loop is varied periodically, it follows from (1.50) that the inductance of the SQUID and therefore the boundary condition of the transmission line resonator is also varied periodically. According to (1.57), this results in a periodic variation of the resonant frequency. From a quantum mechanical point of view, we end up with the pumped harmonic oscillator discussed in Section 1.2.

The pump port is physically separated from the signal port, which is a major advantage of the design developed by T. Yamamoto compared to previous designs [17]. Furthermore, the pump frequency is not a harmonic of the quarter wavelength resonator, increasing the isolation between pump and signal port by orders of magnitude. The latter is important since the amplitude of the pump signal usually is several orders of magnitude larger than the input signal and crosstalk of the pump to the input signal should be held as small as possible.

A general design advantage of all parametric amplifiers featuring a SQUID-terminated transmission line resonator is the possibility to set the resonance frequency during the experiment by applying an easy-to-control external magnetic field. This allows to tune the Josephson parametric amplifier to the frequency of other components of a circuit QED setup, e.g. artificial atoms.

The fact that the periodic variation of the resonant frequency is governed by the periodic variation of the flux through the SQUID loop is the reason why this design class of Josephson parametric amplifiers is called the *flux-driven Josephson parametric amplifiers* in contrast to e.g. the current-driven design developed by AT&T [17].

## 1.5 Generation and detection of squeezed states

In this section, we shall get familiar with the functionality of Josephson parametric amplifiers. Before providing a detailed quantum mechanical analysis, we will first give a rough idea of the general working principle.

#### 1.5.1 Working principle of a Josephson parametric amplifier

The Josephson parametric amplifier is operated in reflection, i.e. the input signal is applied to the same port where the output signal is detected, which will be referenced to as the signal port in what follows.

If an input signal at frequency  $\omega_0 - \omega$  is applied to the signal port of the Josephson parametric amplifier, where  $\omega_0$  is half the pump frequency (cf. Section 1.2), an additional signal at frequency  $\omega_0 + \omega$ , called the *idler mode*, is generated, cf. Fig. 1.13<sup>4</sup>. This can be understood as a result of mixing<sup>5</sup> the pump signal and the input signal [23]:

$$\omega_{\text{pump}} - \omega_{\text{signal}} = 2\omega_0 - (\omega_0 - \omega) = \omega_0 + \omega = \omega_{\text{idler}}$$
(1.72)

Both the signal and the idler mode are amplified by means of the parametric work performed by the pump and reflected back to the signal port where they can be detected.

<sup>&</sup>lt;sup>4</sup>For a detailed quantum mechanical analysis of the idler mode generation we would like to refer to Appendix A.

<sup>&</sup>lt;sup>5</sup>Mixing two signals is understood as a multiplication of these signals. For sinusoidal signals at the frequencies  $\omega_1$  and  $\omega_2$  the result is a superposition of two sinusoidal signals, one at the sum frequency and one at the difference frequency. This follows directly from the theorem  $\sin(\alpha) \cdot \sin(\beta) = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)].$ 



Figure 1.13: Working principle of a Josephson parametric amplifier. The schematic shows the process of idler mode generation and signal mode amplification by means of a pump signal. The amplification factors  $\mathcal{M}$  and  $\mathcal{G}$  will be derived in Section 1.5.2.

#### 1.5.2 Quantum mechanical treatment

For a mathematical analysis of the amplification process, we consider an input signal consisting of two modes, one at  $\omega_0 - \omega$  and one at  $\omega_0 + \omega$ . The calculations presented below basically follow Ref. [47].

The corresponding operator of the electric field reads

$$\hat{A}_{in}(t) = A \left[ \hat{a}(\omega) e^{-i(\omega_0 + \omega)t} + \hat{a}(-\omega) e^{-i(\omega_0 - \omega)t} + \hat{a}^{\dagger}(\omega) e^{i(\omega_0 + \omega)t} + \hat{a}^{\dagger}(-\omega) e^{i(\omega_0 - \omega)t} \right], \quad (1.73)$$

where, classically,  $a(\omega)$  and  $a(-\omega)$  are defined as the amplitudes of the incoming signal at the frequencies  $\omega_0 + \omega$  and  $\omega_0 - \omega$ . Quantum mechanically,  $\hat{a}$  and  $\hat{a}^{\dagger}$  are bosonic annihilation and creation operators satisfying the usual commutation relations

$$\left[\hat{a}(\omega), \hat{a}^{\dagger}(\omega)\right] = \left[\hat{a}(-\omega), \hat{a}^{\dagger}(-\omega)\right] = 1, \qquad (1.74a)$$

$$[\hat{a}(\omega), \hat{a}(\omega)] = [\hat{a}(-\omega), \hat{a}(-\omega)] = 0,$$
 (1.74b)

$$[\hat{a}(\omega), \hat{a}(-\omega)] = [\hat{a}(\omega), \hat{a}^{\dagger}(-\omega)] = 0.$$
(1.74c)

The amplified reflected signal then has the same form as (1.73),

$$\hat{A}_{\text{out}}(t) = A \left[ \hat{b}(\omega) e^{-i(\omega_0 + \omega)t} + \hat{b}(-\omega) e^{-i(\omega_0 - \omega)t} + \hat{b}^{\dagger}(\omega) e^{i(\omega_0 + \omega)t} + \hat{b}^{\dagger}(-\omega) e^{i(\omega_0 - \omega)t} \right].$$
(1.75)

We shall now establish a relationship between the input signal and the reflected signal

by means of a scattering matrix

$$\begin{bmatrix} \hat{b}(\omega) \\ \hat{b}^{\dagger}(-\omega) \end{bmatrix} = \begin{bmatrix} \mathcal{G}(\omega) & \mathcal{M}(\omega) \\ \mathcal{M}^{*}(-\omega) & \mathcal{G}^{*}(-\omega) \end{bmatrix} \begin{bmatrix} \hat{a}(\omega) \\ \hat{a}^{\dagger}(-\omega) \end{bmatrix}.$$
 (1.76)

This allows to write the annihilation and creation operators of the output signal as a linear combination of input signal annihilation and creation operators, introducing the signal amplitude gain  $\mathcal{G}$  and the intermodulation amplitude gain  $\mathcal{M}$ .

From the commutation relations (1.74) it follows

$$\left|\mathcal{G}(\pm\omega)\right|^2 - \left|\mathcal{M}(\pm\omega)\right|^2 = 1 \tag{1.77}$$

and

$$\mathcal{G}(\omega) \cdot \mathcal{M}(-\omega) = \mathcal{G}(-\omega) \cdot \mathcal{M}(\omega).$$
 (1.78)

Equations (1.77) and (1.78) yield

$$\left|\mathcal{G}(\omega)\right|^{2} = \left|\mathcal{G}(-\omega)\right|^{2}, \qquad (1.79a)$$

$$\mathcal{M}(\omega)|^2 = |\mathcal{M}(-\omega)|^2.$$
(1.79b)

A very important result of (1.76 - 1.79) is

$$\left|\mathcal{G}(\omega)\right|^2 - \left|\mathcal{M}(-\omega)\right|^2 = 1,\tag{1.80}$$

indicating that signal and intermodulation gain converge to the same limiting value for large gains, an assumption that holds for well performing parametric amplifiers [19,47].

Up to now, we have not considered the effects of internal and external noise sources in our analysis of the Josephson parametric amplifier. These are best described by a model consisting of an ideal, noiseless and lossless Josephson parametric amplifier, a beam splitter and external noise sources, see Fig. 1.14. The power transmittance<sup>6</sup> of the beam splitter is denoted as  $\sqrt{\eta}$ . As we assume the beam splitter to be non-absorptive, the power reflectance is consequently  $1 - \sqrt{\eta}$ . The different noise sources are modeled as noise coupled in via the beam splitter at the same frequency as the input signal, but uncorrelated in phase. The noise operators  $\hat{c}_{in}$  and  $\hat{c}_{out}$  are also bosonic operators satisfying commutation relations similar to (1.74). The signal at the input port of the ideal Josephson parametric amplifier then reads:

$$\hat{a}(\omega) = \sqrt[4]{\eta} \,\hat{a}_{\rm in}(\omega) + \sqrt{1 - \sqrt{\eta}} \,\hat{c}_{\rm in}(\omega) \tag{1.81}$$

Both the signal and the noise component of the input are now amplified by the ideal Josephson parametric amplifier in terms of (1.76). The output signal including the noise contribution  $\hat{c}_{out}$  is then

$$\hat{b}_{\rm out}(\omega) = \sqrt[4]{\eta} \,\hat{b}(\omega) + \sqrt{1 - \sqrt{\eta}} \,\hat{c}_{\rm out}(\omega). \tag{1.82}$$

<sup>&</sup>lt;sup>6</sup>The reason for the somewhat unusual definition of the power transmittance using a square root will become clear in (1.90), where we will see that  $\eta$  is a physical quantity that can be measured directly.



Figure 1.14: The beam splitter model for the lossy Josephson parametric amplifier.

The output of the Josephson parametric amplifier, a superposition of the amplified signal and idler modes, is a squeezed state, as we will see below. An adequate detector for squeezed coherent radiation is a homodyne detector [48] that multiplies the output of the amplifier by  $2B\cos(\omega_0 t + \vartheta)$ , the so-called local oscillator. In doing so, sum- and difference frequencies are generated in the same way as described on p. 19.

As the sum frequencies usually are filtered away using a low pass filter, the output current of the homodyne detector reads

$$\hat{I} = AB \left[ \hat{b}_{\text{out}}(\omega) e^{-i(\omega t - \vartheta)} + \hat{b}_{\text{out}}(-\omega) e^{i(\omega t + \vartheta)} + \hat{b}_{\text{out}}^{\dagger}(\omega) e^{i(\omega t - \vartheta)} + \hat{b}_{\text{out}}^{\dagger}(-\omega) e^{-i(\omega t + \vartheta)} \right].$$
(1.83)

We shall now consider the classical case where the creation and annihilation operators are complex amplitudes and let the incoming electromagnetic field consist of a signal at frequency  $\omega_0 + \omega$ . Following from (1.76) and neglecting the noise contributions  $c_{in}$  and  $c_{out}$ , the output spectral density  $S(\omega)$  of the homodyne detector reads

$$S(\omega) = \frac{\eta \hbar \omega_0}{2} \left[ |\mathcal{G}(\omega)|^2 + |\mathcal{M}(-\omega)|^2 + e^{2i\vartheta} \mathcal{G}(\omega) \mathcal{M}(-\omega) + e^{-2i\vartheta} \mathcal{G}^*(\omega) \mathcal{M}^*(-\omega) \right] \cdot |a(\omega)|^2.$$
(1.84)

Introducing the signal power gain  $G := |\mathcal{G}(\omega)|^2$ , (1.77) rewrites to  $|\mathcal{M}(\omega)|^2 = G - 1$ , which allows to rewrite the complex number  $\mathcal{G}(\omega)\mathcal{M}(-\omega)$ ,

$$\mathcal{G}(\omega)\mathcal{M}(-\omega) = \sqrt{G}\sqrt{G-1}e^{2i\psi},\tag{1.85}$$

where  $\psi$  is an intrinsic phase of the Josephson parametric amplifier. In absence of a pump signal applied to the Josephson parametric amplifier, there is no signal-to-idler conversion resulting in  $\mathcal{M}(\omega) = 0$  and G = 1.

The quantity F is now defined as the signal power ratio when the pump is turned on relative to when it is turned off and, using (1.84) and (1.85), is given by

$$F(\varphi) = 2G - 1 - 2\sqrt{G\sqrt{G-1}\cos 2\varphi}, \qquad (1.86)$$

with  $\varphi := \vartheta + \psi$ . Obviously, the homodyne detector is phase sensitive, proving that the output signal of the Josephson parametric amplifier is indeed a squeezed state [25]. If one now sets the local oscillator phase  $\vartheta$  in a way that  $\cos(2\varphi) = 1$ , maximum parametric amplification is observed by the detector,

$$F_{\max} = 2G - 1 + 2\sqrt{G}\sqrt{G - 1}.$$
(1.87)

By adjusting the local oscillator phase such that  $\cos(2\varphi) = -1$ , parametric deamplification of the signal is observed,

$$F_{\min} = 2G - 1 - 2\sqrt{G}\sqrt{G - 1}.$$
(1.88)

 $F_{\rm max}$  and  $F_{\rm min}$  obey the relation

$$F_{\max} \cdot F_{\min} = 1. \tag{1.89}$$

Furthermore, the maximally amplified and the maximally deamplified component of the signal differ in local oscillator phase by  $\frac{\pi}{2}$ , i.e. they are in quadrature.

So far we have shown that a Josephson parametric amplifier deamplifies one quadrature of a signal applied to the input port and that this deamplification can be detected with a homodyne detector. Considering now an input signal with more than one frequency component, the spectral density at the input of the homodyne detector reads

$$S(\omega) = \eta F(\varphi) S_{\rm in}(\omega) + (1 - \sqrt{\eta}) \left[\sqrt{\eta} F(\varphi) + 1\right] S_{\rm loss}(\omega), \tag{1.90}$$

in which  $S_{in}(\omega)$  is the spectral density of the input signal at the beam splitter in front of the Josephson parametric amplifier and  $S_{loss}(\omega)$  is the spectral density of the internal and external noise sources.

The quantity  $\eta$  can be determined experimentally. Without any pump signal, (1.86) yields  $F(\varphi) = 1$  and thus, (1.90) simplifies to  $S(\omega) = \eta S_{in}(\omega)$  neglecting noise terms<sup>7</sup>. Considering now that an ideal short exhibits total reflection, the quantity  $\eta$  can be measured by comparing the reflectance of the Josephson parametric amplifier to that of a short.

Now we shall deal with the special case where the input signal frequency is  $\omega_0$  and therefore matches half the pump frequency. This is called the *degenerate-mode operation* of the parametric amplifier. Considering again Fig. 1.13, it can be seen that signal mode and idler mode coincide in the output. As they are at the same frequency and have a fixed phase relative to each other, they will interfere. We note that in the case of degenerate gain no homodyne detector is needed to observe phase dependence.

<sup>&</sup>lt;sup>7</sup>The determination of  $\eta$  should be performed at lowest possible temperature to minimize thermal noise. In order to maximize the signal-to-noise ratio, a coherent state with a defined frequency is chosen as the input signal. This allows to choose a small resolution bandwidth of the detector. Thus, only a small fraction of the thermal noise is coupled into the detector [47].

If the input signal frequency matches half the pump frequency of the resonator, theory (cf. Appendix A) gives the resulting phase-dependent gain of the degenerate-mode operation,

$$G_d = \frac{\left(\frac{\kappa_1^2 - \kappa_2^2}{4} + 4\delta^2 \omega_0^2\right)^2 + 4\delta^2 \kappa_1^2 \omega_0^2 - 4\delta\kappa_1 \omega_0 \left(\frac{\kappa_1^2 - \kappa_2^2}{4} + 4\delta^2 \omega_0^2\right) \sin\left(2\phi\right)}{\left(\frac{\kappa^2}{4} - 4\delta^2 \omega_0^2\right)^2}, \quad (1.91)$$

in which  $\delta$  is the pump amplitude from (1.24),  $\phi$  is the signal phase and the coupling constants are defined by

$$\kappa_1 = \frac{\omega_0}{Q_{\text{ext}}} \tag{1.92a}$$

$$\kappa_2 = \frac{\omega_0}{Q_{\rm int}} \tag{1.92b}$$

$$\kappa = \kappa_1 + \kappa_2. \tag{1.92c}$$

The sinusoidal dependence on the phase  $\phi$  proves that the parametric amplifier is also phase sensitive in the degenerate-mode operation. Adjusting the phase, we get expressions for the minimum and maximum gain:

$$G_d^{\min} = \left(\frac{2\delta\omega_0 - \frac{\kappa_1 - \kappa_2}{2}}{2\delta\omega_0 + \frac{\kappa_1 + \kappa_2}{2}}\right)^2 < 1 \text{ for } \phi = \frac{\pi}{4} + n\pi, \qquad (1.93a)$$

$$G_d^{\max} = \left(\frac{2\delta\omega_0 + \frac{\kappa_1 - \kappa_2}{2}}{2\delta\omega_0 - \frac{\kappa_1 + \kappa_2}{2}}\right)^2 > 1 \text{ for } \phi = \frac{3\pi}{4} + n\pi,$$
(1.93b)

where we have assumed the condition  $(\kappa_1^2 - \kappa_2^2)/4 + 4\delta^2 \omega_0^2 > 0$ . Apparently we get a similar expression as (1.89) for the degenerate case,

$$G_d^{\min} \cdot G_d^{\max} = 1. \tag{1.94}$$

### 1.6 Noise properties of linear amplifiers

As we have discussed in the introduction, signals in circuit quantum electrodynamics are too weak to be measured directly and therefore have to be amplified. In recent years, good progress was made to reduce the noise added by microwave amplifiers, and with the development of HEMT<sup>8</sup> amplifiers the noise temperature of these amplifiers could be brought down to less than 3 Kelvins [49]. At 6 GHz, a noise temperature of 3 K corresponds to approximately 10 noise photons. However, as we will see in this section, there is a physical lower limit for the amplifier noise.

<sup>&</sup>lt;sup>8</sup>High Electron Mobility Transistor

#### 1.6.1 Quantum mechanical limits of amplifier noise properties

Measuring small microwave signals, *linear amplifiers* are predominantly used. A linear amplifier is an amplifier with a linear dependence of the output on the input<sup>9</sup>. This very broad class of amplifiers can be divided into phase-sensitive and phase-insensitive linear amplifiers.

