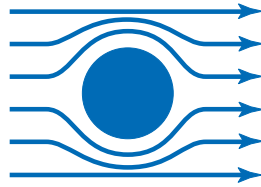


TECHNISCHE
UNIVERSITÄT
MÜNCHEN



WALTHER - MEIßNER -
INSTITUT FÜR TIEF -
TEMPERATURFORSCHUNG



BAYERISCHE
AKADEMIE DER
WISSENSCHAFTEN

Spin Hall Magnetoresistive Noise

Master thesis
Friedrich Paul Witek

Advisor: PD. Dr. Sebastian T.B. Goennenwein
Garching, August 1, 2013

Contents

1	Introduction	1
2	Theory	5
2.1	Spin Hall Magnetoresistance	5
2.2	Introduction to Noise	9
2.3	Thermal Noise and Quantum Fluctuations	10
2.4	From Spin Hall Magnetoresistance to Spin Hall Magnetoresistive Noise	12
2.5	Quantifying White Noise by the Fast Fourier Transform	13
3	Experimental Setup	17
3.1	Sample Fabrication and Characterization	18
3.2	Instruments and Devices	20
3.2.1	Shielding and Wiring	20
3.2.2	Two Dimensional Vector Magnet	20
3.2.3	Low Noise Voltage Amplifier	22
3.2.4	Fast Fourier Transform Spectrum Analyzer	22
4	Angle-Dependent Resistive Noise of YIG Pt Heterostructures	27
4.1	“How low noise are we?” Setup Characterization	27
4.1.1	Limitations by Bandwidth and Background noise	27
4.1.2	Experimental Limit to S/N Ratio	29
4.2	Angle-Dependent Magnetoresistance	33
4.3	Angle-Dependent Resistive Noise Measurements	37
4.4	Biased ADRN	44
5	Summary and Outlook	53
5.1	Summary	53
5.2	Outlook	54
A	Data Post-Processing	57
B	FFT Spectrum Analyzer Implemented on a Computer	61
C	Narrow Bandwidth Detected Noise at Gigahertz	67
	References	73
	Acknowledgment	77

Chapter 1

Introduction

Statistical fluctuations of a physical quantity – commonly referred to as noise – might sometimes be a nuisance for the experimentalist, but might as well provide fascinating information about the underlying physics to him. For instance, A. A. Penzias and R. W. Wilson detected a fluctuating microwave background equivalent to 3 K using a Holmdel Horn antenna. The noise would not even vanish after removing pigeon nests and their droppings from the sensitive antenna. Accidentally, they had discovered the cosmic background microwave radiation, which is one evidence for the Big Bang Theory [1]. Penzias and Wilson were awarded the Nobel Prize in Physics “for their discovery of cosmic microwave background radiation” in 1978. In this work, the well established technique of sensitively detecting fluctuations shall be applied to a rising field of condensed matter physics – spintronics.

Spintronics explores phenomena that interlink the spin and charge degree of freedom in solid state systems [2]. It combines advances in fundamental physics and material science to understand and manipulate spin. The spin degree of freedom fundamentally differs from the charge, because it is a nonconserved quantity in solids due to spin-orbit and hyperfine interaction [3]. Hence understanding the nature of spin injection, transport, relaxation, and accumulation is desirable.

Spintronics is devoted to the study of heterostructures of magnetic and conducting materials. Current applications which can be attributed to the discovery of the tunneling magnetoresistance (TMR) and giant magnetoresistance (GMR) include magnetic storage and sensing [4–6]. Here, the magnetization orientation dependence of resistance of heterostructures involving ferromagnetic conductors allows for sensing magnetic fields. Detecting voltage fluctuations in such devices is a non destructive characterization method measuring spin fluctuations. It sheds light on performance, quality and reliability of TMR as well as GMR structures [7, 8]. Furthermore, the emitted spin noise of a biased TMR structure might be useful in radio frequency devices, such as frequency tunable microwave sources and resonators [9].

In the search for new applications, the injection of spin current and spin-polarized transport have been accomplished, and are now routinely used. They have brought ferromagnetic insulators, such as yttrium iron garnet (YIG), in possible practical applications related to pure spin transport [2]. Ferromagnetic insulators have been successfully used to generate pure spin currents, as they do not allow for charge flow. One established effect to create a pure spin current is the spin pumping effect. Driven at resonance by microwaves, the magnetization relaxes by losing angular momentum to an adjacent normal metal layer, while injecting pure spin current in

the process [10–13]. The inverse effect to spin pumping is the spin transfer torque (STT). An incident absorbed spin current deflects the magnetization or enhances its damping by exerting a torque [14]. Creating such an incident pure spin current is enabled by the Spin Hall effect (SHE) which originates from the relativistic spin orbit coupling. It deflects the electrons perpendicular to the current in a spin dependent way [15–18]. Spin orbit interaction is enhanced in heavy metals. Platinum (Pt) has emerged as one of the metals for driving and detecting spin currents in various heterostructures [19–21].

The recently discovered spin Hall magnetoresistance (SMR) effect enables electrically probing the magnetization direction of a ferromagnetic insulator [22]. It combines the generation of pure spin currents by SHE, their absorption in a ferromagnetic insulator by STT and their back conversion by the inverse spin Hall effect (ISHE). A complete understanding behind the dynamics of this effect is being pursued vigorously. One approach to understand the dynamics behind transport phenomena is to investigate the related fluctuations. So as a first step towards dynamics it is necessary to examine a system in thermal equilibrium. The study of thermal fluctuations in the ferromagnet YIG with adjacent metal Pt was performed in the course of this thesis.

This thesis is structured as follows:

In Chapter 2, the concept of spin currents is introduced and an illustrative explanation of the SMR effect is given. We present the basic ideas behind noise as a measure of the dynamics of a system and conclude that a SMR related thermal noise effect should occur - the spin Hall magnetoresistive noise (SMN). With regard to later measurements, the physical limit to resolving voltage power spectral density (PSD) by fast Fourier transform spectroscopy is derived.

Chapter 3 deals with the YIG|Pt heterostructure sample fabrication and its patterning into a Hall bar mesa structure. Furthermore, the developed low noise experimental setup which is necessary to measure SMN effect is described. Each instrument contributing to the success of the experiment is addressed with particular focus on the fast Fourier transform spectrum analyzer. Additionally, one essential condition for thermal noise measurements, temperature stability, is confirmed.

The experimental results are presented in Chapter 4. It is confirmed that the limitations to low noise detection are pushed to the equipment limit. Thereafter, the Hall bar mesa structure is investigated both along the bar and perpendicular to the bar for SMR. The different measurement geometries are discussed and results quantitatively agree with literature [23]. After characterizing the sample in the dissipative regime, its voltage PSD – the measure of voltage fluctuations – is recorded and discussed as function of frequency as well as of magnetization orientation in thermal equilibrium. The SMN shows the same quantitative magnetization orientation dependence as the SMR as suggested by theory. At the end of Chapter 4, a bias is applied to the sample, resulting in a non-equilibrium spin current flow into the YIG. The voltage PSD does not resolve any additional magnetization dynamics, but confirms the SMN measurement at thermal equilibrium within measuring accuracy.

In Chapter 5, the obtained results are summarized and some ideas for further studies of spin related noise at low frequencies are presented together with some preliminary suggestions on high frequency noise.

Chapter 2

Theory

In the theoretical part of this thesis, the spin Hall magnetoresistance (SMR) effect [24] related to spin current flow in ferromagnetic insulator|normal metal (FMI|NM) heterostructure will be introduced in an illustrative way. We then proceed to electronic noise and motivate the idea of a spin related noise effect, with corresponding insight into noise theory. It is shown that a spin related noise of same relative magnitude as the SMR is expected to occur in the voltage power spectral density (PSD). We will call this noise phenomenon in FMI|NM hybrids the spin Hall magnetoresistive noise (SMN). Last but not least the physical limit to the experimental estimation of the voltage PSD by the fast Fourier transform (FFT) algorithm is derived.

2.1 Spin Hall Magnetoresistance

The SMR has its origin in the conversion of charge to spin current and vice versa at the interface between a ferromagnetic insulator and a normal metal. So we start our discussion with the explanation of spin currents. The angular momentum – spin – of an electron is either $+1/2$ ($\uparrow$) or $-1/2$ ($\downarrow$) with respect to the quantization axis[25]. So the charge current density $\mathbf{J}_q$ can be split into the contributions of spin up electrons $\mathbf{J}_\uparrow$ and spin down electrons $\mathbf{J}_\downarrow$ in a two channel model [26]. The total charge current density is

$$\mathbf{J}_q = \mathbf{J}_\uparrow + \mathbf{J}_\downarrow = -\frac{\sigma}{e} (\nabla\mu_\uparrow + \nabla\mu_\downarrow). \quad (2.1)$$

The charge currents are generated by a gradient of the spin dependent electrochemical potential $\mu_{\uparrow(\downarrow)}$. σ denotes the charge conductivity and $e = |e|$ is the elementary charge. When the same quantity of spin up and spin down electrons flow in the same direction, no net angular momentum is transported with the current and we talk about pure charge currents (cf. Fig. 2.1). The total angular momentum transported is the difference between spin up and spin down currents

$$\mathbf{J}_s = -\frac{\hbar}{2e} (\mathbf{J}_\uparrow - \mathbf{J}_\downarrow). \quad (2.2)$$

$-\mathbf{J}_\uparrow$ is associated with positive angular momentum flow, whereas $-\mathbf{J}_\downarrow$ corresponds to negative angular momentum transport. Consequently, a pure spin current arises

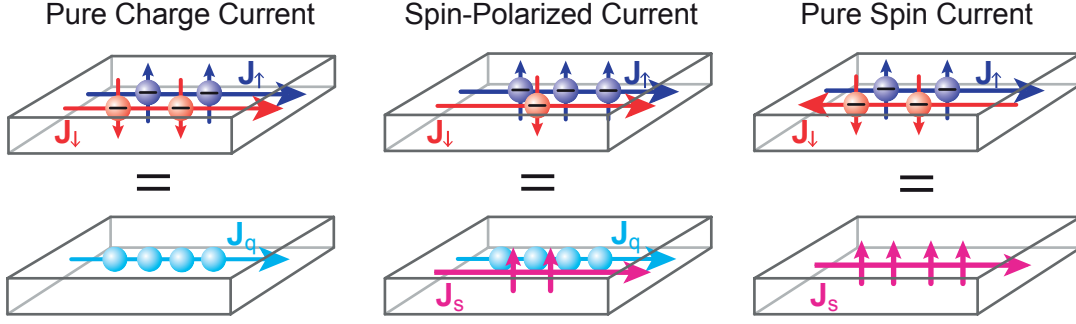


Figure 2.1: The two channel model for spin and charge transport is illustrated. A pure charge current results from the same number of spin up and spin down electrons flowing in the same direction. If one spin species preponderates, both charge and momentum is transported. A pure spin current arises when the same number of both spin species flows in opposite direction. Illustration in analogy to [26].

when $\mathbf{J}_\downarrow = -\mathbf{J}_\uparrow$. The factor $\hbar/(2e)$ in Eq. (2.2) converts the units of charge current density to spin current density and can be understood considering that two electrons are necessary to transfer one quantum of angular momentum $\hbar = h/(2\pi)$. With the spin resolved electrochemical potentials $\mu_{\uparrow(\downarrow)}$, we can write the spin current as a consequence of an imbalance in the spin accumulation $\mu_s = \mu_\uparrow - \mu_\downarrow$:

$$\mathbf{J}_s = -\frac{\hbar\sigma}{2e^2}\nabla\mu_s. \quad (2.3)$$

We define the spin polarization $\mathbf{s}$ of the spin current $\mathbf{J}_s$ as the direction of positive angular momentum $\uparrow$.

Such an imbalance between up and down electron can be caused, for example, by spin orbit interaction. It gives rise to spin dependent scattering effects such as skew scattering [27], side-jump scattering [28] and intrinsic contributions [29] all of which are contributing to the conversion of charge current into spin current and vice versa. In 1999 Hirsch suggested that this conversion is possible in normal metals (NM) [17]. The conversion efficiency of this so called spin Hall effect (SHE) is characterized by the spin Hall angle α_{SH}

$$\mathbf{J}_s = \alpha_{\text{SH}} \left(-\frac{\hbar}{2e} \right) \mathbf{J}_q \times \mathbf{s} \quad (2.4)$$

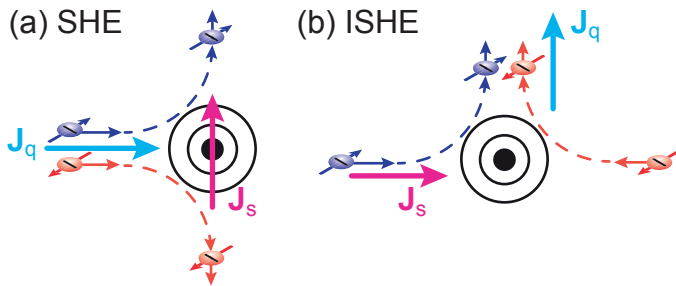


Figure 2.2: (a) The spin Hall effect (SHE) generates a spin current perpendicular to the incident charge current and spin polarization. (b) The inverse spin Hall effect (ISHE) transforms a spin current into a charge current. Both SHE and ISHE arise from spin orbit interaction.

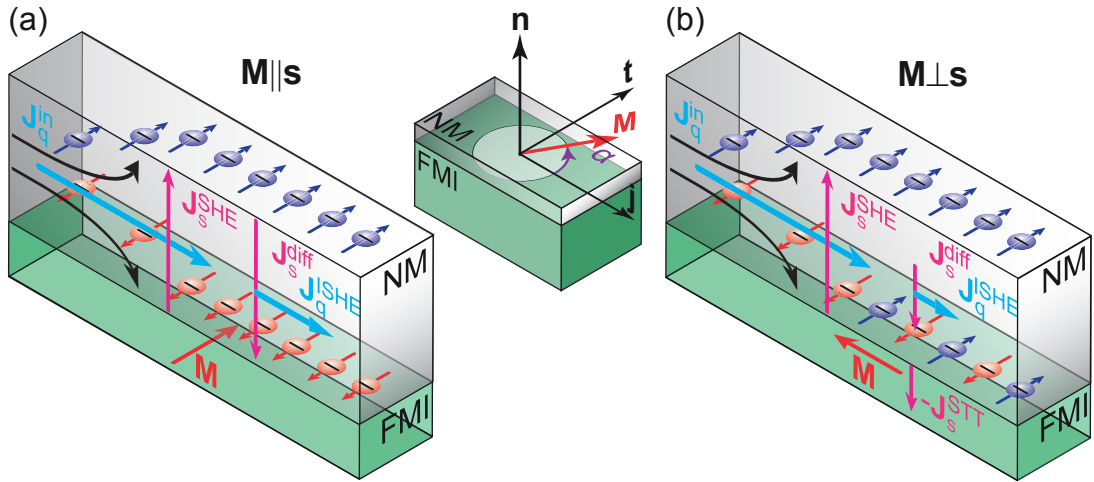


Figure 2.3: Spin and charge currents in a ferromagnetic insulator|normal metal (FMI|NM) heterostructure with spin orbit interaction leads to spin Hall magnetoresistance (SMR). An incident charge current $\mathbf{J}_q^{\text{in}}$ gets partially converted into a spin current $\mathbf{J}_s^{\text{SHE}}$ by the spin Hall effect (SHE). A gradient in spin accumulation arises from $\mathbf{J}_s^{\text{SHE}}$ with magnitude depending on the boundary condition at the FMI|NM interface. For magnetization $\mathbf{M} \parallel \mathbf{s}$ (a) no angular momentum can be transferred by spin transfer torque (STT) into the FMI. The result is a spin diffusion current $\mathbf{J}_s^{\text{diff}}$, which itself is partially converted back into a charge $\mathbf{J}_q^{\text{ISHE}}$ by the inverse spin Hall effect (ISHE). The boundary condition $\mathbf{M} \perp \mathbf{s}$ (b) allows for STT and a finite spin current $\mathbf{J}_s^{\text{STT}}$ propagates into the FMI, thereby reducing the spin accumulation. Hence $\mathbf{J}_s^{\text{diff}}$ and $\mathbf{J}_q^{\text{ISHE}}$ are decreased, resulting in enhanced longitudinal resistivity. By rotating the magnetisation $\mathbf{M}$, the STT mechanism at the FMI|NM interface may continuously be activated or deactivated.

and the direction of spin current is perpendicular to both initial current flow and spin current polarization. The SHE is visualized in Fig. 2.2(a). Here an electron flow with polarization into of the plane is deflected to the top. This leads to a spin accumulation on both sides (top/bottom) of the conductor, as optically observed by Kato *et al.* [16]¹. The same scattering effect can also translate a propagating spin current into a charge current – for this reason it is called inverse spin Hall effect (ISHE) (cf. Fig. 2.2(b))[30]. It is worth mentioning that spin currents can propagate in magnetically ordered materials even in the complete absence of mobile charge carriers. Then $\mathbf{J}_s$ is carried by magnons [31]. However, the physics then is qualitatively different from what is discussed here.

Let us now consider a film of NM in the $\mathbf{j}$ - $\mathbf{t}$ plane (indicated in Fig. 2.3) with spin orbit interaction which hence has a finite spin Hall angle α_{SH} . An incident charge current density $\mathbf{J}_q^{\text{in}}$ is applied as sketched in Fig. 2.3(a). Due to SHE, electrons are deflected towards the top or bottom of the film spin dependently², generating a spin current density $\mathbf{J}_s^{\text{SHE}} \perp \mathbf{J}_q^{\text{in}}, \mathbf{s}$. A spin imbalance is caused. The gradient in spin accumulation μ_s leads to a diffusive spin current $\mathbf{J}_s^{\text{diff}}$ antiparallel to $\mathbf{J}_s^{\text{SHE}}$.

¹More precisely, Kato *et al.* used a semiconductor for their Kerr rotation microscopy experiment.

²The $\mathbf{t}$ -axis is used as quantization for the spin because only spin current along $\mathbf{n}$ is relevant to SMR.

As a consequence of the ISHE, $\mathbf{J}_s^{\text{diff}}$ is partially converted into a charge current $\mathbf{J}_q^{\text{ISHE}} \perp \mathbf{J}_s^{\text{diff}}, \mathbf{s}$, which adds to the incident current $\mathbf{J}_q^{\text{in}}$. We see that when applying an electric field to the sample, the resulting charge current $\mathbf{J}_q$ is inherently a superposition of the initial charge current, with a charge current arising from SHE and ISHE. Both currents inevitably contribute to charge currents in metals with spin orbit interaction.

Now consider that the NM is attached to a ferromagnetic insulator (FMI) with magnetization $\mathbf{M}$ oriented along $\mathbf{t}$ as shown in Fig. 2.3(a). The boundary condition is unchanged compared to the above discussed model according to Brataas *et al.* [32], who showed that $\mathbf{M}$ can only be excited by a spin current if its polarization is perpendicular to $\mathbf{M}$. Hence no spin current can flow across the FMI|NM interface. The situation changes upon rotating $\mathbf{m} = \mathbf{M}/M$ away from $\mathbf{s}$. Now a spin current $\mathbf{J}_s^{\text{STT}}$ is absorbed at the FMI|NM interface by an excitation and deflection of $\mathbf{M}$. Figure 2.3(b) illustrates the consequence of a finite $\mathbf{J}_s^{\text{STT}}$ across the FMI|NM interface. The additional spin flip scattering reduces the gradient in μ_s and thereby $\mathbf{J}_s^{\text{diff}}$ and $\mathbf{J}_q^{\text{ISHE}}$. For $\mathbf{m} \perp \mathbf{s}$ (Fig. 2.3(b)) the net charge current across the NM is decreased in comparison to $\mathbf{m} \parallel \mathbf{s}$ (Fig. 2.3(a)) for a given electric field and thereby the longitudinal resistivity ρ_{long} is increased [23]:

$$\rho_{\text{long}, \mathbf{m} \perp \mathbf{s}} > \rho_{\text{long}, \mathbf{m} \parallel \mathbf{s}}. \quad (2.5)$$

For an exhaustive theoretical discussion the supplementary material to the work of Nakayama *et al.* [22] and the publication by Chen *et al.* [24] are recommended. Nakayama *et al.* solve the spin diffusion equation for the NM film with thickness t_N , width w and length l in presence of spin orbit interaction with boundary conditions for FMI|NM and NM|vacuum interface. The boundary condition at FMI|NM is parametrized by the magnetization orientation angle α relative to the direction of current flow $\mathbf{j}$. The effect is expressed as a longitudinal (ρ_{long}) and transverse (ρ_{trans}) resistivity:

$$\rho_{\text{long}}(\alpha) = \rho_0 + \Delta\rho \cos^2(\alpha) \quad (2.6)$$

$$\rho_{\text{trans}}(\alpha) = \Delta\rho \sin(\alpha) \cos(\alpha). \quad (2.7)$$

$$(2.8)$$

ρ_0 is the magnetization independent part of ρ_{long} and $\Delta\rho$ gives the strength of the SMR effect which is equally observable in ρ_{long} and ρ_{trans} . The Hall bar geometry used for measuring ρ_{long} and ρ_{trans} will be introduced in Ch. 3.1. So far we have discussed the rotation around the surface normal $\mathbf{n}$. Our experimental setup is restricted to this rotation, hence we only mention that in general ρ_{long} is sensitive to the projection of magnetization orientation $\mathbf{m}$ on the $\mathbf{t}$ -axis and ρ_{trans} to both projections on $\mathbf{j}$ and $\mathbf{t}$ [24].

In Eq. (2.9) and (2.10) we define the resistivity. Experimentally the longitudinal and transverse resistivity are accessed by applying a constant current I_q and

detecting the voltage drop V_{long} along $\mathbf{j}$ as well as the voltage V_{trans} in $\mathbf{t}$ direction:

$$\rho_{\text{long}} = \frac{E_{\text{long}}}{J_{\text{q}}} = \frac{V_{\text{long}}}{I_{\text{q}}} \frac{wt_{\text{N}}}{l} = R_{\text{long}} \frac{wt_{\text{N}}}{l} \quad (2.9)$$

$$\rho_{\text{trans}} = \frac{E_{\text{trans}}}{J_{\text{q}}} = \frac{V_{\text{trans}}}{I_{\text{q}}} t_{\text{N}} = R_{\text{trans}} t_{\text{N}}. \quad (2.10)$$

Furthermore it is beneficial to express the SMR effect in terms of resistance for noise investigations which are the experimental task of this thesis. Equations (2.9) and (2.10) are rewritten as longitudinal $R_{\text{long}} = V_{\text{long}}/I_{\text{q}}$ and transverse $R_{\text{trans}} = V_{\text{trans}}/I_{\text{q}}$ resistance to obtain

$$R_{\text{long}} = \frac{l}{wt_{\text{N}}} \rho_0 \left(1 + \frac{\Delta\rho}{\rho_0} \cos^2(\alpha) \right) \quad (2.11)$$

$$R_{\text{trans}} = \frac{1}{t_{\text{N}}} \Delta\rho \sin(\alpha) \cos(\alpha). \quad (2.12)$$

The constant resistance $R_0 := \frac{l}{wt_{\text{N}}} \rho_0$ and the relative SMR $\Delta R/R_0 := \Delta\rho/\rho_0$ are defined. Please note that in contrast to the SMR effect expressed in resistivity the peak to peak SMR in longitudinal resistance is by a factor l/w larger than the peak to peak SMR effect in R_{trans} .

The SMR effect was quantitatively studied by Althammer and coworkers [23]. They used platinum (Pt) as NM and yttrium iron garnet (YIG) as FMI. With this structure it was possible to clearly distinguish the SMR from anisotropic magnetoresistance (AMR) [33, 34]. The YIG|Pt bilayers showed SMR effects with $\Delta\rho/\rho_0 \sim 10^{-4}$ – 10^{-3} with the maximum effect for Pt thickness $t_{\text{Pt}} = 2.5$ nm [23]. Theory predicts that the maximum SMR occurs when the NM thickness corresponds to twice the spin diffusion length [22]. We use similar YIG|Pt samples in the experiment described in Ch. 3.

2.2 Introduction to Noise

The noise will be introduced in a mathematically correct way and its quantization and measurement will be discussed. We do so closely following the introduction of [35].

Generally, noise can be described by a stochastic function $\Phi(t)$ [36]. The time dependent function is given by a value set $\{\Phi(t)\}$ and a probability density function $p(\Phi)$. If $p(\Phi)$ is shaped like a Gaussian distribution, the corresponding noise is also called Gaussian. The time average of the discussed functions is assumed to be zero. Furthermore all stochastic functions we investigate are stationary processes[37], i.e. the probability density function does not change over time. This implies that the average of a stationary probability process does not depend on the time it is acquired. It is equivalent to the Ergodic hypothesis, that averaging over the ensemble is identical to the time average.

In the following we will restrict our study to a stochastic function $\phi(t)$ which is limited to a sampling time τ_s and write w.l.o.g. [36]:

$$\phi(t) = \begin{cases} \Phi(t) & \text{if } t \in (0, \tau_s) \\ 0 & \text{else} \end{cases} \quad (2.13)$$

This step is motivated by the physical fact, that noise will be always investigated over a finite time period $\tau_s \in \mathbb{R}_+$. The so defined stochastic function can be integrated and therefore the Fourier transform of ϕ is well defined. Applying Parseval's theorem we can write [37]:

$$\langle \phi^2 \rangle := \frac{1}{\tau_s} \int_0^{\tau_s} \phi^2(t) dt = \frac{1}{T} \int_{-\infty}^{\infty} |\phi(t)|^2 dt = \frac{1}{\tau_s} \int_{-\infty}^{\infty} |\mathfrak{F}[\phi](f)|^2 df \quad (2.14)$$

with the Fourier transform

$$\mathfrak{F}[\phi](f) := \int_{-\infty}^{\infty} \phi(t) e^{-i2\pi ft} dt. \quad (2.15)$$

Here $\langle \phi^2 \rangle$ can be interpreted as the mean power in a signal ϕ over time $t \in (0, \tau_s)$. From $\phi \in \mathbb{R}$, it follows that $\mathfrak{F}[\phi](-f) = \mathfrak{F}^*[\phi](f)$, i.e. $|\mathfrak{F}[\phi](f)|^2$ is an even function of f . Additionally, it can be concluded from the stationarity of Φ and Eq. (2.14) that $|\mathfrak{F}[\phi](f)|^2 \propto T$. It follows

$$\langle \phi^2 \rangle = \frac{1}{\tau_s} \int_{-\infty}^{\infty} |\mathfrak{F}[\phi](f)|^2 df = \int_0^{\infty} \frac{2}{\tau_s} |\mathfrak{F}[\phi](f)|^2 df. \quad (2.16)$$

We see that the integrand in Eq. (2.16) is a measure of the power per frequency unit. And we define in analogy to Becker [36]:

$$\phi_f^2 := \frac{2}{\tau_s} |\mathfrak{F}[\phi](f)|^2. \quad (2.17)$$

ϕ_f^2 is usually referred to as power spectral density and is often denoted as $S(f)$. Other common terms for $S(f)$ are power spectrum or spectral density. In general the power spectral density (PSD) is determined from a measured variable such as voltage V which will be indicated by $S_V(f)$. The units of voltage PSD are V^2/Hz . The magnitude of the spectrum is normalized to a 1 Hz bandwidth. There is a second popular unit for characterizing noise. The so called noise voltage $\sqrt{S_V}$ in units $V/\sqrt{\text{Hz}}$ is commonly used for characterizing electronic equipment such as amplifiers. From Eq. (2.17) it can easily be seen that voltage power spectral densities, but not noise voltages add.

