Fabrication and Characterization of Josephson Traveling Wave Parametric Amplifiers

Master’s Thesis
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Introduction

In our digital era, microwave amplifiers are one of the most ubiquitous devices around us. They are present practically in all electronic device operated at the frequencies up to several GHz. Amplification as a general process can be characterized by many parameters. Arguably, the most important of them are the amplification gain $G$, bandwidth $BW$, and number of added noise photons $n$. The latter number is of utter importance for detecting weak signals. Unfortunately, even the best available conventional microwave amplifiers add tens or more of noise photons $n \geq 10$ to the input signals during the amplification. This makes very difficult to directly detect single-photon microwave signals. At the same time, quantum theory has demonstrated that the fundamental limit for added noise is only half of photon $n = 1/2$ for phase-insensitive amplification (also known as Standard Quantum Limit, SQL). Such kind of microwave amplifiers eventually have been built by exploiting superconducting materials and Josephson junctions, resulting in a Josephson parametric amplifiers (JPAs) which have convincingly demonstrated operation at the very borderline of SQL [1].

For many years these superconducting amplifiers were merely toys for scientists with rather limited practical importance or applications. This was until the second quantum revolution had stricken. This term denotes two recent decades when scientists realized that (i) many classically unsolvable mathematical problems may be efficiently solved by using quantum computers and (ii) that small-to-medium scale quantum computers can be already built using
available technologies. These quantum computers have demonstrated an outstanding evolution during the last 20 years - from single disconnected quantum bits (qubits) to $\sim 50$ tunable connected qubits with full control. In this respect, superconducting qubits are the most advanced and active branch of quantum information processing science nowadays which has been just recently outlined by the first-ever demonstration of quantum supremacy $^2$. In this and other similar experiments, JPAs play a crucial role in readout of the superconducting qubits, since the latter operate in the GHz range and the corresponding logical quantum states are essentially equivalent to microwave single-photons. An absolute majority of modern qubit readout schemes is based on the JPAs due to their ability to operate at the quantum limit of added noise.

However, these JPAs are not fully ideal in terms of the aforementioned basic parameters. Their gains and added noise numbers are typically excellent, $G \geq 20\, \text{dB}$, $n \simeq 1/2$ but their bandwidths are extremely limited $BW < 5\, \text{MHz}$. This stems from the fact that most of conventional JPAs internally are based on high-quality superconducting resonators which naturally limit the bandwidth. In turn, limited bandwidths severely limit speed and multiplexing capabilities of qubit readouts. In order to solve this challenge, a concept of traveling wave parametric amplifiers (TWPAs) has been introduced $^3$. This approach is based on exploiting a nonlinear transmission line, in contrast to resonators. This trick allows one to open the amplification bandwidth to $BW \sim 4 - 8\, \text{GHz}$ while preserving the gain and noise properties. In our work, we describe design, fabrication, and first preliminary measurements of such a TWPA device. In contrast to earlier devices relying on a 4-wave mixing parametric process $^3$-$^5$, we use a modified 3-wave mixing approach $^6$ for building of a TWPA which potentially allows to solve a specific problem of broadband parametric amplifiers - phase matching between signal and pump tones which, if not addressed, may severely limit the gain to $G \simeq 10\, \text{dB}$ only.

This thesis is structured as follows. First, in chapter $^1$ we provide a short introduction to the theory underlying TWPAs. We give a short overview
of Josephson junctions and superconducting quantum interference devices (SQUIDs) which serve as a central component of superconducting Josephson parametric amplifiers. Afterwards, we give an intuitive introduction to the concept of parametric amplification, as well as the basic concept of the JPA and TWPA. Finally, we discuss differences between TWPAs utilizing the 3-wave mixing process and those utilizing the 4-wave mixing one.

In chapter 2, we discuss a basic design for our 3-wave mixing TWPAs. Next, we introduce our fabrication processes and routines. At the end, we demonstrate a range of parameters to successfully fabricate a 3-wave mixing TWPA prototype.

Chapter 3 describes the sample packaging and preparation for cryogenic microwave measurements. At the end, we also demonstrate first preliminary results of TWPA measurements.