If we write the input signal of an amplifier in quadrature representation,

$$X(t) = X_1 \cos(\omega t) + X_2 \sin(\omega t) = \text{Re}\left[ (X_1 + iX_2) e^{-i\omega t} \right],$$
(1.95)

in which  $X_1$  and  $X_2$  are the amplitudes of the two quadratures, we can define a phaseinsensitive linear amplifier as a linear amplifier exhibiting the same amplification for both quadratures.

It was shown by Caves [50] that the quantum limit for the noise added to the output signal by any linear, phase-insensitive amplifier is given by

$$A \ge \frac{1}{2} \left| 1 - \frac{1}{G} \right|,\tag{1.96}$$

where the amplification G is given in units of number of quanta and A is the number of noise quanta added to the signal by the amplifier. One can see that in the limit of large gains, "half a photon" is added by the amplifier.

A phase-sensitive linear amplifier on the contrary is a linear amplifier responding differently to the two quadratures. Defining gains  $G_1$  and  $G_2$  and noise numbers  $A_1$  and  $A_2$ for the two quadratures, Caves [50] has shown that the *amplifier uncertainty principle* holds for the quantum limit of phase-sensitive linear amplifiers:

$$A_1 A_2 \ge \frac{1}{16} \left| 1 - \frac{1}{\sqrt{G_1 G_2}} \right|^2.$$
 (1.97)

For the special case of a phase-insensitive linear amplifier  $(G_1 = G_2 = G \text{ and } A_1 = A_2 = \frac{1}{2}A)$ , (1.97) reduces to (1.96).

Considering the very important case  $G_1G_2 = 1$ , which we have already seen above for the Josephson parametric amplifier, the amplifier is allowed not to add any noise to either quadrature of the signal. However, the equation  $G_1G_2 = 1$  implies that only one signal quadrature can be amplified, the other needs to be deamplified.

In order to link the unequal treatment of the two quadratures with the squeezed states discussed in Section 1.1.2, we consider a coherent state as the input of a phase-sensitive amplifier, see Fig. 1.15. We arbitrarily choose  $G_1 = 2$  and  $G_2 = \frac{1}{2}$ , fulfilling  $G_1G_2 = 1$ , as the linear amplification factors of the phase-sensitive linear amplifier for the two quadratures. We can see that the output is indeed a squeezed state with reduced uncertainty in the  $X_2$ -Quadrature.

<sup>&</sup>lt;sup>9</sup>In literature, the term "linear amplifier" is sometimes used for phase-insensitive amplifiers. However, in the scope of this work, we will use the definition given above.



Figure 1.15: A coherent state is amplified by a phase-sensitive linear amplifier with the amplification factors  $G_1 = 2$  and  $G_2 = \frac{1}{2}$ . The output is a squeezed state with reduced uncertainty in the  $X_2$ -Quadrature. The amplifier leaves the area of the error contour unchanged, which implies amplification without adding noise.

#### 1.6.2 A hypothetical setup with two Josephson parametric amplifiers

In a gedankenexperiment, we consider a setup like the one depicted in Fig. 1.16. The signal to be amplified is first split by a 50:50 beam splitter. For the signal transmitted through the beam splitter, one signal quadrature is amplified noiselessly in terms of (1.97). For the signal reflected by the beam splitter, the other quadrature is amplified in the same way. If one now joined the amplified signals with another 50:50 beam splitter, one would effectively have amplified both quadratures of the initial signal without adding any noise, violating the uncertainty principle.

In this hypothetical setup, the uncertainty principle is saved by the fact that it would require an ideal three-port beam splitter, that is to say a lossless, matched, reciprocal three-port device. However, it can be shown that such devices do not exist [51].

It can even be shown that an alleged three port microwave beam splitter such as a Wilkinson Power Divider has an internal, hidden fourth port [52] introducing vacuum noise into the system before amplification. Thus, after the signal has passed the first beam splitter, it contains a signal component and a vacuum noise component which are both amplified by the respective parametric amplifier.

The noise properties of the whole setup shown in Fig. 1.16 are thus described by (1.96) and also comply with the definition of a phase-insensitive amplifier as both signal quadratures are treated in the same way.


Figure 1.16: A hypothetical setup with two parametric amplifiers. The uncertainty principle is saved by the fact that ideal three-port beam splitters do not exist at the quantum level.

## 2 Experimental setup

In this chapter, we introduce the experimental setup used to characterize our Josephson parametric amplifiers. We start with a short description of the samples and the corresponding sample holders. We also describe our efforts to optimize the transition from the sample to the microwave feed lines. In Section 2.2 we provide a detailed description of the experimental setup below room temperature. A special feature of our setup is the use of mechanical microwave switches at Millikelvin temperatures, allowing for the independent characterization of two samples in one setup. In a pilot experiment, we have tested whether mechanical switching at cryogenic temperatures is possible at all. The results of this experiment are described in Section 2.2.1. The last part of this chapter introduces the room temperature setup containing the microwave sources necessary to create the pump signal and the input signal for the Josephson parametric amplifier and the detectors used to measure and characterize the output signal of the JPA.

## 2.1 Sample and sample holder

In Section 1.4 we have discussed the constituents necessary to build a flux-driven Josephson parametric amplifier. Looking at the actual sample designed by T. Yamamoto and K. Inomata at NEC, we will see how the individual components were realized. Altogether, we have received 16 samples with four different sets of design parameters from NEC. In Section 2.1.1 we give a short overview of the materials used and the core design parameters. For a detailed description of the samples, we would like to refer to Appendix B.

## 2.1.1 The Josephson parametric amplifier sample

The substrate of the Josephson parametric amplifier sample is a silicon wafer. The ground plain, the center conductor of the transmission line resonator and the pump line are fabricated in niobium technology with a layer thickness of 50 nm. The SQUID Josephson junctions, however, are made from aluminum with an aluminum oxide barrier and are fabricated by shadow evaporation [54]. A microscope image of one of our samples is provided in Fig. 2.1.

The coupling capacitor (see Fig. 2.1(b)) defines the external quality factor of the transmission line resonator. For the two samples we have characterized in the course of our measurements, the design values were  $Q_{\text{ext}} = 30$  and  $Q_{\text{ext}} = 300$ , respectively. Both design and fabrication process were directed to maximize the internal quality factor.



(a) The Josephson parametric amplifier sample. The color transitions were generated by stitching the picture together from single frames. The meandering structure in the center is the transmission line resonator. The red rectangle marks the coupling capacitor. The pump line resides at the right hand side of the image and is marked by the yellow rectangle. The center conductors of the resonator and the pump line are approximately  $10 \,\mu$ m wide. The width of the adjacent gaps is approximately  $6 \,\mu$ m. The structures on the top and bottom are test SQUIDs and single Josephson junctions which were not used in the course of our experiments.



(b) The coupling capacitor  $C_c$ . The finger structure is used to increase the capacity without widening the center conductor.



(c) The SQUID terminating the transmission line resonator can be identified as the horizontal narrow white bands intersecting the vertical blue line to the left. The vertical structure in the center is the pump line.



(d) The pump line. The ground plane is punctuated with many small holes in order to immobilize trapped vortices in the ground plane [53].

Figure 2.1: Microscope images of one of the Josephson parametric amplifiers (Sample Cat. 0-2a, Q30) that we have characterized in the course of this thesis. Figure (a) shows the whole sample, whereas Figs. (b) - (d) show details with greater magnification.

The samples were designed to work best for input signal frequencies around 5.75 GHz. The reason for this is that key components we intend to use in follow-up experiments are designed for this frequency. For the measurements conducted in the course of this thesis, however, we have chosen a setup consisting only of components that allow to characterize the Josephson parametric amplifier over a broad frequency range. The only limiting components are the cryogenic HEMT-amplifier (cf. Section 2.2) working from 3.5 GHz to 8.5 GHz and the base temperature stage circulator (cf. Section 2.2.2) certified from 4.0 GHz to 8.0 GHz [51].

## 2.1.2 The alumina boards

Electrical contact between the sample and other microwave components utilized in the experimental setup is established by means of coaxial microwave feed lines. However, these microwave feed lines are not connected to the JPA sample directly. One reason is that connecting a coaxial microwave plug to a coplanar waveguide is technically demanding and requires soldering or complex bonding. Connecting the microwave plug directly to the sample would therefore be fraught with risk for the latter. Therefore, additional printed circuit boards (PCBs, see Fig. 2.4) are inserted between the sample and the microwave connectors. These PCBs are easier to fabricate and less sensitive than the JPA samples. As their thickness matches the thickness of the sample, establishing electrical contact between the PCB and the sample by bonding is uncritical. The PCBs are made from alumina<sup>1</sup>. On the top is a coplanar waveguide made from gold consisting of a center conductor and ground planes to the left and right, cf. Fig. 2.4. The width of the center conductor and the adjacent gaps are exactly the same as the ones at the edges of the JPA sample, see Fig. 2.1(a). The bottom is completely coated with a gold layer. The ground planes on the top are connected to the bottom by means of vias.

Inserting these alumina boards has additional advantages. The transition between the coaxial microwave feed line and the coplanar waveguide constitutes a discontinuity of the electromagnetic field and is therefore accompanied with an impedance mismatch. This leads to the generation of spurious ground plane modes of the electromagnetic field. These modes decay while propagating along the waveguide and therefore are sufficiently damped before reaching the Josephson parametric amplifier. The vias ensure that the ground planes of the waveguide are sufficiently connected to ground in the vicinity of the connector and in particular ensure that both ground planes of the waveguide are at the same electric potential. This leads to further suppression of ground plane modes.

## 2.1.3 The sample holder

The sample holder has multiple functions. The most important one is to provide a fixation for the sample, the alumina boards and the microwave connectors, see Fig. 2.4. Another task is to enable attaching the sample to the sample rod in the dilution fridge,

<sup>&</sup>lt;sup>1</sup>aluminum oxide,  $Al_2O_3$ 

see Fig. 2.8. A well designed sample holder in addition provides shielding from high-frequency radiation.

We have designed two kinds of sample holders. For the first one, the microwave connector is perpendicular to the top of the sample holder and therefore also perpendicular to the center conductor of the PCB, see Fig. 2.2. We will refer to this design as the *topmount design* or *topmount sample holder*, respectively. For the second sample holder design, referred to as *sidemount*, the microwave connector is parallel to the center strip of the coplanar waveguide, see Fig. 2.3. Using one topmount and one sidemount sample holder allows for stacking two sample holders on the sample rod and therefore enables the space-saving integration of two Josephson parametric amplifiers into the measurement setup, see Fig. 2.8. Furthermore, stacking the sample holders ensures that both JPAs are placed in the center of the field generated by the coil. Both sample holders were fabricated from gold-plated OFHC<sup>2</sup> copper. Fabrication was done by the in-house workshop.

The transition between the microwave feed lines and the alumina boards was realized using V-type microwave connectors [55] in combination with glass beads [56]. These glass beads establish the transition from the connector through the sample holder wall to the alumina board. They are soldered into the housing wall, ensuring a tight fit. This is important as on one end of the glass bead, the microwave connector is screwed into its thread transferring torque to the bead. The other side of the center pin of the glass bead is connected to the alumina board by a solder connection which would break if the glass bead would move or rotate. For a detailed description of the preparation of the sample holders, we refer to Appendix C.

# 2.1.4 The transition from the alumina board to the microwave connector

As mentioned in Section 2.1.2, an impedance mismatch occurs at the transition between the coaxial microwave feed line and the coplanar waveguide. This leads to partial reflection of the signal. The corresponding amplitude reflection coefficient  $\Gamma$  is given by [57]:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{2.1}$$

in which  $Z_L$  is the actual impedance of the transition and  $Z_0 = 50 \Omega$  is the matched impedance.

We have examined the influence of different wire- and ribbon-bonding techniques on the impedance in order to make it as close to  $50 \Omega$  as possible. We have chosen *Time Domain Reflectometry (TDR)* in order to determine the impedances. Here, a signal in the form of a step function is generated at one end of the device under test, in our case a chain consisting of a microwave cable, the microwave connector, the alumina board and a bond between the latter two. Impedance mismatches now lead to partial reflection of the step

<sup>&</sup>lt;sup>2</sup>Oxygen-Free High-Conductive



(a) Top view (left) and bottom view (right) of the topmount sample holder. The rectangular notches provide fixation for two PCBs and one sample in between. The heightened rim ensures that PCBs, sample and bonds are not touched by the lid. The microwave connectors are mounted in the big holes at the bottom. The holes in the side provide fixations for e.g. thermometers.



(b) The lid for the topmount sample holder.

Figure 2.2: CAD drawing of the topmount design. One major design advantage is the easy-to-fabricate lid without any accurately fitting flanges. The lid can be omitted if another sample holder is placed on top of the topmount sample holder as it is the case in our setup.



(a) The sidemount sample holder. The sample and the PCBs are placed in the rectangular notches in the same way as in the topmount sample holder. The plugs however are mounted sideways in the flanges.



(b) The corresponding lid. The notches have to be fabricated accurately in order to insure tight fit with the flanges.

Figure 2.3: CAD drawing of the sidemount sample holder design. One design advantage is that the microwave connectors are mounted sideways, allowing for stacking multiple sample holders. One disadvantage compared to the topmount design is the need for a more precise manufacturing to ensure high accuracy for fitting the flanges.



Figure 2.4: The topmount sample holder with sample and alumina boards. a The Josephson parametric amplifier sample. b Alumina board ground plane with screw hole and vias. c Alumina board center conductor. d Microwave glass bead center pin.

signal. These reflected signal components are detected time resolved. The amplitude of the reflected component gives quantitive information about the impedance mismatch. If the dielectric constant and the magnetic permeability, and therefore the speed of light, for the component under test are known, the position of an impedance mismatch can be determined as well. If the speed of light is not known, the position can also be identified by changing the electromagnetic environment, e.g. by shortening the coplanar waveguide with a conducting pencil, and monitoring the change in the TDR signal. Figure 2.5 shows TDR data for four combinations of wire and ribbon bonds between the alumina board and the center pin of the glass bead. As the impedance diverges at open ends, the transition point between pin and coplanar waveguide could be determined measuring the impedance when neither the pin nor the ground were connected to the alumina board, cf. Fig. 2.5(a).

Figure 2.5(b) shows the case where the pin was connected to the center conductor by one single bond wire and the ground connection was only established via the sample holder and the alumina board vias. Comparing the results of this case to the results of Fig. 2.5(e), where ribbons were used to connect the pin to the center conductor and to establish a direct connection to the ground planes, the impedance mismatch at the transition point was reduced from  $6.2 \Omega$  to  $2.3 \Omega$ , following Eq. (2.1). The amplitude reflection at the transition was reduced from 5.8 % to 2.2 %. We will see in Section 2.2 why reducing the reflection is of prime importance.

Figures 2.5(b) and 2.5(c) provide a direct comparison between a wire bond and a ribbon bond. As the impedance mismatch is reduced by more than  $1\Omega$ , it becomes clear why ribbon bonding should be the method of choice. However, the ribbons used for the tests described here were hand-cut from a piece of gold ribbon and bonded to the pin and the board using the head of a wire bonder. Although this method was sufficient for a test, it seemed not reliable enough to be used in the actual experiment at low temperatures. Therefore, we have decided to use standard wire bonds in our experiment except for the transition between glass bead center pin and alumina board where we have used a solder connection.

After the installation of the glass beads, the microwave connectors and the alumina boards and soldering the glass bead center conductor to the alumina board center conductor, but before bonding the alumina board ground plane to the sample holder and inserting the samples, we have measured the impedance of the sample holders, again using TDR. The results are shown in Fig. 2.6. These results were also used to decide which of the sample holder ports to use for the signal or pump line, respectively. As mentioned above, reflections in the signal line should be held as small as possible. We have therefore used the port that showed a smaller impedance mismatch as signal port. For the signal port of the sidemount and topmount sample holder, we have measured amplitude reflections of 2.3% and 3.2%, respectively. We note that TDR utilizes a broadband 30 GHz signal for the impedance measurements. In our experiments, however, we have only used signals from 3 - 6 GHz. Therefore, the impedance mismatches measured with TDR can be considered as worst case scenarios.

## 2.2 The cryogenic setup

The cryogenic setup comprises all components that are operated below room temperature, in particular the sample, the cryogenic amplifier and microwave components that are operated at or near base temperature. We have designed our cryogenic setup so that it meets three major demands:

- Capability to fully characterize two Josephson parametric amplifiers.
- Possibility to compare the reflection of each of the amplifiers to that of a short in order to determine  $\eta$ , cf. Section 1.5.2.
- Possibility to calibrate the input line and the output line including the amplifier chain.

Our setup meeting all of the above demands is shown in Fig. 2.7. The input signal is passing a series of attenuators at the different temperature stages before it reaches a cryogenic circulator at base temperature. This circulator directs the input signal to two cryogenic switches arranged in series. These switches allow to guide the input signal either to one of the samples or to a short. As the Josephson parametric amplifier is operated in reflection, i.e. input and output port are identical, the output signal is propagating back to the circulator. As the circulator treats signals differently with respect to their directed into the output line. At the 4-K stage, a cryogenic HEMT<sup>3</sup> amplifier amplifies the signal before it enters the room temperature part of the setup. The pump signal also passes a series of attenuators before it is split by a Wilkinson power divider at

<sup>&</sup>lt;sup>3</sup>High Electron Mobility Transistor



Figure 2.5: Impedances corresponding to different bonding techniques are shown for the transition between the connector pin and the alumina board. In the small figures (a) - (e), the yellow circle depicts the pin and the yellow stripes depict the center conductor and the ground planes of the coplanar waveguide on the alumina board. Figure (a), where the pin is not connected to the alumina board at all, acts as a reference. The open end causes the impedance to diverge. The black arrow in the top figure therefore marks the end of the pin.