2.3 Thermal Noise and Quantum Fluctuations

After introducing the voltage PSD S_V we estimate which kind of fluctuations will dominate a room temperature measurement with ferromagnetic systems such as a

FMI|NM heterostructure. According to fluctuation dissipation theorem (FDT) the voltage PSD across a resistor R to a matched load is given by the energy $E(\omega, T)$ as a function of temperature T and frequency ω [38]

$$\frac{S_V(f)}{4R} = E(\omega, T). \quad (2.18)$$

The factor $1/(4R)$ enters because the power density transferred to a load R' is

$$\langle I^2 \rangle R' = \frac{\langle V^2 \rangle R'}{(R + R')^2} \quad (2.19)$$

which at match ($R' = R$) is $\langle V^2 \rangle / (4R)$. The energy of a quantum mechanical oscillator with zero point energy $hf/2$ and Bose-Einstein statistics is

$$E(\omega, T) = \frac{hf}{2} + hf \frac{1}{e^{\frac{hf}{k_B T}} - 1} = \frac{hf}{2} \coth \left(\frac{hf}{2k_B T} \right). \quad (2.20)$$

Now the coth is considered for the classic limit and the quantum mechanical limit, $hf \ll k_B T$ and $hf \gg k_B T$ respectively.

$$hf \ll k_B T \quad S_V(f) = 4k_B T R \quad (2.21)$$

$$hf \gg k_B T \quad S_V(f) = 2hfR \quad (2.22)$$

For room temperature we recognize the well known Nyquist noise [39, 40] – also called Johnson noise or thermal noise – which arises from the thermal excitation of the charge carriers. It describes the voltage power emitted by a resistor R at temperature T per frequency unit Δf :

$$S_{V,N}(R, T) = 4k_B T R. \quad (2.23)$$

Nyquist noise is frequency independent and consequently referred to as white noise. The actual voltage power $\langle V^2 \rangle$ emitted by the resistor depends on the bandwidth B under investigation:

$$\langle V^2 \rangle = 4k_B T R B. \quad (2.24)$$

When we interpret the energy $hf = eV$ to be equal to the electron energy, the quantum mechanical limit of voltage noise is equivalent to shot noise [41] – often referred to as Schottky noise.

$$hf \gg k_B T \quad S_V(f) = 2eV R = 2eIR^2 \quad (2.25)$$

The electron energy exceeds the thermal energy ($eV \gg k_B T$), for example in vacuum diodes, pn-diodes, or pnp-transistors, and shot noise occurs³. In the study of

³A more common form of writing shot noise is the current power spectral density $S_I = S_V/R^2 = 2eI$.

FMI, magnons transport angular momentum. Their energy level is of the order of the ferromagnetic resonance frequency which lies in the GHz regime [42]. So their quantized transport becomes relevant to noise at temperatures below $50 \text{ mK} \hat{=} 1 \text{ GHz}$. This might be the main reason why spin current shot noise is yet a purely theoretical concept [43]. Concluding, the noise in SMR type bilayer structures at room temperature will be dominated by classical thermal fluctuations.

2.4 From Spin Hall Magnetoresistance to Spin Hall Magnetoresistive Noise

We now pick up the idea from the previous section that thermal noise is relevant for FMI|NM heterostructures. This way the spin Hall magnetoresistive noise is derived from FDT. We apply the formula for Nyquist noise Eq. (2.23) on the SMR resistance Eq. (2.11):

$$S_V = S_{V,0} + \Delta S_V \cos^2(\alpha) \quad (2.26a)$$

$$S_{V,0} = 4k_B T \frac{l}{wt_N} \rho_0 \quad (2.26b)$$

$$\Delta S_V = 4k_B T \frac{l}{wt_N} \Delta \rho. \quad (2.26c)$$

Hence the SMN is expected to show the same qualitative characteristic $\cos^2(\alpha)$ behavior as well as the identical quantitative SMN signal $\Delta S_V / S_{V,0} = \Delta \rho / \rho_0$. According to Althammer *et al.* [23] a maximum SMR effect of $\sim 10^{-3}$ is expected in appropriate YIG|Pt hybrid structures.

The SMN was rigorously derived by Kamra *et al.* [44, 45] starting from the thermal fluctuations of the magnetization to the voltage fluctuations which are experimentally detected. The basic idea of the derivation shall be presented below. First all contributions to time dependent spin current I_s across the FMI|NM interface are estimated. They arise from angular momentum transport to $\mathbf{M}$ and emitted angular momentum by $\mathbf{M}$ [46]. Based on the Wiener-Khintchine theorem [47, 48], the power spectrum of the spin current is connected to its autocorrelation function

$$\rho_{(I_s I_s)}(\tau) = \langle I_s(t) I_s(t + \tau) \rangle \quad (2.27)$$

by

$$S_{I_s}(f) = 2 \int_{-\infty}^{\infty} \rho_{(I_s I_s)}(\tau) e^{-2\pi i f \tau} d\tau. \quad (2.28)$$

After calculating the correlations according to the macrospin model [46], the spin current PSD is converted via ISHE into a voltage PSD. Kamra *et al.* showed that this derivation leads to the same result as the above approach (Eq. 2.26) for frequencies below ferromagnetic resonance.

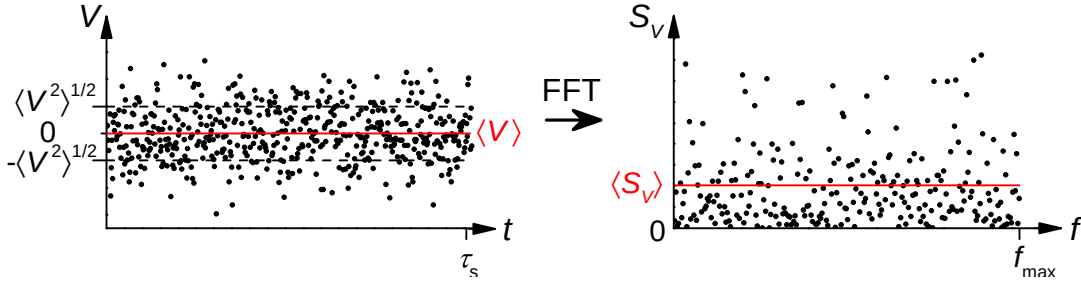


Figure 2.4: A voltage signal with white noise, expectation value $\langle V \rangle = 0$, and variance $\text{var}(V) = \langle V^2 \rangle$ is fast Fourier transformed (FFT) in order to estimate the voltage power spectral density S_V .

2.5 Quantifying White Noise by the Fast Fourier Transform

We now address the possibility to experimentally detect SMN by measuring the voltage PSD with fast Fourier transform (FFT) spectroscopy. The focus is on the physical limitation to estimating voltage PSD by FFT spectroscopy. We go through the derivation to have a rigorous argument⁴. Figure 2.4 depicts a voltage signal with white noise $V(t)$ and average $\langle V \rangle = 0$. The signal is fast Fourier transformed to obtain the PSD spectrum $S_V(f)$. $S_V(f)$ intrinsically fluctuates around its expectation value $\langle S_V \rangle$. The magnitude of this fluctuation is described by the signal to noise ratio (S/N), which is

$$S/N = 1. \quad (2.29)$$

for the white, stationary voltage noise signal $V(t)$ and shall be derived below. This physical limit can be interpreted the following way: In order to have a spectrum with a decent S/N ratio, it is necessary to average each frequency bin $S_{V,r}$ over n_{AV} spectra. The accuracy of the resulting spectrum obeys the law of large numbers, so S/N can be improved to

$$S/N = \sqrt{n_{AV}}. \quad (2.30)$$

In other words the measuring time τ_{AV} to obtain a certain accuracy is determined by the number of averaged spectra and the time it takes to acquire one spectrum τ_s :

$$\tau_{AV} = \tau_s \times n_{AV}. \quad (2.31)$$

The measurement time depends on the desired frequency resolution $\Delta f = 1/\tau_s$ and measuring accuracy $S/N = \sqrt{n_{AV}}$. The sampling rate gives the maximum resolved frequency and it does not influence the physical limit to accuracy.

⁴In references [37, 49, 50], the derivation is describes in general, but not applied to this particular case.

Now, equation (2.29) shall be derived. In Eq. (2.17) we defined the power spectral density as the Fourier transform of a statistical variate. Experimentally the voltage power spectral density S_V can be accessed by the FFT algorithm performed on a time trace with n_s samples, acquired in time τ_s . The voltage PSD as defined in Eq. (2.17) is

$$S_V(f) = \frac{2}{\tau_s} \left| \int_{-\infty}^{\infty} V(t) e^{-i2\pi ft} dt \right|^2. \quad (2.32)$$

When changing to discrete values in time $t_k = k\tau_s/n_s$ and frequency $f_r = r/\tau_s$, $k, r = 0, \dots, n_s - 1$, also the infinitesimal time interval $dt \rightarrow \tau_s/n_s$ takes a finite value⁵. So $S_V(f_r)$ is calculated from the time trace $V(t_k)$

$$S_V(f_r) = \frac{2\tau_s}{n_s^2} |\mathfrak{F}_{\text{DFT}}[V](f_r)|^2 \quad (2.33)$$

with the discrete Fourier transform (DFT) [51]

$$\mathfrak{F}_{\text{DFT}}[V](f_r) = \sum_{k=0}^{n_s-1} V(t_k) e^{-i\frac{2\pi}{n_s}rk}. \quad (2.34)$$

The FFT algorithm is an elegant way of computing the DFT. For Eq. (2.16) only positive frequencies of the Fourier transform spectrum are relevant. Hence the spectrum reaches up to $f_{\text{max}} = n_s/(2\tau_s)$.

Now we look at the expectation value of S_V and expand Eq. (2.33). For simplicity we rewrite $V_k := V(t_k)$ and $S_{V,r} := S_V(f_r)$.

$$\langle S_{V,r} \rangle = \frac{2\tau_s}{n_s^2} \left\langle \left(\sum_{k=0}^{n_s-1} V_k e^{-\frac{2\pi i}{n_s}rk} \right) \left(\sum_{j=0}^{n_s-1} V_j e^{-\frac{2\pi i}{n_s}rj} \right)^* \right\rangle \quad (2.35)$$

$$= \frac{2\tau_s}{n_s^2} \sum_{kj} \langle V_k V_j \rangle e^{\frac{2\pi i r}{n_s}(j-k)} \quad (2.36)$$

The double sum stands for all combinations of $k, j = 0, \dots, n_s - 1$. V is assumed to have white noise, hence V_k and V_j are not correlated and for this reason $\langle V_k V_j \rangle = \langle V^2 \rangle \delta_{kj}$. Here δ_{kj} denotes the Kronecker-delta. That simplifies Eq. (2.36) to

$$\langle S_{V,r} \rangle = \frac{2\tau_s}{n_s^2} \sum_k \langle V^2 \rangle \quad (2.37)$$

$$= \frac{2\tau_s}{n_s} \langle V^2 \rangle \quad (2.38)$$

⁵Note that the spacing between samples is τ_s/n_s and that the frequencies are equally spaced by $\Delta f = 1/\tau_s$

For thermal noise of a resistor R at T , $\langle V^2 \rangle$ is given by Eq. (2.24). For an inspected bandwidth $B = f_{\max} = n_s/(2\tau_s)$, we see that the DFT correctly estimates the thermal noise power.

$$\langle S_{V,r} \rangle = 4k_B T R \quad (2.39)$$

The variance - also called mean square deviation - of an estimator X is given by [49]

$$\text{var}(X) = \langle X^2 \rangle - \langle X \rangle^2. \quad (2.40)$$

$\langle S_{V,r}^2 \rangle$ needs to be determined, and in analogy to Eq. (2.36) we write

$$\langle S_{V,r}^2 \rangle = \frac{4\tau_s^2}{n_s^4} \sum_{kjmn} \langle V_k V_j V_m V_n \rangle e^{\frac{2\pi i r}{n_s}(j-k+n-m)}. \quad (2.41)$$

$\langle V_k V_j V_m V_n \rangle$ is evaluated according to Isserlis' theorem [52]. V_k , V_j , V_m , and V_n are stochastically independent and we obtain:

$$\begin{aligned} \langle V_k V_j V_m V_n \rangle &= \langle V^2 \rangle^2 (\delta_{kj} \delta_{mn} + \delta_{km} \delta_{jn} + \delta_{kn} \delta_{jm}) \\ \langle S_{V,r}^2 \rangle &= \frac{4\tau_s^2}{n_s^4} \langle V^2 \rangle^2 \left(\sum_{km} 1 + \sum_{kj} e^{\frac{2\pi i r}{n_s}(2j-2k)} + \sum_{kj} 1 \right) \\ &= \frac{4\tau_s^2}{n_s^4} \langle V^2 \rangle^2 (n_s^2 + 0 + n_s^2) = \frac{8\tau_s^2}{n_s^2} \langle V^2 \rangle^2 \end{aligned} \quad (2.42)$$

In the second sum a complex vector of length 1 is rotated in the imaginary plane and hence the sum evaluates to zero. The figure of merit for the accuracy of a measure X is its signal to noise (S/N) ratio given by [37]

$$\text{S/N} = \frac{\langle X \rangle}{\sqrt{\text{var}(X)}}. \quad (2.43)$$

Using Eq. (2.38), (2.40), and (2.42) the S/N ratio of each $S_{V,r}$ of a voltage power spectrum, which was measured from a voltage with white noise, is obtained

$$\text{S/N} = 1. \quad (2.44)$$

Chapter 3

Experimental Setup

The measurement task at hand is to experimentally show that the spin Hall magnetoresistive noise (SMN) alters the Nyquist noise power of a ferromagnetic insulator|normal metal (FMI|NM) heterostructure depending on the magnetization orientation of the FMI. In the following the setup for detecting angle-dependent¹ resistive noise (ADRN) is described, which is used to quantitatively prove that Eq. (2.26) holds.

Figure 3.1 illustrates the schematic circuit diagram for the setup. The sample is mounted on a chip carrier system extending into the magnetic field of a two dimensional vector magnet (2D-VM). The voltage signal of the sample can be probed by two copper wires leading to a low noise voltage amplifier. The voltage amplifier enlarges the signal, which is then fed into a fast Fourier transform (FFT) spectrum analyzer. The FFT analyzer returns the measured noise signal spectrally resolved as voltage power spectral density (PSD) data to a computer. The main challenges for resolving the predicted SMN effect of order 10^{-3} are low noise detection and thermal stability. The first is achieved by a conductive box partially enclosing the setup. The second challenge is handled by maintaining a closed laboratory environment during a measurement and by a special sequence of magnetic field orientations.

In this chapter we discuss the setup equipment in detail. Starting with the sample, the details about the instruments required for low noise voltage PSD measurements are outlined.

Why did we chose this particular room temperature setup instead of a liquid He magnet cryostat system, which has been used for magnetotransport measurements of similar samples [23, 53]? This is a particular interesting question, because temperature stability is crucial for thermal noise measurements. The drawback of a temperature stabilized system, such as a cryostat, is that the temperature control needs to be decoupled from the sample shielding. In the available cryostat, this was not the case and the temperature control couples additional background noise into the measurement circuit, and is consequently inapplicable to our task.

setup-photo.png

¹In the following we use “angle-dependent” synonymously for “as a function of external magnetic field orientation”.

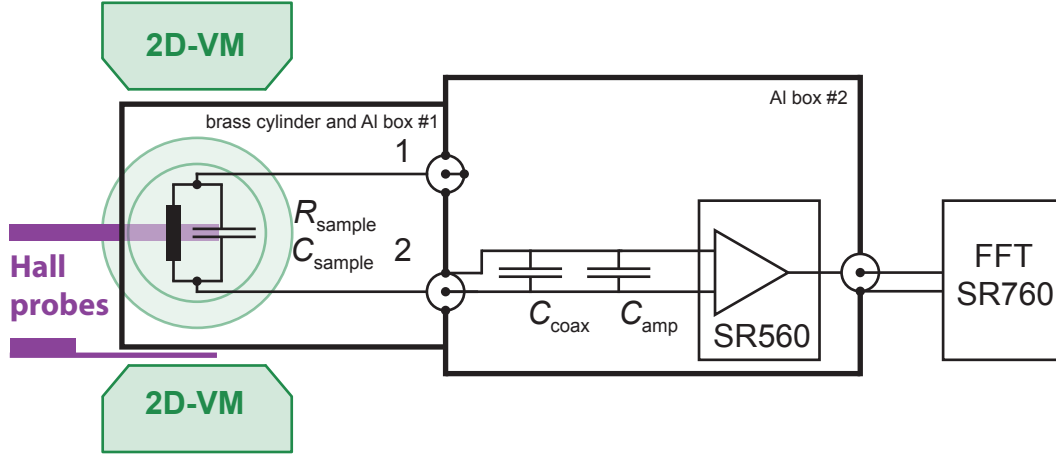


Figure 3.1: Schematic circuit diagram for measuring angle-dependent resistive noise.

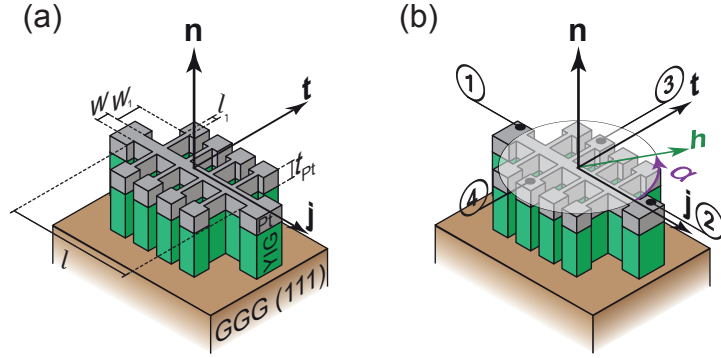
3.1 Sample Fabrication and Characterization

In our experimental setup yttrium iron garnet ($\text{Y}_3\text{Fe}_5\text{O}_{12}$, YIG) was used as a FMI and platinum (Pt) is utilized as NM. The YIG|Pt heterostructure was grown on (111)-oriented gadolinium gallium garnet (GGG) substrate in analogy to Althammer *et al.*[23] (see Fig. 3.2). YIG thin films with $d_{\text{YIG}} \approx 60$ nm are epitaxially grown via pulsed laser deposition. Polycrystalline platinum layers (thickness $t_{\text{Pt}} = 2.7$ nm and 2.2 nm) are grown in-situ on top of the YIG films using electron beam evaporation, at room temperature. We use high-resolution X-ray diffractometry (HRXRD) in a four circle diffractometer to determine the thickness of the nonmagnetic metal layers². Hall bar mesa structures were patterned out of the plain YIG/Pt films using photolithography and argon ion beam milling. The Hall bar structure was investigated under a microscope and photographed. The image was used to determine width w and length l of the Hall bar to 0.08 mm and 1 mm, respectively (Fig. 3.2). The feed lines have dimensions $w_1 \approx 0.02$ mm and $l_1 \approx 0.06$ mm. The sample is glued to a copper block (ca $5 \times 5 \times 2$ mm) as a heat reservoir. Four bond pads are connected by aluminum bond wires to a chip carrier system, see Fig. 3.2(b).

Now we characterize the electrical and thermal properties of the YIG|Pt sample with thickness $t_{\text{Pt}} = 2.7$ nm. To do so, we apply a current using a Keithley K2400 source meter and detect the voltage over contacts 1 and 2 utilizing a Keithley K2182 (Fig. 3.2(b)). We will refer to these kind of voltage measurements along bond pads 1-2 as a “longitudinal” voltage measurement, whereas “transverse” measurements imply that we used contacts 3-4. The sample is not heated, since the applied currents below 50 μA do not carry enough power as verified by the $V(I_q)$ -characteristic up to 50 μA , Fig. 3.3(a). Not heating the sample by applying a current is of importance in order to compare resistance and voltage PSD measurements. We used a liquid He magnet cryostat system for investigating the temperature dependence $R(T)$, see Fig. 3.3(b) in 1 K steps. A linear equation was fitted to the two point probe voltage

²We explicitly thank the PhD student S. Meyer for growing and precharacterizing the samples.

Figure 3.2: (a) The YIG|Pt heterostructure is patterned into a Hall bar mesa with dimensions w, w_1, l, l_1 , and t_{Pt} and definition of the coordinate system $\mathbf{j}, \mathbf{t}$ and $\mathbf{n}$ according to Ch. 2.1. (b) The in-plane magnetic field orientation angle α with respect to $\mathbf{j}$ and nomenclature of the used bondpads are defined. Illustrated in analogy to [23].



measurements at constant current $I_q = 40 \mu\text{A}$ and yielded:

$$R(T) = R_0 \times [(1 + A(T - 273.15 \text{ K}))]. \quad (3.1)$$

with the resistance $R_0 = 1956.4 \Omega$ at 273.15 K and the constant $A = 7.9 \times 10^{-4} / \text{K}$. In analogy to [54], the temperature dependence of the resistance was normalized to the ice-point resistance. Only the linear term of the normally quadratic description is considered here, because we only measured in a small thermal range $286 \text{ K} < T < 303 \text{ K}$. The parameter A deviates from the linear term of IEC 751/DIN EN 60751 norm ($A = 3.9083 \times 10^{-3} / \text{K}$) and of [55] ($A = 3.9673 \times 10^{-3} / \text{K}$) by a factor of 4.9. Both sources refer to bulk Pt, while our sample is only a thin film, such that presumably its thermal sensitivity is decreased. We can not directly use the Pt sample as a calibrated temperature sensor with calibration curve Eq. (3.1), because the sample bonds had to be renewed after the $R(T)$ measurement. Still it is safe to assume that the sample temperature characteristic is linear and is described by the dimensionless factor $A = 7.9 \times 10^{-4}$. This allows to convince our selves of the thermal stability of the setup in Ch. 3.2.2.

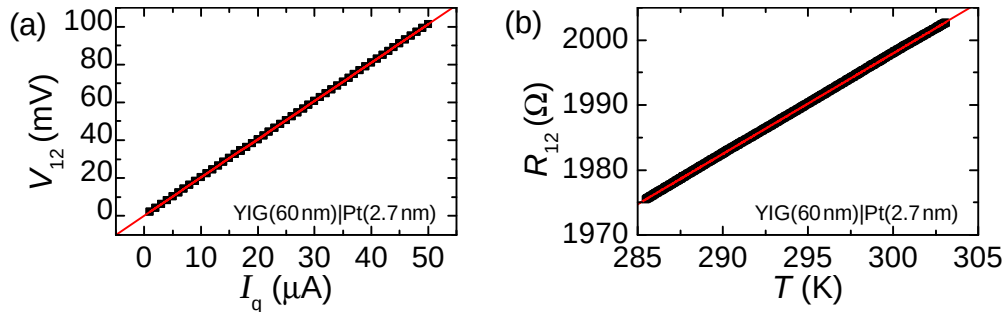


Figure 3.3: (a) Voltage-current characteristic of YIG(60 nm)|Pt(2.7 nm). The constant resistance implies a constant sample temperature T . (b) Temperature dependence of Pt resistance for YIG(60 nm)|Pt(2.7 nm) (black). The linear fits are depicted red solid lines.

3.2 Instruments and Devices

Now the details of the setup shown in Fig. 3.1 will be discussed instrument by instrument.

3.2.1 Shielding and Wiring

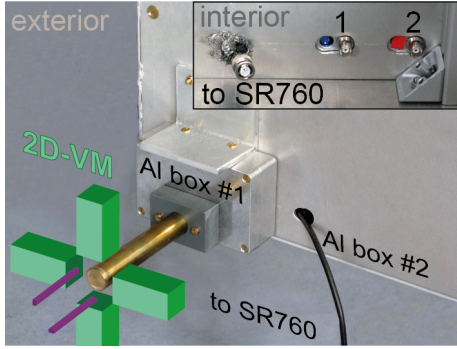


Figure 3.4: Exterior and interior view of the shielding box are depicted.

The shielding principle is similar to the one of a coaxial cable - the signal ground is the same as the shield ground [56]. The most sensitive parts of the measurement setup are consequently enclosed by aluminum to shield electromagnetic radiation, especially 50Hz and higher orders of 50Hz which stem from line frequency and laboratory equipment. Therefore, a big aluminum shielding box (Al box #2) contains the voltage amplifier SR560 (cf. Figs. 3.1, 3.4). A smaller aluminum box (Al box #1) plus cylindrical brass sample casing is conductively mounted on one side of Al box #2 (electrical contact $R_{Al} \leq 1 \Omega$). All casings must be disconnected from the surrounding conductors, so the box rests on a wooden basis and SR760 input is disconnected by $1 M\Omega$ (float) from its casing/line ground. Still it is highly recommended to make sure that the electromagnet is disconnected from the SR760 casing, so that no signal is directly coupled into the SR760.

Before discussing the setup in further detail, we here take a quick look at the wiring which is designed to minimize the parasitic capacity $C_{\text{parasitic}} = C_{\text{sample}} + C_{\text{coax}} + C_{\text{amp}}$. Starting at the chip carrier system, two soldered copper wires (twisted pair) can be connected to different pads. They lead to two coax plugs on the wall of Al box #2 (Fig. 3.4). For noise measurements plug (1) is shorted and thereby the signal ground is connected to shield ground. The other wire leads through a male-male BNC connector straight into the SR560. This setup not only provides good shielding but also has little parasitic capacitance, because no coax cables are used between sample and amplifier. After the signal is amplified, it is fed into the FFT spectrum analyzer.