Finally, we summarize all experimental achievements and give a short outlook into the future of Josephson parametric amplifiers.
Chapter 1

Theory

In this chapter we present a theoretical background for our work. In the first section, we provide a short introduction into Josephson physics. We discuss the Josephson equations and their connection to the Josephson inductance. Then, we briefly discuss flux quantization and how it is connected to direct current superconducting quantum interference devices (SQUIDs). In the next section, we introduce superconducting Josephson parametric amplifiers, by describing the principle of parametric amplification. Subsequently, we discuss the concept of TWPAs which are considered an evolution of JPAs and highlight their similarities and differences. Finally, we look into most conventional types of TWPAs exploiting the 3-wave mixing and 4-wave mixings processes. We consider the physics underlying both types as well as their respective advantages and disadvantages.

1.1 Superconducting quantum interference

In order to derive a theory of Josephson junctions and SQUIDs one must rely on the macroscopic quantum model of superconductivity. According to this model, Cooper pairs of electrons in a superconductor can be described by
a single macroscopic wave function [7]

\[ \psi(r, t) = \sqrt{n(r, t)}e^{i\theta(r, t)}, \]  

(1.1)

here \( n(r, t) \) describes the local density of superconducting electrons and \( \theta(r, t) \) denotes the macroscopic phase. Josephson effects can be observed in a system of weakly coupled superconductors. Weak coupling can be achieved either by connecting two superconductors through a insulating barrier or creating a constriction in a bulk superconductor. These points of weak coupling are called Josephson junctions, systems that make use of Josephson junctions and the effect of flux quantization in superconductors in concurrence are called SQUIDs.

1.1.1 Josephson junctions

In general, the to-be-discussed Josephson effects occur due to tunneling of Cooper pairs through the barrier. Although it was believed that tunneling of Cooper pairs is very improbable, Brian D. Josephson predicted in 1962 [8] that these pairs tunnel coherently which makes this process much more probable. This prediction was confirmed at the same year by Rowel [9]. In the following, we state the fundamental equations and effects of Josephson physics. For the reader interested in the derivation or a more detailed discussion of these elements we recommend the course material on Applied Superconductivity by R. Gross [7].

One can define a gauge invariant phase difference between the two bulk superconductors which is defined as [10]

\[ \varphi = \theta_2(r, t) - \theta_1(r, t) - \frac{2\pi}{\Phi_0} \int_1^2 \mathbf{A}(r, t) \cdot d\mathbf{l}, \]  

(1.2)
where $\Phi_0 = 2.067 \times 10^{-15}$ Wb is the magnetic flux quantum and $\mathbf{A}$ is the magnetic vector potential. The integration path leads from one bulk superconductor to another while crossing the tunnel barrier (see Fig. 1.1).

Josephson has calculated the supercurrent across the junction to be equal to

$$I_s(\varphi) = I_c \sin(\varphi),$$

(1.3)

where $I_c$ is the critical current and thus the maximally possible supercurrent through the junction. This is called the first Josephson equation. Using the London equations one can derive the second Josephson equation

$$\frac{\partial \varphi}{\partial t} = \frac{2\pi}{\Phi_0} V,$$

(1.4)

which describes the time-evolution of $\varphi$ in relation to the voltage $V$ across the junction. A particular consequence of the Josephson equations is the fact that the Josephson current varies sinusoidally in time if a constant voltage is applied across the junction. In terms of conventional electrical components a Josephson junction can be understood as a non-linear inductance. The Josephson
inductance can be estimated from the Josephson equations

\[ L = \frac{\Phi_0}{2\pi I_c \cos(\varphi)} = L_c \frac{1}{\cos(\varphi)}. \]  

(1.5)

This Josephson inductance can remarkably become negative and also depends on the Josephson phase across the junction.