Figure 2.6: Impedance properties of the (a) sidemount sample holder and (b) topmount sample holder. In both figures, the black curve was recorded when the sample holder was disconnected from the cable going from the TDR probe to the microwave connector in the sample holder, therefore marking the position of the microwave connector.

base temperature and sent to the pump ports of the samples. In the following sections we provide a detailed description of the components used below room temperature. The images provided in Figs. 2.8 and 2.9 show how the base temperature components and the samples were mounted to the sample rod of the cryostat.

## 2.2.1 The cryogenic switches

The demand on our setup of being able to compare the reflection of the Josephson parametric amplifiers to that of a short requires microwave switches working at cryogenic temperatures. In a pilot experiment we have shown that a modified version of the Agilent N1810UL coaxial switch [58] is working at low temperatures down to 1.2 K. The N1810UL is a single-pole double-throw<sup>4</sup> switch certified for signals ranging from dc to 20 GHz. One major advantage of this switch, making it suitable for low-temperature operation, is the fact that it is stable in both switch positions, so a current only needs to be applied when switching between these two positions and not for holding one of them. This property is referred to as *latching*. To switch between the two states, a current pulse of 100 mA needs to be applied to the switch coils for about 20 ms.

We have designed and built a pulse driver, that, at the push of a button, sends the required current pulse to the switch coils and thus changes its state. As the switch coils change resistance with temperature, the voltage applied to the switch coils needs to be adjustable, which was taken into account designing the driver. The pulse driver was

<sup>&</sup>lt;sup>4</sup>A single-pole double-throw (SPDT) switch is a three-port changeover switch where one port, called the COM (common) port, can be connected to one of the remaining ports.



Figure 2.7: The cryogenic setup. The input signal is attenuated passing through the different temperature stages. At the base temperature stage, a circulator directs the input signal towards the cryogenic switches which guide the input signal either to one of the JPAs or towards the short. The output signal, after passing the circulator, is amplified at the 4-K stage before it enters the room temperature stage. The pump signal is also attenuated before it reaches the base temperature stage where it is distributed equally on both samples by a Wilkinson power divider.



Figure 2.8: Front view of the setup at base temperature. a Sample rod. b Sidemount sample holder with sample Cat. 2-1c, Q300. c Topmount sample holder with sample Cat. 0-2a, Q30. d Thermometer monitoring the sample temperature.
e Circulator (cf. Section 2.2.2). f Wilkinson power divider (cf. Section 2.2.3). g Heatable attenuator (cf. Section 2.2.5). In the picture, the thermalization is not in place. h Heatable attenuator thermometer. i 5-port switch N1812UL. j 3-port switch N1810UL. k Sample heater to stabilize the sample temperature.



Figure 2.9: Rear view of the setup at base temperature. a Sample rod. b Superconducting coil used to set the resonant frequency of the resonator. c 5-port switch N1812UL. d 3-port switch N1810UL e Calibration short used to determine η. f Circulator (cf. Section 2.2.2).

designed so that it can also be computer operated by means of TTL signals. An image of the pulse driver is provided in Fig. 2.10.

The key element of the driver is a retriggerable monostable multivibrator [59] with an upstream debouncing circuit consisting of two cross-coupled NAND-gates. On the push of a button, the output of the debouncing circuit is a rising edge. An invertor, consisting of a NAND Schmitt-trigger gate, converts the rising edge into a falling edge. The retriggerable monostable multivibrator, then delivers the rectangular pulse required for changing the switch state to a Darlington transistor. The latter amplifies the pulse and provides the current needed to operate the cryogenic switch. The height of the rectangular pulse can be regulated by an adjustable voltage regulator setting the voltage of the collector of the Darlington transistor. The pulse length can be set by choosing a suitable combination for the external capacitance and resistance of the monostable multivibrator. For the latter, we have used another potentiometer that allows to fine adjust the pulse length to the desired value of 20 ms.



Figure 2.10: The pulse driver. All operation controls and circuits are implemented twice so that two switches can be controlled independently. **a** Switch to disconnect the switch driver from the switch feed lines. **b** Voltmeter displaying the switching voltage. **c** Measuring range shifter for the voltmeter. **d** Potentiometer to adjust the switching voltage. **e** BNC connector. An oscilloscope can be connected to analyze the switching pulse. The measured voltage has to be divided by  $10 \Omega$  in order to determine the pulse current. **f** Buttons (blue) to trigger a switching pulse. The top row LED lamps flash up if a switching pulse was triggered. The bottom row LED lamps indicate the switch state.

This circuit was implemented twice, with one circuit for each switching direction. The above mentioned debouncing circuit is of utmost importance as it prevents the generation of multiple sequent current pulses. These would not harm the switch itself, but as energy of approximately 15 mJ is dissipated to the base temperature stage of the cryostat per pulse, every additional pulse would unnecessarily heat the cryostat.

For the test of the switch, we have mounted it in a He-4-cryostat with vacuum insert (cf. Fig. 2.11), cooled it down to its base temperature of 1.2 K and measured the transmission through the switch for both switch positions. For the measurement we have used an HP8722D vector network analyzer set to an output power of -10 dBm and an IF-bandwidth of 100 Hz. The measured transmission curves in the frequency range from 50 MHz to 10 GHz are shown in Fig. 2.12. The COM port of the switch is here denoted as port "3".



Figure 2.11: Testbed for the Agilent N1810UL coaxial switch.

We see that the switch has very good transmission properties at cryogenic temperatures. The transmission is sufficiently flat over the whole frequency spectrum and shows an insertion loss smaller than the room temperature specifications of 0.37 dB at 1 GHz and 0.52 dB at 10 GHz. The negative transmission values result from inaccuracies in the available cold calibration data for the microwave feed lines. Comparing both transmission curves in Fig. 2.12, one can see that some peaks and dips appear in both curves at the same frequencies. We therefore can exclude noise as the origin of these peaks. Possible reasons include inaccuracies in the calibration data of the microwave feed lines as they were recorded in a separate cooldown, features of the microwave feed line and adapters connected to the COM-port and the COM-port itself. The most important result for us, however, is the fact that switching is possible at cryogenic temperatures and that we do not see any changes of the transmission properties after several switching cycles.

We have therefore concluded that the Agilent N1810UL coaxial switch is suitable for cryogenic applications and have decided to integrate the 3-port switch and its 5-port sister model, the N1812UL in our setup. How the switches are actually used in the setup can be seen in Fig. 2.7. The 3-port N1810UL allows for switching between the two



Figure 2.12: Insertion loss of the Agilent N1810UL microwave switch at 1.2 K. The green line denotes the room temperature specification.

Josephson parametric amplifiers and therefore makes it possible to characterize both of them independently which is one of the major requirements on our setup.

The 5-port N1812UL was installed in order to measure the reflection of the Josephson parametric amplifier relative to that of a short. Wiring the outputs of the 3-port switch to the COM ports of the 5-port switch makes it possible to do so for every Josephson parametric amplifier independently, see Fig. 2.7. The short used in our setup is a special calibration short. As both an ideal short and a perfect (lossless) Josephson parametric amplifier with the pump turned off exhibit total reflection, comparing the reflectance of a real (lossy) Josephson parametric amplifier to that of a short gives direct access to the quantity  $\eta$  discussed in Section 1.5.2.

#### 2.2.2 The circulators

As described in Section 1.5.1, the input signal is applied to the same port of the Josephson parametric amplifier where the output signal is detected. To separate the output signal from the input signal, a device is needed that treats signals differently with respect to their direction of propagation and therefore breaks the time-reversal symmetry of the electromagnetic field. This property is denoted as *directivity*. Examples for directive microwave devices are directional couplers and circulators. The latter have the advantage of almost lossless transmission in comparison to the coupling losses of the former while still providing isolations of approx. 20 dB. Therefore, we are using a circulator where breaking the time-reversal symmetry is achieved by means of a piece of ferrite placed in the vicinity of the microwave lines inside the circulator [57]. To avoid the effects of magnetic fields leaking out of the circulator, we have used a shielded model. The unitary, nonreciprocal scattering matrix of an ideal 3-port circulator is given by

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$
 (2.2)

Apparently, power can only be transferred from port 1 to 2, 2 to 3 and 3 to 1. At the same time, power transfer between any other port combinations is suppressed. It can now be understood why signal reflections at the sample holder should be held as small as possible. The reflected fraction of the signal is not only unavailable for amplification in the Josephson parametric amplifier, but, even worse, is directly coupled into the output line, distorting the output signal of our sample. The circulator does not only direct the input signal from the input line to the sample and the output signal from the sample into the amplification chain, but also isolates the sample from noise generated by a 50  $\Omega$  load at the 600 mK stage. This load is mounted to another circulator. This circulator is used to prevent noise from the amplifier from reaching the base temperature stage. Port 2 is terminated with the 50  $\Omega$  matched load. Amplifier noise entering the circulator from port 1 via the output line is directed to port 2 where it is absorbed, whereas the signal entering the circulator via port 3 is guided to the output line at port 1. For a detailed description of the cryogenic HEMT amplifier we refer to [51].

## 2.2.3 The pump line

Also the pump signal has to be fed to the two Josephson parametric amplifiers. Our solution here was to use a single pump line down to the base temperature stage, where the pump signal is split by a Wilkinson power divider [60] and distributed equally on both samples.

We have chosen the Wilkinson power divider to split the pump signal as we know that it is working reliably at cryogenic temperatures. An advantage of this solution compared to separate feed lines is that for a given signal fed into the pump line at room temperature, both samples are supplied with identical pump power levels.

#### 2.2.4 The attenuator configuration

Neither the pump signal nor the input signal can be sent down to the Josephson parametric amplifier directly as the 300 K thermal noise would obscure the input signals. Furthermore, the center conductor of the microwave cables has to be thermally coupled to the respective temperature stage.

Thus, attenuators are used at the various temperature stages of the cryostat. As these attenuators not only damp the noise, but also the signal, the latter has to be increased accordingly before being fed into the input line. Attenuators do not only damp the signal, but also add thermal noise themselves corresponding to the temperature stage at which they are placed.

The attenuator configuration of both the signal and the pump line are chosen such that the noise after the last attenuator, and thus at the signal and pump port of the Josephson parametric amplifier, is minimized. In doing so, several boundary conditions had to be taken into account.

- The maximum available input signal at the beginning of the microwave lines is limited by the maximum output power of the signal sources and the insertion loss of microwave components that may be placed in the respective line. In our case, this maximum available input power at the top end of the cryostat was measured to be 18.6 dBm.
- The pump line attenuator configuration had to be designed such that -30 dBm of pump power are available at the pump port of the sample. The experiments conducted by T. Yamamoto were performed at pump powers up to this level. In addition, a loss of 3 dB had to be taken into account for the power divider.
- The insertion loss of the microwave cables itself had to be considered.
- Only certain attenuator values are available. To widen the scope, we have used up to two attenuators for every temperature stage.
- The power dissipated at each temperature stage must not outvalue the cooling capacity of each stage. The maximum allowed values are given in Tab. 2.1.

Stage	Temperature	Max. dissipation	
	(mK)	$(\mu \mathbf{W})$	
1K-pot	1200	20000	
Still	600	500	
Step exchanger	50	200	
Sample	30	50	

Table 2.1: Maximum allowed values for the power dissipation at each stage.

The thermal noise in units of quanta added by each attenuator is given by [51]

$$S = \frac{1}{2} + \frac{1}{\exp\left(\frac{h \cdot \nu}{k_B \cdot T}\right) - 1}$$
(2.3)

in which  $h = 6.626 \cdot 10^{-34}$  Js is the Planck constant,  $k_B = 1.381 \cdot 10^{-23}$  J/K is the Boltzmann constant, T is the temperature of the attenuator and  $\nu$  is the signal or the pump frequency, respectively.

The dissipated power in  $\mu W$  is given by

$$P_{\text{diss},\,\mu\text{W}} = 1000 \cdot \left( 10^{\frac{P_{\text{in},\,\text{dBm}}}{10}} - 10^{\frac{P_{\text{in},\,\text{dBm}} - A_{\text{dB}}}{10}} \right) [\mu\text{W}]$$
(2.4)

in which  $P_{\text{in, dBm}}$  is the input power of the attenuator in dBm and  $A_{\text{dB}}$  is the attenuation in dB.

With available attenuators of 1, 3, 6, 10, 20 and  $30 \,\mathrm{dB}$  and an assumed insertion loss of the microwave cables of 7 dB, a brute force Maple algorithm<sup>5</sup> was used to find the optimal attenuator configuration with respect to lowest noise, but taking all boundary conditions into account. The results, which were subsequently implemented in our setup, can be found in Tab. 2.2 for the signal line and in Tab. 2.3 for the pump line.

$\mathbf{Stage}$	Temperature	Attenuation	dissipated Power	Noise
	(mK)	(dB)	$(\mu \mathbf{W})$	(quanta)
He-Bath	4200	6	2.98	289
1K-pot	1200	20	0.99	7.25
Still	600	20	0.01	2.94
Step Exchanger	100	3	< 0.005	2.04
Sample	30	30	< 0.005	0.502

Table 2.2: Attenuator configuration of the input line. The values were calculated for a signal frequency of 5.75 GHz and a signal power of -24 dBm before the first attenuator. With these values, the input signal power reaching the sample was calculated to be -110 dBm.

Stage	Temperature (mK)	Attenuation (dB)	dissipated Power $(\mu \mathbf{W})$	Noise (quanta)
He-Bath	4200	23	$7.21 \cdot 10^{4}$	10.3
1K-pot	1200	0	0.00	10.3
Still	600	3	181	6.34
Step Exchanger	100	6	136	2.10
Sample	30	6	34.23	1.03

Table 2.3: Attenuator configuration of the pump line, calculated for a pump frequency of 11.5 GHz and a pump power of 18.6 dBm before the first attenuator. The calculated pump power at the sample is -29.4 dBm.

## 2.2.5 The heatable attenuator

In order to be able to calibrate both the amplification chain and the input line, we have equipped the 30 dB attenuator of the signal line at the base temperature stage with a heater and a thermometer which allows us to stabilize the temperature of this attenuator to values ranging from base temperature to 800 mK. The temperature is controlled by a PICOWATT TS-530A temperature controller together with an AVS-47A resistance

<sup>&</sup>lt;sup>5</sup>Programmed and optimized by E. P. Menzel

bridge. Both devices are controlled by a LabView programme<sup>6</sup>. The attenuator is not mounted to the sample rod directly, thus avoiding direct thermal contact. This allows to stabilize the temperature of the attenuator independently of the sample rod temperature as long as the temperature difference between them is not too large. A schematic of the thermal coupling of the heatable attenuator to the different temperature stages is provided in Fig. 2.13.

The temperature dependence of the thermal noise of a resistor is well-known [52]. One can now calibrate the amplification chain applying neither an input signal nor a pump signal to the sample and thus only detecting the thermal noise at the top end of the amplification chain. Comparing the measured amplified thermal noise to the well-known input signal provides a good calibration of the amplification chain. If one now applies a signal to the signal input line, again with the pump turned off, and detects the amplified signal at the top end of the amplification chain, one is able to also calibrate the input line, as the output line characteristics are known. We will discuss the results of this experiment in Section 3.6.

As mentioned above, the aim of decoupling the 30 dB attenuator from the sample rod is to vary its temperature independent of the sample temperature. But as described in Fig. 2.13, a microwave cable still causes weak thermal coupling between attenuator and sample. In order to find out how far we can heat the attenuator until the sample also starts to heat up, we have set the attenuator temperature to values between 30 mK and 740 mK and stabilized it for about 30 minutes at each temperature. We have then measured the sample temperature. The results are shown in Fig. 2.14(a). It can be seen that the sample temperature rises to about 50 mK for high attenuator temperatures. To evaluate these data, we have calculated the thermal noise in units of quanta at the sample by means of Eq. (2.3) with a frequency of 5.639 GHz. The results are shown in Fig. 2.14(b). We see that the thermal noise differs only marginally from the vacuum limit of 0.5. We therefore conclude that we can indeed vary the temperature of the 30 dB attenuator without generating additional thermal noise at the sample stage.

## 2.2.6 The superconducting coil

The superconducting coil (cf. Fig. 2.9) is used to expose the samples to a magnetic field. This magnetic field in particular intersperses the SQUID terminating the transmission line resonator of the Josephson parametric amplifier with a magnetic flux.

The coil itself consists of a superconducting wire. Also the cryogenic section of the feed lines is superconducting. A current is now applied to the coil by means of a current source at room temperature. In order to take advantage of one of the most important properties of superconductivity, namely the current conduction without resistance, the current source can be bypassed by a *persistent current switch* located in the Helium bath. The switch is equipped with a heater. If the persistent current switch is normal

<sup>&</sup>lt;sup>6</sup>Programmed by the author during his employment as working student



Figure 2.13: Thermal coupling of the 30 dB attenuator. The attenuator is coupled weakly to the 50 mK stage and to the base temperature stage by the microwave cables. Using a well tempered silver ribbon, the dominating thermal coupling is established to the second step exchanger.



Figure 2.14: Sample temperature (left) and thermal noise at the sample (right) plotted against the attenuator temperature. Even if the sample temperature rises to about 50 mK when heating the attenuator to 740 mK, the thermal noise at the sample stage differs only marginally from the vacuum limit.

conducting, the current through the coil is determined by the current source. If the heater is switched off, i.e. the persistent current switch is superconducting, the current through the coil at the moment of transition will be constant in time even if the room temperature current source is switched off as the electric circuit consisting of the coil, parts of the feed lines and the persistent current switch is all superconducting and therefore maintains the supercurrent flowing through it. As the current noise caused by the current source is decoupled from the coil by the switch, the flux noise in the SQUID will be minimized.