3.2.2 Two Dimensional Vector Magnet

A two dimensional vector magnet (2D-VM) consisting of two iron yokes and 4 copper coils provides the magnetic field for rotating the magnetization orientation of the YIG film. Two Hall probes, one for $\mathbf{j}$ and $\mathbf{t}$ field component, respectively, are used

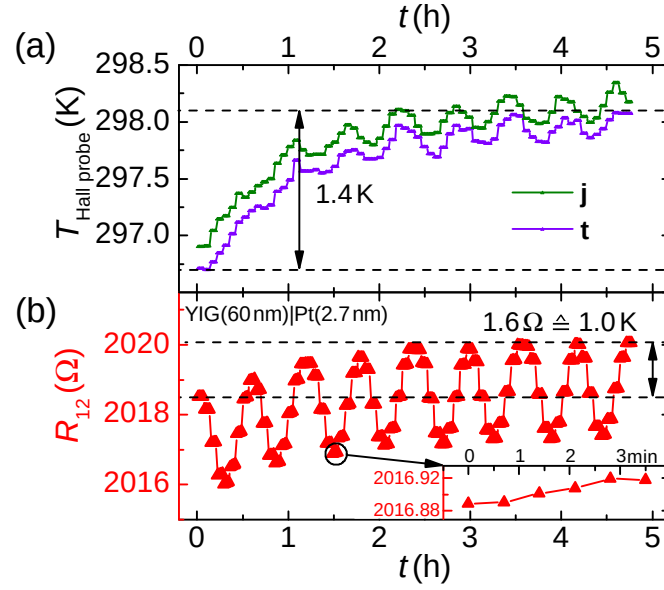


Figure 3.5: The evolution of room temperature measured by Hall probes for **j** and **t** field component (a) and resistance evolution of YIG(60 nm)|Pt(2.7 nm) are depicted as function of time (b). While the T and R data were taken simultaneously, the magnetic field $\mu_0 H = 60$ mT was repeatedly rotated from $\alpha = 0^\circ$ to 360° in 22.5° steps. Each angle was maintained for 4 min. Resistance at 90° of the second rotation is displayed in insert to (b). The Hall probes indicate a thermal increase of 1.4 K. The sample heated up by 1.0 K.

to control the magnetic field via gaussmeters. After reaching the set field, the control is turned off to avoid field oscillations during the measurement. Additionally, the magnet temperature is detected by the Hall probes. There is no additional temperature control, meaning the sample is at laboratory temperature (referred to as room temperature). This electromagnet allows us to change the field orientation **h** and magnitude $\mu_0 H$ within a couple of seconds. The orientation **h** is consistent with the direction of a compass needle and occurs under the magnetic field orientation angle α with respect to the **j**-axis (Fig. 3.2).

Longterm temperature stability is essential for ADRN measurements and it is achieved by keeping the laboratory closed during a measurement and by a special sequence of magnetic field rotation, which shall be discussed now. The magnetic field is rotated from $\alpha = 0^\circ$ to 360° field in 22.5° steps, then the rotation is repeated. The field $\mu_0 H = 60$ mT dwells on each angular position for 4 min.

The Pt film of sample YIG(60 nm)|Pt(2.7 nm) and the two Hall probes were used as temperature probe to prove the thermal stability of the magnetic field rotation sequence. A 40 μ A current was applied to the sample and the voltage drop was measured every 40 s. The 360° rotation was undergone four times.

The magnet temperature recorded by the Hall probes (components **j** and **t**) showed temperatures differing by 0.2 K, because they are not calibrated (cf. Fig. 3.5). The room temperature as well as the resistance show that the room heats up during one measurement due to the heat emitted by the 2D-VM power supplies. According to

the gaussmeters, the room heats up by approximately 1.4 K over 5 h. The sample resistance increased by 1.6Ω , which corresponds to a temperature increase of 1.0 K ³.

Besides the longterm temperature increase, the magnet temperatures showed oscillations, which arise from two different power supplies used for **j** and **t** direction. Their differing heat emission causes oscillations in magnet temperature. The sample resistance oscillation due to thermal changes are small compared to the oscillations due to spin Hall magnetoresistance effect, which will be discussed in detail in Ch. 4.2. The insert in Fig. 3.5(b) depicts constant resistance and consequently constant temperature at one field orientation ($\Delta T < 7 \times 10^{-3} \text{ K}$). After two 360° rotations the sample reaches a thermal steady state. We conclude that the sample is poorly thermally coupled to the laboratory and that the sequence of field rotation is well suited for temperature sensitive ADRN measurements.

3.2.3 Low Noise Voltage Amplifier

The setup is not only sensitive to temperature, but also to background noise and to amplifier noise. To reach the specifications of the low noise voltage amplifier SR560 from Stanford Research we use by two 65 Ah car batteries instead of line power. All of which are incorporated by the shielding. The incoming signal is dc-coupled to the amplifier. No filter was applied to the signal for unbiased measurements. The input signal is amplified by 10^3 in “low noise” gain mode and connects through the 50Ω output into the FFT spectrum analyzer.

3.2.4 Fast Fourier Transform Spectrum Analyzer

In this section the basics and setting options of the Stanford Research fast Fourier transform (FFT) spectrum analyzer SR760 are explained[57]. Understanding the SR760 in detail gives us the chance to later implement its functionality on a A/D converter and a graphic processing unit (GPU). We will follow the trace of the signal from the SR760’s input to the FFT spectrum returned to a computer.

First, the input signal passes through an analog filter which attenuates all frequency components above 128 kHz by 90 dB, in order to make sure that Nyquist’s theorem is satisfied. Then it samples the incoming voltage signal with sampling frequency $f_s = 256 \text{ kHz}$ and $n_s = 1024$ sampling points. The acquisition takes $\tau_s = n_s/f_s = 4 \text{ ms}$. According to Nyquist’s theorem [58] the highest resolvable frequency of the Fourier spectrum is

$$f_{\max} = f_s/2 = 128 \text{ kHz} \quad (3.2)$$

and the resolvable line width is

$$\Delta f = 1/\tau_s = 250 \text{ Hz}. \quad (3.3)$$

³For this estimation first the resistance at ice-point $R_0 = 1.982 \text{ k}\Omega$ is calculated from Eq. (3.1) assuming that the sample was at $T = 296.5 \text{ K}$. Afterwards Eq. (3.1) is used again to calculate the temperature change from resistance change.

The time trace is fast Fourier transformed by the SR760 processor, which is limited to perform FFTs on the 1024 sampled values. The FFT is a highly efficient procedure for computing the discrete Fourier transform (DFT) of a time series $V(t_k) \in \mathbb{R}$ taken at times $t_k = k/f_s$, $k = 0, \dots, n_s - 1$ [51]:

$$\mathfrak{F}_{\text{DFT}}[V](f_r) \equiv \sum_{k=0}^{n_s-1} V(t_k) e^{\frac{2\pi i r k}{n_s}}. \quad (3.4)$$

This results in $n_f = n_s/2$ complex Fourier coefficients $\mathfrak{F}_{\text{DFT}}[V](f_r)$ at frequencies $f_r = r\Delta f$, $r = 0, \dots, n_f - 1$. The other half of the Fourier coefficients is interpreted as negative frequencies and does not carry additional information. For noise measurements we are especially interested in voltage PSD (Eq. (2.17)) in units of V^2/Hz . The SR760 has a PSD mode which returns the magnitude of the Fourier spectrum (the square root of the FFT times its complex conjugate normalized to a bandwidth of 1 Hz). The returned magnitude is compensated for the actual linewidth and window function by the SR760 implemented software [57]. The combination of PSD mode with unit settings Volts RMS leads to spectra in $\text{V}/\sqrt{\text{Hz}}$. It needs to be squared by the Labview routine which communicates between computer and SR760. The voltage power spectral density is calculated from the DFT by (cf. Eq. (2.33)):

$$S_V(f_r) = \frac{2\tau_s}{n_s^2} |\mathfrak{F}_d[V](f_r)|^2 = \frac{2}{n_s^2 \Delta f} |\mathfrak{F}_d[V](f_r)|^2. \quad (3.5)$$

The analog filter has a flat response from dc to 100 kHz, so only the first 400 of the 512 frequency values are usable and hence returned by the SR760. So the maximum returned bandwidth B , also called span, is 100 kHz with resolution $\Delta f = 250 \text{ Hz}$. The PSD is resolved in steps of $\approx 7 \times 10^{-14} \text{ V}^2/\text{Hz}$ by the SR760⁴. To measure spans smaller than 100 kHz, τ_s has to be increased. This is achieved by digitally filtering the incoming data samples (at 256 kHz) to limit the bandwidth. This is similar to the analog anti-aliasing filter at the input, except the digital filter's cutoff can be changed. Still 1024 samples are fast Fourier transformed and the first 400 values are returned, so that new spans of $B_{\text{new}} = 100 \text{ kHz}/a$, $a \in \mathbb{N}$ are accessible. It takes $\tau_{s,\text{new}} = a \times 4 \text{ ms}$ for acquisition. The consequence is that for covering the bandwidth of 100 kHz with reasonable resolution numerous spectra with $B < 100 \text{ kHz}$ need to be stitched together.

Heterodyning is used for moving the span away from dc to f_{start} . Heterodyning is the process of multiplying the incoming signal by a sine wave. The resulting spectrum is shifted by the multiplied frequency. This is incorporated to the digital filter and allows to shift the span in the range of 0 Hz to 100 kHz.

The SR760 supports root mean square (RMS) averaging. It computes the average of the squared magnitudes (FFT times its complex conjugate). The number of

⁴The amplitude of the SR760 output has a resolution of $\pm 0.02\%$ of the full scale. For full scale of about $3.5 \times 10^{-10} \text{ V}^2/\text{Hz}$ the absolute accuracy is $7 \times 10^{-14} \text{ V}^2/\text{Hz}$. This is 0.2% of the noise power of a $2 \text{ k}\Omega$ -resistor at room temperature amplified by 60 dB. Resolving $\text{SMR} \approx 0.1\%$ is still possible.

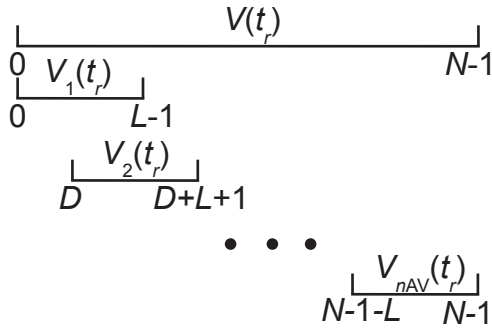


Figure 3.6: The concept of overlapping time segments for computing an average power spectrum is illustrated. A time record of N samples is divided into K segments of length L and with starting points D units apart [60].

averages is limited to $n_{AV} \leq 3 \times 10^4$. The average over spectra reduces fluctuations in the spectrum if they are statistical.

Last we introduce two additional features of the SR760: windowing and overlapping. The necessity of windowing arises from the finite-time interval of τ_s and therefore of the limited orthogonal basis (the frequency bins). From the continuum of possible frequencies, only those which coincide with the basis will project onto a single frequency bin; all other frequencies will exhibit non zero projections on the entire basis set. This is often referred to as spectral leakage and is the result of processing finite-duration records. Although the amount of leakage is influenced by the sampling period, leakage is not caused by sampling [59].

Windows are weighting functions applied to data to reduce the spectral leakage associated with finite observation intervals. From one viewpoint, the window is applied to data (as a multiplicative weighting) to reduce the order of the discontinuity at the boundary of the periodic extension. The SR760 provides four different window functions (Uniform = *no* window, Blackman-Harris (BMH), Flattop and Hanning), each providing different noise bandwidth and processing gain. We will not discuss the choice of window functions theoretically, as it does not have an influence on our application. For us windowing is of interest because when combined with the concept of overlapping the S/N ratio is improved at constant measurement time [60].

The SR760 can average over n_{AV} PSD spectra, which is useful because the signal to noise (S/N) ratio⁵ increases with the law of large numbers by $\sqrt{n_{AV}}$. As mentioned earlier, the processor is working to full capacity, when the span is set to 100 kHz. When the span is now decreased the acquisition of a single time record increases, leaving the processor with nothing to do. To exploit the free processor capacity the SR760 can use the concept of overlapping time records. This is well explained by Welch in [60] and is illustrated in Fig. 3.6. If we interpret the whole data set acquired during the averaging process as $V(t_k)$, it is split into segments of length L which starting point D units apart from each other. For $D < L$ the segments overlap and therefore the numbers of averages that can be performed on the trace $V(t_k)$ is enlarged. Welch furthermore shows in his paper, that when applying an overlap of $o = D/L = 50\%$ (i.e. $D = L/2$) and using a so called Welch windowing, the S/N ratio needs to be corrected by a factor 18/11 and is still proportional to

⁵Notice that we refer to the noise power under investigation as signal and to the fluctuation of that signal as noise.

$\sqrt{n_{AV}}$:

$$S/N = \frac{18}{11} \sqrt{n_{AV}} \quad (3.6)$$

Consequently, it is desirable to split the time trace into 50 % overlapping sequences, apply a window function on each sequence, average over all transformed spectra, and hence gain $\approx \sqrt{2}$ on the S/N ratio without overlap while sampling for the same amount of time τ_{AV} .

Chapter 4

Angle-Dependent Resistive Noise of YIG|Pt Heterostructures

In the third part of this thesis we address the experimental confirmation that both spin Hall magnetoresistance (SMR) and spin Hall magnetoresistive noise (SMN) are connected by the fluctuation dissipation theorem (FDT) and that both effects are of the same magnitude. First, the experimental setup is characterized concerning bandwidth limitation and signal to noise (S/N) ratio, to ensure that SMN can be resolved. Then, we quantitatively study the dissipative SMR effect in magnetotransport measurements. The analysis is confirming the former studies of Althammer *et al.* [23] and Nakajama *et al.* [22]. Afterwards the related thermal fluctuations in the spin current are investigated by observing voltage fluctuations in the charge channel. This will be done measuring in longitudinal and transverse geometry. We present voltage power spectral density (PSD) data in thermal equilibrium and in a biased sample as a function of the magnetization orientation. It is found that the SMN quantitatively behaves like the longitudinal part of the SMR effect. When applying a current in longitudinal direction, the transverse SMR effect is detected when measuring the transverse voltage as function of magnetization orientation, whereas the transversely measured voltage PSD is the same as an unbiased voltage PDS measurement. This allows to infer that the transverse SMR effect is a purely resistive effect. The presented measurements are in good quantitative agreement with the model described in Ch. 2.4.

4.1 “How low noise are we?” Setup Characterization

After introducing the setup for angle-dependent resistive noise (ADRN) measurements in Ch. 3, we now check its performance. The setup will be characterized in terms of the accessible bandwidth and the S/N ratio achievable using the SR760 FFT spectrum analyzer.

4.1.1 Limitations by Bandwidth and Background noise

In order to detect Nyquist voltage noise, which is directly proportional to resistance (cf. Eq. (2.23)), one would like to have large resistance for large signal. However, larger resistance also implies a larger RC-time constant of the circuit connected to

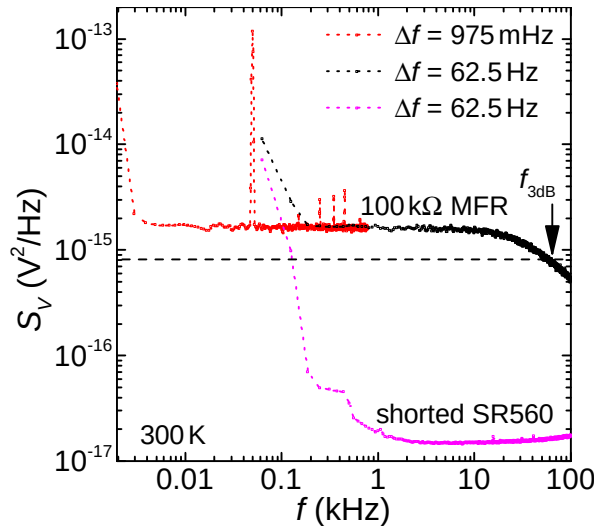


Figure 4.1: The voltage PSD spectra of a 100 kΩ metal film resistor (MFR) ($2 \text{ Hz} \leq f \leq 780 \text{ Hz}$, resolution $\Delta f = 975 \text{ mHz}$ – red; $62.5 \text{ Hz} \leq f \leq 100 \text{ kHz}$, $\Delta f = 62.5 \text{ Hz}$ – black) and of a 0 Ω terminated amplifier SR560 ($62.5 \text{ Hz} \leq f \leq 100 \text{ kHz}$, $\Delta f = 62.5 \text{ Hz}$ – magenta) are shown.

the low noise voltage amplifier SR560 input, i.e. limits measurement bandwidth below the maximum bandwidth (100 kHz) of the SR760. The relevant capacitance $C_{\text{parasitic}}$ consists of the capacitance of the sample C_{sample} , the coax cables C_{coax} and of the amplifier input C_{amp} (cf. Fig. 3.1):

$$C_{\text{parasitic}} = C_{\text{sample}} + C_{\text{coax}} + C_{\text{amp}}. \quad (4.1)$$

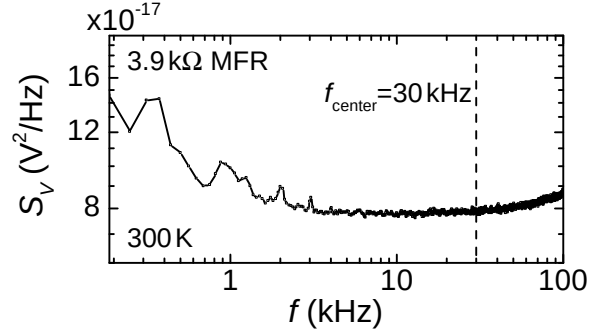
Depending on the type of amplifier the input capacitance can be given by the differential input capacitance, the relative input capacitance or a combination of the two [35]. In the case of a SR560 $C_{\text{amp}} \approx 25 \text{ pF}$ [61]. The resulting RC bandwidth for a 100 kΩ resistor is:

$$f_{\text{3dB}} = \frac{1}{2\pi RC} \approx 64 \text{ kHz}. \quad (4.2)$$

For characterizing the setup, a 100 kΩ metal film resistor (MFR)¹ is mounted on the chip carrier system and the voltage power spectrum is detected by the SR760. The spectrum analyzer SR760 always returns 400 voltage PSD values and its highest resolvable frequency is $f_{\text{max}} = 100 \text{ kHz}$, as discussed in Ch. 3.2.4. So it is necessary to acquire multiple S_V -spectra in order to have a resolution $\Delta f < 250 \text{ Hz}$. For the red spectrum in Fig. 4.1 with $1.95 \text{ Hz} < f < 781.95 \text{ Hz}$ two spectra with span $B = 390 \text{ Hz}$ and start frequency $f_{\text{start}} = 1.95 \text{ Hz}$ and 391.95 Hz were recorded one after the other. For a spectrum from DC to 100 kHz with 62.5 Hz resolution four spectra with span $B = 25 \text{ kHz}$ are connected, as in Fig. 4.1 black and magenta). A resolution of $\Delta f = 975 \text{ mHz}$ is used to resolve 50 Hz line frequencies and $\Delta f = 62.5 \text{ Hz}$ is

¹A metal film resistor is used, because it possesses a very low noise floor compared to differently fabricated resistors, i.e. it only exhibits Nyquist noise [62].

Figure 4.2: Spectrum of a 3.9 k Ω metal film resistor taken with resolution $\Delta f = 62.5$ Hz and $n_{AV} = 15 \times 10^3$ averages. The center frequency of below measurements is highlighted by the dashed line.



the resolution used for ADRN measurements below. 50 Hz peaks are visible to the 13th order² and they are suppressed to 10^{-13} V²/Hz or lower. In Fig. 4.1 the noise power of the 100 k Ω MFR drops by 3 dB (indicated by the dashed black line) at 64 kHz. This shows according to Eq. (4.2), that wiring keeps the parasitic capacitance at its unavoidable minimum – the input capacitance of the SR560. Furthermore, the SR560’s input voltage noise was detected by shorting the circuit at the chip carrier system³. The complete background of the setup is depicted in magenta in Fig. 4.1. Note that the spectrum is not perfectly white and the noise power rises for frequencies $\gtrsim 50$ kHz, which is investigated in Appendix B. The measured average over the bandwidth $15 \leq f \leq 35$ kHz is $S_{SR560} = 1.52 \times 10^{-17}$ V²/Hz, consistent with the data sheet < 1.6 V²/Hz [61]. This noise floor is equivalent to a 1 k Ω -resistor at room temperature (300 K).

We conclude: As the amplifier’s input voltage noise S_{SR560} always adds to the signal, the sample resistance is required to be $R_{\text{sample}} > 1$ k Ω , in order to provide a voltage PSD signal which is larger than S_{SR560} . Furthermore, in order to prevent RC bandwidth limitation, the sample resistance should be less than $R_{\text{sample}} < 100$ k Ω . This requirement is well met by Hall bar mesa structures patterned from YIG|Pt ($R(t_{Pt} = 2.7$ nm) ≈ 2 k Ω , $R(t_{Pt} = 2.2$ nm) ≈ 4 k Ω). Additionally, 50 Hz peaks in the white noise power spectrum and inexplicable behavior of the SR560 (cf. Appendix B) should be avoided and hence, measurements should be performed in the bandwidth $5 \text{ kHz} \leq f \leq 50 \text{ kHz}$.

4.1.2 Experimental Limit to S/N Ratio

Now we want to study how accurately we can detect the voltage PSD of a resistor, using the S/N ratio as a figure of merit. The S/N ratio was introduced for variate X with samples x_i ($i = 1, \dots, n$), expectation value $\langle X \rangle$, and variance $\text{var}(X)$ in Eq. (2.43)

$$\frac{S}{N} = \frac{\langle X \rangle}{\sqrt{\text{var}(X)}} \approx \frac{\bar{x}}{s_X} \quad (4.3)$$

²This is not an issue, as there is sufficient bandwidth above 650 Hz. These 50 Hz peaks also lead to the sharp increase in spectra with $\Delta f = 62.5$ Hz, because the noise power is collected in few bins.

³Shorting means an input of thermal noise of a 0 Ω resistor, i.e. vanishing Nyquist voltage noise.

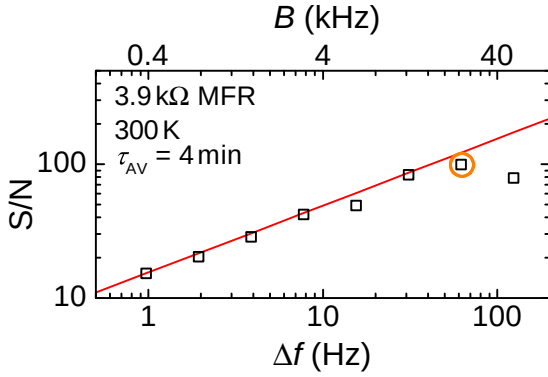


Figure 4.3: Signal to noise ratio (S/N) of S_V as a function of line width Δf for constant $\tau_{AV} = 4$ min and Blackman-Harris window function for fast Fourier transform computation. The $S/N \propto \sqrt{\Delta f}$ behavior is indicated by the red line. The orange circle points out the settings used for angle-dependent resistive noise measurements in Ch. 4.3.

and we estimate $\langle X \rangle / \sqrt{\text{var}(X)}$ by the sample mean $\bar{x}$ and the sample standard deviation s_X for a large number of samples [63]. They are calculated as

$$\mu = \frac{1}{n} \sum_{i=1}^n x_i \quad (4.4a)$$

$$s_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}. \quad (4.4b)$$

The S/N ratio thus obtained corresponds to the accuracy with which a white noise power such as the SMN can be detected (Ch. 2.5) and hence the S/N ratio will be used to select the best settings of the SR760 for an ADRN measurement.

We choose a 3.9 kΩ MFR in order not to be affected by RC bandwidth limitation. The corresponding noise spectrum recorded with line width $\Delta f = 62.5$ Hz resolution, exhibits the expected noise power of

$$S_{\text{SR560}} + S_{V,N}(3.9 \text{ k}\Omega, 300 \text{ K}) \approx 8.0 \times 10^{-17} \text{ V}^2/\text{Hz},$$

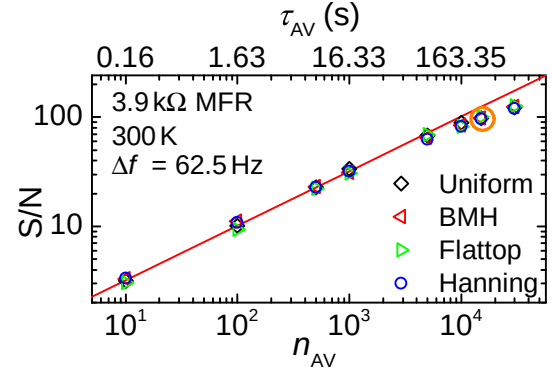
see Fig. 4.2. The voltage PSD spectrum is not perfectly white because of the SR560's behavior above 50 kHz (Appendix B) and the 50 Hz and higher orders of the line frequency (Fig. 4.1).

The S/N ratio will now be calculated from PSD spectra measured with a 3.9 kΩ MFR with different settings of the SR760 using Eq. (4.4b), to determine the optimal settings for high S/N ratio at little measurement time. All spectra taken for S/N calculations in the framework of this study are centered at 30 kHz. For $5 \text{ kHz} < f < 50 \text{ kHz}$ the spectrum (Fig. 4.2) is white and every PSD value can be interpreted as a sample $S_V(f_i)$ of the same variate $X = S_V$. Furthermore one can show that two Fourier coefficients are uncorrelated when a stationary periodic random process is investigated⁴ [37]. So the S/N ratio for one set of settings can be determined from one spectrum using $S_V(f_i)$ ($i = 1, \dots, 400$)⁵ samples applying Eq. (4.4b) and (4.3).

⁴A stationary random process with sample functions $X(t)$ is said to be periodic if the sample function is periodic.

⁵Note that every spectrum returned by the SR760 consists of 400 frequency bins as discussed in Ch. 3.2.4.