1.1.2 Direct current superconducting quantum interference devices (dc-SQUID)

In the following we provide a short overview of the most important equations describing the behavior of a direct current superconducting quantum interference device (SQUID). For a more detailed explanation and derivation of the equations we refer the reader to Ref. \[7\] In SQUIDs the physics of Josephson junctions interact with the effect of flux quantization. Fluxoid quantization was found in 1961 by Doll and Näbauer \[11\] at Walther-Meißner-Institut in Munich and Deaver and Fairbank in Stanford \[12\]. According to the macroscopic quantum model the phase and amplitude of the macroscopic wave function are required to be well defined at all points in a superconductor, thus for a superconducting ring the phase change along a closed contour has to be \( \Delta \theta = 2\pi n \), where \( n \) is an integer. Using this, one can derive the formula describing fluxoid quantization

\[ \oint_C (\Lambda J_z) \cdot dl + \int_F B \cdot ndF = n\Phi_0, \]  

(1.6)

where \( C \) is a closed contour inside a superconductor and \( F \) is the area encircled by contour \( C \). This says that the sum of the external magnetic flux and the magnetic flux created by a superconducting loop current has to be a integer multiple of the flux quantum \( \Phi_0 = \hbar/2e = 2.068 \times 10^{-15} \) Wb. A dc-SQUID
1.1 Superconducting quantum interference consists of two parallel Josephson junctions connected on a superconducting loop.

![Diagram of a dc-SQUID](image)

**Figure 1.2:** Scheme of a direct current superconducting quantum interference device (dc-SQUID). The superconductor is depicted in gray and the Josephson junctions in blue. The total current across the dc-SQUID $I_\Sigma$ splits into currents $I_1$ and $I_2$ across the different junctions. A current circulating in the loop $I_{circ}$ results from a difference of $I_1$ and $I_2$.

In the following, we will assume dc-SQUIDs to consist of two Josephson junction with identical critical currents $I_c$. The superconducting current through the dc-SQUID can be calculated using Kirchhoff’s law. Fluxoid quantization suggests, that $\varphi_1$ and $\varphi_2$ are not independent from each other and must be related to each other like

$$\varphi_2 - \varphi_1 = \frac{2\pi \Phi}{\Phi_0},$$

(1.7)

where $\Phi$ denotes the total flux through the loop consisting of the external flux $\Phi_{\text{ext}}$ and the flux created by the loop current $\Phi_L = L_{\text{loop}} I_{\text{circ}}$. Here $I_{\text{circ}}$ denotes the current circulating in the loop and $L_{\text{loop}}$ denotes the geometric self-inductance of the loop not considering the Josephson inductance of the
junctons. It helps to introduce the parameter
\[ \beta_L = \frac{2L_{\text{loop}}I_{\text{circ}}}{\Phi_0}, \tag{1.8} \]
which estimates a ratio of the magnetic flux due to the circulating current and \( \Phi_0/2 \).

**Negligible screening** (\( \beta_L \ll 1 \)):

In the case of negligible screening the self-induced flux will be small compared to \( \Phi_0/2 \) and it can simply be left out of the calculations, i.e. \( \Phi \approx \Phi_{\text{ext}} \). The dc-SQUID then behaves like a Josephson junction with a critical current
\[ I_s^m = 2I_c \left| \cos \left( \frac{\Phi_{\text{ext}}}{\Phi_0} \right) \right|, \tag{1.9} \]
which is adjustable by the external magnetic field. As for a single Josephson junction, one can then define an inductance value for the whole dc-SQUID based on its critical current
\[ L_{SQUID} = \frac{\Phi_0}{4\pi I_c \left| \cos \left( \frac{\Phi_{\text{ext}}}{\Phi_0} \right) \right|}. \tag{1.10} \]

**Large screening** (\( \beta_L \gg 1 \)):

The case of large screening implies that \( LI_c \gg \Phi_0 \). Therefore, the inductance of the Josephson junctions contribute less to the behavior of the dc-SQUID which acts more like a simple superconducting loop. The approximation \( \Phi \approx \Phi_{\text{ext}} \) cannot be used anymore and has to be replaced with \[ \Phi = \Phi_{\text{ext}} - LI_c \sin \left( \frac{\Phi}{\Phi_0} \right) \cos \left( \varphi + \frac{\Phi}{\Phi_0} \right), \tag{1.11} \]
which has to be solved self-consistently. It turns out that for big $\beta_L$ there are multiple valid $\Phi$ for a fixed $\Phi_{\text{ext}}$ which leads to a hysteretic dc-SQUID \[13\].