## 2.3 The room temperature setup

In this section we shall introduce the room temperature part of our experimental setup. We will give a short description of the microwave sources and detectors and motivate the other components used. A schematic of the room temperature setup is provided in Fig. 2.15. All detectors and sources are connected to a 10 MHz Stanford FS725 Rubid-ium frequency standard in order to assure frequency and phase lock between the devices. The major advantage of our setup is that one can switch sources and detectors without unplugging and plugging back in, resulting in a reproducibility of the transmission properties better than 0.03 dB. Another advantage is that the switch used can be controlled remotely with a LabVIEW program.

#### 2.3.1 The input line

As described in Section 2.2, the role of the input line is to send a microwave signal down to the Josephson parametric amplifier which can then be amplified by the latter. In our setup, we have two ways of generating this input signal. One is to use the output port of the Rohde & Schwarz ZVA24 Network Vector Analyzer (ZVA). The signal power can not only be set by adjusting the ZVA output power, but also using two step attenuators with 110 dB in 10 dB steps and 11 dB in 1 dB steps, respectively. They were originally installed in order to test a more complex setup, but were left in place as the ZVA only offers an output power range from +16 dBm to -44 dBm. With the step attenuators in place, the output power can now be set arbitrarily from +13 dBm to -165 dBm. The switch placed after the step attenuators allows to utilize another input signal source, the Rohde & Schwarz SMF Microwave Signal Generator (SMF 100A). Inside the shielded room, the signal is attenuated by 40 dB before entering the cryogenic part of the setup.

#### 2.3.2 The pump line

The pump signal is generated by the Agilent E8267D PSG Microwave Vector Signal Generator (PSG). As discussed, the pump signal frequency is close to twice the signal frequency. In order to suppress crosstalk of pump frequency subharmonics, we have placed four 7.15 GHz highpass filters after the PSG in order to attenuate subharmonics by at least 106 dB. The phase of the pump signal can be controlled in two ways. One is the digital phase shifter of the PSG microwave source. Another is a mechanical motor-driven phase shifter. This was originally integrated as we intended to use the ZVA output signal simultaneously for both the JPA input signal and the pump utilizing a frequency doubler. However, we realized that the mechanical phase shifter strongly increases the measurement time, so we stuck to the digital phase shifter in the course of our measurements.



Figure 2.15: The room temperature setup. The input signal can either be generated by the SMF microwave source or the ZVA network vector analyzer. The output signal is amplified by about 73 dB (including the cryogenic amplifier) before it is split by a Wilkinson power divider, allowing for the utilization of two different detectors, the ZVA network vector analyzer and the FSP spectrum analyzer, without unplugging and plugging back in. The pump signal is generated by the PSG microwave source. Pump frequency subharmonics are suppressed by means of four high-pass filters. All sources and detectors are frequency-locked by a 10 MHz phase reference.

## 2.3.3 The output line

When the output signal of the Josephson parametric amplifier leaves the cryostat, it is already amplified by the cryogenic HEMT amplifier [51] (cf. Section 2.2) by 25 dB. At the room temperature stage, it is further amplified by two JS2 amplifiers with serial numbers 1448282 and 599349. The amplifiers are placed inside the shielded room as one wants to keep the cable length before the amplifiers as short as possible in order to not diminish the signal-to-noise ratio (SNR). Furthermore, spurious signals that may couple into the amplifiers are reduced inside the shielded room. At 6 GHz, the JS2 amplifiers amplify the signal by 23 dB and 25 dB, respectively. Between the amplifiers, circulators are placed in order to suppress reflections between subsequent amplifiers. After leaving the shielded room, the signal is split by a Wilkinson Power Divider. This allows to utilize two detectors without unplugging and plugging back in as mentioned above.

One detector is the Rohde & Schwarz FSP7 Spectrum Analyzer (FSP). A spectrum analyzer is able to measure the frequency-domain representation of a signal, i.e. its time-averaged power density versus the frequency [51,61]. The RF input signal is first down-converted by an oscillator whose frequency is tuned by a sweep generator. The resulting IF signal is amplified and filtered by a low-pass filter in order to set the resolution bandwidth and to remove possible intermodulation products from the signal. In order to achieve a wide dynamic range, the signal is subsequently amplified by a logarithmic response amplifier before it is detected by a diode detector. At last, a video filter with a variable bandwidth is used to display the signal.

The other detector we have used is the ZVA network analyzer [51, 62], which can measure the complex S-parameters of a device under test (DUT). A known signal, usually a sweep over a frequency band of interest, is generated by the ZVA. Part of this signal is used as reference and the other is sent to the DUT. The signal going to the DUT passes a directional coupler, so that each port of the vector network analyzer can be used as input or output port or both at the same time. Both the measurement signal and the reference signal are down-converted, filtered and amplified. In this stage, phase locking between reference and measured signal is assured. All signals are then converted to digital signals, analyzed and displayed. The bandwidth of the downconverted signal, called the intermediate frequency (IF) bandwidth, is an important quantity for measurements with a network analyzer as it determines the bandwidth of white noise coupled into the receiver. However, for small bandwidths, the measurement time is inverse proportional to the IF-bandwidth.

If the FSP is used as detector, the input signal can either be generated by the SMF or the ZVA as a spectrum analyzer only measures absolute quantities. If the ZVA is used as detector, the input signal also needs to be generated by the ZVA so that a reference signal is provided to the network vector analyzer.

# **3** Experimental results

After the discussion of the theoretical foundations and the measurement setup, we shall now turn towards the experiments we have conducted with our Josephson parametric amplifiers. As mentioned in Section 2.1, we have received 16 samples from NEC. We have chosen the samples Cat. 2-1c, Q300 and Cat. 0-2a, Q30 for our measurements. Their distinguishing feature is the design value for the external quality factor of the resonator of 300 and 30, respectively. Our decision was based on the fact that these two samples have the smallest and largest external quality factors among the samples we have received. For further details of the sample nomenclature, we refer to Appendix B.

The main aspect of our measurements was to characterize the samples, i.e. to find out if they are actually working, if they meet the design parameters and to find good working points concerning frequencies and input and pump power levels. For both samples, we have therefore started with a detailed analysis of the tunable resonator as this is the main constituent of a flux-driven Josephson parametric amplifier. Subsequently, we have investigated the gain properties of the samples. We also have looked at the amplifier bandwidth. As discussed in Section 2.2.1, our setup is also capable of determining the quantity  $\eta$ , which we will discuss in Section 3.5.

Unless otherwise stated, all power levels mentioned in this chapter refer to the output powers of the respective microwave sources or the input powers of the respective detectors and *not* to the power that is coupled into or coupled out of the sample. In the last section of this chapter, we will provide calibration data for the input and output line which allow to deduce the power levels at the sample.

## 3.1 Characterization of the resonator

As discussed in the theory section, the SQUID-terminated resonator is the central building block of the Josephson parametric amplifier. Hence, our first experiments were aiming at determining the properties of the resonator for both samples and checking the agreement with the design values. The most interesting property for us is the flux dependence of the resonant frequency. Further important properties are the internal and external quality factor.

## 3.1.1 Flux-dependence of the resonant frequency

In a first experiment, we have measured the reflection of the resonator at different magnetic flux values. We have therefore applied an input signal  $A_{\rm in}$  to the resonator and have detected the reflected signal  $\Gamma \cdot A_{in}$ . As described in Section 1.4.1, this allowed for the determination of the resonant frequency. In order to check if the SQUID terminating the transmission line resonator is actually working, we have measured the resonant frequency at different flux values and have indeed observed a periodic flux dependence. The flux was generated by the magnetic coil mounted in the vicinity of the samples (cf. Fig. 2.9).

The room temperature measurement setup differed from the one presented in Section 2.3. The measurements were conducted using the ZVA network analyzer set to an output power of -14 dBm and an IF-bandwidth of 100 Hz. The input power was chosen such that the resonator is operated in the linear regime. Both ports of the ZVA were connected to the input and output line feedthroughs of the shielded room via a 2 m SMA cable each, where port 2 was used as the signal source and port 1 as receiver. Furthermore, the room temperature circulators were not installed. The persistent current switch (cf. 2.2.6) was heated during the measurements as we have conducted measurements for 601 flux values. Switching the persistent current switch on and off for every flux value would have been too time consuming. For all measurements described in this section, the sample temperature was stabilized at 30 mK.

The results for sample Cat. 2-1c, Q300 are shown in Fig. 3.1. The phase was recorded in *phase deviation* mode. Here, the network vector analyzer calculates a linear fit to the raw phase data of the first trace. This linear fit now acts as a reference for the first and all further traces. The phase deviation that is displayed by the network vector analyzer is the deviation of the measured phase from the reference. At resonant frequency, we expect to observe a dip in magnitude and a phase shift by  $2\pi$  on going from  $\omega \ll \omega_{\rm res}$  to  $\omega \gg \omega_{\rm res}$ . The magnitude dip, however, is barely visible suggesting high internal quality factors as discussed in Section 1.4.1. The  $2\pi$  phase shift is clearly visible, indicating that the resonator is operated in the overcoupled regime. For the flux values where the phase shift is almost unrecognizable, the resonator is operated in the undercoupled regime and the phase assumes the form shown in Fig. 1.11. The reason for the transition between over and undercoupled regime is the resonant frequency dependence of the internal and external quality factor as described in Section 1.4.1.

In order to improve data quality, we have turned towards the calibration of the setup. As described in Section 2.3, a vector network analyzer compares input and output signal and delivers the complex S-parameters of the device under test. In our case, however, the comparison of the ZVA input and output signal does not give the reflection coefficient of the resonator, but does give the transmission coefficient of the whole setup. To this end, we have set the flux such that the resonator was operated in the undercoupled regime and the resonant frequency was outside of the measurement range. The corresponding trace detected by the network analyzer thus only contained features of the measurement setup and none of the resonator so that we could use this trace as calibration for all further measurements on sample Cat. 2-1c, Q300<sup>1</sup>. In order to reduce noise, we have performed the calibration with a reduced IF-bandwidth of 10 Hz. Also, the persistent

<sup>&</sup>lt;sup>1</sup>The calibration procedure had to be repeated only when the room temperature setup was changed. For sample Cat. 0-2a, Q30, separate calibration data were recorded.

current switch heater was switched off in order to minimize flux noise. After calibration, we have repeated the measurement of the flux-dependent resonant frequency. The results are shown in Fig. 3.2. Looking at the lower panel of Fig. 3.2, the phase shift<sup>2</sup> is clearly observable and again is well separated from the undercoupled regime. Also the dip in magnitude is better recognizable in some flux ranges.

We also observe that, as expected, the resonant frequency is periodic with the external magnetic flux. The theoretical dependence derived in Section 1.4.1,

$$\omega_r(\Phi) = \frac{\omega_0}{\sqrt{1 + \frac{L_{c,SQUID}(\Phi)}{L}}}$$
(1.57)

and

$$L_{c,\text{SQUID}}(\Phi) = \frac{\Phi_0}{4\pi I_c \left| \cos\left(\pi \frac{\Phi}{\Phi_0}\right) \right|}$$
(1.50)

allows to write the flux axis in units of  $\frac{\Phi}{\Phi_0}$ , where  $\Phi$  is the external magnetic flux and  $\Phi_0$  is the flux quantum, considering that the resonant frequency is  $\Phi_0$ -periodic and shows a maximum for  $\Phi = 0$ . We find that the change in coil current necessary to change the flux through the SQUID by  $\Phi_0$  is 138  $\mu$ A.

The same measurements, with and without calibration, though with a wider flux range and with the room temperature circulators in place for the calibrated measurement, were performed for sample Cat. 0-2a, Q30. The results are shown in Figs. 3.3 and 3.4. Apparently, a larger coil current,  $385 \,\mu$ A, is needed compared to sample Cat. 2-1c, Q300 in order to change the flux through the SQUID by one flux quantum. This can be explained by the fact that sample Cat. 0-2a, Q30 is mounted on top of sample Cat. 2-1c, Q300 and therefore is farther away from the coil.

Now that we have seen that the properties of the resonators of both samples show a good qualitative agreement with theory, we shall now turn towards a quantitative analysis of the resonator. The complex reflection coefficient  $\Gamma$ , defined as the amplitude of the signal coupled out of the resonator relative to the amplitude of the resonator input signal has a theoretical value of

$$\Gamma = \frac{\left(\omega - \omega_L\right)^2 + i\kappa_2\left(\omega - \omega_L\right) + \frac{\kappa_1^2 - \kappa_2^2}{4}}{\left(\omega - \omega_L + i\frac{\kappa_1 + \kappa_2}{2}\right)^2},\tag{1.68}$$

where  $\kappa_1 = \frac{\omega_L}{Q_{\text{ext}}}$ ,  $\kappa_2 = \frac{\omega_L}{Q_{\text{int}}}$  and  $\omega_L$  is the (flux-dependent) loaded resonant frequency (cf. Eq. (A.79) in Appendix A for  $\epsilon = 0$ ,  $\alpha = 2$ ,  $\langle c_{\text{in}} \rangle = 0$ ).

<sup>&</sup>lt;sup>2</sup>The network analyzer was set to measure the *unwrapped phase*, that is to say the actual phase difference is displayed even if it is larger than  $360^{\circ}$ .



Figure 3.1: Magnitude (top figure) and phase (bottom figure) of the resonator reflectance for sample Cat. 2-1c, Q300. The measurement was conducted with the ZVA network analyzer with no calibration applied. The dip in magnitude is barely visible, suggesting high internal quality factors. The expected phase shift of  $2\pi$ , however, is clearly visible for most flux values. For some small flux regions, the  $2\pi$  phase shift is no longer visible as the resonator is operated in the undercoupled regime (cf. Fig. 1.11). As described in the text, this measurement can be used to gauge the flux axis and to find flux values where the network analyzer can be calibrated for further measurements.



Figure 3.2: Magnitude (top figure) and phase (bottom figure) of the resonator reflectance after calibration for sample Cat. 2-1c, Q300. In the magnitude plot, the resonator turns out more clearly as the calibration removes all features of the remaining setup from the measured traces. In the phase plot, the phase is no longer referenced to the first trace, but to the calibration trace. The overcoupled regime with the  $2\pi$  phase shift and the undercoupled regime with the hardly pronounced phase shift are both clearly identifiable and well separated. As described in the text, the axis of abscissa is now gauged to units of flux quanta.



Figure 3.3: Uncalibrated magnitude (top figure) and phase (bottom figure) of the resonator reflectance for sample Cat. 0-2a, Q30. The phase data were again recorded in phase deviation mode. As described in the text, the sample is mounted farther from the coil compared to sample Cat. 2-1c, Q300 (cf. Fig. 2.8). Therefore, a larger coil current needs to be applied in order to intersperse the sample with the same magnetic flux.



Figure 3.4: Calibrated resonator reflectance for sample Cat. 0-2a, Q30. Again, the  $2\pi$  phase shift is clearly visible and can be well separated from the undercoupled regime.

The magnitude measured by the calibrated network analyzer is given by  $|\Gamma|$  and the phase is given by  $\arctan \frac{\text{Im}\Gamma}{\text{Re}\Gamma}$ . We have performed a least mean square fit simultaneously for both magnitude and phase with the fitting parameters  $Q_{\text{ext}}$ ,  $Q_{\text{int}}$  and the loaded resonant frequency  $\omega_L$ . In this way, the loaded resonant frequency was determined for every flux value. The resonant frequencies found in doing so are represented by the black squares in Figs. 3.5(a) and 3.5(b).

Subsequently, another fit was performed in order to check if the flux dependence of the resonant frequency follows the theoretical relationship described by Eqs. (1.50) and (1.57). The results are shown in Fig. 3.5(a) for sample Cat. 2-1c, Q300 and Fig. 3.5(b) for sample Cat. 0-2a, Q30. The fit shows very good agreement with theory. The deviation of the fitted curve from the measured data is less than 0.5% for both samples in the frequency range around 5.75 GHz.

However, if we wanted to set the resonator to a specific resonant frequency, we could not just take the corresponding coil current value determined by the fit, as the deviations are still in the order of several MHz. For all following experiments, we thus had to adjust the coil current to the desired resonant frequency using the  $2\pi$  phase shift visible in the phase trace of the network analyzer.

Furthermore, the curves shown in Figs. 3.5(a) and 3.5(b) are not symmetric with respect to the coil current. We suspect that leakage fields from components of the cryogenic setup, e.g. the switches or the cryogenic circulators, may expose the samples to an additional flux contribution that is not generated by the coil. We also cannot exclude external magnetic flux, created somewhere outside of the cryostat, as the source of the spurious flux contributions.



Figure 3.5: Flux-dependence of the resonant frequency for (a) sample Cat. 2-1c, Q300 and (b) sample Cat. 0-2a, Q30. The black squares indicate the resonant frequency determined from the simultaneous fit of magnitude and phase data presented in Figs. 3.2 and 3.4. For the red line, we have fitted the flux-dependent resonant frequencies to the theoretical dependence described by Eqs. (1.50) and (1.57). We find that the measured resonant frequencies are well described by the fit. The flux is given in units of coil current as this is the only parameter we can control directly in order to adjust the flux.