Figure 4.4: Signal to noise ratio (S/N) of S_V over the number of averages n_{AV} with constant resolution $\Delta f = 62.5$ Hz. Measurements are displayed for all four window functions supported by the FFT analyzer (Uniform, Blackman-Harris (BMH), Flattop, Hanning). The equivalent averaging time τ_{AV} is quoted on the upper abscissa. $S/N \propto \sqrt{\tau_{AV}}$ is depicted by the red line. The settings used for angle-dependent resistive noise measurements are highlighted by an orange circle.



S/N can be interpreted as relative power density deviation of each frequency bin from the approximately white expectation value.

Firstly, the optimal span is determined keeping the overlap zero. Therefor, the number of averages n_{AV} is adjusted so that the averaging time is kept constant at $\tau_{AV} = n_{AV}\tau_s = 4$ min. We choose 4 min as averaging time, because it fits the magnetic field orientation sequence from Ch. 3.2.2. From Fig. 4.3 the optimal span $B = 25$ kHz is selected. Figure 4.3 also shows that the S/N ratio is proportional to $\sqrt{\Delta f}$. The proportionality constant is $\sqrt{\tau_{AV}} = \sqrt{240}$ s. The S/N ratio for resolution $\Delta f = 125$ Hz deviates from this trend, because the inspected bandwidth $B = 50$ kHz (i.e., $5 \text{ kHz} \leq f \leq 55 \text{ kHz}$) includes colored features for $f > 50$ kHz in the spectrum (Ch. 4.1.1).

Secondly, the law of large numbers is confirmed by measuring spectra with different numbers of averages n_{AV} , which is equivalent to increasing the average time τ_{AV} (Fig. 4.4). We do so for all window functions of the SR760 to determine, which of them provides the highest S/N ratio for given n_{AV} and $B = 25$ kHz. Apparently, different windows can not be told apart. For ADRN measurements we decide to use Blackman-Harris (BMH) as it has the full dynamic range of the analyzer [57].

In order to evaluate Fig. 4.3 and 4.4 simultaneously, S/N is plotted against n_{AV} in Fig. 4.5. Blackman-Harris was used as window function for acquiring these S/N data. We find that both S/N data obtained at constant frequency resolution $\Delta f = 62.5$ Hz (black open squares) and at constant averaging time $\tau_{AV} = 4$ min (magenta circles) behave according to $S/N = \sqrt{n_{AV}}$. The analysis of Fig. 4.3 was that $S/N = \sqrt{\tau_{AV}\Delta f} = \sqrt{n_{AV}}$, which confirms that result ($\tau_{AV} = n_s\tau_s$ and $\Delta f = \tau_s^{-1}$). This is in perfect agreement with the statistical limit $S/N = \sqrt{n_{AV}}$ for estimating white

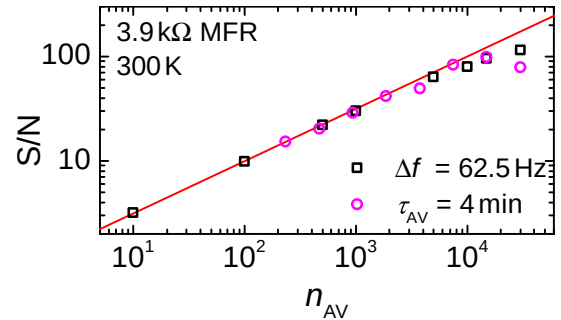


Figure 4.5: Signal to noise ratio (S/N) of S_V as function of number of averages n_{AV} for constant resolution $\Delta f = 62.5$ Hz (black open squares) and constant averaging time $\tau_{AV} = 4$ min (magenta circles). The red line depicts $S/N = \sqrt{n_{AV}}$.

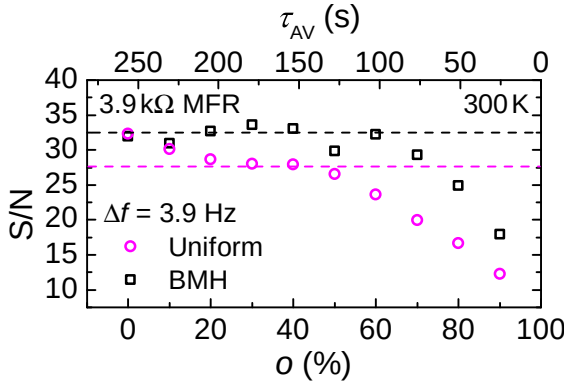


Figure 4.6: Signal to noise ratio (S/N) of S_V plotted versus overlap o for Uniform (magenta circles) and Blackman-Harris (BMH, black open squares) for $\Delta f = 3.9$ Hz and $n_{AV} = 1000$. On the top x-axis the equivalent integration τ_{AV} time is indicated.

noise by FFT analysis as derived in Ch. 2.5 and proves that the setup operates at the physical accuracy limit. Note also that besides the acquisition time τ_{AV} each measurement takes an additional 6 s for communication between computer and SR760.

Finally, we focus on the influence of overlapping time sequences for computing an average FFT spectrum. Due to processor speed, overlapping can only be applied to spans smaller than 25 kHz by the SR760. Consequently, we use a narrow span $B = 1.56$ kHz to be able to utilize overlaps up to $o = 90\%$. Figure 4.6 shows that with decreasing integration time $\tau_{AV} = o n_s \tau_s$ and constant number of averages, the S/N ratio for a uniform window decreases, whereas for BMH windowed sequences the integration time could be reduced by 60 % without influencing the accuracy. This can be understood as follows: the window function literally suppresses information at the end of each sequence, which can be reused in the FFT of the next overlapping sequence. For additional theoretical explanation, the publication by P. Welch [60] is recommended.

Summarizing, the FFT analyzer is set to PSD mode, Volt RMS units, $B = 25$ kHz, BMH window, number of averages $n_{AV} = 15 \times 10^3$ and start frequency $f_{start} = 10$ kHz or 20 kHz for best S/N ratio performance in shortest possible acquisition time of ADRN measurements⁶. As measuring one spectrum takes 4.1 min with these settings, they also fit the magnetic field rotation sequence from Ch. 3.2.2. For resolving a signal change with relative magnitude 10^{-8} a S/N ratio in the order 10^{8+1} is required. Unfortunately, no overlapping is applicable in this bandwidth by the SR760, so in order to improve S/N from 10^2 (for a single bin) to 10^4 for detecting relative SMN magnitude 10^{-3} , the 400 frequency bins of each spectrum plus spectra of following rotations will be averaged. According to the law of large numbers about 25 rotations of the magnetic field are required to result in $S/N = 10^2 \sqrt{400 \times 25} = 10^4$. Consequently, an ADRN measurement with magnetic field orientation angle $0^\circ \leq \alpha \leq 360^\circ$ and 22.5° resolution takes about 29 h including magnetic field rotations and data communication.

⁶The start frequency is adjusted to minimize the number of maxima in the voltage PSD spectrum which are caused by sources of stray radiation.

4.2 Angle-Dependent Magnetoresistance

For observing the spin Hall magnetoresistance in a YIG|Pt sample⁷, the longitudinal and transverse resistance (R_{long} and R_{trans}) are to be recorded as a function of the magnetization orientation. Since the angle between current direction and magnetization shall be changed, we call the measurement angle-dependent. The measurement geometry is depicted in Fig. 4.7(a). We utilized the magnetic field of a 2D-VM field to rotate the magnetization orientation of the YIG film around its surface normal $\mathbf{n}$ ⁸ (cf. Fig. 4.7). The direction of the magnetic field is indicated by the unit vector $\mathbf{h}$, its magnitude is $\mu_0 H$. The longitudinal and transverse resistance could not be measured simultaneously like in [23], because only two electrical connections lead into Al box #1. Instead, we measure R_{long} and R_{trans} sequentially. For R_{long} the voltage drop V_{long} across the longitudinal contacts is measured while applying a current $I_q = 40.5 \mu\text{A}$ along the $\mathbf{j}$ axis as depicted in Fig. 4.7(a). As described in Ch. 3.2.2, both source meter and nanovoltmeter are connected to the BNC sockets of the small Al box (cf. Fig. 3.1). For the transverse measurement, a 9 V block battery combined with a $220 \text{ k}\Omega$ series resistance is utilized for applying the same amount of longitudinal current⁹ (cf. Fig. 4.7(a)). The transverse voltage V_{trans} was again detected by a nanovoltmeter connected to the BNC sockets. The longitudinal and transverse resistance measurements can be analyzed simultaneously, because the same current was applied along $\mathbf{j}$. The measurement results are displayed in Fig. 4.7(b).

For the ADMR experiments (red solid triangles) depicted in Fig. 4.7(b), we observe a maximum in R_{long} for $\mathbf{h}$ parallel ($\alpha = 0^\circ$) and antiparallel ($\alpha = 180^\circ$) to the direction of current flow $\mathbf{j}$. Minima occur at $\mathbf{h}$ perpendicular to the direction of applied current $\mathbf{t}$ ($\alpha = 90^\circ$ and $\alpha = 270^\circ$). The data qualitatively reproduce a $\cos^2(\alpha)$ shape, which was predicted for the angle-dependence of ρ_{long} as function of magnetization orientation, hence it is safe to assume that the magnetization of the YIG layer is always oriented parallel to the external magnetic field [23]. This allows to directly use α as the magnetization orientation. Thus R_{long} is expected to follow a $\cos^2(\alpha)$ dependence in accordance to Eq. (2.6), which is nicely reproduced by our ADMR data.

Now we turn our attention to R_{trans} (green diamonds in Fig. 4.7(d)). Maxima and minima occur in a 180° periodicity and are shifted by 45° relative to R_{long} . A $R_{\text{trans}} \propto \sin(2\alpha)$ behavior is predicted by the presented model, Eq. (2.12). The peak to peak amplitude (PPA) ΔR_{long} and ΔR_{trans} is 2.6Ω and 0.21Ω , respectively. So the PPA of R_{long} is

$$\Delta R_{\text{long}} = \frac{l}{w} \Delta R_{\text{trans}} = \frac{1 \text{ mm}}{80 \mu\text{m}} 0.21 \Omega = 2.6 \Omega \quad (4.5)$$

⁷All angle-dependent measurements shown in this chapter are done on the same sample with $t_{\text{Pt}} = 2.7 \text{ nm}$, if not indicated differently.

⁸The magnetic field magnitude was $\mu_0 H = 60 \text{ mT}$ for all measurements presented in Ch. 4.

⁹Battery and series resistor were used as current source because only two electrical contacts lead into Al box #1. Also, note that the sample resistance and the series resistance add to $222 \text{ k}\Omega$, i.e. $V_{\text{battery}}/R_{\text{total}} = 40.5 \mu\text{A}$.

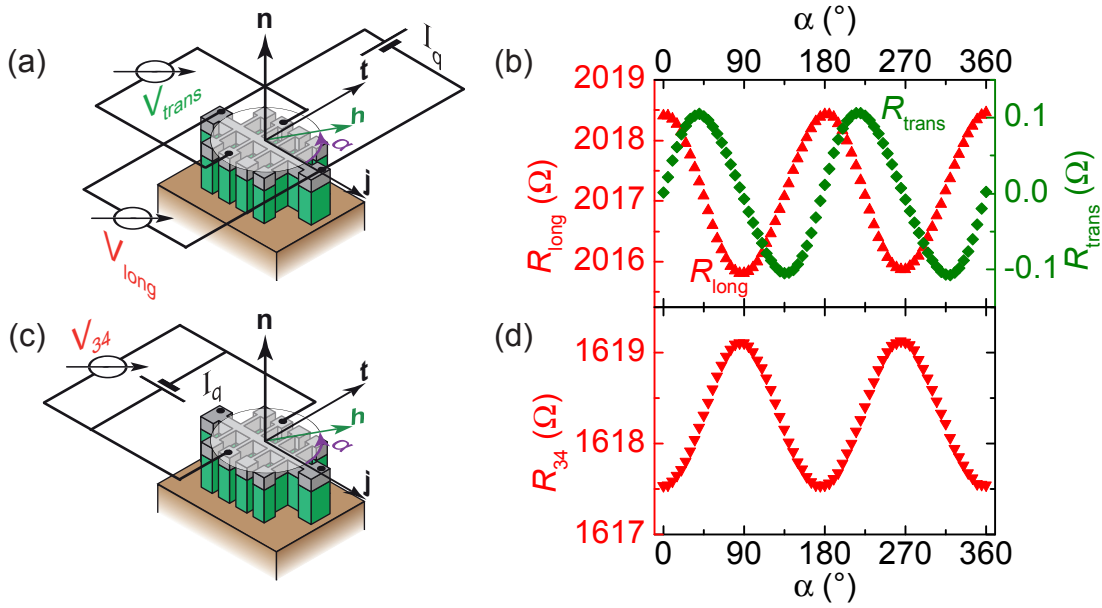


Figure 4.7: Magnetotransport measurement of YIG(60 nm)|Pt(2.7 nm) with in plane magnetization rotation. (a, c) depict the ADMR measurement scheme used for detecting resistance R_{long} , R_{trans} (current flowing along j) and R_{34} (current applied along t). (b) Evolution of R_{long} (red solid triangles) and R_{trans} (green diamonds) are plotted as a function of magnetic field orientation α with magnitude $\mu_0 H = 60$ mT. The angle $\alpha = 0$ is defined as magnetic field along j -direction. The analysis according to Eq. (2.11) and (2.12) gives a relative SMR effect of $\Delta\rho/\rho_0 = 1.3 \times 10^{-3}$ with $\rho_0 = 430.6$ n Ω m. (d) R_{34} plotted versus magnetic field orientation α with same properties as in (b). The relative SMR effect observed in R_{34} is 1.0×10^{-3} , smaller as compared to (b), because the current path through the Hall bar is not straight, when current is applied along t .

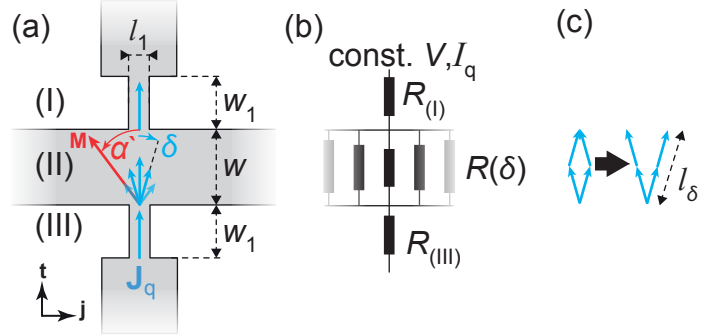
by a factor l/w larger than the PPA of R_{trans} , as suggested by theory Eq. (2.11) and (2.12). Fitting Eq. (2.6) and (2.7) to R_{long} and R_{trans} gives a constant resistivity $\rho_0 = 430.6$ n Ω m and a relative SMR magnitude $\Delta\rho/\rho_0 = 1.3 \times 10^{-3}$. Similar values were found for comparable samples by Althammer *et al.* [23].

Looking back at the study of thermal stability for magnetic field rotations in section 3.2.2, the same sample YIG(60 nm)|Pt(2.7 nm) was used and showed time dependent resistance oscillations, while the magnetization was rotated. The periodic resistance change as a function of time in Fig. 3.5 corresponds to a relative SMR effect of $\Delta\rho/\rho_0 = 1.3 \times 10^{-3}$. As the measurement started with $\alpha = 0^\circ$ at $t = 0$ s a repeated $\cos^2(\alpha)$ shape is superimposed on the resistance increase due to sample heating.

To have a full description of the dissipative effects of the sample, we also investigate the transverse resistance¹⁰ R_{34} which is measured by applying a current across the sample in t direction. The setup is sketched in Fig. 4.7(c). The measured resistance plotted in Fig. 4.7(d) follows a $\cos^2(\alpha - 90^\circ)$ shape. A resistance maximum

¹⁰The transverse resistance is named $R_{34} = V_{34}/I_q$, where both V_{34} and I_q are measured/applied over contacts 3-4 to clearly distinguish it from R_{trans} . $R_{trans} = V_{trans}/I_q$ is an entirely different quantity, where I_q is applied along j direction and V_{trans} is the voltage measured along t .

Figure 4.8: (a) Sketch of the not straight current flow along $\mathbf{t}$. The dimensions are $w = 80 \mu\text{m}$, $w_1 = 60 \mu\text{m}$, and $l_1 = 20 \mu\text{m}$, the magnetization orientation α' with respect to $\mathbf{t}$ and the angle of current spread δ in part (II). (b) In part (II) the resistance is modeled as parallel connection of resistors $R(\delta) \propto l_\delta$. (c) Concerning resistance, a bend current path is equivalent to a straight current path with same length, so $l_\delta \propto 1/\cos(\delta)$.



occurs when the magnetization is aligned with the direction of current flow ($\alpha = 90^\circ$, $\alpha = 270^\circ$) and a minimum occurs for magnetization orientation perpendicular to current flow ($\alpha = 0^\circ$, $\alpha = 180^\circ$) which agrees both with the above measurement (Fig. 4.7(a, b)) and the theory as the direction of current flow is shifted by 90° to the $\mathbf{t}$ -axis. Interestingly, the relative SMR amplitude is 1.0×10^{-3} on top of a constant $R_0 = 1617.5 \Omega$, which is less than the relative SMR amplitude observed in longitudinal direction. This can be intuitively be understood as follows. For simplification the magnetization orientation angle relative to $\mathbf{t}$ $\alpha' = \alpha - 90^\circ$ is introduced. We split the path of current flow into three regions, see Fig. 4.8(a). In (I) and (III) the current path is straight and the resistivity is given by Eq. (2.6). In part (II) the angle between direction of current flow and magnetization orientation might deviate from α' by a current spread angle δ . Consequently, the SMR effect for a current with current spread δ is:

$$\rho_{(II)}(\alpha, \delta) = \rho_0 + \Delta\rho \cos^2(\alpha' + \delta). \quad (4.6)$$

In the following an expression for the distribution function of the spread angle δ is derived. We associate every current spread angle δ with a resistance $R(\delta)$, Fig. 4.8(b). Considering current paths with triangular shape, they can be simplified to linear current paths when the second half of each path is interpreted as a linear extension of the respective path with $-\delta$ (depicted in Fig. 4.8(c)). The resistance $R(\delta)$

$$R(\delta) = \frac{\rho l_\delta}{dA} \propto l_\delta \quad (4.7)$$

of each path is proportional to the length of the current path l_δ . Here dA denotes the small area through which each current flows through. dA is considered constant for all current spread angles δ , whereas the length l_δ of each path is

$$l_\delta = \frac{w}{\cos(\delta)}. \quad (4.8)$$

The current through each path is given by $I_\delta = V_{\text{II}}/R(\delta)$ (here V_{II} is the voltage drop across part (II)). I_δ is proportional to $\cos(\delta)$, because

$$I_\delta \propto \frac{1}{R(\delta)} \propto \frac{1}{l_\delta} \propto \cos(\delta). \quad (4.9)$$

Consequently, the distribution function of the current will have a $\cos(\delta)$ shape. The current $I = B \int_{-\pi/2}^{\pi/2} \cos(\delta) d\delta$ is constant through the structure, hence we find the normalization factor of the distribution to be $B = I_q/2$. For calculating the resistivity in part (II) we integrate Eq. (4.6) weighting each angle δ with the relative amount of current, which propagates along it¹¹.

$$\rho_{\text{II}}(\alpha) = \rho_0 + \Delta\rho \int_{-\pi/2}^{\pi/2} \frac{1}{2} \cos(\delta) \cos^2(\alpha' + \delta) d\delta. \quad (4.10)$$

This integral can be solved analytically and the result is:

$$\rho_{\text{II}}(\alpha) = \rho_0 + \Delta\rho \left[\frac{1}{3} + \frac{1}{3} \cos(\alpha')^2 \right] \quad (4.11)$$

The first term in the squared brackets is an angle independent contribution, which is negligible compared to ρ_0 . In the second term we recognize the $\cos^2(\alpha')$ term, but with a reducing factor $1/3$. Parts (I) and (III) contribute to the total resistance by $2R_{\text{I}} = 2\rho_0 w_1/(t_{\text{Pt}} l_1) = 1.0 \text{ k}\Omega$. Consequently, the other $0.6 \text{ k}\Omega$ of $R_{34} \approx 1.6 \text{ k}\Omega$ comes from part (II) and the contact pads. We expect that the parts contribute to the total SMR effect according to their contribution to the total resistance, so

$$\frac{1 \times 1 \text{ k}\Omega + 0.33 \times 0.6 \text{ k}\Omega}{1.6 \text{ k}\Omega} \Delta\rho = 0.7 \Delta\rho \quad (4.12)$$

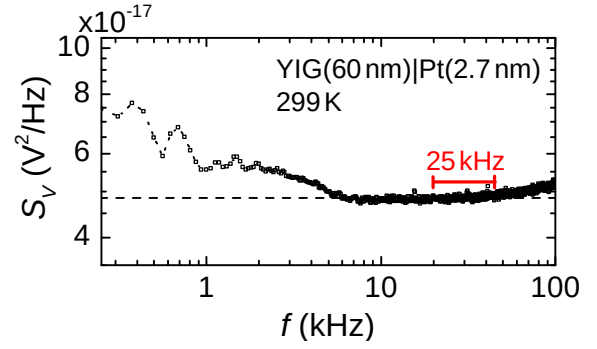
a reduction of 70 % in relative SMR is estimated from R_{long} to R_{34} . The experimental value for the relative SMR effect across 3-4 (1.0×10^{-3}) is 77 % of $\Delta\rho/\rho_0 = 1.3$ measured in the longitudinal direction. This simple model¹² is in good agreement with the measurement, so we interpret the decreased SMR signal in the transverse geometry as a consequence of the not straight current flow in the Hall bar structure in part (II).

In conclusion, the ADMR measurements on $\text{YIG}(60 \text{ nm})|\text{Pt}(t_{\text{Pt}}=2.7 \text{ nm})$ agree with the spin Hall magnetoresistance effect measured in [22, 23, 53, 64] and explained in detail by [24]. The SMR was presented as the dissipative consequence of

¹¹Correctly speaking it is necessary to average over the conductance, because the resistance is modeled as a parallel circuit. As the SMR effect is small ($\rho_0 \gg \Delta\rho$) this lengthier approach leads to the same result as the presented one and hence is not discussed.

¹²In this model we assumed that the current paths are straight lines, which is a raw assumption. For a rigorous approach one would start with deriving the two dimensional electric field in part (II) and then solve the spin diffusion equation in analogy to the SMR theory published by Nakayama *et al.* [22].

Figure 4.9: Voltage PSD spectrum with resolution $\Delta f = 62.5$ Hz for the YIG(60 nm)|Pt(2.7 nm) sample in thermal equilibrium, recorded between contacts 1-2 as indicated in Fig. 4.10(a). The magnetization was oriented along the Hall bar ($\alpha = 0^\circ$) by a magnetic field $\mu_0 H = 60$ mT. The red bar indicates the spectral range over which was averaged for the ADRN measurement in Fig. 4.10(b).



magnetization dynamics. It arises from a pure spin current created by the spin Hall effect and depends on the absorption or reflection of this spin current at the YIG|Pt interface. The dissipative SMR effect is detected in a resistivity change caused by a back-conversion of the deflected spin current via the inverse spin Hall effect. In the experiments as well as theory, the spin current is absorbed when the spin current polarization is perpendicular to the magnetization orientation, i.e. parallel or antiparallel to the charge flow, leading to a maximum in resistance. We investigated two directions of charge current flow, along the $\mathbf{j}$ -direction and $\mathbf{t}$ -direction, respectively, and both are consistent with the model.

4.3 Angle-Dependent Resistive Noise Measurements

In the previous section the dissipative SMR effect was investigated and its dependence on the magnetization orientation was discussed. Let us proceed with the investigation of the SMN effect whose theory was introduced in Ch. 2.4. The magnetization orientation will be controlled as above by the 2D-VM and is assumed to point parallel to the magnetic field orientation α . Before discussing the noise power as a function of magnetic field orientation, we take a look at the longitudinal voltage PSD spectrum of the sample across contacts 1-2 in thermal equilibrium while the magnetization points along $\alpha = 0^\circ$. The geometry is visualized in Fig. 4.10(a).

Firstly, the voltage PSD spectrum¹³ of YIG(60 nm)|Pt(2.7 nm) sample (Fig. 4.9) with bandwidth $250 \text{ Hz} \leq f \leq 100 \text{ kHz}$ and resolution $\Delta f = 62.5 \text{ kHz}$ shows the same features as discussed in Ch. 4.1.1. It is white between 5 kHz and 50 kHz. Secondly, there are maxima in the voltage PSD spectrum at 15.625 kHz and 31.312 kHz. We expect white noise from the sample and conclude that these maxima are caused by sources of stray radiation, such as laboratory equipment. Thirdly, it was found in Ch. 4.1.2 that the optimal bandwidth for best S/N ratio performance in shortest possible acquisition time is 25 kHz. Consequently, the frequency range $20 \text{ kHz} < f < 45 \text{ kHz}$ is chosen for angle-dependent resistive noise (ADRN) measurements, including only one maximum at 31.312 kHz in the otherwise white PSD spectrum. In order to cut out the maximum, the frequency bins $31.18 \text{ kHz} < f_{\text{delete}} < 31.38 \text{ kHz}$ are deleted from the spectrum consisting of 400 voltage PSD values. The mean noise

¹³The acquisition of a spectrum ($250 \text{ Hz} \leq f \leq 100 \text{ kHz}$) was described in detail in section 4.1.1.

power of the remaining $n_p = 396$ voltage PSD points corresponds to

$$S_{V,N}(2018 \Omega, 298.6 \text{ K}) + S_{\text{SR560}} = 4.848 \times 10^{-17} \text{ V}^2/\text{Hz}$$

and is indicated by the dashed line in Fig. 4.9.

Now, longitudinal voltage PSD is investigated as function of magnetic field orientation angle α . In section 4.1.2, it was found that averaging over 25 rotations of the magnetic field are required to obtain an accuracy of the PSD of $S/N \approx 10^4$. Hence, for ADRN measurements the frequency range $20 \text{ kHz} < f < 45 \text{ kHz}$ was averaged over $n_{\text{rep}} = 29$ repeated rotations of $0^\circ \leq \alpha \leq 360^\circ$ in 22.5° steps. At each field orientation a voltage PSD spectrum is acquired with the optimal setting determined in Ch. 4.1.2 for an averaging time $\tau_{\text{AV}} = 4 \text{ min}$ ($B = 25 \text{ kHz}$, $n_{\text{AV}} = 15 \times 10^3$, and $f_{\text{start}} = 20 \text{ kHz}$). The measurement at each magnetic field orientation took 246 s, because of the time needed to set the field and to communicate with the FFT spectrum analyzer. Furthermore, this sequence of magnetic field orientations provides thermal stability for the sample according to Ch. 3.2.2. After data acquisition, the frequency bins $31.18 \text{ kHz} < f_{\text{delete}} < 31.38 \text{ kHz}$ were cut from every spectrum the same way as described above¹⁴.

The averaging performed in order to obtain an angle-dependent voltage PSD signal with an accuracy better than 10^{-3} might appear tedious, but is essential to the success of the measurement. Each acquired voltage PSD value $\tilde{S}_{V,i}(f, \alpha)$ was measured at the i^{th} rotation ($i = 1, \dots, n_{\text{rep}}$) at frequency f (there are n_p frequencies left after deleting maxima in the white noise spectrum, which are caused by stray radiation emitters) and at magnetization orientation angle α . The frequency and angle-dependent voltage PSD values $\tilde{S}_{V,i}(f, \alpha)$ are averaged over all voltage PSD spectra which were taken at the same magnetization orientation α . Additionally, the voltage PSD is reduced by the amplifier's input voltage noise $S_{\text{SR560}} = 1.52 \times 10^{-17} \text{ V}^2/\text{Hz}$. The angle-dependent voltage PSD $S_V(\alpha)$ is obtained:

$$S_V(\alpha) = \frac{1}{n_{\text{rep}} n_p} \sum_{i=1}^{n_{\text{rep}}} \sum_f \tilde{S}_{V,i}(f, \alpha) - S_{\text{SR560}}. \quad (4.13)$$

Here n_{rep} is the number of magnetization rotations, n_p is the number of voltage PSD values in each spectrum after deleting maxima from the white noise spectrum ($n_p \leq 400$), and $\sum_f$ sums over n_p frequencies. The uncertainty of the angle-dependent voltage PSD is given by the standard error of the mean [37]

$$\sigma_{(S_V)} = \frac{\sigma_{(\tilde{S}_V)}}{\sqrt{n_{\text{rep}} n_p}} \quad (4.14a)$$

$$\begin{aligned} \sigma_{(\tilde{S}_V)} &= \sqrt{\frac{1}{n_p n_{\text{rep}} - 1} \sum_{i=1}^{n_{\text{rep}}} \sum_f (S_V(\alpha) - \tilde{S}_{V,i}(f, \alpha))^2} \\ &= 4.4 \times 10^{-21} \text{ V}^2/\text{Hz} \end{aligned} \quad (4.14b)$$

¹⁴In Appendix A, the data post-processing is described step by step.

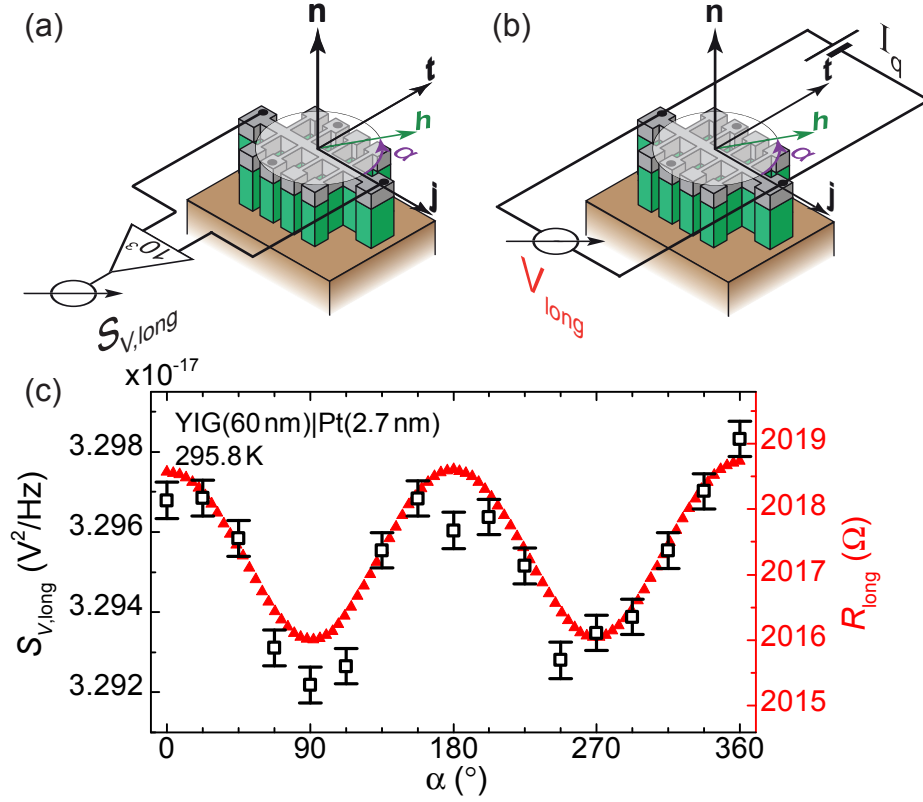


Figure 4.10: Unbiased angle-dependent resistive noise (ADRN) measurement along the $\mathbf{j}$ -direction. (a) The unbiased ADRN measurement scheme for detecting the voltage power spectral density $S_{V,\text{long}}$. (b) Setup sketch for detecting measuring resistance R_{long} . (c) Evolution of $S_{V,\text{long}}$ (open squares) and R_{long} (red solid triangles) as a function of magnetic field orientation α , with field magnitude $\mu_0 H = 60 \text{ mT}$. For $\alpha = 0$, the magnetic field is parallel to $\mathbf{j}$. The field was rotated from 0° to 360° in 22.5° steps for ADRN. The voltage PSD for each field orientation was averaged over $n_{\text{rep}} = 29$ separate measurements (29 rotations) and results in an accuracy of $\pm 4.4 \times 10^{-21} \text{ V}^2/\text{Hz}$ indicated by the error bars. The R_{long} data were acquired shortly before $S_{V,\text{long}}$ in an ADMR measurement. The $S_{V,\text{long}}$ axis and the R_{long} axis are scaled by $S_{V,\text{long}} = 4k_B(295.8 \text{ K})R_{\text{long}}$. An analysis according to Eqs. (2.26, 2.11) gives a relative spin Hall magnetoresistive noise and spin Hall magnetoresistance of $\Delta S_V/S_{V,0} = (1.4 \pm 1) \times 10^{-3}$ and $\Delta R/R_0 = 1.3 \times 10^{-3}$ with $S_{V,0} = 3.2928 \times 10^{-17} \text{ V}^2/\text{Hz}$, and $R_0 = 2.0160 \text{ k}\Omega$. Apparently, the angle-dependent fluctuation change corresponds to the angle-dependent dissipation change.

where $\sigma_{(\tilde{S}_V)}$ is the sample standard deviation defined in Eq. (4.4b). Equation (4.14a) is simply the law of large numbers. For the longitudinal ADRN measurement, each of the $n_{\text{rep}} = 29$ spectra consists of $n_p = 396$ voltage PSD values, and hence 11484 values were averages for one $S_{V,\text{long}}(\alpha)$ value. The result of the averaged voltage PSD $S_{V,\text{long}}(\alpha)$ is plotted Fig. 4.10(c) as a function of magnetization orientation α . A maximum occurs when the contact direction is aligned with the magnetization ($\alpha = 0^\circ$ and 180°) and a minimum is observed for a 90° angle between contact direction and magnetization. For the spin Hall magnetoresistive noise (SMN), we expect a $\cos^2(\alpha)$ shape (cf. Eq. (2.26)) which is reproduced by the data. The constant resistive noise $S_{V,0} = 3.2928 \times 10^{-17} \text{ V}^2/\text{Hz}$ and the relative SMN amplitude $\Delta S_V/S_{V,0} = (1.4 \pm 0.1) \times 10^{-3}$ were obtained by fitting Eq. (2.26) to $S_{V,\text{long}}$. Evidently, the detection of the SMN was enabled by the excellent thermal stability.

For a quantitative comparison between ADRN and ADMR measurements, the longitudinal resistance R_{long} was measured shortly before $S_{V,\text{long}}$. The current was applied by a Keithley K2400 sourcemeter and a Keithley K2182A nanovoltmeter was used for voltage detection. This is necessary because Pt resistance is sensitive to thermal changes. R_{long} is plotted into the same graph as $S_{V,\text{long}}$. The voltage PSD scale relates to the Nyquist noise power of the resistance on the right y -axis by $S_{V,\text{long}} = 4k_B T R_{\text{long}}$ at $T = 295.8 \text{ K}$ indicating a direct correspondence between the noise and resistive measurement. Note, however, that this temperature was determined from matching noise power to measured resistance. The average temperatures measured by the two Hall probes of the 2D-VM over the 35 h long experiment are $T_{\text{Hall probe,j}} = 295.8 \text{ K}$ and $T_{\text{Hall probe,t}} = 296.5 \text{ K}$, respectively. The sample temperature determined from $S_{V,\text{long}}$ and R_{long} thus agrees with the measured magnet temperature.

In Fig. 4.10 not only maxima and minima of the noise power fit maxima and minima of the angle-dependent resistance, respectively, but also the amplitude of the SMN and the SMR are the same within the measuring accuracy. The angle-dependent behavior of the fluctuation described by the SMN model corresponds in shape and magnitude to the angle-dependent behavior of the dissipation explained by the SMR model not only in theory but also in experiment. When the crossproduct of spin current fluctuation polarization and fluctuation direction points in the direction of the contacts, the voltage noise which arises from inverse spin Hall effect is hence detected in the resistive noise power of the sample. So when the magnetization orientation and contact direction are parallel ($\mathbf{j} \parallel \mathbf{h}$) the spin current fluctuations can be detected in charge voltage fluctuations.

To corroborate, that SMN and SMR behave qualitatively and quantitatively the same way, we now study the voltage PSD along the transverse contacts (see Fig. 4.12(a)). Following the procedure for ADRN measurement instructions, the voltage PSD spectrum was measured (Fig. 4.11) and the result agrees with the thermal noise of 1618Ω resistor at $T = 298 \text{ K}$. The temperature difference between the longitudinal and the transverse ADRN measurement arises from a change in laboratory temperature. A change in laboratory temperature is confirmed by measuring the magnet temperature with the Hall probes, which indicate an increase by 3 K . The sum of amplifier background $S_{\text{SR560}} = 1.52 \text{ V}^2/\text{Hz}$ and of thermal noise is

Figure 4.11: Voltage PSD spectrum with resolution $\Delta f = 62.5$ Hz from the transverse contacts as indicated in Fig. 4.12(a). A magnetic field $\mu_0 H = 60$ mT was utilized to align the magnetization along the Hall bar ($\alpha = 0^\circ$). The ADRN measurement in Fig. 4.12(b) was averaged over the spectral range $20 \text{ kHz} < f < 35 \text{ kHz}$ illustrated by the red bar.

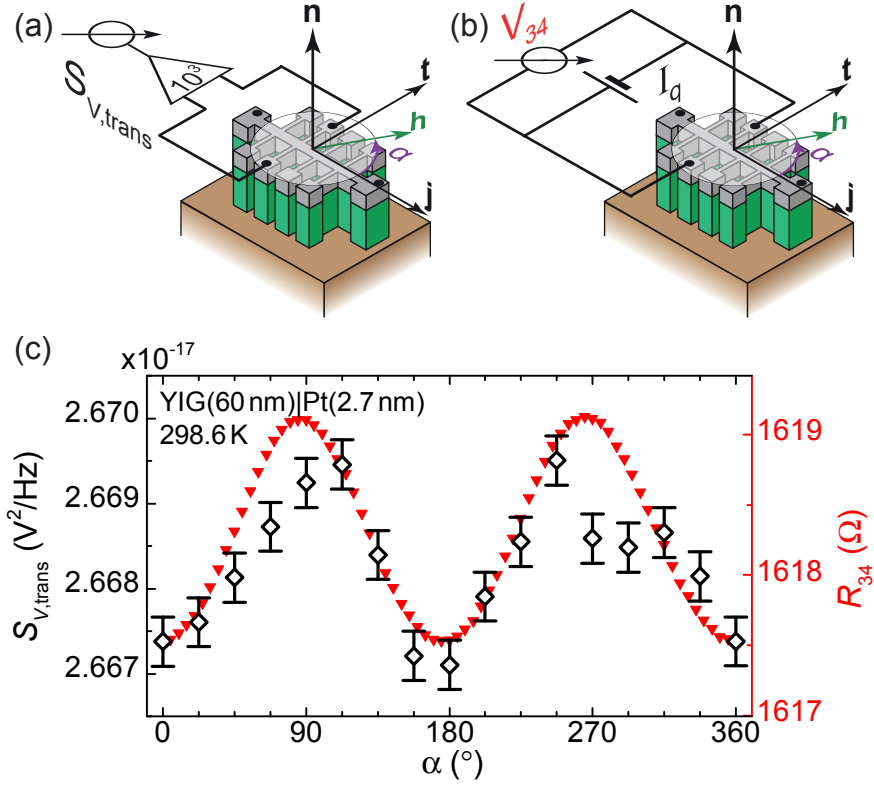
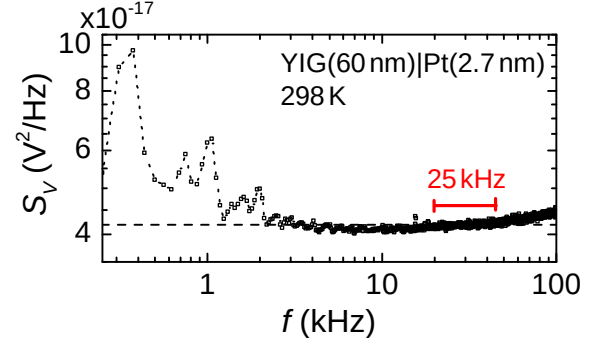


Figure 4.12: Unbiased resistive noise measurement along $\mathbf{t}$ -axis. (a) depicts the scheme used for unbiased ADRN measurement, determining the transverse power spectral density $S_{V,\text{trans}}$. (b) The evolution of $S_{V,\text{trans}}$ (black open diamonds) and R_{34} (red solid triangles) is plotted as a function of field orientation α while applying an external magnetic field $\mu_0 H = 60$ mT. The field was rotated in 17 steps from 0° to 360° for ADRN. An accuracy of $\pm 2.9 \times 10^{-21} \text{ V}^2/\text{Hz}$ is achieved by $n_{\text{rep}} = 54$ field rotations. $S_{V,\text{trans}}$ was measured shortly after R_{34} . The resistive noise power converts into resistance at $T = 298.6$ K. SMN and SMR are determined from fits to Eqs. (2.26, 2.6) where the magnetization orientation $\alpha \mapsto \alpha - 90^\circ$. Results are $\Delta S_V/S_{V,0} = (7 \pm 1) \times 10^{-4}$, $\Delta R/R_0 = 1.0 \times 10^{-3}$ with $S_{V,0} = 2.6674 \times 10^{-17} \text{ V}^2/\text{Hz}$, $R_0 = 1.6175 \text{ k}\Omega$.

indicated by the dashed line. Furthermore, the spectrum exhibits peaks caused by 50 Hz radiation from the grid up to 3 kHz and rises for frequencies $f > 50$ kHz due to the amplifier (cf. Appendix B).

Now that the contact direction is rotated by 90° the setup is sensitive to the voltage fluctuations in $\mathbf{t}$ -direction. Hence we expect maxima in the thermal noise power at $\alpha = 90^\circ$ and $\alpha = 270^\circ$ and minima at $\alpha = 0^\circ$ and $\alpha = 180^\circ$. This time $n_{\text{rep}} = 54$ rotations of the magnetic field were carried out in the same manner as above and frequencies (31.3115 ± 0.125) kHz and (41.0625 ± 0.125) kHz were cut out. For this ADRN measurement the accuracy is calculated from sample standard deviation $\sigma_{(\tilde{S}_V)} = 4.2 \times 10^{-19} \text{ V}^2/\text{Hz}$ by

$$\sigma_{(S_V)} = \frac{1}{\sqrt{389 \times 59}} \sigma_{(\tilde{S}_V)} = 2.9 \times 10^{-21} \text{ V}^2/\text{Hz}. \quad (4.15)$$

The open black diamonds represent $S_{V,\text{trans}}$ in Fig. 4.12(b) and show minima for $\alpha = 0^\circ$ and $\alpha = 180^\circ$, in agreement with theory. A maximum at 90° is prominent, whereas the maximum at 270° is distorted. The resistance measured over 3-4 contacts (Fig. 4.7(d)) is again plotted into the same graph with a scale, that converts into the thermal noise scale at $T = 298.6$ K. The derived sample temperature well fits the mean magnet temperature detected by the Hall probes ($T_{\text{Hall probe,j}} = 298.2$ K and $T_{\text{Hall probe,t}} = 298.4$ K). The fluctuations follow the angle-dependence of the dissipation in the range of thrice the standard deviation $\sigma_{(S_V)}$. The deviation around 270° is attributed to statistics. Due to this deviation the relative SMN effect $\Delta S_V/S_{V,0} = 7 \times 10^{-4}$ also differs from the relative SMR effect $\Delta R/R_0 = 1.0 \times 10^{-3}$ by three standard deviations. In comparison to the longitudinal angle-dependent voltage PSD $S_{V,\text{long}}$ we observe a smaller SMN effect. As an explanation we draw from the argumentation from section 4.2 for the resistance R_{34} and argue with the FDT that it is also valid for noise.

A second Hall bar mesa structure was fabricated from YIG(60 nm)|Pt($t_{\text{Pt}} = 2.2$) as a reference sample. It was investigated for SMN along the Hall bar structure. The longitudinal voltage PSD $S_{V,\text{long}}$ was measured (cf. Fig. 4.13(a)). The sequence of field orientations was changed to cover 360° with a 15° resolution. The sequence consisted of $n_{\text{rep}} = 19$ rotations. The SR760 acquired spectra with bandwidth 25 kHz and start frequency $f_{\text{start}} = 10$ kHz. In data post-processing, two maxima in the power spectrum at $15.6763 \text{ kHz} \pm 125 \text{ Hz}$ and at $31.3115 \text{ kHz} \pm 160 \text{ Hz}$ were removed. The setup for detecting R_{long} is illustrated in Fig. 4.13(b). Figure 4.13(c) displays $S_{V,\text{long}}$ (black open squares) and the angle-dependent resistance R_{long} (red solid triangles). The resistance scale converts to the voltage PSD scale by means of thermal noise at $T = 291.5$ K. This temperature might underestimate the true sample temperature, because the Hall probes detected 296.0 K. The deviation might arise from an overestimation of the amplifier's input voltage noise. Nevertheless, the $\cos^2(\alpha)$ shape of the SMN is reproduced by the ADRN data and the amplitude of the angle-dependent change in $S_{V,\text{long}}$ agrees with the amplitude of the SMR effect in angle-dependent resistance ($\Delta S_V/S_{V,0} = 1.1 \times 10^{-3}$, $\Delta R/R_0 = 1.0 \times 10^{-3}$). The ADRN measurement of YIG(60 nm)|Pt($t_{\text{Pt}} = 2.2$) comes to the same result as the

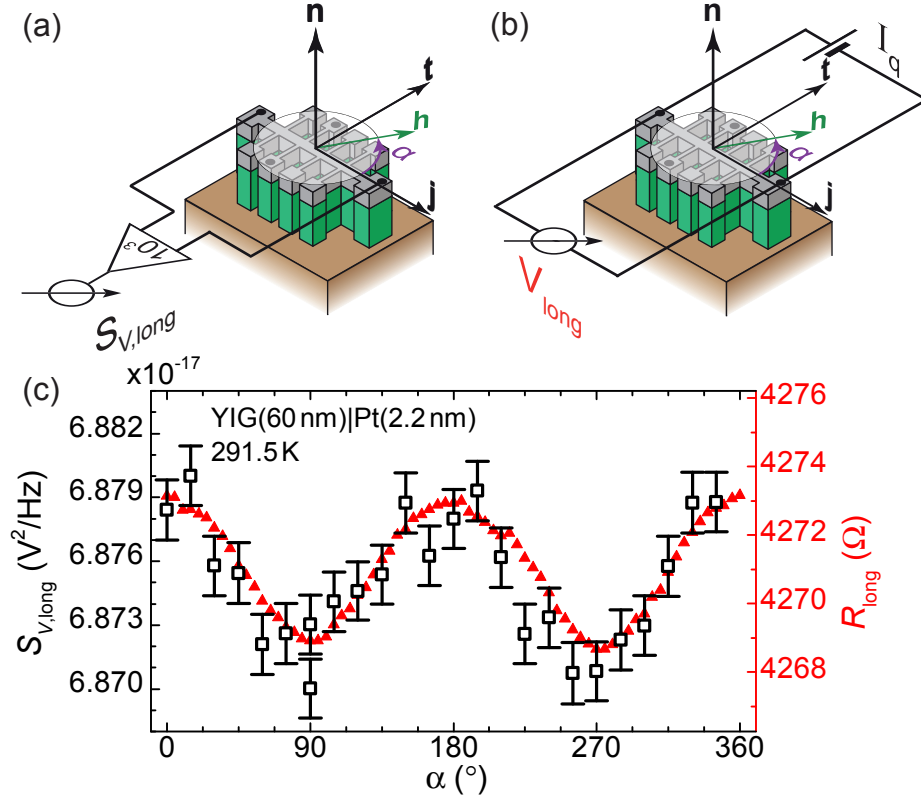


Figure 4.13: Unbiased longitudinal ADRN measurement of YIG(60 nm)|Pt($t_{\text{Pt}} = 2.2$ nm). (a) shows the detection scheme for the longitudinal voltage PSD, $S_{V,\text{long}}$. The longitudinal resistance R_{long} is measured according to sketch (b). (c) $S_{V,\text{long}}$ (black open squares) and R_{long} (red solid triangles) are plotted as a function of magnetic field orientation α . When $\mathbf{h}$ points along $\mathbf{j}$, α is 0° . The magnetization was rotated $n_{\text{rep}} = 19$ times from 0° to 360° in 15° steps. Voltage PSD scale corresponds to Nyquist noise of resistance scale at $T = 291.5$ K. Fitting Eqs. (2.26, 2.6) to data gives $\Delta S_V/S_{V,0} = 1.1 \times 10^{-3}$, $S_{V,0} = 6.8716 \times 10^{-17} \text{ V}^2/\text{Hz}$ and $\Delta R/R_0 = 1.0 \times 10^{-3}$, $R_0 = 4.2690 \text{ k}\Omega$.

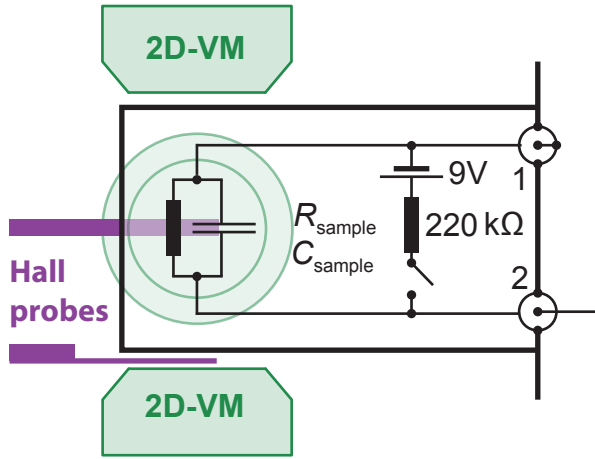


Figure 4.14: Schematic circuit diagram for biased angle-dependent resistive noise measurements. The sample is biased by a 9 V block battery with an additional 220 kΩ series resistor to generate a charge current across the sample.

first sample ($t_{\text{Pt}} = 2.7 \text{ nm}$), that fluctuation dissipation theorem holds.

Note that the changed sequence of magnetic field rotations still provides the thermal stability required for ADRN measurements.

In this section the experimental results of unbiased ADRN measurements were presented for the longitudinal and the transverse contact geometry in two different samples. The spin Hall magnetoresistive noise model is nicely reproduced, as noise power maxima were observed whenever the magnetization direction pointed along the direction of contacting, allowing the detection of thermal spin current fluctuations via the inverse spin Hall effect. The amplitude of the SMN effect is consistent with the SMR effect studied on the same sample. Our results thus are quantitatively consistent with fluctuation dissipation theorem.

4.4 Biased ADRN

So far we have investigated thermally excited spin current fluctuations. Now we are aiming for the noise of a driven spin current. To do so we generate a spin current by means of the inverse spin Hall effect. Therefore, we apply a charge current to the sample and measure the voltage power spectrum.

In a first step towards, the measurement of the voltage noise connected with a spin current, we study a 3.9 kΩ metal film resistor to check on the setup properties for biased samples. A conventional 9 V-block battery in series¹⁵ with a metal film resistor $R_{\text{serial}} = 220 \text{ k}\Omega$ is used as a low noise current source (sketch see Fig. 4.14), following the rule of thumb that R_{series} should exceed $20 \times R_{\text{sample}}$ [35]. Battery and series resistor are placed inside Al box #1 to minimize pickup noise and are connected to the chip carrier system. The actual voltage of the block battery was detected by a Fluke multimeter before each measurement and was used to calculate the current $I_q = V_{\text{battery}}/R_{\text{total}}$. The voltage PSD spectrum is measured for an unbiased ($I_q = 0 \text{ A}$) and a biased ($I_q = 41 \mu\text{A}$) metal film resistor $R = 3.9 \text{ k}\Omega$. For biased

¹⁵To prevent applying high voltages to the sample, the series resistor is placed in front of the positive terminal.