#### 3.1.2 Determination of the quality factors

We shall continue our quantitative analysis of the resonator with the determination of the internal and external quality factor. As we have seen in Section 1.4.1, the internal quality factor determines the depth of the dip in the reflection magnitude at resonant frequency. However, we have observed above that neither of our samples shows a clearly pronounced dip in the relevant frequency range of 5.0 GHz to 5.8 GHz. Thus, the internal quality factor cannot be determined accurately fitting the theoretical value of  $|\Gamma|$  to the measured reflection magnitude data. In order to estimate at least a lower limit of  $Q_{\rm int}$ , we have set the resonator to a resonant frequency of 5.75 GHz and recorded a single magnitude and phase trace with the ZVA network analyzer set to an IF-bandwidth of 1 Hz, thus reducing noise. We have then again performed a simultaneous fit of the magnitude and phase trace using only the resonant frequency and the external quality factor as fit parameters. The internal quality factor was fixed to three different values. The results are shown in Fig. 3.6 for sample Cat. 2-1c, Q300. Comparing the fitted curves to the actual data, we conclude that the internal quality factor is in the range between 5000 and 10000. At the same time, we can exclude internal quality factor values as low as 1000.

The dip-peak-structure in the magnitude could result from spurious signals coupled from the input line directly into the output line as a result of the non-ideal isolation of the circulator. However, so far the origin of this structure is not understood in detail. We also suspected that the ZVA network analyzer might not separate the two signal quadratures exactly by 90°. Calculations and simulations have shown that this could not explain the dip-peak-structure.

The characteristic  $2\pi$  phase shift however is clearly visible and allows to extract reliable values for the external quality factor, see Fig. 3.7. For reasonable values of  $Q_{\text{int}}$ , the fit yields an external quality factor of approximately 249 for 5.75 GHz. Only for the unreasonably small value of  $Q_{\text{int}}$  we get a different result for  $Q_{\text{ext}}$ . We therefore conclude that in order to determine the external quality factor, which actually can be determined by fitting the reflection magnitude.

As described in Section 3.1.1, a simultaneous fit of phase and magnitude was performed at different flux values with the fitting parameters  $Q_{\text{int}}$ ,  $Q_{\text{ext}}$  and  $\omega_L$  in order to determine the frequency dependence of the resonant frequency. Now we are interested in the resonant frequency dependence of the external quality factor for both samples. Fig. 3.8 shows the results. Sample Cat. 2-1c, Q300 was designed to an external quality factor of 300 at 5.75 GHz. However, we see that the real value is about 50 lower than the design value. For sample Cat. 0-2a, Q30, the design value of 30 is hit almost exactly at 5.75 GHz.


Figure 3.6: Blue trace: Measured reflection magnitude of sample Cat. 2-1c, Q300. The resonant frequency is set to  $5.75 \,\text{GHz}$ . Instead of a pronounced dip, the curve exhibits a dip-peak-structure. The reason for this is unknown. Red curves: Magnitude fits with assumed internal quality factors of 5000, 10000 and 1000, respectively. Depending on whether one interprets the full or half height of the dip-peak-structure as the determining factor, the measured curve is best described by internal quality factors of 5000 or 10000. If  $Q_{\text{int}}$  would be as low as 1000, we expect to see a distinct dip in the reflection magnitude.



Figure 3.7: Blue trace: Measured reflection phase of sample Cat. 2-1c, Q300 corresponding to Fig. 3.6. Red curves: Phase fits with assumed internal quality factors of 5000, 10000 and 1000, respectively. For reasonable values of  $Q_{\rm int}$ , the external quality factor is found to be approximately 249.



Figure 3.8: Resonant frequency dependence of the external quality factor. The external quality factors and the corresponding resonant frequencies were determined by a simultaneous fit of magnitude and phase with the fitting parameters Q<sub>int</sub>, Q<sub>ext</sub> and ω<sub>L</sub>. The horizontal lines indicate the design values at 5.75 GHz.
(a) Sample Cat. 2-1c, Q300. (b) Sample Cat. 0-2a, Q30.

#### 3.2 The degenerate Gain

In order to determine the phase-dependent degenerate gain of our samples, we have made use of the measuring method presented by T. Yamamoto *et al.* in [19] in order to compare the results. Here, the degenerate gain is not measured simply by applying an input signal at half the pump frequency to the JPA and measuring the output signal at the input signal frequency. Instead, the input signal is subjected to an amplitude modulation before it is applied to the sample. To understand the concept of amplitude modulation, we consider a signal of the form

$$A_{\rm in} = A_0 \cos\left(\omega_{\rm mod} t + \phi\right) \tag{3.1}$$

with amplitude  $A_0$ , frequency  $\omega_{\text{mod}}$  and phase  $\phi$ . A constant component  $A_c$  is now added to the signal before it is multiplied by a carrier signal at frequency  $\omega_c$ , yielding

$$A_{\text{mod}} = [A_c + A_0 \cos(\omega_{\text{mod}}t + \phi)] \cdot \cos(\omega_c t)$$
  
=  $A_c \cos(\omega_c t) + \frac{A_0}{2} \cos((\omega_c - \omega_{\text{mod}})t - \phi) + \frac{A_0}{2} \cos((\omega_c + \omega_{\text{mod}})t + \phi).$  (3.2)

The modulated signal therefore consists of three frequency components. One at frequency  $\omega_c$  and two components where the frequency is increased or decreased by  $\omega_{\text{mod}}$ , respectively. As follows from Eq. (3.2), the three signal components have a fixed phase relation with respect to each other. The modulation depth is defined as

$$m = \frac{A_0}{A_c}.\tag{3.3}$$

For our measurements, the modulation frequency  $\omega_{\text{mod}}$  was set to 10 kHz, the carrier frequency  $\omega_c$  to the respective resonant frequency of the resonator and the modulation depth to m = 1. The pump signal frequency was set to  $2\omega_c$ .

The JPA input signal component at frequency  $\omega_c + \omega_{\text{mod}}$  undergoes signal amplification as described in Section 1.5.1. The input signal component at frequency  $\omega_c - \omega_{\text{mod}}$  leads to the creation of an idler mode also at frequency  $\omega_c + \omega_{\text{mod}}$ . The total signal detected at frequency  $\omega_c + \omega_{\text{mod}}$  thus is the sum of two components with a fixed phase relation. Depending on the pump phase, they will interfere constructively or destructively.

In the course of our measurements, the phase dependent degenerate gain was determined comparing the signal level at frequency  $\omega_c + \omega_{\text{mod}}$  when the pump was turned on relative to when it was turned off. The signal levels were detected with the FSP7 spectrum analyzer set to a resolution bandwidth of 10 Hz and a video bandwidth of 1 Hz.

In order to find a good working point with high gain of sample Cat. 0-2a, Q30, we have measured the phase-dependent degenerate gain for different resonant frequencies of the resonator. The input signal was generated by the SMF signal generator set to an output power of -20 dBm. The pump signal was generated by the PSG signal source with an output power of +20 dBm. The PSG was also used to digitally vary the phase of the pump signal in steps of 2°. As can be seen in Fig. 3.9, we have achieved a maximum gain of



Figure 3.9: The phase-dependent degenerate gain for sample Cat. 0-2a, Q30 at different resonant frequencies. The maximum achievable gain decreases for increasing resonant frequencies.

14.0 dB for a resonant frequency of 5.06 GHz. At our desired working point of 5.75 GHz, the maximum gain goes down to as low as 1.6 dB. This can be explained considering Fig. 3.10, which is a close-up of Fig. 3.5(b). We have seen in Section 1.2 that parametric amplification is achieved by periodically varying the resonant frequency. The stronger the frequency is varied, the more amplification is gained. In our samples, however, we are not changing the resonant frequency directly, but the flux through the SQUID. At a given resonant frequency  $\omega_L$ , the frequency variation  $\Delta \omega$  is given by

$$\Delta\omega \approx \left. \frac{d\omega}{d\Phi} \right|_{\omega_L} \cdot \Delta\Phi \tag{3.4}$$

in which the derivative represents the slope of the curve shown in Fig. 3.10. Apparently, these slopes - and therefore the frequency modulation - become larger for smaller resonant frequencies. A quantitative comparison of the slopes and the maximum achievable amplifications and deamplifications is provided in Tab. 3.1.

In order to achieve higher gains for our desired working point around 5.75 GHz, we would have to apply more power to the JPA pump port. However, even with a PSG output power of +20 dBm, the fridge could not be held at its base temperature but was already warming up to approximately 70 - 80 mK. Thus, applying more pump power was not



Figure 3.10: The efficiency of the conversion of pump power applied to the JPA into signal amplification depends on the slope of the resonant frequency dependence on the magnetic flux. For small resonant frequencies, the slopes become larger. The consequence is higher gain for smaller resonant frequencies. The data shown here correspond to sample Cat. 0-2a, Q30.

desirable. We have therefore characterized the degenerate gain operating the JPA at 5.06 GHz. The sample temperature was stabilized at 90 mK.

One very important property of an amplifier is its 1 dB compression point. As the output power of any amplifier is physically limited, the input signal will be less amplified if it is too large. We have therefore applied input signals of various power levels to the JPA and measured the phase-dependent gain as described above. The 1 dB compression point is defined as the signal power at which the gain is reduced by 1 dB in comparison to smaller output powers where the gain is power independent. The data shown in Fig. 3.11 were taken at a pump level of  $+20 \,\mathrm{dBm}$ , whereas the pump was set to  $+10 \,\mathrm{dBm}$  for the data shown in Fig. 3.13. In order to find the 1 dB compression point, we have extracted the maximum gain for each signal power and plotted it against the input power. For a pump power of  $+20 \,\mathrm{dBm}$ , we find that the amplifier is linear over at least three orders of magnitude and has its 1 dB compression point for an input signal power of -8.1 dBm, see Fig. 3.12. Comparing this result to the measurements for a pump power of  $+10 \, \text{dBm}$ , see Fig. 3.14, we find on the one hand that the maximum amplification has decreased as a result of the reduced pump power. On the other hand, the amplifier is now linear over nearly five orders of magnitude and the 1 dB compression point is shifted to  $+7.5 \,\mathrm{dBm}$ . This can be understood considering that the output power is determining the 1 dB compression point. If the amplification is reduced, the 1 dB compression point is reached at higher input powers. For both pump power values, the deviations in the gain for input powers lower than -30 dBm have to be attributed to noise, cf. Figs. 3.11 and 3.13.

Res. frequency	slope	Max. deg. gain	Min. deg. gain
(GHz)	$\left(\frac{\mathrm{GHz}}{\Phi/\Phi_0}\right)$	(dB)	(dB)
5.060	-16.9	14.0	-14.3
5.356	-10.4	6.23	-6.11
5.750	-3.65	1.58	-1.75
5.850	-2.56	1.34	-1.34

Table 3.1: Slopes of the resonant-frequency-dependence on the coil current and the corresponding values for the maximum amplification and deamplification for sample Cat. 0-2a, Q30. The slopes were extracted from Fig. 3.10, whereas the gains were extracted from Fig. 3.9. It can be seen that the gain decreases significantly if the absolute value of the slope decreases. We also note that the maximum amplification approximately equals the maximum deamplification, therefore fulfilling the equation  $G_d^{\min}[dB] + G_d^{\max}[dB] = 0$ , cf. Eq. (1.94).

For sample Cat. 2-1c, Q300, we have first measured the phase-dependent degenerate gain at a resonant frequency of 5.7513 GHz as this is close to our desired working point of 5.75 GHz. The setup was not changed compared to the measurement of sample Cat. 0-2a, Q30. The input signal power level was set to -13.4 dBm. The PSG pump signal source was used again to vary the phase in steps of  $2^{\circ}$ . The results can be found in Fig. 3.15 for two different pump powers. Whilst the expected dependence of the gain on phase and pump power can be observed, the maximum achievable gains of only 3.7 dB and 2.6 dB at pump powers of +17 dBm and +14 dBm, respectively, fall short of our expectations.

We have therefore decreased the resonant frequency in order to enter a regime where the slope of the resonant-frequency-flux-curve is steeper, see Fig. 3.5(a). The setup was left unchanged, only the step size of the phase sweep was increased to  $10^{\circ}$ . Figure 3.16 shows the results for a resonant frequency of 5.6389 GHz. Contrary to the other measurements of the degenerate gain, we have not amplitude modulated the input signal, i.e. the input signal consisted of one frequency component at half the pump frequency. The gain was determined for two different input signal power levels, -13.4 dBm and -23.4 dBm and two pump powers each, namely +20 dBm and +17 dBm. As the gain only depends on the pump power, we can conclude that the JPA has not entered the nonlinear regime for any of our power combinations. The maximum gains that can be extracted from Fig. 3.16 are 20.7 dB for a pump power of +20 dBm and 12.2 dB for a pump power of +17 dBm. Apparently, reducing the pump power by 3 dB already diminishes the gain by 8.5 dB.

For both samples and all studied resonant frequencies, we see that the maximum amplification approximately equals the maximum deamplification, fulfilling the equation

$$G_d^{\min}[dB] + G_d^{\max}[dB] = 0.$$
(1.94)



Figure 3.11: Phase dependent gain of sample Cat. 0-2a, Q30 for different input signal power levels. The resonant frequency was set to 5.06 GHz and the pump power to +20 dBm. For clarity, the traces were shifted in phase so that the minima coincide.



Figure 3.12: Gain maxima extracted from Fig. 3.11 plotted versus the corresponding input power levels. The lines connecting the black squares are provided as a guide to the eye. The 1 dB compression point of the amplifier operated at 5.06 GHz was found to be about -8.1 dBm for a pump power of +20 dBm.



Figure 3.13: Phase dependent gain of sample Cat. 0-2a, Q30 for different input signal power levels, with the resonator set to 5.06 GHz. The pump power was reduced to +10 dBm. The traces were shifted in phase in order to get coincident minima.



Figure 3.14: Determination of the 1 dB compression point for a pump power of +10 dBm. The gain maxima were extracted from Fig. 3.13 and plotted against the input power. Compared to Fig. 3.12, we find that the 1 dB compression point is shifted to a higher input power level of +7.5 dBm.



Figure 3.15: Phase-dependent gain of sample Cat. 2-1c, Q300 set to a resonant frequency of 5.7513 GHz. The measurement was performed for two different pump power levels.



Figure 3.16: Phase-dependent gain of sample Cat. 2-1c, Q300 set to a resonant frequency of 5.6389 GHz. The traces were shifted in phase so that the minima coincide. The lines connecting the data points are provided as a guide to the eye. Four different combinations of pump and input signal power levels were studied. For the chosen input signal levels, the degenerate gain only depends on the pump power.

## 3.3 Signal and Intermodulation gain

If an input signal at frequency  $\omega_0 + \Delta \omega$ , and thus differing from half the pump frequency  $\omega_0$ , is applied to a Josephson parametric amplifier, the output signal will consist of two components as discussed in Section 1.5.1. In addition to the signal mode at input signal frequency  $\omega_0 + \Delta \omega$  the idler mode at frequency  $\omega_0 - \Delta \omega$  will be generated.

The signal gain was determined comparing the output level at signal frequency when the pump was turned on relative to when it was turned off. The pump-off level at signal frequency also served as reference level for the determination of the intermodulation gain at idler mode frequency. However, for small pump powers the idler mode level will be smaller than the reference level, resulting in negative gain values. Contrary to the measurement of the degenerate gains, negative gain values here do *not* imply deamplification, they just express that there is only little signal-to-idler conversion.

For both samples, we have determined the signal and intermodulation gain at our desired working point of 5.75 GHZ. In addition, we have performed measurements at resonant frequencies of 5.06 GHz for sample Cat. 0-2a, Q30 and 5.639 GHz for sample Cat. 2-1c, Q300, respectively. For all measurements, the input signal frequency was 10 kHz higher than half the pump frequency. The pump signal itself was set to twice the resonant frequency and created by the PSG signal source. As for the detector, we have used the FSP7 spectrum analyzer set to a resolution bandwidth of 10 Hz and a video bandwidth of 1 Hz.

The results for sample Cat. 0-2a, Q30 can be seen in Fig. 3.17(a) for a resonant frequency of 5.06 GHz and in Fig. 3.17(b) for a resonant frequency of 5.75 GHz. At 5.06 GHz, the signal gain is, depending on the input signal power, between 4.4 dB and 5.0 dB, whereas the intermodulation gain is between 2.8 dB to 3.7 dB. According to theory, signal gain and intermodulation gain should obey the relation

$$\left|\mathcal{G}(\omega)\right|^{2} - \left|\mathcal{M}(-\omega)\right|^{2} = 1 \tag{1.77}$$

where  $\mathcal{G}$  and  $\mathcal{M}$  are (linear) amplitude amplification factors of signal and idler mode, respectively. For the measurement at 5.06 GHz, the difference  $|\mathcal{G}(\omega)|^2 - |\mathcal{M}(-\omega)|^2$ , calculated separately for the four different input power levels, gives values between 0.68 (-43.4 dBm) and 0.91 (-23.4 dBm) which is in sufficient agreement with theory.

At a resonant frequency of 5.75 GHz, where we have already seen a very low degenerate gain, there is almost no signal and intermodulation gain at all. Furthermore, we also note that the JPA gain seems to saturate at pump power levels as low as +17 dBm. We feel that the data quality is not sufficient to say this with certainty. For the observed low gains, the SQUID design may be responsible. Enlarging the SQUID and therefore increasing the coupling to the pump line may help to realize higher gains.

Figure 3.18(a) shows signal and intermodulation gain for sample Cat. 2, Q300 for a resonant frequency of 5.639 GHz. First we note that both signal and intermodulation gain converge to the same limiting value in the limit of large gains, therefore fulfilling (1.77). The measured gain maximum is  $15.1 \,\mathrm{dB}$ , but as the amplifier is not yet saturated at a pump power of  $100 \,\mathrm{mW}$ , we assume that higher gains would be possible by applying more pump power. We also see that the gains are similar for all four input powers applied. Therefore, the amplifier is linear over at least three orders of magnitude.

At 5.75 GHz, see Fig. 3.18(b), the gains are much smaller which is not surprising comparing the degenerate gains for 5.639 GHz (Fig. 3.16) and 5.75 GHz (Fig. 3.15). We can also see here that similar gains are achieved for both input power levels and that we have not yet reached the compression point.