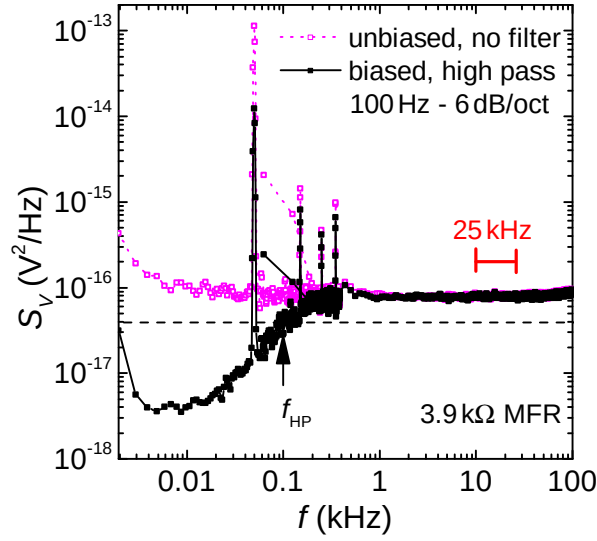


Figure 4.15: Voltage PSD spectra of a $3.9\text{ k}\Omega$ metal film resistor without bias (magenta open squares, dotted line) and with bias $V_{\text{bias}} = 0.16\text{ V}$ (black solid squares, solid line). A high pass filter at 100 Hz with slope 6 dB/oct was inserted before the amplifier for the biased measurement. The 3 dB frequency (f_{HP}) is indicated. The mean voltage PSDs over 25 kHz with $f_{\text{start}} = 10\text{ kHz}$ are $(7.92 \pm 0.02) \times 10^{-17}\text{ V}^2/\text{Hz}$ and $(7.80 \pm 0.02) \times 10^{-17}\text{ V}^2/\text{Hz}$, respectively.

measurements a high pass filter with 6 dB/oct roll off and 3 dB -frequency at 100 Hz is inserted before the amplifier to prevent the SR760 from overloading¹⁶. In Fig. 4.15, the unbiased S_V spectrum is indicated by magenta open squares and dashed lines, whereas the biased S_V spectrum is indicated by black solid squares and solid lines. The analog filter attenuates the noise power signal by 3 dB (black dashed line) at 100 Hz . Furthermore we marked the frequency range $10\text{ kHz} < f < 35\text{ kHz}$, which will be used for the biased ADRN measurements, with a red bar. The mean noise powers in this range are $(7.92 \pm 0.02) \times 10^{-17}\text{ V}^2/\text{Hz}$ and $(7.80 \pm 0.02) \times 10^{-17}\text{ V}^2/\text{Hz}$ for unbiased and biased sample, respectively. After correcting for the input voltage noise power of the SR560, these values correspond to thermal noise of a resistor at $T = 297\text{ K}$ and $T = 292\text{ K}$, respectively. The measurement range at 10 kHz is 6.6 octaves away from f_{HP} and therefore its noise power is not influenced by the high pass filter. A temperature difference of 5 K between those two measurements is unlikely, because both measurements were performed within half an hour. Repeating each measurement resulted in an identical spectrum, whereas reconnecting the chip carrier system lead to noise power variations in the order of $\pm 1 \times 10^{-18}\text{ V}^2/\text{Hz}$. Sometimes the unbiased and sometimes the biased experiment lead to higher voltage PSD. It is unclear what effect is responsible for these deviations. Concluding, the voltage power spectrum of both the unbiased and biased resistor $3.9\text{ k}\Omega$ correspond to Nyquist noise. It is unclear why the noise power fluctuates when replugging the battery.

We now turn to biased voltage PSD measurements of $\text{YIG}(60\text{ nm})|\text{Pt}(t_{\text{Pt}} = 2.7\text{ nm})$. The sample is mounted to the chip carrier system as in the previous chapter. The current longitudinally driven through the sample is $I_q = 40.5\text{ }\mu\text{A}$ as shown

¹⁶The SR560 amplifier provides build in analog filters

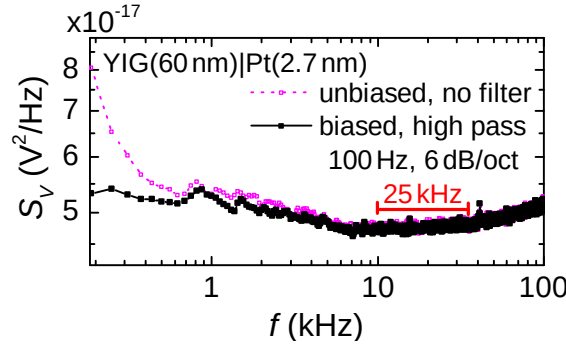


Figure 4.16: Comparison between unbiased (magenta open symbols, dotted line) and biased (black solid symbols, solid line) voltage PSD spectra with $\Delta f = 62.5$ Hz from contacts 1-2 as indicated in Fig. 4.10(a). The current $I_q = 40.5 \mu\text{A}$ is applied as depicted by Fig. 4.17(d). The magnetization points along the magnetic field in $\mathbf{j}$ -direction with field strength $\mu_0 H = 60$ mT. The ADRN results in Fig. 4.17(d) were obtained by averaging voltage PSD spectra with start frequency $f_{\text{start}} = 10$ kHz and span 25 kHz illustrated by the solid red bar.

in Fig. 4.17(d), with the magnetization pointing along $\mathbf{j}$. First, voltage PSD spectra of the unbiased and biased YIG|Pt sample were acquired ($\Delta f = 62.5$ Hz, $n_{\text{AV}} = 15 \times 10^3$). The spectra (Fig. 4.16) match above 7 kHz. The average over a 25 kHz range (indicated by the red bar) give noise powers of $4.815 \times 10^{-17} \text{ V}^2/\text{Hz}$ and $4.778 \times 10^{-17} \text{ V}^2/\text{Hz}$. This corresponds to sample temperatures of $T = 295.6$ K and 292.3 K, respectively, reconnecting lead to the same behavior as for metal film resistors. Like for the metal film resistor $3.9 \text{ k}\Omega$, it is unclear where this fluctuation stems from. From this measurement we can not conclude whether the noise of the driven spin current changed the noise characteristics.

Consequently, we do not turn the spin current induced by the SHE on and off by connecting and disconnecting the power supply. Instead, we switch the spin current across the YIG|Pt by rotating the magnetization in YIG. The different measurement geometries are depicted in Fig. 4.17. The left column refers to angle-dependent noise power measurements and the right side to angle-dependent resistance measurements. The measurement results are plotted in the center column. The magnetization orientation α and the coordinate system ($\mathbf{j}$, $\mathbf{t}$, $\mathbf{n}$) are defined in Fig. 4.17(a). Figure 4.17(c) reviews the angle-dependent magnetoresistance measurement setup, where the current is applied along $\mathbf{j}$ and voltages longitudinal V_{long} and transverse V_{trans} to the Hall bar are measured. For the biased longitudinal ADRN measurement, the voltage PSD $S_{V_{\text{long}}}^{(1)}$ of the bias voltage is detected, see Fig. 4.17(d). The bias is applied by a 9 V block battery with a series $220 \text{ k}\Omega$ metal film resistor (cf. Fig. 4.14) for geometries (d, f, g, i). In the biased transverse ADRN measurement, the voltage fluctuations across the Hall probe were recorded, while the current I_q is applied along $\mathbf{j}$. Furthermore, ADMR measurements for R_{long} and R_{34} (Fig. 4.17(f, i)) were performed shortly before ADRN measurements to be compared with the biased longitudinal and transverse ADRN measurements, respectively. The 9 V block battery plus series resistor served as a current source and the voltage was detected over the BNC connects by a nanovoltmeter. Afterwards the nanovoltmeter was replaced by

the amplifier SR560 and the FFT analyzer SR760. This way the contacts on the chip carrier system remained untouched.

So we drive a constant current of $I_q = 41.0 \mu\text{A}$ through the longitudinal contacts while rotating the magnetization orientation with the 2D-VM in the same manner as for unbiased ADRN measurements in Ch. 4.3. The differences in the settings between biased and unbiased ADRN measurements are that the preamplifier applies a high pass (at 100 Hz by 6 dB/oct) to prevent overloading the SR760 and that the start frequency is changed to $f_{\text{start}} = 10 \text{ kHz}$ for minimizing the number of distinct maxima in the white noise spectrum, which are caused by sources of stray radiation¹⁷. There are such maxima at 15.686 kHz and 31.311 kHz which needed to be deleted in the bandwidth $\pm 160 \text{ Hz}$ and $\pm 125 \text{ Hz}$, respectively. Hence 389 voltage PSD values of each spectrum remain for averaging.

Let us first comment on the biased longitudinal ADRN results in Fig. 4.17(e) and then discuss the result. Afterwards, the biased transverse ADRN results are addressed. The power spectral density $S_{V,\text{long}}^{(I)}$ plotted as solid black squares versus the magnetization orientation angle α shows a maximum for $\alpha = 0^\circ$ and $\alpha = 180^\circ$ ($\mathbf{m} \parallel \mathbf{j}$). An accuracy of $\pm 5 \times 10^{-21} \text{ V}^2/\text{Hz}$ was achieved by averaging the spectra of $n_{\text{rep}} = 19$ field rotations. A minimum of $S_{V,\text{long}}^{(I)}$ is located at $\alpha = 90^\circ$ and at $\alpha = 270^\circ$ ($\mathbf{m} \perp \mathbf{j}$). The indicated $\cos^2(\alpha)$ behavior is in good agreement with the model that the spin current is only flowing into the YIG when the crossproduct of spin current polarization and magnetization orientation is unequal to zero. Consequently, the spin current noise only enhances the electric noise via ISHE when a spin current flows, i.e. if $\mathbf{m}$ is not perpendicular to $\mathbf{j}$. The amplitude of the spin current noise is given by SMN model. For a quantitative analysis the biased ADRN data is compared to the angle-dependent resistance R_{long} (red solid triangles). Using Eq. (2.23), we estimated the sample temperature $T = 303.3 \text{ K}$. This temperature deviates from the temperatures detected by the Hall probes (295.7 K and 295.0 K). We saw in Ch. 3.2.2 that the sample heats by about 1.4 K during an ADRN measurement. So the offset in noise power is unlikely to stem from setup heating after ADMR was measured. It might have the same cause as the unexplainable fluctuations which was observed when reconnecting unbiased and biased setups. Focusing on the relative magnitude of SMN effect, we fit ADRN data as above using Eq. (2.26) and ADMR using Eq. (2.6). The result $\Delta S_V/S_{V,0} = 8 \times 10^{-4}$ with the constant term $S_{V,0} = 3.3385 \times 10^{-17} \text{ V}^2/\text{Hz}$ does not agree with the SMR effect $\Delta\rho/\rho_0 = 1.3 \times 10^{-3}$ within the measuring accuracy $\pm 1.6 \times 10^{-4}$. The voltage PSD exhibits a the same 180° periodicity as angle-dependence as the SMR. The maxima in $S_{V,\text{long}}^{(I)}$ appear broader and the minima narrower than the $\cos^2(\alpha)$ shape of R_{long} . The altered behavior is most likely due to statistics. Less likely, the fluctuations of thermal equilibrium are affected by the spin current driving the magnetization dynamics to influence the low frequency spin current fluctuations.

To further investigate the noise characteristics of S_V , also the transverse resistive noise was probed while applying a current along the longitudinal direction

¹⁷In comparison to the unbiased ADRN measurements additional instruments in other setups in the same laboratory were turned on.

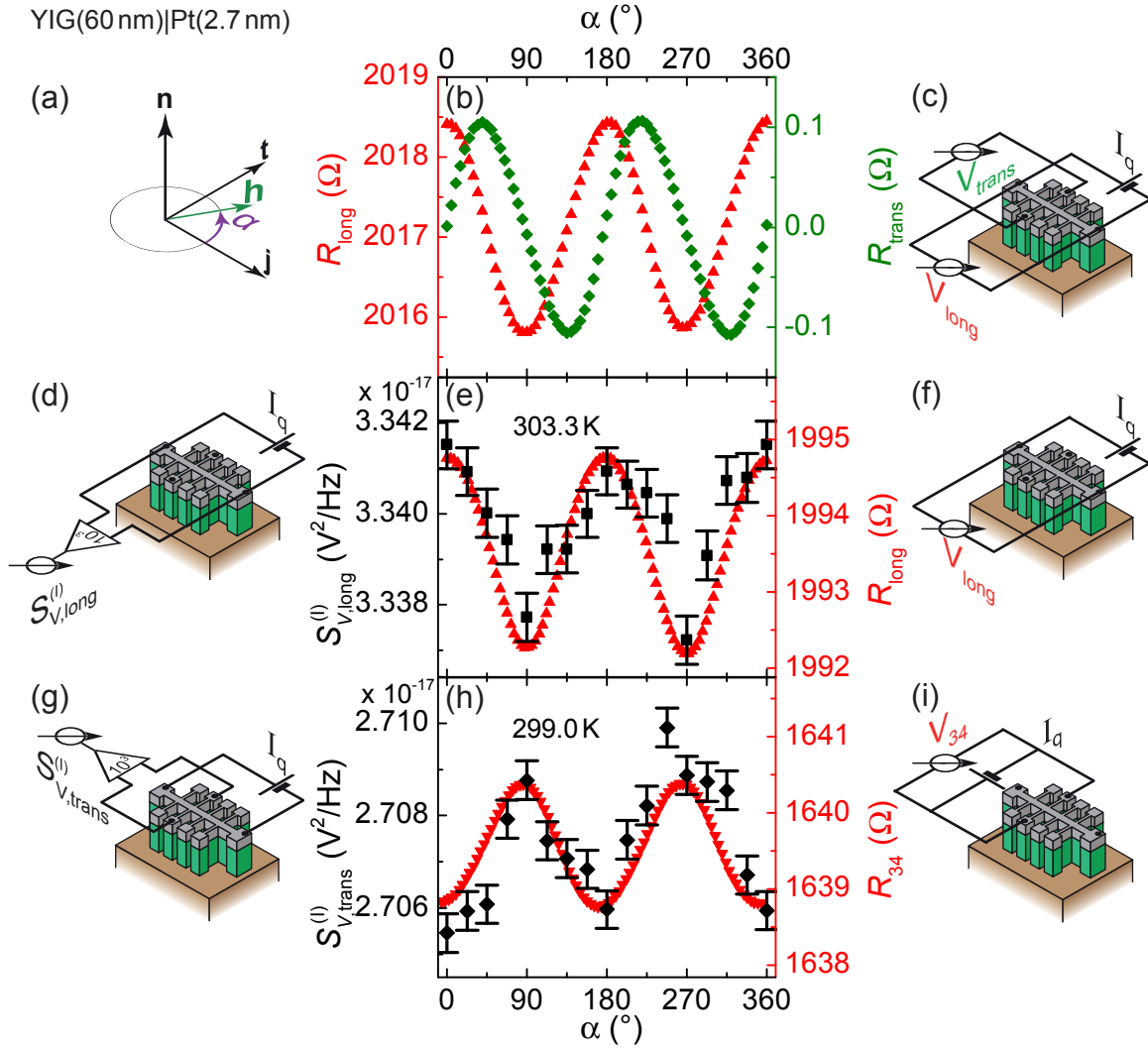


Figure 4.17: Biased angle-dependent resistive noise measurement (ADRN). (a) Coordinate system. All data is plotted as function of magnetic field orientation α at magnitude $\mu_0 H = 60$ mT. (b) Angle-dependent magnetoresistance data. Resistance R_{long} (red solid triangles) and R_{trans} (green solid diamonds) are from Fig. 4.7 and detected as sketched in (c). The longitudinal power spectral density (PSD) $S_{V,\text{long}}^{(I)}$ is measured, with a current $I_q = 41.0 \mu\text{A}$ driven along $\mathbf{j}$ as depicted in (d). $S_{V,\text{long}}^{(I)}$ (black solid squares in (e)) was recorded shortly after R_{long} (red solid triangles in (e)). R_{long} was measured as depicted in (f). (g) shows the setup for detecting biased transverse PSD $S_{V,\text{trans}}^{(I)}$, with a current $I_q = 41.0 \mu\text{A}$ applied along $\mathbf{j}$. (h) $S_{V,\text{trans}}^{(I)}$ (black solid diamonds) was measured shortly after R_{34} (red solid triangles). (i) Measurement geometry for acquiring R_{34} with current along $\mathbf{t}$. The field rotations were performed the same way as in the unbiased ADRN measurements with $n_{\text{rep}} = 19$ repetitions for $S_{V,\text{long}}^{(I)}$ and $n_{\text{rep}} = 30$ for $S_{V,\text{trans}}^{(I)}$, leading to a relative accuracy of $\pm 1.6 \times 10^{-4}$ (error bars). PSD and resistance axis are scaled by $S_V^{(I)} = 4k_B T R$ with $T = 303.3$ K for $S_{V,\text{long}}^{(I)}$ and $T = 299.0$ K for $S_{V,\text{trans}}^{(I)}$. Analysis according to Eqs. (2.26, 2.6) results in longitudinal relative SMN and SMR effects $\Delta S_V/S_{V,0} = 8 \times 10^{-4}$ and $\Delta \rho/\rho_0 = 1.3 \times 10^{-3}$ with $S_{V,0} = 3.3385 \times 10^{-17} \text{ V}^2/\text{Hz}$, $R_0 = 1.9948 \text{ k}\Omega$; for transverse setup $\Delta S_V/S_{V,0} = \Delta \rho/\rho_0 = 1.0 \times 10^{-3}$ with $S_{V,0} = 2.7084 \times 10^{-17} \text{ V}^2/\text{Hz}$, $R_0 = 1.6388 \text{ k}\Omega$.

($I_q = 41.0 \mu\text{A}$). Figure 4.17(g) depicts the connection scheme, where the current was driven through contacts 1-2 and the voltage noise signal between 3-4 was detected. We examined the voltage PSD spectrum. It exhibits the same features as its corresponding unbiased measurement, Fig. 4.16 (so this spectrum is not displayed). $n_{\text{rep}} = 30$ rotations of the magnetic field were performed to achieve an accuracy $\pm 4.2 \times 10^{-21} \text{ V}^2/\text{Hz}$. Hence it allows us to resolve a $\cos^2(\alpha - 90^\circ)$ dependence of the voltage PSD $S_{V,\text{trans}}^{(I)}$ (black solid diamonds in Fig. 4.17(e)). Note that although the number of magnetization rotations $n_{\text{rep}} = 19$ for the longitudinal measurement was increased to $n_{\text{rep}} = 30$ for the transverse measurement, the relative accuracy of the two biased ADRN measurements remained the same, because the transverse PSD signal is smaller than the longitudinal PSD signal in comparison to the amplifier's input voltage noise power S_{SR560} .

A drift in voltage PSD from $\alpha = 0^\circ$ to $\alpha = 360^\circ$ is caused by a continuous rise in magnet temperature by 0.02 K per rotation cycle over the total measurement period. This corresponds to a voltage PSD increase of $4 \times 10^{-21} \text{ V}^2/\text{Hz}$ for each cycle. The increase in sample temperature is consistent with the thermal change indicated by the Hall probes $\Delta T = 0.6 \text{ K}$ over all $n_{\text{rep}} = 30$ magnetization rotations.

Maxima at $\alpha = 90^\circ$ and at $\alpha = 180^\circ$ occur when the magnetization direction are aligned with contacts 3-4 for detecting $S_{V,\text{trans}}^{(I)}$ ($\mathbf{m} \parallel \mathbf{t}$), whereas minima arise at $\alpha = 0^\circ$ and at $\alpha = 180^\circ$ when the magnetization orientation is perpendicular to the probed contacted 3-4 ($\mathbf{m} \perp \mathbf{t}$). This behavior is neither reproduced by the longitudinal nor the transverse SMR effect (cf. Fig. 4.17(b)), but by the resistance measured over contacts 3-4 (R_{34}). So we compare the magnitude of voltage PSD change to the resistance along the transverse direction, which was recorded using a block battery ($I_q = 41.0 \mu\text{A}$) and a nanovoltmeter. The setup for detecting R_{34} is shown in Fig. 4.17(i). To compare ADRN and ADMR data, we determine the sample temperature from Eq. (2.23) with $S_{V,\text{trans}}^{(I)} = 2.7088 \times 10^{-17} \text{ V}^2/\text{Hz}$ and $R_0 = 1.6404 \text{ k}\Omega$ to be $T = 299.0 \text{ K}$. The Hall probes indicate $T = 295.1 \text{ K}$. The 4 K deviation between sample and magnet temperature are equivalent to a $3.6 \times 10^{-19} \text{ V}^2/\text{Hz}$ or 22Ω deviation in noise power or resistance, respectively. The amplifier's input noise power was inspected and it remained unchanged. It was tested with a Fluke multimeter that the contact resistance of BNC connects changes by less than 1Ω upon changing the connection from nanovoltmeter to amplifier plus SR760. So the cause of the increased noise power remains ambiguous. The resistance axis in Fig. 4.17(h) is chosen in a way, which corresponds to the respective Nyquist noise at 299.0 K . The red solid triangles depict the resistance data in Fig. 4.17(h). Repeating the analysis from above, $S_{V,\text{trans}}^{(I)}$ quantitatively reproduces the SMR effect very well ($\Delta S_V/S_{V,0} = (1.0 \pm 0.2) \times 10^{-3}$ and $\Delta \rho/\rho_0 = 1.0 \times 10^{-3}$). This result is also consistent with the unbiased ADRN measurement.

A fluctuation in the transverse SMR component with $\sin(\alpha)\cos(\alpha)$ shape (green diamonds in Fig. 4.17(b)) adds to the thermal SMN in transverse direction with $\cos^2(\alpha - 90^\circ)$ shape. In the resistance regime, the transverse SMR changes by

YIG(60 nm) Pt(2.2 nm)	$\Delta S_V/S_{V,0}$ ($\times 10^{-3}$)	$S_{V,0}$ ($\times 10^{-18} \text{ V}^2/\text{Hz}$)	$\Delta\rho/\rho_0$ ($\times 10^{-3}$)	R_0 (k Ω)
unbiased longitudinal	1.4 ± 0.1	32.9	1.3	2.02
unbiased transverse	0.7 ± 0.1	26.7	1.0	1.62
biased longitudinal	0.8 ± 0.2	33.4	1.3	1.99
biased transverse	1.0 ± 0.2	27.1	1.0	1.64

Table 4.1: Parameters $\Delta S_V/S_{V,0}$, $S_{V,0}$, $\Delta\rho/\rho_0$, and R_0 obtained from fits to experimental ADRN and ADMR data for YIG(60 nm)|Pt(2.2 nm).

$\approx \pm 0.1 \Omega$ ¹⁸, which is $\pm 6 \times 10^{-5}$ relative change on R_{34} . The transverse noise power thus would need to be measured with an accuracy of $\pm 1 \times 10^{-5}$ in order to resolve the angle-dependent change in noise power caused by the transverse SMR effect. So the presented biased ADRN measurements with a spin current introduced by ISHE can not resolve a change the noise characteristic of YIG|Pt as a function of magnetization orientation between biased and unbiased sample, because the measurement accuracy is $\pm 2 \times 10^{-4}$.

In summary, we studied the angle-dependence of the voltage noise power of a YIG|Pt sample with a spin current across the YIG|Pt interface. An applied charge current gets converted into a spin current by the SHE which flows across the YIG|Pt interface depending on the magnetization orientation of the YIG. The measured voltage noise power is influenced via the ISHE by the fluctuations of a spin current polarized perpendicular to the line connecting the measurement contacts. Thus, longitudinal and transverse biased ADRN measurements allowed us to study the fluctuations of a spin current with polarization $\mathbf{s} \perp \mathbf{j}$ and $\mathbf{s} \perp \mathbf{t}$, respectively. According to the SMN model the driven spin current exhibits the same thermal fluctuations as a spin current in thermal equilibrium fluctuating around zero, which was nicely reproduced by our measurement. The fit results are summarized in Tab. 4.1.

Longitudinal bias and longitudinal probing gave us insight into the spin current fluctuations polarized along $\mathbf{t}$. The $\cos^2(\alpha)$ shape of $S_{V,\text{long}}^{(I)}$ qualitatively agrees with the unbiased longitudinal ADRN measurement and with longitudinal ADMR measurements. The quantitative deviation of longitudinal biased and probed ADRN measurement likely stems from statistics. The longitudinal bias and transverse probing ADRN measurement provided information on the fluctuations of a spin current with polarization in the longitudinal direction ($\mathbf{j}$). The observed $S_{V,\text{trans}}^{(I)}$ showed quantitatively the same angle-dependent behavior ($\cos^2(\alpha - 90^\circ)$) as the unbiased transverse ADRN and ADMR measurements (R_{34}). The experimental accuracy ($\pm 2 \times 10^{-4}$) does not allow to resolve the angle-dependent change in noise power caused by the transverse SMR effect, which is expected to be in the order 6×10^{-5} .

The frequency range $f < 50 \text{ kHz}$ of the voltage PSD across a biased YIG|Pt Hall bar mesa structure was investigated in different geometries. The measurement

¹⁸Note that it is ambiguous whether a negative transverse resistance causes a decrease in noise power, or whether $|R_{\text{trans}}|$ is relevant for noise power and a $\sin^2(\alpha) \cos^2(\alpha)$ like behavior is added to the SMN.

results are consistent with fluctuation dissipation theorem, with the SMN model describing the fluctuation effect and the SMR model explaining the dissipative effect.

Chapter 5

Summary and Outlook

In this thesis, magnetoresistive noise in ferromagnetic insulator|normal metal (FMI|NM) heterostructures was investigated theoretically and experimentally. To this end, a low noise setup for detecting voltage power spectral density (PSD) was developed, manufactured and characterized. This chapter summarizes the work done during this thesis and its main results, and gives an outlook on further studies which are based on the present results.

5.1 Summary

In Chapter 2, the spin Hall magnetoresistive noise (SMN) was derived from the spin Hall magnetoresistance (SMR) applying the fluctuation dissipation theorem. The spin Hall magnetoresistive noise describes thermal noise in a ferromagnetic insulator|normal metal heterostructure as a function of magnetization orientation. It was predicted that SMN has the same quantitative dependence on magnetization orientation as SMR. Afterwards, the signal to noise (S/N) ratio was derived for the case of white noise detected by a FFT spectrum analyzer. The signal (S) is the expected voltage power spectral density (PSD) of the white noise and the noise (N) refers to the fluctuation of the measured PSD from the expected value. The S/N ratio is given by the number of averaged spectra which are calculated from separate time traces, $S/N = \sqrt{n_{AV}}$.

Chapter 3 describes the sample growth of the YIG|Pt heterostructure, which was done by S. Meyer, and the structuring of Hall bar mesa structures. The sample was characterized for its thermal and electrical properties at room temperature. The sample is ohmic and has a linear resistance-temperature characteristic, for current $0 \mu A \leq I_q \leq 50 \mu A$ and temperature $286 K < T < 303 K$ respectively. The temperature characteristic was used to show the thermal stability of the setup. Then, the FFT spectrum analyzer SR760 and other instruments for low noise detection were explained.

Chapter 4 deals with the experimental study of the SMN. It was demonstrated that the setup reaches the physical limit to S/N ratio for detecting white voltage noise power derived in Ch. 2. The necessary integration time was calculated for resolving a 10^{-3} relative SMN signal. Angle-dependent magnetoresistance measurements (ADMR) were performed and analyzed to quantify the SMR effect in the YIG(60 nm)|Pt(2.7 nm) sample used in this thesis. A SMR effect of $\Delta\rho/\rho =$

1.3×10^{-3} was observed, which is in agreement to literature [23]. The relative SMR $\Delta\rho/\rho$ of the resistance detected transversely to the Hall bar was 23 % smaller than the relative SMR observed in the longitudinally detected resistance, which could be understood by analyzing the bent current path. For noise characterization of the YIG(60 nm)|Pt(2.7 nm) sample, the voltage PSD was recorded in longitudinal and transverse direction and analyzed. The detected noise powers correspond to thermal noise of the sample. The angle-dependent resistive noise (ADRN) is detected by averaging spectra with a bandwidth of 25 kHz and a start frequency of 10 kHz. A SMN effect of same angle-dependence and relative amplitude as SMR is observed. In addition, the longitudinal ADRN measurement of a reference sample YIG(60 nm)|Pt(2.2 nm) was presented. The change in noise power corresponds to the resistance change as a function of magnetization orientation.

Furthermore, a charge current bias was applied along the longitudinal direction of the Hall bar structure YIG(60 nm)|Pt(2.7 nm) and the voltage PSD was recorded in longitudinal and transverse configuration. Given the measurement accuracy of $\pm 2 \times 10^{-4}$, the biased ADRN measurements yield a SMN effect with an amplitude consistent with unbiased measurements. The contribution of the transverse SMR (6×10^{-5}) could not be resolved.

We conclude that the fluctuation dissipation theorem holds and both experimentally and theoretically the spin Hall magnetoresistive noise and the spin Hall magnetoresistance effect have the same quantitative behavior for frequencies below 50 kHz. Thus, the first and important step to characterize magnetoresistance by voltage noise in FMI|NM heterostructures was done in this thesis.

5.2 Outlook

Based on the presented results and the work of two Bachelor theses, two approaches for future experiments are suggested for investigating the influence of magnetoresistance on noise in FMI|NM heterostructures. First, a wide-band measurement up to MHz shall be addressed which enables high precision ADRN measurements. Second, a narrow-band noise detection technique is introduced for measuring noise power in the GHz regime. This section gives an outlook on future projects, together with some preliminary calculations.

Detecting SMN up to MHz

We now pick up the results of A. Schade¹, who implemented a FFT spectrum analyzer using an analog digital converter card, a computer, and a graphics processing unit [66]. We discuss how this FFT spectrum analyzer can be used to increase the S/N ratio of angle-dependent resistive noise measurements so that the influence of transverse SMR effect on the noise power can be resolved.

The FFT spectrum analyzer provides a bandwidth up to 100 MHz. Its S/N ratio is limited by the physical limitation from Ch. 2.5. According to Appendix B, the

¹Schade was supervised by the author of this thesis.

S/N ratio of an angle-dependent magnetoresistance measurement as performed in Ch. 4 can be written as follows:

$$S/N = \sqrt{\frac{B_{\text{meas}}\tau_{\text{meas}}}{1 + \tilde{\tau}_{\text{blind}}}}. \quad (5.1)$$

The noise power in the bandwidth B_{meas} is averaged for an ADRN measurement which takes the time τ_{meas} . $\tilde{\tau}_{\text{blind}}$ is the blind time². A. Schade provides a look up table for the blind time of his FFT implementation [66] (cf. Table B.1).

Now numbers shall be used for Eq. 5.1 to calculate the S/N ratio for a measurement time of 1.5 h per magnetization orientation of an ADRN measurement³. The measurement bandwidth B_{meas} for YIG/Pt samples as used in this thesis is constricted by the RC-limitation of the sample resistance and the input capacitance of the used voltage amplifier. A state of the art low noise voltage amplifier, for example, the Femto voltage amplifier model HVA-10M-60-B, allows for a measurement bandwidth of $B_{\text{meas}} \approx 2.2$ MHz. The sampling rate of the FFT spectrum analyzer would need to be set to 50 MHz in order to avoid aliasing effect and thus the blind time is 1.88. Thus one expects a

$$S/N = 6.4 \times 10^4 \quad (5.2)$$

This S/N ratio is equivalent to an measurement accuracy of $\pm(S/N)^{-1} = 1.5 \times 10^{-5}$. The transverse SMR effect is expected to induce a relative voltage noise power change of 6×10^{-5} as shown in Ch. 4.4. Hence, noise power measurements with a bandwidth B_{meas} in the MHz regime should be able to resolve the transverse influence of SMR in the noise power as function of magnetic field orientation.

Samples with lower resistance and amplifiers with lower input impedance are desirable for extending the bandwidth of this noise detection technique. Future noise measurements in the megahertz regime might enable the quantitative study of the magnetoresistive noise of FMI|NM heterostructures with a S/N ratio which is one order of magnitude higher than presented in this thesis with the same measurement time.

Noise Detected Magnetization Dynamics at Gigahertz

It would be desirable to measure noise power at gigahertz frequencies, because the dynamics behind the spin current flow across the FMI|NM interface might become relevant for the SMR effect. We suggest to use the impedance matching technique applied by K. Baumgärtl [68] to measure noise power at gigahertz frequencies. In Appendix C, the impedance matching, high-frequency noise theory and a first step to measuring noise in a narrow bandwidth around 5 GHz are presented.

²The blind time is defined as $\tilde{\tau}_{\text{blind}} = \tau/\tau_s - 1$, with the time needed for completing the measurement of one spectrum (τ) and the time required for acquiring voltage data ($\tau_s = n_s/f_s$).

³An extended discussion is written in Appendix B.

We are confident that the results presented here contribute to both the theoretical and experimental understanding of the spin Hall magnetoresistive noise and spin Hall magnetoresistance effect. Yet they also showed that many more experiments will be necessary to create a full understanding of the spin Hall magnetoresistive noise at frequencies smaller than gigahertz and at microwave frequencies. We therefore hope to encourage further investigations of magnetoresistive noise in ferromagnetic insulator|normal metal heterostructures.

Appendix A

Data Post-Processing

Here, we give an additional insight into the data post-processing for ADRN analysis. We begin with a voltage PSD spectrum recorded by the FFT spectrum analyzer SR760 and end with the plot of angle-dependent resistive noise as displayed in the thesis (cf. Fig. 4.10).

The first step in data processing is performed by the LabView program controlling the instruments. After the SR760 returned the PSD spectrum in units $V/\sqrt{\text{Hz}}$ the spectrum is divided by the amplification of the SR560. Furthermore, each frequency bin is squared to obtain PSD spectra in units V^2/Hz , and stored in TDMS format.

One of 510 PSD spectra acquired during the unbiased longitudinal ADRN measurement is plotted as solid black squares in Fig. A.1(a). A Mathematica program is used for the data post-processing. First, data is loaded from the TDMS-file. Then the subroutine ADRNprocessing is called. It has four input parameters (#1 full data from TDMS file, #2 filename, #3 a list of frequencies $f_{\text{filter},i}$ and bandwidths $B_{\text{filter},i}$, $i = 1, \dots, s$, in Hz^1 , #4 a string for additional information in the output filename). ADRNprocessing creates a list w_j , $j = 1, \dots, 400$ to cut or keep each voltage PSD $S_V(f_j)$, with entries $w_j \in \{0, 1\}$. The frequency bins which shall be deleted are

¹The list is structured as follows: $\{\{f_{\text{filter},1}, B_{\text{filter},1}\}, \dots, \{f_{\text{filter},s}, B_{\text{filter},s}\}\}$.

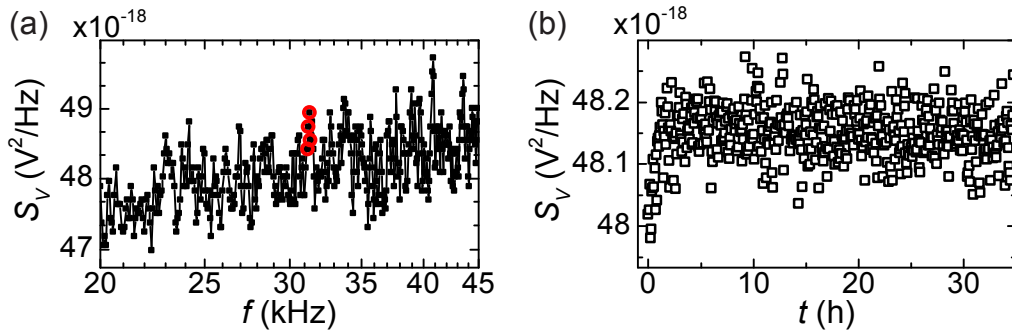


Figure A.1: (a) PSD spectrum from unbiased longitudinal ADRN measurement. The voltage PSD values in red circles are deleted by the data post-processing subroutine ADRNprocessing. (b) Time evolution of the voltage PSD meas averaged by ADRNprocessing.

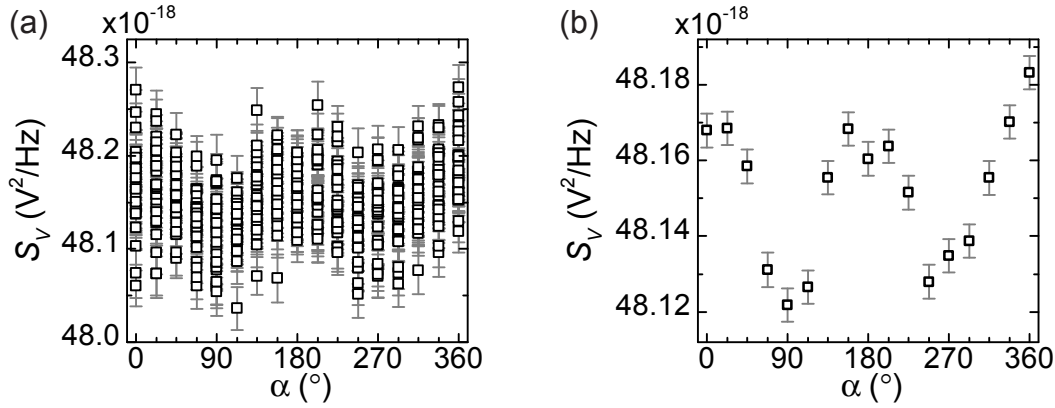


Figure A.2: (a) PSD meas as a function of field orientation. (b) Angle-dependent PSD without subtracting SR560 background.

given by the input list $\sharp 3$:

$$w_j = \begin{cases} 0 & \text{if } nf_{\text{filter},1} - B_{\text{filter},1} \leq f_j \leq nf_{\text{filter},1} + B_{\text{filter},1}, \\ & \vdots \\ & nf_{\text{filter},s} - B_{\text{filter},s} \leq f_j \leq nf_{\text{filter},s} + B_{\text{filter},s}, \quad n \in \mathbb{N} \\ 1 & \text{else} \end{cases} \quad (\text{A.1})$$

Each voltage PSD $S_V(f_j)$ is multiplied by the entry w_j , respectively, effectively deleting $S_V(f_j)$ when $w_j = 0$. For the presented example, there is $f_{\text{filter}} = 31.312$ kHz and $B_{\text{filter}} = 125$ Hz. In Fig. A.1(a) the filtered frequency bins are indicated by red circles. Afterwards the mean of the remaining $n_p = \sum_{j=1}^{400} w_j$ PSD values is determined (here $n_p = 396$). The output of ADRNprocessing is a .dat-file with a table listing all settings of each spectrum, the average PSD, and the sample standard deviation from the mean. For convenience this file is called file1.

From the time evolution of these mean PSD we can extract the temperature trend of the sample, Fig. A.1(b). Usually the first magnetization rotation is not taken into account for further analysis because of a substantial drift in temperature. A copy of file1 is created and the mean voltage PSDs for the first rotation are deleted from the table (the file is marked as “_manip”). At this point the angle-dependence of the voltage PSD can already be seen in Fig. A.2(a), where mean voltage PSD is plotted as function of field orientation α from file1_manip. There are 29 averages for each field orientation. The accuracy of the mean PSD is indicated by the error bars and is calculated from $\sigma_{\text{PSD}}/\sqrt{n_p}$. The PSD means are distributed according to a Gaussian with standard deviation indicated by the error bars $\pm\sigma_{\text{PSD}}/\sqrt{n_p}$.

A second Mathematica subroutine – called ADRNcirculating – has the filename of file1_manip, the number of repeated rotation n_{rep} , the number of selected α values, and an offset correction angle (used for defining $\alpha = 0$) as input. After reading data from file1_manip, ADRNcirculating averages all PSD means linearly and all sample standard deviations quadratically which were measured at the same magnetic field orientation. Additionally, the accuracy of the angle-dependent PSD is calculated by $\sigma_{\text{PSD}}/\sqrt{n_p n_{\text{rep}}}$. The experimental settings, PSD value and its accuracy

are returned in an other file. The resulting angle-depending PSD values are displayed in Fig. A.2(b) as function of field orientation. To obtain Fig. 4.10 the amplifier's input noise power was subtracted.

Appendix B

FFT Spectrum Analyzer Implemented on a Computer

In this Appendix the implementation of the fast Fourier transform (FFT) on a computer is addressed. It is done to present a measurement device for detecting voltage power spectral density (PSD) up to the MHz range. With the larger accessible bandwidth the Stanford Research voltage amplifier SR560 is characterized up to 5 MHz. Afterwards, the signal to noise ratio accomplished by the Stanford Research FFT spectrum analyzer SR760 is compared to the computer FFT implementation when using the SR560. It is shown that the SR560 limits the setup used in this thesis. Then, we discuss a state of the art low noise voltage in order to improve on the SR560.

The SR760 is a FFT-based spectrum analyzer designed by Stanford Research. It consists of an analog digital (AD) converter front end and a digital signal processor that performs frequency analysis on digitalized data. By the availability of a high frequency AD converter PCIe hardware and the advent of graphics processing unit (GPU) computing, a computer can be readily turned into a FFT spectrum analyzer. In the course of a bachelor thesis, A. Schade implemented such a spectrum analyzer in LabView making use of OpenCL [66]. In the setup a AD converter PCIe-card GaGe Razor fabricated by GaGe Applied Inc. is used in combination with a Nvidia GTX 680 graphics card. Sampling rates are provided up to 200 MHz and traces up to length 10^9 samples can be recorded. This improves upon the maximum bandwidth of the SR760 by three orders of magnitude. It supports a digitalization depth of 16 bit at ± 100 mV input range. Its input impedance can be adjusted to $1\text{ M}\Omega$ to be equivalent to the SR760, which is important so that the same input voltage is recorded by the AD converter card and by the SR760.

Essentially, the voltage PSD S_V is calculated from the voltage time trace recorded during the time span τ_s by the GaGe according to Eq. (2.33). The discrete Fourier transform is calculated by a FFT algorithm on the GPU. For a detailed discussion on the programming we refer to A. Schade's thesis [66]. The voltage PSD spectrum will reach from DC to half the sampling frequency.

$$0 \leq f \leq \frac{f_s}{2} \quad (\text{B.1})$$

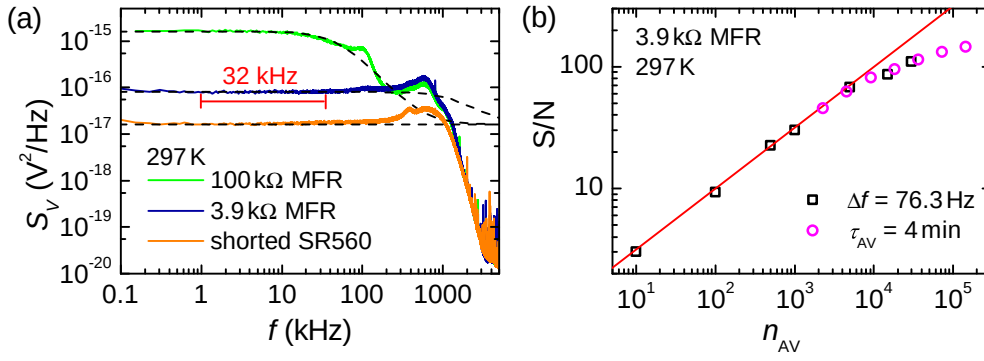


Figure B.1: (a) Voltage power spectral density (PSD) as function of frequency for a 100 kΩ metal film resistor (MFR, green), a 3.9 kΩ MFR (blue), and a $< 0.4 \Omega$ termination at the SR560 input (orange). 10^3 spectra were averaged to obtain each shown PSD spectrum. The dashed lines indicate the expected Nyquist noise powers. (b) Signal to noise ratio (S/N) is displayed over the number of averaged spectra n_{AV} . The 3.9 kΩ MFR was investigated within the bandwidth $1 \text{ kHz} \leq f \leq 33 \text{ kHz}$ (indicated in (a) by the red bar) in order to calculate S/N once for constant frequency resolution $\Delta f = 76.3 \text{ Hz}$ (black squares) and once for constant averaging time $\tau_{AV} = 4 \text{ min}$ (magenta circles). The theoretical limit to S/N ($S/N = \sqrt{n_{AV}}$) is inserted as a red line.

Here s denotes the sampling rate of the AD converter card. The frequency resolution depends on the number of samples n_s acquired for the time trace.

$$\Delta f = \frac{f_s}{n_s} \quad (\text{B.2})$$

The computer FFT implementation replaces the SR760 in the setup, which was used for angle-dependent resistance noise (ADRN) measurements (Ch. 3). The setup characterization from section 4.1.2 is repeated with a 100 kΩ and a 3.9 kΩ metal film resistor (MFR). Additionally, the SR560 was terminated with a resistor $R_{\text{short}} < 0.4 \Omega$ to investigate the amplifier's input noise.

The expected noise power is given by the thermal noise of the sample and the amplifier's input noise $S_{\text{SR560}} = 1.6 \times 10^{-17} \text{ V}^2/\text{Hz}$

$$\begin{aligned} S_V(f) &= 4k_B T R |G(f)|^2 + S_{\text{SR560}} \\ |G(f)|^2 &= \frac{1}{1 + (f/f_{3\text{dB}})^2} \\ f_{3\text{dB}} &= \frac{1}{2\pi RC}. \end{aligned} \quad (\text{B.3})$$

The frequency response of the sample and amplifier is given by $|G(f)|$ [49]. The capacity C of the measurement setup is 25 pF (cf. Ch. 4.1.1). Figure B.1(a) shows the measured PSD spectra for $R = 100 \text{ k}\Omega$, $3.9 \text{ k}\Omega$, and $< 0.4 \Omega$ in green, blue, and orange, respectively. The expected noise power at 297 K is plotted into Fig. B.1(a) with black dashed lines according Eq. (B.3). All three spectra behave according to Eq. (B.3) below 50 kHz. The detected noise power does not roll off according to RC

bandwidth limitation for frequencies bigger than 50 kHz. The S_V detected for $f > 50$ kHz is higher than the expected PSD from Eq. (B.3) in all three measurements. The behavior of the measured spectra can not be compensated by subtracting the amplifier's input noise. The deviation of the spectra are thus not due to a colored amplifier's input noise. The behavior of the amplifier can not be explained by a frequency dependent amplification, because the noise power spectra of 3.9 k Ω and 100 k Ω cross at 200 kHz and at 1.1 MHz. Consequently, this SR560¹ does not meet its specifications [61]. Hence the reliable bandwidth of SR560 is from DC to 50 kHz. The behavior for higher frequencies remains ambiguous. Note that the tapering for $f > 50$ kHz in spectra acquired by the SR760 (Fig. 4.1, 4.2, 4.9, 4.11, and 4.16) are caused by the SR560 and are no features of the used samples. For frequencies bigger than 50 kHz the behavior of the amplifier can be explained neither by a colored amplifier's input noise power nor by a frequency depending amplification.

Following the signal to noise (S/N) characterizing of the SR760 (Ch. 4.1.2), the 3.9 k Ω MFR is investigated in the bandwidth $1 \text{ kHz} \leq f \leq 33 \text{ kHz}$ and the S/N ratio was determined according to Eq. (4.3) and (4.4b). Therefor, a sampling rate $f_s = 10 \text{ MHz}$ is used. First, the number of samples per time trace n_s was kept constant $n_s = 2^{17}$ so that the frequency resolution was fixed at 76.3 Hz and the number of averages was varied (black squares in Fig. B.1(b)). For a second S/N characterization, a constant average time $\tau_{AV} = 4 \text{ min}$ was kept by adjusting $\tau_{AV} = n_{AV}n_s/f_s$ and sweeping the frequency resolution Δf . The result is plotted as function of the number of averages in magenta circles in Fig. B.1(b). The theoretically predicted behavior of the S/N ratio, $S/N = \sqrt{n_{AV}}$, is plotted in Fig. B.1(b) as a red line (cf. Ch. 2.5). The S/N ratio well fits the theoretical limit to accuracy for $n_{AV} < 10^4$. The deviation of detected S/N ratio from the theoretical curve might originate from colored features arising in the investigate bandwidth for $f < 5 \text{ kHz}$.

Now the performance of SR760 and computer FFT implementation is addressed. For ADRN measurements presented in this thesis the voltage power spectral density was determined by averaging spectra over a frequency bandwidth (Appendix A). First, the SR760 acquired n_{AV} spectra and averaged each frequency bin in RMS mode. Then, a number of frequency bins, which were influenced by sources of stray radiation, were deleted and the remaining n_p frequency bins were averaged. Last, all voltage PSD values which were acquired at the same magnetic field orientation (n_{rep} rotations of the magnetic field are performed) are averaged. Hence the signal to noise ratio achieved is given by Eq. (2.29) and the law of large numbers:

$$S/N = \sqrt{n_{AV}n_pn_{rep}} \quad (\text{B.4})$$

The measurement is limited by the bandwidth in which the noise power is white and over which thus can be averaged. Furthermore the measurement time τ_{meas} limits the S/N ratio. Due to the performance of the SR560, the measurement bandwidth B_{meas} is $5 \text{ kHz} < f < 50 \text{ kHz}$ (Ch. 4.1.1), i.e. $B_{meas} = 45 \text{ kHz}$.

¹An additional SR560 was tested and showed the same characteristics.

	f_s (MHz)							
	200	100	50	25	10	5	2	1
$\log_2(n_s)$ 24	7.53	3.78	1.88	0.949	0.382	0.191	0.0761	0.0377
23	7.92	3.96	1.96	0.986	0.403	0.203	0.0801	0.0394
22	8.23	4.07	2.05	0.989	0.414	0.208	0.0830	0.0411
21	7.12	3.56	1.76	0.882	0.352	0.179	0.0707	0.0352
20	7.92	3.93	1.93	0.945	0.386	0.195	0.0780	0.0380
19	8.92	4.01	2.01	0.984	0.397	0.199	0.0794	0.0392
18	8.41	4.18	2.33	1.10	0.449	0.222	0.0911	0.0445
17	8.65	4.39	2.24	1.11	0.438	0.220	0.0888	0.0448
16	10.2	4.92	2.76	1.21	0.634	0.259	0.107	0.0535
15	12.0	6.12	3.03	1.54	0.620	0.323	0.137	0.0697
14	17.8	9.03	4.33	2.19	0.864	0.508	0.217	0.104
13	28.2	14.0	6.95	3.44	1.44	0.674	0.291	0.172
12	47.6	23.5	11.3	5.80	2.39	1.22	0.480	0.249
11	82.9	42.4	21.2	10.2	4.20	2.09	0.946	0.455
10	162	80.9	40.1	19.5	7.91	3.99	1.68	0.850
9	306	154	75.1	38.3	15.3	7.90	3.29	1.63
8	602	302	150	74.5	29.6	15.3	6.21	3.21

Table B.1: The blind time $\tilde{\tau}_{\text{blind}}$ of the FFT implementation is listed as a function of the number of samples per time trace n_s and sampling rate f_s . $\tilde{\tau}_{\text{blind}} < 1$ is marked green, $1 < \tilde{\tau}_{\text{blind}} < 10$ is yellow, and $\tilde{\tau}_{\text{blind}} > 10$ is red. From [66].

The computer FFT implementation to the SR760 Eq. (B.4) shall be expressed in terms of

$$B_{\text{meas}} = n_p \Delta f = n_p \frac{f_s}{n_s} \quad (\text{B.5})$$

$$\tau_{\text{meas}} = n_{\text{AV}} n_{\text{rep}} \tau_s (1 + \tilde{\tau}_{\text{blind}}) = n_{\text{AV}} n_{\text{rep}} \frac{n_s}{f_s} (1 + \tilde{\tau}_{\text{blind}}) \quad (\text{B.6})$$

Here $\tilde{\tau}_{\text{blind}}$ is the blind time, defined as $\tau_{\text{blind}} = \tau/\tau_s - 1$, with the time needed for realizing the measurement of a spectrum (τ) and the time required for acquiring voltage data (τ_s). The SR760 is a real time FFT spectrum analyzer, i.e. its $\tilde{\tau}_{\text{blind}} = 0$. The performance of the computer FFT implementation can be read from the Table B.1 [66]. Equations (B.5, B.6) are inserted into Eq. (B.4) to obtain

$$S/N = \sqrt{\frac{B_{\text{meas}} \tau_{\text{meas}}}{1 + \tilde{\tau}_{\text{blind}}}} \quad (\text{B.7})$$

The S/N ratio achieved by the SR760 is determined by the optimal bandwidth $B_{\text{meas}} = 25 \text{ kHz}$ (Ch. 4.1.2) and is $S/N = \sqrt{(25 \text{ kHz}) \tau_{\text{meas}}}$. The sampling rate f_s of the GaGe has to be set to 10 MHz in order to prevent aliasing and the minimum blind time for $f_s = 10 \text{ MHz}$ is $\tilde{\tau}_{\text{blind}} = 0.382$ for $n_s = 2^{24}$ samples. The performance of the SR560 yields $B_{\text{meas}} = 45 \text{ kHz}$. Thus, $S/N = \sqrt{(32 \text{ kHz}) \tau_{\text{meas}}}$ gives only a small

improvement of 13 % to the S/N ratio achievable by the setup with SR760.