For a consistency check, we perform a calculation in order to find the maximum degenerate gain that we expect for given signal and intermodulation gain. We therefore assume that signal and idler mode frequency are so close to half the pump frequency that the respective gains can be considered frequency-independent. If the signal frequency converges towards half the pump frequency, signal and idler mode will interfere, resulting in the phase-dependent degenerate gain. The maximum of the latter is reached if signal and idler mode interfere constructively, i.e. their amplitudes add. With the signal gain  $G_{sig}$  and the intermodulation gain  $G_{int}$ , we get for the maximum degenerate gain  $G_{deg}$ 

$$G_{\rm deg}[\rm dB] = 20 \log_{10} \left( 10^{\frac{G_{\rm sig}[\rm dB]}{20}} + 10^{\frac{G_{\rm int}[\rm dB]}{20}} \right).$$
(3.5)

In the limit of large gains, signal and idler gain converge to the same limiting value G,

$$G_{\rm deg}[dB] = 20 \log_{10} \left( 10^{\frac{G[dB]}{20}} + 10^{\frac{G[dB]}{20}} \right) = 20 \log_{10} \left( 2 \cdot 10^{\frac{G[dB]}{20}} \right)$$
$$= 6 \, dB + G[dB]. \tag{3.6}$$

Therefore, the degenerate gain is expected to be 6 dB larger than signal and intermodulation gain. Comparing the results of Fig. 3.18(a), where we have seen a signal and intermodulation gain of 14.5 dB for sample Cat. 2-1c, Q300 at 5.639 GHz and an input power of -13.4 dBm to Fig. 3.16, where we have found a degenerate gain of 20.7 dB for the same parameters, one can see that the difference indeed is approximately 6 dB. For the other measurements of signal and intermodulation gains, Eq. (3.5) gives a good quantitative explanation for the small gains.



(a) Signal and intermodulation gain at 5.06 GHz. There is an indication that the amplifier saturates at  $50\,\mathrm{mW}$  of pump power.



(b) Signal and intermodulation gain at 5.75 GHz. Note that the ordinate is rescaled above the break.

Figure 3.17: Signal and intermodulation gain for sample Cat. 0-2a, Q30 for the resonant frequencies 5.06 GHz (top) and 5.75 GHz (bottom). The squares connected with solid lines denote the signal gain, whereas the squares connected with dotted lines denote the intermodulation gain. In both cases, the lines are provided as a guide to the eye.



(a) Signal and intermodulation gain for 5.639 GHz. For large gains, signal and intermodulation gain converge on each other as predicted by theory. The amplifier is linear over at least three orders of magnitude.



(b) Signal and intermodulation gain for 5.75 GHz.

Figure 3.18: Signal and intermodulation gain for sample Cat. 2-1c, Q300 for the resonant frequencies 5.639 GHz (top) and 5.75 GHz (bottom). The squares connected with solid lines denote the signal gain, whereas the squares connected with dotted lines denote the intermodulation gain. In both cases, the lines are provided as a guide to the eye.

#### 3.4 Bandwidths

The bandwidth of an amplifier is another characteristic quantity important for its application. We have determined the signal and intermodulation bandwidth for sample Cat. 2-1c, Q300 with the resonant frequency set to 5.6391 GHz. For sample Cat. 0-2a, Q30, we have not seen significant signal and intermodulation gain at any resonant frequency and have therefore refrained from determining the bandwidth. The same applies to sample Cat. 2-1c, Q300 at 5.75 GHz. For the measurement, the pump signal was generated by the PSG signal generator and its frequency was fixed. The input signal was delivered by the ZVA network vector analyzer. For the measurement of the signal gain, the ZVA was set to measure at input signal frequency. For the intermodulation gain however, the ZVA was set to measure at idler frequency, i.e. at a frequency differing from the ZVA output signal frequency. The ZVA itself was set to an IF-bandwidth of 3 Hz and a 2 MHz step size between the measuring frequencies. Figure 3.19 shows the signal gain for different input signal frequencies and two different pump power levels and the corresponding Lorentzian fits. In Fig. 3.20 we present the corresponding intermodulation gains. The ZVA was set to deliver an output power of 12 dBm that was attenuated by 24 dB by the subsequent step attenuators, cf. Fig. 2.15. We have chosen Lorentzian fits as the JPA bandwidth should be determined by the resonator bandwidth, cf. Eq. (A.79) in Appendix A. For the latter, we know that it is described by a Lorentzian curve. Considering the figures, we find that it describes our measurement data very well.

The 3 dB bandwidth of an amplifier is defined as the full width at half maximum (FWHM) of the frequency-gain-curve. As it can be seen in both figures, the bandwidth depends on the maximum gain, increasing for smaller gains. If a signal with white (i.e. frequency independent) spectrum is to be amplified, the output power in the whole bandwidth of the amplifier will be optimal if the product of gain and bandwidth is maximal. We have calculated this *gain-bandwidth-product* for both signal and intermodulation gain in dependence of the pump power. From Figs. 3.19 and 3.20, we have extracted values for the maximum signal and intermodulation gains and the corresponding bandwidths. As it can be seen in Tab. 3.2, the gain-bandwidth-product decreases significantly if the pump power is reduced. The same holds for the intermodulation gain, see Tab. 3.3. Thus, the decrease in gain is not compensated by the increasing bandwidth. If this Josephson parametric amplifier is used to amplify signals with white spectrum, one would therefore choose maximum gain over bandwidth.

Comparing the bandwidth of our samples to commercially available, classical HEMT amplifiers points out one major disadvantage of our Josephson parametric amplifiers. These HEMT amplifiers offer bandwidths of several GHz, that is three orders of magnitude more than the bandwidth of our JPAs. This of course is compensated by the fact that these HEMT amplifiers inevitably add noise to the signal as discussed in Section 1.6. We also note that the small bandwidth is irrelevant for the amplification of narrow-band signals. Furthermore, the resonant frequency of the resonator, and therefore the amplifier, can be tuned over a frequency range of some GHz.



Figure 3.19: Signal bandwidth for sample Cat. 2-1c, Q300 set to a resonant frequency of 5.639 GHz measured for two different pump power levels. The squares mark the data points whereas the continuous lines mark the Lorentzian fits. The 3 dB bandwidth, marked by the black arrows, increases if the gain is decreased.



Figure 3.20: Intermodulation bandwidth for sample Cat. 2-1c, Q300 operated at 5.639 GHz. The data points are represented by the squares. The Lorentzian fit (continuous line) suggests an intermodulation gain higher than the signal gain in Fig. 3.19. As we only have few data points around resonant frequency, the maximum gain is associated with a relatively high degree of uncertainty. The arrows indicate the 3 dB bandwidth.

Pump power	Max. gain	3-dB bandwidth	Gain-bandwidth-product
(dBm)	(lin. units)	(MHz)	(MHz)
20 17	$24.6 \\ 4.29$	$3.13 \\ 6.34$	77.0 27.2

Table 3.2: Signal bandwidth and gain-bandwidth-product for sample Cat. 2-1c set to a resonant frequency of 5.639 GHz extracted from Fig. 3.19. A decrease in gain is not compensated by an increasing bandwidth in the gain-bandwidth-product.

Pump power	Max. gain	3-dB bandwidth	Gain-bandwidth-product
(dBm)	(lin. units)	(MHz)	(MHz)
20	28.8	2.71	78.0
17	3.70	5.82	21.5

Table 3.3: Intermodulation bandwidth and gain-bandwidth-product for sample Cat. 2-1c operated at 5.639 GHz extracted from Fig. 3.19. Again, increasing the bandwidth by decreasing the maximum gain does not increase the gain-bandwidth product.

### 3.5 Determination of $\eta$

The quantity  $\eta$ , as discussed in Section 1.5.2, is defined as the (power) reflectivity of the Josephson parametric amplifier with the pump turned off relative to the reflectivity of a short at resonant frequency. We have therefore set the 5-port switch in our setup such that input signals are directed to the calibration short (cf. Fig. 2.7) instead of the sample. With this switch position, calibration data were recorded for the ZVA network vector analyzer. Switching the 5-port switch back so that incoming signals are directed towards the sample again now allowed for a direct measurement of  $\eta$ . However, unlike the short, the Josephson parametric amplifier sample is not connected to the switch directly. From the switch, a microwave cable is leading to the sample holder, where a PCB is inserted in front of the Josephson parametric amplifier sample, see Fig. 3.21.



Figure 3.21: The quantity  $\eta$  is determined for the entity consisting of the microwave cable, the alumina board and the JPA sample.

Choosing the switch output port as reference plane,  $\eta$  now describes the reflectance of the interconnected system consisting of the cable, the alumina board and the actual JPA. In this system, reflections in particular can occur at the interface between two components as a result of impedance mismatch, cf. Section 2.1.4. If the sample is to be used in future experiments, the value determined for  $\eta$  only stays valid if the sample is not removed from the sample holder and if the sample holder is connected to the rest of the setup using the same microwave cable again. Figure 3.22 and Tab. 3.4 show the results determined for sample Cat. 0-2a, Q30 at three different resonant frequencies. As one can see, at two of these resonant frequencies we have found values slightly exceeding 1. However, the Josephson parametric amplifier physically cannot exhibit larger reflectance than a short. We suspect that either noise or calibration artifacts are responsible for this. We have recorded the calibration data with an IF-bandwidth of 10 Hz, which may have been to large.

In order to determine  $\eta$  for sample Cat. 2-1c, Q300 we have recorded new calibration data. The results for  $\eta$  are shown in Fig. 3.23 and Tab. 3.5. Sources of error for the values of  $\eta$  for both samples are noise and also potential unequal transmission properties of the different switching states of the 5-port switch. Considering the resonant frequencies where we have seen significant gains, the values for  $\eta$  are 0.93 for sample Cat. 0-2a, Q30 at 5.05 GHz and 0.92 for sample Cat. 2-1c, Q300 at 5.64 GHz.



Figure 3.22: The quantity  $\eta$  was determined for three different resonant frequencies for sample Cat. 0-2a, Q30. The resonant frequencies were determined from the characteristic  $2\pi$  phase shift in the resonator reflectance.

<b>Resonant frequency</b>	$\eta$
(GHz)	
4.80	1.02
5.05	0.93
5.74	1.01

Table 3.4: Values for the quantity  $\eta$  for sample Cat. 0-2a, Q30 extracted from Fig. 3.22. We believe that either noise or calibration artifacts are responsible for values exceeding 1.



Figure 3.23: For sample Cat. 2-1c, Q300 we have determined  $\eta$  for four different resonant frequencies. The resonant frequencies were determined from the characteristic  $2\pi$  phase shift in the resonator reflectance.

Resonant frequency (GHz)	$\eta$
5 55	0.86
5.55	0.00
5.04	0.92
5.75	0.89
5.86	0.72

Table 3.5: Values of  $\eta$  extracted from Fig. 3.23 for sample Cat. 2-1c, Q300.

## 3.6 Calibration of input and output line

As described in Section 2.2.5, we have equipped the 30 dB input line attenuator at base temperature with a heater and a thermometer so that we can control its temperature from base temperature to approximately 800 mK independent of the sample temperature. Now we want to use this in order to calibrate input and output line. The thermal state generated by the heated attenuator is directed through the circulator towards the sample where it is reflected. There is no pump signal applied to the sample. The reflected signal is again passing the circulator before it is amplified in the output line.

Depending on the attenuator temperature T, the output signal power P obeys the equation [51]

$$P = \eta \cdot G \cdot B \cdot \left(\frac{h\nu}{e^{\frac{h\nu}{k_B(T+\delta T)}} - 1} + \frac{1}{2}h\nu + k_B T_N\right)$$
(3.7)

in which  $h = 6.626 \cdot 10^{-34}$  Js is the Planck constant,  $k_B = 1.381 \cdot 10^{-23}$  J/K is the Boltzmann constant,  $\nu$  is the frequency, B is the detector bandwidth, G is the power amplification factor of the amplification chain and  $T_N$  is the amplification chain noise temperature. As the signal generated by the attenuator is reflected at the sample, the quantity  $\eta$  has to be taken into account. In our case, the signal was reflected at sample Cat. 2-1c, Q300 set to a resonant frequency of 5.6365 GHz. Near this frequency, we have found  $\eta = 0.92$ , cf. Tab. 3.5. The parameter  $\delta T$  takes into account that the electronic temperature of the resistors inside the attenuator may differ from the measured attenuator temperature [52]

In Eq. (3.7), the first two terms describe the Bose-Planck-distribution of the noise of a resistor. The third term takes into account that not only the amplified thermal state will be detected, but also noise that is added by the amplifiers or coupled into the amplification chain. In our case, the output signal was detected by the FSP spectrum analyzer set to the center frequency of 5.6365 GHz and a resolution bandwidth of 100 kHz. The temperature was varied from 30 mK to 800 mK in steps of 10 mK. The measured temperature-dependent output power is shown in Fig. 3.24.

We have then performed a fit with the fit parameters G,  $\delta T$  and  $T_N$ . For the amplification factor G, we have determined a value of  $G = 6.99 \cdot 10^5$ , corresponding to a gain of 58.4 dB. In addition, the fit gives the noise temperature of the amplification chain, 13.38 K. The quantity  $\delta T$  was determined to be -28.7 mK.

After letting the attenuator cool down to base temperature again, we have applied an input signal at 5.6365 GHz to the input line and detected the reflected signal with the spectrum analyzer. The input signal was generated by the PSG microwave source. An additional cable with an insertion loss of 0.8 dB was inserted between the input line and the PSG. The total transmission measured was -68.0 dB. As we know the amplification from the attenuator to the detector and the insertion loss of the additional cable, we can calculate the total attenuation of the input line. A signal generated by the SMF signal



Figure 3.24: In order to calibrate the input and output line, the heatable attenuator is set to temperatures from 30 mK to 800 mK. The thermal state generated by the attenuator is detected by the spectrum analyzer. By means of Eq. (3.7), the gain of the output line can be determined. Measuring the transmission through input and output line subsequently gives the input line attenuation.

generator, as used in our measurements, will be attenuated by 127.2 dB after having passed through the 30 dB base temperature attenuator. Provided that the switches and the microwave cables from the last attenuator to the sample all have small insertion loss, it is safe to assume a total attenuation of approximately 127 dB from the signal source to the sample.

We cannot determine the attenuation of the pump line in the same way. But we can give an estimation considering that the attenuators mounted in the input line have a total attenuation of 109 dB. Therefore, the microwave cables at and below room temperature have a total insertion loss of about 18 dB. As the microwave cables used for the pump line are approximately of the same length, and as we have attenuators and a power divider with a total attenuation of 41 dB installed, we can estimate that the pump signal leaving the signal source is attenuated by about 59 dB before it reaches the pump port of the sample.

# **Summary and Outlook**

The detection of weak quantum microwave signals presently is a fundamental challenge in the field of circuit quantum electrodynamics. One approach to the problem is to use phase-sensitive amplifiers that, in principle, allow for noiseless amplification. A very promising representative of this class is the flux-driven Josephson parametric amplifier. From our co-operation partners at NEC in Japan we have received a set of JPA samples with different design parameters.

In the course of this thesis, we have characterized two of these samples with respect to their resonator characteristics, gains, bandwidths and reflection coefficients. To this end, we have designed and implemented a highly flexible measurement setup allowing for the independent characterization of two Josephson parametric amplifier samples in the same cooldown.

In the first chapter of this thesis, we have discussed the theoretical background of the flux-driven Josephson parametric amplifier. In chapter 2 we have provided a detailed description of our measurement setup. We have demonstrated that mechanical switching between microwave cables is possible at cryogenic temperatures and that it is possible to create thermal states up to 800 mK at the base temperature stage of the cryostat without significant heating of the sample.

The two Josephson parametric amplifier samples we have characterized differ in the design value of the external quality factor of the resonator. We have determined the flux dependence of the resonant frequency of the coplanar waveguide resonator and observed good agreement with theory. The resonant frequency dependence of the external quality factor was measured and a lower limit for the internal quality factor could be provided. The phase dependent degenerate gain for both samples at different resonant frequencies was determined and gains of 14.0 dB for sample Cat. 0-2a, Q30 at 5.06 GHz and 20.7 dB for sample Cat. 2-1c, Q300 at 5.639 GHz were observed. However, we have found significantly smaller gains for our desired working point at 5.75 GHz. The decreasing gains for larger resonant frequencies could be explained considering the slope of the resonant frequency dependence on the magnetic flux. Sample Cat. 0-2a, Q30 exhibited a signal gain of 5.0 dB and an intermodulation gain of 3.7 dB at 5.06 GHz, whereas sample Cat. 2-1c, Q300 set to 5.639 GHz showed a signal and intermodulation gain of 15.1 dB. We have seen that signal and intermodulation gains are consistent with the corresponding degenerate gains. For sample Cat. 2-1c, Q300 the signal and intermodulation bandwidth at maximum amplification was determined to be 3.13 MHz and 2.71 MHz, respectively. It also turned out that decreasing the gain is not compensated by the increasing bandwidth.

For both samples, we have compared the reflection of the JPA with no pump signal applied to the reflection of a short and could therefore determine the quantity  $\eta$  at different resonant frequencies. The possibility to create thermal states up to 800 mK at base temperature allowed for a precise calibration of input and output line.

All in all, we have proved that both samples are performing very well and that we have chosen a highly flexible measurement setup suitable for the independent characterization of two samples and for the determination of the reflection coefficient  $\eta$  in a single cooldown. Despite the high pump power levels, the sample stage of the cryostat could be held near base temperatue. The measured data are all mutually consistent and the large gains observed for sample Cat. 2-1c, Q300 suggest that squeezed states with a high degree of squeezing can be created.