The S/N ratio can be increased when a state of the art low noise amplifier is used, for example, the Femto voltage amplifier model HVA-10M-60-B. The Femto amplifier provides the necessary 10^3 amplification up to 10 MHz, 12 pF input capacity, and $8.1 \times 10^{-18} \text{ V}^2/\text{Hz}$ input voltage noise power [67]. The 3 dB frequency due to RC bandwidth limitation of a sample ($R_{\text{sample}} \approx 2 \text{ k}\Omega$) and amplifier ($C = 12 \text{ pF}$) is at $f_{3\text{dB}} \approx 6.6 \text{ MHz}$ (Eq. (4.2)). For an ADRN measurement the frequency bandwidth which is attenuated by less than $|G|^2 = -10 \text{ dB}$ can be used for averaging PSD spectra as in Ch. 4. Hence, a measurement bandwidth of

$$B_{\text{meas}} = \sqrt{\frac{1 - |G|^2}{|G|^2}} f_{3\text{dB}} = 2.2 \text{ MHz} \quad (\text{B.8})$$

can be used for measuring ADRN. In order to prevent aliasing, $f_s = 50 \text{ MHz}$ is used, hence the lowest blind time is $\tilde{\tau}_{\text{blind}} = 1.88$ for $n_s = 2^{24}$ [66] (cf. Table B.1). It is realistic to measure for $\tau_{\text{meas}} = 1.5 \text{ h}$ per angle-dependent voltage noise power value. Thus, one expects

$$\text{S/N} = 6.4 \times 10^4. \quad (\text{B.9})$$

This is an improvement of one order of magnitude on the S/N ratio of 5×10^3 of the ADRN measurements presented in this thesis².

The computer FFT implementation is a valuable tool to replace the SR760 by a spectrum analyzer with a bandwidth up to 100 MHz. The amplifier SR560 was proved insufficient as voltage amplifier above 50 kHz.

²In Ch. 4, The signal to noise ratio is calculated from the measurement accuracy of 2×10^{-4} by $\text{S/N} = (2 \times 10^{-4})^{-1}$

Appendix C

Narrow Bandwidth Detected Noise at Gigahertz

The impedance matching technique realized by K. Baumgärtl shall be briefly described with regard to narrow bandwidth impedance matched noise power measurements. Afterwards, noise power at microwave frequencies shall be addressed. In the end, a first noise power measurement of an impedance matched network with impedance matching at 5 GHz is presented.

One particular issue of microwave frequency measurements is the power reflection at the transition from one impedance Z_0 to another resistance $R > Z_0$. The reflection coefficient Γ gives the amount of reflected voltage amplitude $\Gamma = (R - Z_0)/(R + Z_0)$. In order to prevent power reflection all devices in a circuit need to be impedance matched. In the cause of his bachelor thesis, K. Baumgärtl, who was supervised during this thesis, implemented an impedance matching technique using the printed circuit board (PCB) technology to match resistors with $k\Omega$ resistance to a cable with an impedance of $50\ \Omega$ [68].

The following results have been obtained by K. Baumgärtl: The impedance matching network consists of a resonator which is capacitively coupled by C_k to the input line (1) and a sample resistance R_{sample} as load (cf. Fig. C.1(a)). Figure C.1(a) illustrates that the resonance frequency ω_0 is given by the length of the $\lambda/2$ micro strip resonator. A connection to the middle of the microstrip serves as bias tee to apply a direct current through the sample. The input impedance of the loaded micro strip at resonance becomes real [68, 69]

$$Z_{\text{in}}(\omega_0) \approx \frac{1}{R_{\text{sample}}(\omega_0 C_k)^2}. \quad (\text{C.1})$$

It inverts the sample resistance. From Eq. (C.1) it is apparent that for given R_{sample} and resonance frequency ω_0 the coupling capacity C_k can be adjusted so that the network matches $Z_0 = 50\ \Omega$.

The $\lambda/2$ micro strip resonator is realized on a printed circuit board (PCB) Rogers RT/duroid®5880. A PCB consists of a dielectric layer placed between two conducting plates. One plate can be structured by photolithography and wet etching. The linear design sketched in Fig. C.1(a) was changed to fit the requirements of an electro magnet setup to Fig. C.1(b). The microstrip resonator is designed to be resonant at

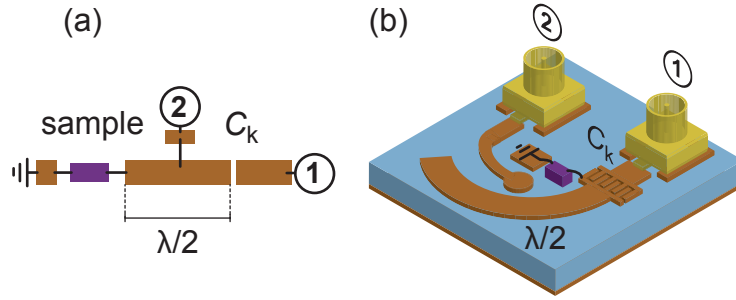


Figure C.1: (a) The impedance matching network consists of a sample connected to ground, a $\lambda/2$ resonator, and a coupling capacitor [69]. The $\lambda/2$ long resonator determines the resonance frequency. The capacitance C_k adjusts the resonator to impedance match to contact 1. Contact (2) serves as bias tee. (b) The impedance matching network for a sample ($R_{\text{sample}} = 8.2 \text{ k}\Omega$) is matched by an interdigital capacitor to the 50Ω impedance at contact (1) for resonance frequency $f_0 = 5 \text{ GHz}$ (detailed description see text).

5 GHz. A eight finger condensation with interdigital capacitor couples the resonator to contact (1). The capacity was designed to match a $2 \text{ k}\Omega$ resistor to an impedance of 50Ω . The sample is placed in the middle of the board and bonded by Al wires to the interdigital capacitor and the ground plane. The ground plane is contacted by a soldered Al wire through the dielectric to ground. In order to shield the resonant circuit from radiation and to prevent it from radiating into the environment, it is placed inside a copper box. Now the contribution of K. Baumgärtl is closed. We will use a $8.2 \text{ k}\Omega$ surface mount resistor on the PCB in the copper box for noise power measurements.

Before discussing the measurement result, an introduction to noise at microwave frequencies shall be given according to the Chapter “noise and active RF components” from [70]. The thermally emitted voltage noise amplitude is given by Nyquist noise, cf. Eq. (2.23):

$$V_n = \sqrt{4k_B T R B}. \quad (\text{C.2})$$

Where $k_B = 1.38 \times 10^{-23} \text{ J/K}$ is the Boltzmann constant, T is the temperature of the device under test with resistance R , and B the measurement bandwidth. For noise measurements at microwave frequencies the power emitted by a matched load is of interest:

$$P_n = \frac{V_n^2}{4R} = k_B T B. \quad (\text{C.3})$$

When considering the noise power N_0 from any arbitrary white noise source, the noise power can be written in terms of an associated temperature T_e of the matched load R (cf. Fig. C.2).

$$T_e = \frac{N_0}{k_B B} \quad (\text{C.4})$$



Figure C.2: The equivalent noise temperature T_e of an arbitrary white noise source, DUT.

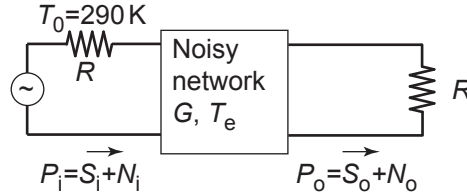


Figure C.3: Determining the noise figure of a noisy network, such as an amplifier.

In order to be able to detect low level signals, amplifiers are necessary. Unfortunately amplifiers do not only amplify the input signal (i.e. amplify incoming noise), but are themselves noise sources. For determining the noise figure of a noisy network, consider Fig. C.3. The noise figure describes the ratio between signal to noise ratio at the input S_i/N_i and output S_o/N_o of the noisy network.

$$F := \frac{S_i/N_i}{S_o/N_o} \geq 1 \quad (\text{C.5})$$

$$F = 1 + \frac{T_e}{T_0} \quad (\text{C.6})$$

Where the input noise is given by $N_i = k_B T_0 B$ from a matched load at room temperature $T_0 := 290$ K. The output noise of an amplifier with an impedance matched load at temperature T_0 is

$$N_o = k_B G B (T_0 + T_e), \quad (\text{C.7})$$

where G is the amplifier gain which is given in power units.

The Kuhne amplifier, KU LNA BB 202 A (WMI index: 30.12.2010), is used for the following experiments. According to the data sheet its gain is $G = 30.95$ dB and its noise figure is $F = 2.942$ dB at 5 GHz. First, the amplifier input is terminated by 50Ω as sketched in Fig. C.4(a). The input noise power is $N_i = k_B T B$ with the resistor at room temperature $T = T_0$. The output noise power N_o was investigated by a Rohde & Schwarz signal analyzer, FSV30 10 Hz ... 30 GHz. The resolution bandwidth was set to $B = 50$ Hz for center frequency $f_{\text{center}} = 5$ GHz and span 3 GHz.

For estimating N_o we calculate the equivalent amplifier temperature from Eq. (C.6) considering that F is given in dB

$$T_e[\text{K}] = (10^{F[\text{dB}]/10} - 1)T_0[\text{K}] = 0.968 T_0[\text{K}] = 281 \text{ K} \quad (\text{C.8})$$

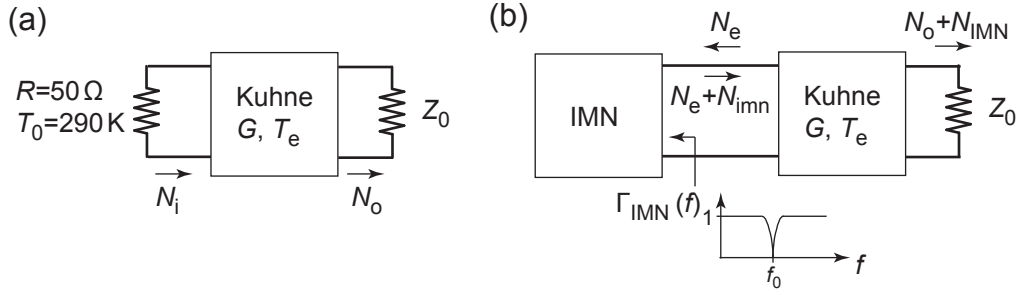


Figure C.4: Setups for noise power measurements. (a) The Kuhne amplifier is terminated by $50\ \Omega$. (b) The impedance matching network (IMN) with resistor $R_{\text{sample}} = 8.2\ \text{k}\Omega$, resonance frequency $f_0 = 5\ \text{GHz}$, and reflection coefficient $\Gamma(f)$ is connected to Kuhne amplifier.

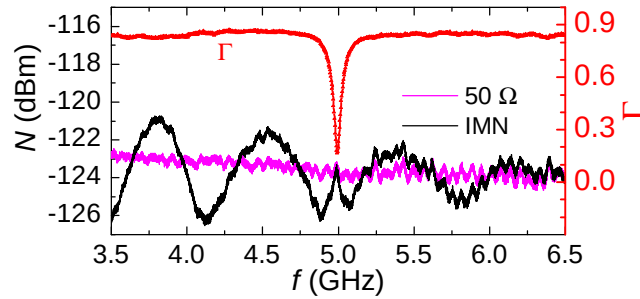


Figure C.5: Noise power N as function of frequency f . The Kunhe amplifier was terminated with a $50\ \Omega$ load (magenta, cf. Fig. C.4(a)) and with the impedance matching network (black, cf. Fig. C.4(b)). The reflection coefficient Γ of the impedance matching network was measured as function of frequency by a vector network analyses and is plotted in red.

and the gain

$$G[\] = 10^{G[\text{dB}]/10} = 1244. \quad (\text{C.9})$$

Hence

$$N_o[\text{mW}] = Gk_B B(T_0 + T_e) = 4.90 \times 10^{-13} \text{ mW} \quad (\text{C.10})$$

$$N_o[\text{dBm}] = 10 \log(N_o[\text{mW}]) = -123 \text{ dBm} \quad (\text{C.11})$$

The measured power of the signal analyzer in Fig. C.5, magenta plot, nicely reproduces $N_o = -123\ \text{dBm}$.

When studying the impedance matching network from K. Baumgärtl as depicted in Fig. C.4(b), the impedance matching network matches only at $5\ \text{GHz}$ and reflects incident power for other frequencies. This is described by the reflection coefficient $\Gamma(f)$. Hence, the noise power N_e emitted by the amplifier will be reflected for most

frequencies and a noise power output N_o of

$$N_o = Gk_B B(T_e + T_e) = 4.83 \times 10^{-13} \text{ mW} \quad (\text{C.12})$$

$$= -123 \text{ dBm} \quad (\text{C.13})$$

is expected, which can not be distinguished from a 50Ω termination Eq. (C.11). The noise power of the impedance matching network at resonance is $N_{\text{IMN}} = k_B T B$ with temperature equal to room temperature $T = T_0$.

$$N_{\text{IMN}} = Gk_B B(T_0 + T_e) = 4.90 \times 10^{-13} \text{ mW} \quad (\text{C.14})$$

$$= -123 \text{ dBm} \quad (\text{C.15})$$

Measuring the noise power of the impedance matching network results in a power spectrum plotted in Fig. C.5 (black). The magnitude of the noise power corresponds to Eqs. (C.13) and (C.15). The velocity ratio of the SMP to SMA cable ($l_{\text{cable}} = 4.5 \text{ cm}$) used for connecting impedance matching network and Kuhne amplifier is 70 % according to the data sheet [71]. Hence, a standing wave with $f = 0.7c/l_{\text{cable}} = 4.7 \text{ GHz}$ is expected, c is the speed of light in vacuum. The interference in the broad noise power thus stems from standing waves which form before the reflecting impedance matching network when not at resonance. The noise power emitted at the matching point 5 GHz can be told apart from the broad noise power, because it is located at an interference minimum. However, it is unclear whether the impedance matching network is driven by the amplifier noise or whether it is emitting noise from the built in sample (surface mount resistor $R_{\text{sample}} = 8.2 \text{ k}\Omega$).

Amplifiers which are cooled to 4 K should be used to measure thermal noise power so that the emitted noise of the amplifier N_e is by -18 dB smaller than the thermal noise power emitted by the impedance matching network.

References

- [1] A. A. Penzias and R. W. Wilson, *Astrophysical Journal* **142**, 419 (1965).
- [2] Y. Tserkovnyak, A. Brataas, G. E. W. Bauer, and B. I. Halperin, *Reviews of Modern Physics* **77**, 1375 (2005).
- [3] I. Žutić, J. Fabian, and S. Das Sarma, *Reviews of Modern Physics* **76**, 323 (2004).
- [4] J. S. Moodera, L. R. Kinder, T. M. Wong, and R. Meservey, *Physical Review Letters* **74**, 3273 (1995).
- [5] G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn, *Physical Review B* **39**, 4828 (1989).
- [6] M. N. Baibich, J. M. Broto, A. Fert, F. N. Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas, *Physical Review Letters* **61**, 2472 (1988).
- [7] G. C. Han, Y. K. Zheng, Z. Y. Liu, B. Liu, and S. N. Mao, *Journal of Applied Physics* **100**, 063912, 063912 (2006).
- [8] K. Klaassen, X. Xing, and J. van Peppen, *IEEE Transactions on Magnetics* **41**, 2307 (2005).
- [9] A. M. Deac, A. Fukushima, H. Kubota, H. Maehara, Y. Suzuki, S. Yuasa, Y. Nagamine, K. Tsunekawa, D. D. Djayaprawira, and N. Watanabe, *Nature Physics* **4**, 803 (2008).
- [10] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, *Physical Review Letters* **88**, 117601 (2002).
- [11] O. Mosendz, J. E. Pearson, F. Y. Fradin, G. E. W. Bauer, S. D. Bader, and A. Hoffmann, *Physical Review Letters* **104**, 046601 (2010).
- [12] K. Ando, S. Takahashi, J. Ieda, Y. Kajiwara, H. Nakayama, T. Yoshino, K. Harii, Y. Fujikawa, M. Matsuo, S. Maekawa, and E. Saitoh, *Journal of Applied Physics* **109**, 103913, 103913 (2011).
- [13] F. D. Czeschka, L. Dreher, M. S. Brandt, M. Weiler, M. Althammer, I. Imort, G. Reiss, A. Thomas, W. Schoch, W. Limmer, H. Huebl, R. Gross, and S. T. B. Goennenwein, *Physical Review Letters* **107**, 046601 (2011).
- [14] Z. Wang, Y. Sun, Y.-Y. Song, M. Wu, H. Schultheiß, J. E. Pearson, and A. Hoffmann, *Applied Physics Letters* **99**, 162511, 162511 (2011).
- [15] M. I. Dyakonov and V. I. Perel, *Journal of Experimental and Theoretical Physics Letters* **13**, 467 (1971).

- [16] Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, *Science* **306**, 1910 (2004).
- [17] J. E. Hirsch, *Physical Review Letters* **83**, 1834 (1999).
- [18] E. Saitoh, M. Ueda, H. Miyajima, and G. Tatara, *Applied Physics Letters* **88**, 182509 (2006).
- [19] M. Weiler, M. Althammer, M. Schreier, J. Lotze, M. Pernpeintner, S. Meyer, H. Huebl, R. Gross, A. Kamra, J. Xiao, Y.-T. Chen, H. Jiao, G. E. W. Bauer, and S. T. B. Goennenwein, arXiv:1306.5012 (2013).
- [20] K. Uchida, S. Takahashi, K. Harii, J. Ieda, W. Koshibae, K. Ando, S. Maekawa, and E. Saitoh, *Nature* **455**, 778 (2008).
- [21] S. Takahashi and S. Maekawa, *Science and Technology of Advanced Materials* **9**, 014105 (2008).
- [22] H. Nakayama, M. Althammer, Y.-T. Chen, K. Uchida, Y. Kajiwara, D. Kikuchi, T. Ohtani, S. Geprägs, M. Opel, S. Takahashi, R. Gross, G. E. W. Bauer, S. T. B. Goennenwein, and E. Saitoh, *Physical Review Letters* **110**, 206601 (2013).
- [23] M. Althammer, S. Meyer, H. Nakayama, M. Schreier, S. Altmannshofer, M. Weiler, H. Huebl, S. Geprägs, M. Opel, R. Gross, D. Meier, C. Klewe, T. Kuschel, J.-M. Schmalhorst, G. Reiss, L. Shen, A. Gupta, Y.-T. Chen, G. E. W. Bauer, E. Saitoh, and S. T. B. Goennenwein, *Physical Review B* **87**, 224401 (2013).
- [24] Y.-T. Chen, S. Takahashi, H. Nakayama, M. Althammer, S. T. B. Goennenwein, E. Saitoh, and G. E. W. Bauer, *Physical Review B* **87**, 144411 (2013).
- [25] W. Gerlach and O. Stern, *Zeitschrift für Physik A Hadrons and Nuclei* **9**, 353 (1922).
- [26] F. Czeschka, *Spin currents in metallic nanostructures*, PhD thesis, Technische Universität München (2011).
- [27] N. F. Mott, *Proceedings of the Royal Society of London. Series A* **124**, 425 (1929).
- [28] L. Berger, *Physical Review B* **2**, 4559 (1970).
- [29] R. Karplus and J. M. Luttinger, *Physical Review* **95**, 1154 (1954).
- [30] O. Mosendz, V. Vlaininck, J. E. Pearson, F. Y. Fradin, G. E. W. Bauer, S. D. Bader, and A. Hoffmann, *Physical Review B* **82**, 214403 (2010).
- [31] R. Karplus and J. M. Luttinger, *Physical Review* **95**, 1154 (1954).
- [32] A. Brataas, Y. Tserkovnyak, G. E. W. Bauer, and B. I. Halperin, *Physical Review B* **66**, 060404 (2002).
- [33] W. Thomson, *Proceedings of the Royal Society of London* **8**, 546 (1856).
- [34] T. McGuire and R. Potter, *IEEE Transactions on Magnetism* **11**, 1018 (1975).

- [35] S. T.B. Goennenwein, *Spinabhängigkeit von elektronischem Rauschen in amorphem Silizium*, Diploma thesis, Technische Universität München (1999).
- [36] R. Becker, *Theorie der Wärme* (Springer Verlag, 1985).
- [37] W. B. Davenport and W. L. Root, *An introduction to the theory of random signals and noise* (John Wiley & Sons, 1987).
- [38] L. Landau and E. M. Lifshitz, *Statistical physics part 1* (Pergamon Press, 1980).
- [39] H. Nyquist, Physical Review **32**, 110 (1928).
- [40] J. B. Johnson, Physical Review **32**, 97 (1928).
- [41] W. Schottky, Annalen der Physik **362**, 541 (1918).
- [42] S. Chikazumi, *Physics of ferromagnetism* (Oxford University Press, 1997).
- [43] J. Foros, A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, Physical Review Letters **95**, 016601 (2005).
- [44] A. Kamra, J. Xiao, and G. E. W. Bauer, “Spin current mediated thermal noise in a spin seebeck configuration”, private communication, Dec. 2012.
- [45] A. Kamra, J. Xiao, and G. E. W. Bauer, “Spin current mediated thermal noise in Pt|YIG structure”, private communication, Nov. 2012.
- [46] J. Xiao, G. E. W. Bauer, S. Maekawa, and A. Brataas, Physical Review B **79**, 174415 (2009).
- [47] N. Wiener, Acta Mathematica **55**, 117 (1930).
- [48] A. Khintchine, Mathematische Annalen **109**, 604 (1934).
- [49] C. Kittel, *Elementary statistical physics* (John Wiley & Sons, 1958).
- [50] R. Müller, *Rauschen* (Springer, 1990).
- [51] W. Cochran, J. W. Cooley, D. Favin, H. Helms, R. Kaenel, W. Lang, G. Maling, D. Nelson, C. Rader, and P. D. Welch, Proceedings of the IEEE **55**, 1664 (1967).
- [52] L. Isserlis, Biometrika **11**, 185 (1916).
- [53] C. Hahn, G. de Loubens, O. Klein, M. Viret, V. V. Naletov, and J. Ben Youssef, Physical Review B **87**, 174417 (2013).
- [54] H. B. Callen, Physical Review **73**, 1349 (1948).
- [55] D. Flynn and M. O’Hagan, Journal of Research of the National Bureau of Standards. Section C: Engineering and Instrumentation **71C**, 255 (1967).
- [56] A. Rich, Analog Devices **AN-347**.
- [57] *Model SR760FFT Spectrum Analyzer*, Stanford Research Systems (2007).
- [58] H. Nyquist, Transactions of the American Institute of Electrical Engineers **47**, 617 (1928).
- [59] F. Harris, Proceedings of the IEEE **66**, 51 (1978).
- [60] P. D. Welch, IEEE Transactions on Audio and Electroacoustics **15**, 70 (1967).

- [61] *Model SR560 Low-noise preamplifier*, Stanford Research Systems (2011).
- [62] *Low level measurements handbook* (Keithley Instruments, 2004).
- [63] T. Fließbach, *Statistische Physik* (Springer, 2010).
- [64] N. Vlietstra, J. Shan, V. Castel, J. Ben Youssef, and B. J. van Wees, arXiv:1301.3266 (2013).
- [65] M. Ross, *Spin dynamics in an antiferromagnet*, Diploma thesis, Technische Universität München (2013).
- [66] A. Schade, *Fast-Fourier-transform-based electrical noise measurements*, Bachelor thesis, Technische Universität München (2013).
- [67] *10 MHz low-noise voltage amplifier HVA-10M-60-B*, Femto ().
- [68] K. Baumgärtl, *Impedanzangepasste Widerstandsmessungen bei Frequenzen im Gigahertz-Bereich*, Bachelor thesis, Technische Universität München (2013).
- [69] D. Fang, T. D. Skinner, H. Kurebayashi, R. P. Campion, B. L. Gallagher, and A. J. Ferguson, *Applied Physics Letters* **101**, 182402, 182402 (2012).
- [70] D. Pozar, *Microwave engineering* (John Wiley & Sons, 2005).
- [71] *Cable specification, RTK-Flex405-SPC-FEP*, Rosenberger (2008).

Acknowledgements

This thesis was made possible by the support, encouragement, and knowledge of members of the Walther-Meißner-Institut. In particular I would like to thank:

Prof. Dr. Rudolf Gross, for the opportunity to accomplish this work at the Walther-Meißner-Institut. His intuitive way of explaining physics and his dedication to teaching are truly encouraging.

Dr. Sebastian T.B. Goennenwein, for his excellent supervision and numerous extensive discussions which enabled the experimental success of this thesis. For his kind introduction into the “Magnetiker” group and for his confidence in me to supervise two bachelor students.

Sibylle Meyer, for supervising my work, for introducing me to the spin Hall magnetoresistance effect, and her patience with countless requests for help.

Akashdeep Kamra, for giving me a deep insight into the theoretical physics of noise power and for his patience with answering numerous questions on the topic.

Dr. Hans Huebl, for his discussions about shielding, low level measurements and microwave frequency technology. For his lecture on magnetism and spintronics, which was both educating and entertaining.

Alexander Schade, for his enthusiastic attitude of working and broad knowledge about programming which lead to the success of his bachelor thesis.

Korbinian Baumgärtl, for his ambiguous way of working during his bachelor thesis which benefited from the input of *Tim D. Skinner* from Cavendish Laboratory, University of Cambridge.

For the experimental support I thank in particular: *Dr. Matthias Althammer*, *Micheal Schreier*, *Matthias Pernpeintner*, and *Christoph Zollitsch*.

For the general support and the harmonic atmosphere in the “Magnetiker” group I thank: *Niklas Roschewsky*, *Stephan Klimesch*, *Johannes Lotze*, *Philipp Ross*, *Mathias Frank*, *Dr. Stephan Geprägs*, *Dr. Matthias Opel*, and *Rasmus Holländer*.

The technical staff at the WMI, most notably *Helmut Thies*, who helped manufacturing shields and boxes making noise experiments possible in the first place.

A special thanks goes to the private support that I received from my family and friends.

Persönliche Erklärung

Ich versichere, dass ich diese Masterarbeit selbstständig angefertigt, sowie alle Stellen, die wörtlich oder annähernd wörtlich aus Veröffentlichungen entnommen wurden, als solche kenntlich gemacht und nur die angegebenen Hilfsmittel verwendet habe. Die Arbeit hat in dieser Form keiner anderen Prüfungsinstitution vorgelegen.

Ort, Datum

Unterschrift