The next step will be to analyze these squeezed states generated by the Josephson parametric amplifier with the cross-correlation detection scheme introduced by E. P. Menzel *et al.* [63]. This method is primarily designed to characterize propagating microwaves at the quantum level. The key element of the cross-correlation detection scheme is the utilization of two independent amplification chains. In both chains, linear, phase-insensitive off-the-shelf HEMT amplifiers are used that obscure the signals with noise much larger than the signal itself. However, cross-correlation measurements and massive averaging allow for the recovery of, in principle, all statistical signal moments. With the knowledge of all statistical moments, the Wigner function can be reconstructed using the inverse Radon transform [64,65]. The Wigner function completely characterizes a quantum mechanical state.

In proof-of-principle experiments, the resolution limit of the cross-correlation detection scheme was determined to be less than  $10^{-3}$  photons on average (poa) for the mean value, 1-2 poa for the variance and 10-20 poa for the third central moment. The method has already been successfully demonstrated for coherent states and weak thermal states.

As the dual-path scheme is capable of removing amplifier noise from the signal, it is well-suited for the measurement of the noise temperatures of our Josephson parametric amplifiers. Applying a squeezed state, it shall be proved that the dual-path scheme is also capable of reconstructing non-classical<sup>1</sup> signals.

<sup>&</sup>lt;sup>1</sup>Non-classical in the sense that a squeezed state cannot be described using classical electromagnetics.

# A Quantum analysis on the flux-driven parametric amplifier

The quantum analysis on the flux-driven parametric amplifier was performed by T. Yamamoto, Y. Nakamura and K. Koshino. Parts of it are imprinted below by courtesy of T. Yamamoto.

### A.1 The Hamiltonian and the equation of motion

We start from an equation of motion for a harmonic oscillator,

$$\frac{d^2q}{d^2t} + \Omega_0^2 q = 0, (A.1)$$

and introduce a modulation of  $\Omega_0$ , namely,  $\Omega_0 \to \Omega_0 [1 + \delta \cos(\alpha \Omega_0 t)]$ . Then,

$$\frac{d^2q}{d^2t} + \Omega_0^2 \left[1 + 2\delta \cos\left(\alpha \Omega_0 t\right)\right] q = 0, \tag{A.2}$$

where we neglected  $\delta^2$  term. The Hamiltonian which gives this equation of motion is

$$\mathcal{H} = \frac{p^2}{2m} + \frac{m}{2}\Omega_0^2 \left[1 + 2\delta\cos\left(\alpha\Omega_0 t\right)\right] q^2.$$
(A.3)

Introducing the creation and annihilation operators as follows,

$$q = \frac{a + a^{\dagger}}{2} \sqrt{\frac{2\hbar}{m\Omega_0}} \tag{A.4}$$

$$p = \frac{a - a^{\dagger}}{2i} \sqrt{2\hbar m \Omega_0} \tag{A.5}$$

we arrive at the Hamiltonian of the parametrically-modulated harmonic oscillator,

$$\mathcal{H} = \hbar \Omega_0 \left[ a^{\dagger} a + 2\delta \cos\left(\alpha \Omega_0 t\right) \left(a + a^{\dagger}\right)^2 \right].$$
(A.6)

Now the Hamiltonian of the system including signal and loss ports [24] is given by ( $\epsilon = 2\delta$ )

$$\mathcal{H} = \hbar\Omega_0 \left[ a^{\dagger}a + \epsilon \cos\left(\alpha\Omega_0 t\right) \left(a + a^{\dagger}\right)^2 \right] + \int d\omega \left[ \hbar\omega b(\omega)^{\dagger} b(\omega) + i\hbar\sqrt{\frac{\kappa_1}{2\pi}} \left(a^{\dagger}b(\omega) - b(\omega)^{\dagger}a\right) \right] + \int d\omega \left[ \hbar\omega c(\omega)^{\dagger}c(\omega) + i\hbar\sqrt{\frac{\kappa_2}{2\pi}} \left(a^{\dagger}c(\omega) - c(\omega)^{\dagger}a\right) \right].$$
(A.7)

The coupling constants  $\kappa_1$  and  $\kappa_2$  are related to the quality factors of the cavity as follows:

$$\kappa_1 = \frac{\Omega_0}{Q_{\text{ext}}} \tag{A.8}$$

$$\kappa_2 = \frac{\Omega_0}{Q_{\text{int}}}.\tag{A.9}$$

From the Heisenberg equation of motion for a,

$$\frac{da}{dt} = \frac{1}{i\hbar} \left[ a, \mathcal{H} \right], \tag{A.10}$$

we obtain,

$$\frac{da}{dt} = -i\Omega_0 a - 2i\Omega_0 \epsilon \cos\left(\alpha\Omega_0 t\right) \left(a + a^{\dagger}\right) + \sqrt{\frac{\kappa_1}{2\pi}} \int d\omega b(\omega) + \sqrt{\frac{\kappa_2}{2\pi}} \int d\omega c(\omega). \quad (A.11)$$

Here we used following relations.

$$\left[a, a^{\dagger}\right] = 1 \tag{A.12}$$

$$\left[a, a^{\dagger}a\right] = a \tag{A.13}$$

$$[a, (a+a^{\dagger})^2] = 2(a+a^{\dagger})$$
 (A.14)

From the Heisenberg equation of motion for  $b(\omega)$ , we obtain

$$\frac{db(\omega)}{dt} = -i\omega b(\omega) - \sqrt{\frac{\kappa_1}{2\pi}}a.$$
(A.15)

By solving this differential equation, we have

$$b(\omega) = e^{-i\omega(t-t_0)}b_0(\omega) - \sqrt{\frac{\kappa_1}{2\pi}} \int_{t_0}^t e^{-i\omega(t-t')}a(t')dt',$$
 (A.16)

where  $b_0(\omega)$  means  $b(\omega)$  at  $t = t_0$ . Similarly,

$$c(\omega) = e^{-i\omega(t-t_0)}c_0(\omega) - \sqrt{\frac{\kappa_2}{2\pi}} \int_{t_0}^t e^{-i\omega(t-t')}a(t')dt'.$$
 (A.17)

Substituting Eqs. (A.16) and (A.17) into Eq. (A.11), we have

$$\frac{da}{dt} = -i\Omega_0 a - 2i\Omega_0 \epsilon \cos(\alpha \Omega_0 t)(a + a^{\dagger}) 
+ \sqrt{\frac{\kappa_1}{2\pi}} \int d\omega e^{-i\omega(t-t_0)} b_0(\omega) - \frac{\kappa_1}{2\pi} \int d\omega \int_{t_0}^t e^{-i\omega(t-t')} a(t') dt' 
+ \sqrt{\frac{\kappa_2}{2\pi}} \int d\omega e^{-i\omega(t-t_0)} c_0(\omega) - \frac{\kappa_2}{2\pi} \int d\omega \int_{t_0}^t e^{-i\omega(t-t')} a(t') dt'.$$
(A.18)

We define input field operators by

$$b_{\rm in}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega e^{-i\omega(t-t_0)} b_0(\omega) \tag{A.19}$$

$$c_{\rm in}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega e^{-i\omega(t-t_0)} c_0(\omega). \tag{A.20}$$

Using the relation

$$\int d\omega e^{-i\omega(t-t')} = 2\pi\delta(t-t'), \qquad (A.21)$$

and

$$\int_{t_0}^t dt' \delta(t - t') = \frac{1}{2},$$
(A.22)

Eq. (A.18) reads

$$\frac{da}{dt} = \left(-i\Omega_0 - \frac{\kappa}{2}\right)a - 2i\Omega_0\epsilon\cos(\alpha\Omega_0 t)(a+a^{\dagger}) + \sqrt{\kappa_1}b_{\rm in}(t) + \sqrt{\kappa_2}c_{\rm in}(t), \qquad (A.23)$$

where

$$\kappa = \kappa_1 + \kappa_2 \tag{A.24}$$

Equation (A.15) can also be solved in terms of the final condition at  $t_1 > t$ . Namely,

$$b(\omega) = e^{-i\omega(t-t_1)}b_1(\omega) + \sqrt{\frac{\kappa_1}{2\pi}} \int_t^{t_1} e^{-i\omega(t-t')}a(t')dt'.$$
 (A.25)

Taking the same procedure and defining output operators by

$$b_{\rm out}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega e^{-i\omega(t-t_1)} b_1(\omega) \tag{A.26}$$

$$c_{\text{out}}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega e^{-i\omega(t-t_1)} c_1(\omega), \qquad (A.27)$$

we have

$$\frac{da}{dt} = \left(-i\Omega_0 + \frac{\kappa}{2}\right)a - 2i\Omega_0\epsilon\cos(\alpha\Omega_0 t)(a+a^{\dagger}) + \sqrt{\kappa_1}b_{\rm out}(t) + \sqrt{\kappa_2}c_{\rm out}(t).$$
(A.28)

Subtracting Eq. (A.28) from Eq. (A.23), we obtain

$$b_{\rm in}(t) - b_{\rm out}(t) = \sqrt{\kappa_1}a \tag{A.29}$$

$$c_{\rm in}(t) - c_{\rm out}(t) = \sqrt{\kappa_2}a. \tag{A.30}$$

# A.2 Gain

Now we return to Eq. (A.23).

$$\frac{da}{dt} + W(t)a + V(t)a^{\dagger} = F(t), \qquad (A.31)$$

$$W(t) = i\Omega_0 + \frac{\kappa}{2} + 2i\epsilon\Omega_0\cos(\alpha\Omega_0 t)$$
(A.32)

$$V(t) = 2i\Omega_0 \epsilon \cos(\alpha \Omega_0 t) \tag{A.33}$$

$$F(t) = \sqrt{\kappa_1} b_{\rm in}(t) + \sqrt{\kappa_2} c_{\rm in}(t) \tag{A.34}$$

By taking the Hermite conjugate of Eq. (A.31), we obtain (we will drop the explicit expression of time dependence for W(t), V(t), and F(t))

$$\frac{da^{\dagger}}{dt} = -W^* a^{\dagger} - V^* a + F^{\dagger}. \tag{A.35}$$

Also from Eq. (A.31),

$$a^{\dagger} = \frac{1}{V} \left( F - \frac{da}{dt} - Wa \right) \tag{A.36}$$

Taking the time derivative in both sides of Eq. (A.31),

$$\frac{d^2a}{dt^2} + \frac{dW}{dt}a + W\frac{da}{dt} + \frac{dV}{dt}a^{\dagger} + V\frac{da^{\dagger}}{dt} = \frac{dF}{dt}$$
(A.37)

Substituting Eqs. (A.35) and (A.36) into Eq. (A.37), we obtain

$$\frac{d^2a}{dt^2} + \left[W + W^* - \frac{1}{V}\frac{dV}{dt}\right]\frac{da}{dt} + \left[\frac{dW}{dt} - \frac{W}{V}\frac{dV}{dt} + |W|^2 - |V|^2\right]a = \\ = \frac{dF}{dt} + \left(W^* - \frac{1}{V}\frac{dV}{dt}\right)F - VF^{\dagger}. \quad (A.38)$$

If we consider a classical signal, namely  $a = \langle a \rangle$ ,  $\langle b \rangle = E e^{-i\beta\Omega_0 t}$ , and  $\langle c \rangle = 0$ , and apply rotating wave approximation in Eq. (A.31) ( $\alpha \approx 2, \beta \approx 1$ ),

$$W(t) = i\Omega_0 + \frac{\kappa}{2} \tag{A.39}$$

$$V(t) = i\Omega_0 \epsilon e^{-i\alpha\Omega_0 t} \tag{A.40}$$

$$F(t) = \sqrt{\kappa_1} E e^{-i\beta\Omega_0 t} \tag{A.41}$$

we have

$$\frac{d^2 \langle a \rangle}{dt^2} + \left[\kappa + i\alpha \Omega_0\right] \frac{d\langle a \rangle}{dt} + \left[\frac{\kappa^2}{4} + \Omega_0^2 (1 - \epsilon^2 - \alpha) + i\alpha \Omega_0 \frac{\kappa}{2}\right] \langle a \rangle = 
= \sqrt{\kappa_1} E \left[\frac{\kappa}{2} + i(\alpha - \beta - 1)\Omega_0\right] e^{-i\beta\Omega_0 t} - i\epsilon \Omega_0 \sqrt{\kappa_1} E^* e^{-i(\alpha - \beta)\Omega_0 t}.$$
(A.42)

Now we consider the homogeneous equation for Eq. (A.42), namely

$$\frac{d^2\langle a\rangle}{dt^2} + \left[\kappa + i\alpha\Omega_0\right]\frac{d\langle a\rangle}{dt} + \left[\frac{\kappa^2}{4} + \Omega_0^2(1 - \epsilon^2 - \alpha) + i\alpha\Omega_0\frac{\kappa}{2}\right]\langle a\rangle = 0.$$
(A.43)

The solution is of the form

$$\langle a \rangle_g = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}, \tag{A.44}$$

where

$$\lambda_1 = -\frac{\kappa + i\alpha\Omega_0}{2} + \Omega_0\sqrt{\epsilon^2 - \left(\frac{\alpha}{2} - 1\right)^2} \tag{A.45}$$

$$\lambda_2 = -\frac{\kappa + i\alpha\Omega_0}{2} - \Omega_0\sqrt{\epsilon^2 - \left(\frac{\alpha}{2} - 1\right)^2}.$$
(A.46)

Next we consider the special solution of Eq. (A.42). It is given by the following,

$$\langle a \rangle_s = -y_1 \int \frac{f(t)y_2}{\Delta} dt + y_2 \int \frac{f(t)y_1}{\Delta} dt,$$
 (A.47)

where

$$y_1 = e^{\lambda_1 t} \tag{A.48}$$

$$y_2 = e^{\lambda_2 t}$$

$$\Delta = y_1 y_2' - y_2 y_1'$$
(A.49)

$$= (\lambda_2 - \lambda_1)e^{(\lambda_1 + \lambda_2)t}$$
(A.50)

$$f(t) = \sqrt{\kappa_1} E\left[\frac{\kappa}{2} + i(\alpha - \beta - 1)\Omega_0\right] e^{-i\beta\Omega_0 t} - i\epsilon\Omega_0\sqrt{\kappa_1} E^* e^{-i(\alpha - \beta)\Omega_0 t}.$$
 (A.51)

From these equations, we obtain

$$\langle a \rangle_s = A_s e^{-i\beta\Omega_0 t} + A_i e^{-i(\alpha-\beta)\Omega_0 t}, \qquad (A.52)$$

where

$$A_s = \frac{\sqrt{\kappa_1 E} \left[\frac{\kappa}{2} + i(\alpha - \beta - 1)\Omega_0\right]}{\left[\frac{\kappa}{2} + i(1 - \beta)\Omega_0\right] \left[\frac{\kappa}{2} + i(\alpha - \beta - 1)\Omega_0\right] - \epsilon^2 \Omega_0^2}$$
(A.53)

$$A_{i} = \frac{i\epsilon\Omega_{0}\sqrt{\kappa_{1}}E^{*}}{\left[\frac{\kappa}{2} - i(1-\beta)\Omega_{0}\right]\left[\frac{\kappa}{2} - i(\alpha-\beta-1)\Omega_{0}\right] - \epsilon^{2}\Omega_{0}^{2}}.$$
 (A.54)

Thus, from Eqs. (A.44) and (A.52), the solution of Eq. (A.42) is

$$\langle a \rangle = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + A_s e^{-i\beta\Omega_0 t} + A_i e^{-i(\alpha-\beta)\Omega_0 t}.$$
 (A.55)

From Eqs. (A.45) and (A.46), the real part of  $\lambda_2$  is always negative, while that of  $\lambda_1$  can be positive when  $\frac{\kappa}{2} < \Omega_0 \sqrt{\epsilon^2 - (\frac{\alpha}{2} - 1)^2}$ , which gives the threshold of pump power for the divergence,

$$\epsilon > \sqrt{\left(\frac{\kappa}{2\Omega_0}\right)^2 + \left(\frac{\alpha}{2} - 1\right)^2}.$$
(A.56)

Here, we define the critical value of  $\epsilon$  as follows,

$$\epsilon_c = \frac{\kappa}{2\Omega_0} = \frac{1}{2} \left( \frac{1}{Q_{\text{ext}}} + \frac{1}{Q_{\text{int}}} \right). \tag{A.57}$$

Now, let us consider the case where  $\lambda_1$  is negative (below threshold). In this case, the steady state solution of Eq. (A.42) is  $\langle a \rangle_s$ . From Eq. (A.29),

$$\langle b \rangle_{\text{out}} = (E - \sqrt{\kappa_1} A_s) e^{-i\beta\Omega_0 t} - \sqrt{\kappa_1} A_i e^{-i(\alpha - \beta)\Omega_0 t}.$$
 (A.58)

In the case of non-degenerate-mode operation ( $\alpha \neq 2\beta$ ), we can define the signal and the intermodulation gains ( $G_s$  and  $G_i$ , respectively), which are given by

$$G_s \equiv \left| \frac{E - \sqrt{\kappa_1} A_s}{E} \right|^2 = \left| 1 - \kappa_1 \frac{\frac{\kappa}{2} + i(\alpha - \beta - 1)\Omega_0}{\left[\frac{\kappa}{2} + i(1 - \beta)\Omega_0\right] \left[\frac{\kappa}{2} + i(\alpha - \beta - 1)\Omega_0\right] - \epsilon^2 \Omega_0^2} \right|^2 \quad (A.59)$$

$$G_i \equiv \left| \frac{\sqrt{\kappa_1} A_i}{E^*} \right|^2 = \left| \frac{\kappa_1 \epsilon \Omega_0}{\left[ \frac{\kappa}{2} + i(1-\beta)\Omega_0 \right] \left[ \frac{\kappa}{2} + i(\alpha-\beta-1)\Omega_0 \right] - \epsilon^2 \Omega_0^2} \right|^2.$$
(A.60)

In the case of degenerate-mode operation ( $\alpha = 2\beta$ ), equation (A.58) becomes

$$\langle b \rangle_{\text{out}} = (E - \sqrt{\kappa_1} A_s - \sqrt{\kappa_1} A_i) e^{-i\beta\Omega_0 t}.$$
 (A.61)

The phase-dependent gain  $G_d$  is given by

$$G_{d} \equiv \left| \frac{E - \sqrt{\kappa_{1}} A_{s} - \sqrt{\kappa_{1}} A_{i}}{E} \right|^{2} = \left| 1 - \kappa_{1} \frac{\frac{\kappa}{2} + i(\beta - 1)\Omega_{0} + i\epsilon\Omega_{0}e^{-2i\theta}}{\frac{\kappa^{2}}{4} + (1 - \beta)^{2}\Omega_{0}^{2} - \epsilon^{2}\Omega_{0}^{2}} \right|^{2}, \quad (A.62)$$

where  $E = |E| e^{i\theta}$ . Let us consider a special case,  $\alpha = 2\beta = 2$ . Equation (A.62) leads,

$$G_{d} = \frac{\left(\frac{\kappa_{1}^{2} - \kappa_{2}^{2}}{4} + \epsilon^{2} \Omega_{0}^{2}\right)^{2} + \epsilon^{2} \kappa_{1}^{2} \Omega_{0}^{2} - 2\epsilon \kappa_{1} \Omega_{0} \left(\frac{\kappa_{1}^{2} - \kappa_{2}^{2}}{4} + \epsilon^{2} \Omega_{0}^{2}\right) \sin 2\theta}{\left(\frac{\kappa^{2}}{4} - \epsilon^{2} \Omega_{0}^{2}\right)^{2}}$$
(A.63)

From this formula,  $G_d$  for  $\epsilon = 0$  is given by

$$G_d^0 = \left(\frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2}\right)^2. \tag{A.64}$$

The minimum gain is achieved when  $\theta = \frac{\pi}{4} + n\pi$ , and given by

$$G_d^{\min} = \left(\frac{\epsilon\Omega_0 - \frac{\kappa_1 - \kappa_2}{2}}{\epsilon\Omega_0 + \frac{\kappa_1 + \kappa_2}{2}}\right)^2.$$
 (A.65)

The maximum gain is achieved when  $\theta = \frac{3\pi}{4} + n\pi$ , and given by

$$G_d^{\max} = \left(\frac{\epsilon\Omega_0 + \frac{\kappa_1 - \kappa_2}{2}}{\epsilon\Omega_0 - \frac{\kappa_1 + \kappa_2}{2}}\right)^2.$$
 (A.66)

Note that in Eqs. (A.65) and (A.66), the condition  $(\kappa_1^2 - \kappa_2^2)/4 + \epsilon^2 \Omega_0^2 > 0$  is assumed. First we define the field operator A as follows,

$$a = e^{-i\frac{\alpha}{2}\Omega_0 t} A. \tag{A.67}$$

The Fourier transform of A is defined by

$$\mathcal{F}[A(t)] \equiv A(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt A(t) e^{i\omega t}.$$
 (A.68)

Taking the Hermite conjugate,

$$\mathcal{F}^{\dagger}[A(t)] \equiv A^{\dagger}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt A^{\dagger}(t) e^{-i\omega t}.$$
 (A.69)

On the other hand, the Fourier transform of  $A^{\dagger}$  is given by

$$\mathcal{F}[A^{\dagger}(t)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt A^{\dagger}(t) e^{i\omega t}.$$
 (A.70)

Comparing Eqs. (A.69) and (A.70), we obtain

$$A^{\dagger}(-\omega) = \mathcal{F}[A^{\dagger}(t)]. \tag{A.71}$$

Now we start from Eq. (A.23). Substituting Eq. (A.67) into Eq. (A.23), we obtain

$$\frac{dA}{dt} + \left[ \left( 1 - \frac{\alpha}{2} \right) i\Omega_0 + \frac{\kappa}{2} \right] A + i\epsilon\Omega_0 A^{\dagger} = e^{i\frac{\alpha}{2}\Omega_0 t} F(t).$$
(A.72)

Taking the Hermite conjugate of both sides, we obtain

$$\frac{dA^{\dagger}}{dt} + \left[-i\left(1-\frac{\alpha}{2}\right)\Omega_0 + \frac{\kappa}{2}\right]A^{\dagger} - i\epsilon\Omega_0 A = e^{-i\frac{\alpha}{2}\Omega_0 t}F^{\dagger}(t).$$
(A.73)

The Fourier transform of Eq. (A.72) and (A.73) leads to

$$\begin{pmatrix} -i\omega - i\left(\frac{\alpha}{2} - 1\right)\Omega_0 + \frac{\kappa}{2} & i\epsilon\Omega_0 \\ -i\epsilon\Omega_0 & -i\omega + i\left(\frac{\alpha}{2} - 1\right)\Omega_0 + \frac{\kappa}{2} \end{pmatrix} \begin{bmatrix} A(\omega) \\ A^{\dagger}(-\omega) \end{bmatrix} = \begin{bmatrix} F(\omega + \frac{\alpha}{2}\Omega_0) \\ F^{\dagger}(-\omega + \frac{\alpha}{2}\Omega_0) \end{bmatrix},$$
(A.74)

where

$$F(\omega) = \sqrt{\kappa_1} b_{\rm in}(\omega) + \sqrt{\kappa_2} c_{\rm in}(\omega).$$
 (A.75)

From this equation, we obtain

$$A(\omega) = \frac{-i\omega + i\left(\frac{\alpha}{2} - 1\right)\Omega_0 + \frac{\kappa}{2}}{-\left(\omega + i\frac{\kappa}{2}\right)^2 + \left(\frac{\alpha}{2} - 1\right)^2\Omega_0^2 - \epsilon^2\Omega_0^2}F\left(\omega + \frac{\alpha}{2}\Omega_0\right) + \frac{-i\epsilon\Omega_0}{-\left(\omega + i\frac{\kappa}{2}\right)^2 + \left(\frac{\alpha}{2} - 1\right)^2\Omega_0^2 - \epsilon^2\Omega_0^2}F^{\dagger}\left(-\omega + \frac{\alpha}{2}\Omega_0\right).$$
(A.76)

From Eqs. (A.29) and (A.67),

$$e^{i\frac{\alpha}{2}\Omega_0 t}b_{\text{out}}(t) = e^{i\frac{\alpha}{2}\Omega_0 t}b_{\text{in}}(t) - \sqrt{\kappa_1}A(t), \qquad (A.77)$$

which leads,

$$b_{\text{out}}\left(\omega + \frac{\alpha}{2}\Omega_0\right) = b_{\text{in}}\left(\omega + \frac{\alpha}{2}\Omega_0\right) - \sqrt{\kappa_1}A(\omega).$$
(A.78)

From Eqs. (A.76) and (A.78), we obtain

$$b_{\text{out}}\left(\omega + \frac{\alpha}{2}\Omega_{0}\right) = J_{b}(\omega)b_{\text{in}}\left(\omega + \frac{\alpha}{2}\Omega_{0}\right) + K_{b}(\omega)b_{\text{in}}^{\dagger}\left(-\omega + \frac{\alpha}{2}\Omega_{0}\right) + J_{c}(\omega)c_{\text{in}}\left(\omega + \frac{\alpha}{2}\Omega_{0}\right) + K_{c}(\omega)c_{\text{in}}^{\dagger}\left(-\omega + \frac{\alpha}{2}\Omega_{0}\right), \quad (A.79)$$

where

$$J_b(\omega) = 1 + \kappa_1 \frac{-i\omega + i\left(\frac{\alpha}{2} - 1\right)\Omega_0 + \frac{\kappa}{2}}{\left(\omega + i\frac{\kappa}{2}\right)^2 - \left(\frac{\alpha}{2} - 1\right)^2\Omega_0^2 + \epsilon^2\Omega_0^2}$$
(A.80)

$$K_b(\omega) = -\frac{i\epsilon\kappa_1\Omega_0}{\left(\omega + i\frac{\kappa}{2}\right)^2 - \left(\frac{\alpha}{2} - 1\right)^2\Omega_0^2 + \epsilon^2\Omega_0^2}$$
(A.81)

$$J_c(\omega) = \sqrt{\kappa_1 \kappa_2} \frac{-i\omega + i\left(\frac{\alpha}{2} - 1\right)\Omega_0 + \frac{\kappa}{2}}{\left(\omega + i\frac{\kappa}{2}\right)^2 - \left(\frac{\alpha}{2} - 1\right)^2\Omega_0^2 + \epsilon^2\Omega_0^2} \tag{A.82}$$

$$K_c(\omega) = -\frac{i\epsilon\sqrt{\kappa_1\kappa_2}\Omega_0}{\left(\omega + i\frac{\kappa}{2}\right)^2 - \left(\frac{\alpha}{2} - 1\right)^2\Omega_0^2 + \epsilon^2\Omega_0^2}.$$
(A.83)

# B Nomenclature and design parameters of the samples

The Josephson parametric amplifier samples were delivered in a gel-pack, see Fig. B.1. The sample names arise from the row (a - d) and column (1 - 4) designations. All samples in one row belong to the same sample category, where Cat.-0-samples are found in row a, Cat.-1-samples in row b and so forth. Table B.1 shows the design parameters for the samples.



Figure B.1: The JPA samples were delivered in a gel-pack. The samples highlighted with a yellow frame were characterized in the course of our measurements.

Cat.	$Q_{\rm ext}$	$C_c$	$I_c$ /Josephson junction	Josephson junction size	Squid loop size
		$(\mathrm{fF})$	$(\mu A)$	$(\mu { m m}^2)$	$(\mu m^2)$
0	30	94	1.0	$0.44 \ge 0.38$	4.2 x 2.4
1	100	51	1.0	$0.44 \ge 0.38$	$4.2 \ge 2.4$
2	300	30	1.6	$0.44 \ge 0.38$	$4.2 \ge 2.4$
3	300	42	1.6	$0.44 \ge 0.38$	$4.2 \ge 2.4$

Table B.1: Design values for the different JPA sample categories.

# C Preparation of a sample holder with Anritsu V102M microwave connectors

This instruction manual on the preparation of a sample holder with Anritsu V102M microwave connectors and Anritsu V100 glass beads is based on the author's experience. The connector, the glass bead and the alumina boards were assembled exactly as presented below.

# C.1 Integrity check and cleaning

- 1. Check the dimensions of the sample holders. Check dimensions and positions of holes and pockets.
- 2. Carefully remove chips and burrs from holes, threads and faces with tweezers, a scalpel or a toothpick.
- 3. Check all via holes for clearance.
- 4. Check for a tight fit of the sample holder lid.
- 5. Clean the sample holder in an ultrasonic bath with acetone and isopropanol for five minutes each. During and after cleaning, do not touch the sample holder with bare fingers!
- 6. Check if samples and PCBs fit in their designated pockets. If necessary, carefully enlarge pockets with a scalpel or a small milling cutter. Repeat the cleaning in the ultrasonic bath afterwards if enlarging the pockets was necessary.
- 7. Store the sample holder in a closable container to avoid collecting dust.

# C.2 Gold plating

- 8. Have the sample holder gold-plated. Make sure the plating is not too thick.
- 9. Check via holes and threads again for clearance and burrs that may have developed during the plating. Remove burrs if necessary.
- 10. Check if samples and PCBs still fit in their designated pockets.

## C.3 Installation of the glass beads

- 11. If the PCBs have to be installed before the plugs are mounted, insert them into their designated pocket and carefully screw them.
- 12. Set hot plate to  $200 \pm 10^{\circ}$ C. Use an external thermometer to control the temperature of the hot plate. A metal plate is recommended for better heat transfer.
- 13. Insert the glass bead long-end first into the holding fixture.
- 14. Flux the glass bead evenly. Do not flux the pin.
- 15. Use the holding fixture to hand-screw the bead into the mounting hole. Do not remove the holding fixture until the bead is soldered!
- 16. Repeat the last steps until all glass beads are mounted.
- 17. Check that all pins are centered in their designated via holes at the backside of the interface. Else remove the glass bead and repeat steps 15-17.
- 18. Insert solder into the soldering access holes and cut it flush with the top of the holes.
- 19. Place the sample holder on the hot plate.
- 20. After the solder melts, leave the sample holder on the hot plate for another 15 20 seconds before removing it.
- 21. Let the sample holder cool down to room temperature.
- 22. Remove the holding fixtures and check the quality of the soldering. A little amount of solder has to be visible all around the glass bead when looking from where the holding fixture was.
- 23. Clean the sample holder in an ultrasonic bath with a compound of 1:1 acetone and ethanol for one minute to remove flux remnants. Flush with isopropanol subsequently.
- 24. If necessary, carefully remove residual flux remnants with tweezers or a toothpick and redo step 23.

# C.4 Installation of the Anritsu V102M V Male Sparkplug Connector

- 25. Use centering fixture to stabilize the sparkplug insert.
- 26. Carefully hand-screw the plug into the housing without damaging the glass bead pin.
- 27. Make sure the pin is centered in the plug.
- 28. Use the Anritsu torque wrench to tighten the plug. The torque wrench has to click two times.
- 29. Check again for the centering of the pin at both sides of the plug.

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## Acknowledgments

First of all, I would like to express my gratitude to Prof. Dr. Rudolf Gross for giving me the opportunity to join the qubit group at the Walther Meißner Institute first as a working student and then as a diploma student. His words of encouragement and advice and the instructive discussions with him have always been highly motivating to me.

Dr. Achim Marx has to be thanked for many things. When I joined the WMI as a working student, he already has entrusted me with responsible tasks in the qubit lab. During my thesis period, he always was ready to provide support whenever we had problems with the cryostat or the measurements, even late at night or at the weekend. His willingness to help and his people skills have impressed me again and again.

I am especially thankful to Edwin P. Menzel. I have been working closely with Edwin during the last year and would like to thank him for always being available for questions, for his thorough instructions to the lab and the measurement instruments and his patient explanations. His motivation, his efficient way of working, his vast knowledge, not only in physics, but also in chemistry and electro technology and his great experience made the results of this work possible in the first place. Every result of this thesis is as much his as it is mine.

Dr. Frank Deppe is to be thanked for fruitful discussions and his helpful suggestions on our measurements. It is due to his good contacts to the group of Yasunobu Nakamura at NEC in Japan that they have provided us with the Josephson parametric amplifier samples which laid the groundwork for this thesis.

Again I would like to thank Prof. Dr. Rudolf Gross, Dr. Achim Marx, Dr. Frank Deppe and Edwin Menzel for rescheduling the modification works on the cryostat, allowing me to measure the data presented in this thesis in time.

Elisabeth Hoffmann, Thomas Niemczyk, Fredrik Hocke, Max Häberlein, Manuel Schwarz, Georg Wild, Edwin Menzel, Dr. Hans Huebl, Dr. Frank Deppe, Dr. Achim Marx and Prof. Dr. Rudolf Gross are to be thanked for the great working atmosphere in the WMI qubit group. Their helpfulness and their willingness to share their knowledge always was motivating and encouraging to me. Fredrik Hocke provided his cryostat for the pilot experiments on the cryogenic switches. As a working student, Max Häberlein has developed the framework of the measurement software used throughout our experiments.

Yasunobu Nakamura, Tsuyoshi Yamamoto and Kunihiro Inomata from NEC have designed and fabricated our Josephson parametric amplifier samples. Tsuyoshi Yamamoto in addition has provided theory support and has attended the analysis of the measurement results.

Dr. Christian Probst, Dr. Kurt Uhlig, Dr. Karl Neumaier and Sepp Höß have developed and constructed crucial parts of the cryostat. Their many years of experience in the field are of great value for the research work done at the WMI. I thank Dr. Karl Neumaier for providing the thermometers used in our experiments.

The WMI workshop is to be thanked for the accurate fabrication of our sample holders and the housing of the mechanical phase shifter. I thank all of them for their efforts. Robert Müller, head of the diploma student's workshop, was always willing to share his vast experience and knowledge when I had technical questions. For this, I would like to thank him very much.

Karen Helm-Knapp has gold-plated our sample holders and has never gotten tired to fabricate yet another version of the printed circuit boards for the pulse driver. I would like to thank her for her patience.

Ulrich Guggenberger has developed crucial parts of the circuits of the pulse driver. Every time I talked to him I was deeply impressed by his profound knowledge and experience in electro technology.

Our working student Garlich Fischbeck has layouted and equipped the pulse driver's printed circuit boards. He also is to be thanked for his help with the assembly of the measurement setup.

My office partners Bea Botka, Manuel Schwarz and Reinhard Rossner are to be thanked for the good working atmosphere in the office and for the cheerful chit-chat.

I would like to thank Jinming Lu and Sergej Truschin for true friendship. They were always there for me when I needed someone to talk to when things were not going so well. And they always reminded me that there is life beyond the lab.

Finally I would like to thank my parents Margot and Werner Baust. Without their support, neither my physics studies nor this diploma thesis would have been possible. What I have achieved in life I owe to them. I am aware that I have not made life easier for my parents during the last six years, bringing home all my sorrows, fears and worries. When they hold these pages in their hand, I hope that they will realize that it was worth it. Thank you so much.

## Erklärung

Mit der Abgabe der Diplomarbeit versichere ich, dass ich die Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Garching, den 10. August 2010

Alexander Theodor Baust