1.2 Superconducting Josephson parametric amplifiers

Wherever one has to measure extremely weak microwave signals (e.g. single-photon signals, squeezed microwave states) one needs to use microwave amplifiers. According to the Caves theorem, there exists a fundamental lower limit on the added noise in amplifiers \[14\]. It states that a phase-insensitive, or non-degenerate, amplifier has to add at least some noise

$$A \geq \frac{1}{2} \left| 1 - \frac{1}{G} \right|,$$  \[1.12\]

quantified by the noise photon number $A$. Here, $G$ denotes the phase-insensitive power gain of the amplifier. It is apparent that for high gains at least half of a photon has to be added by the amplifier as noise. The biggest advantage of superconducting parametric amplifiers over conventional amplifiers is their noise properties which can reach the fundamental quantum noise limit, provided by Eq.\[1.12\]. This section provides an insight to parametric amplification in general and considers specific of superconducting parametric amplifiers based on Josephson junctions, namely, the Josephson parametric amplifiers (JPAs) and Josephson traveling wave parametric amplifiers (TWPAs). As the names suggest these types of amplifiers use Josephson junctions and SQUIDs as the main circuit elements. Although, there are different types of traveling wave parametric amplifiers, in the following we will only refer to those using Josephson junctions and SQUIDs to implement parametric amplification.
1.2.1 Principle of parametric amplification

Parametric amplification is observed in systems where a distinct parameter, e.g. frequency, is modulated time-periodically by an external force. This external force is often called the pump. During parametric amplification small excitations of the oscillatory system grow exponentially due to the energy transfer from the pump to the signal. A simple mechanical pendulum can be used as an example to explain parametric amplification. The pendulum consists of a massless string suspending a point mass. Losses due to friction are omitted in the following discussion. The differential equation describing this system is

\[ \ddot{\theta} + \frac{g}{l} \sin(\theta) = 0, \]  

(1.13)

where \( \theta \) is the deflection angle, \( g \) is the gravity constant and \( l \) is the pendulum length. Assuming small \( \theta \) and subsequently \( \sin(\theta) \approx \theta \), this equation yields

\[ \omega_0 = \sqrt{\frac{g}{l}}, \] 

(1.14)

for the natural angular frequency \( \omega_0 \). Time-modulating the frequency \( \omega_0 \rightarrow \omega_0 + \Delta \omega \sin(2\omega_p t) \) with \( \omega_p \approx 2\omega_0 \) yields the differential equation

\[ \ddot{\theta} + \omega_0^2 \left( 1 + \frac{2\Delta \omega}{\omega_0} \sin(2\omega_p t) \right) \theta = 0. \]  

(1.15)

Here, the term of the order \((\Delta \omega)^2/\omega_0^2\) was neglected which is valid only for small \( \Delta \omega \) and the small-angle approximation for \( \theta \) was used. One can find the solution to this differential equation

\[ \theta(t) = \omega_0 e^{\alpha t} \cos(\omega_0 t) \] 

(1.16)

with the constant \( \alpha \) being dependent on the detuning \((\omega_p^2 - \omega_0^2)/\omega_0^2\) and modulation depth \( \Delta \omega/\omega_0 \). For additional information on this topic we refer the reader to Ref. [15]. This system demonstrates clearly the principle of para-
metric amplification, i.e. the transfer of power between two frequency modes through the intrinsic parametric interaction.

1.2.2 Josephson parametric amplifiers

Josephson parametric amplifiers feature Josephson junctions or dc-SQUIDs as nonlinear inductances. A typical structure of a JPA is a $\lambda/4$-resonator with the frequency $\omega_0$ which is terminated with the dc-SQUID to the ground. In this case, the pump power can be provided by flux modulation through the dc-SQUID via inductive coupling to a separate microwave antenna (see Fig. 1.3). In the following, we consider this case of a flux-driven JPA exploiting the 3-wave mixing parametric process and Coplanar Waveguide (CPW) geometry for the resonator.
Chapter 1 Theory

Figure 1.3: Scheme of a JPA consisting of a CPW $\lambda/4$-resonator and a dc-SQUID. The dc-SQUID is biased with an external magnetic flux $\Phi_{dc}$ and flux-driven through the inductive coupling to another CPW carrying the pump tone. The symbol $M$ denotes the mutual inductance of the dc-SQUID-loop and the pump inductance.

The inductance of the dc-SQUID is modulated periodically with the pump frequency $2\omega_0$ by inductively coupling the SQUID-loop to a second transmission line, called the pump line. The signal tone enters the resonator through a coupling capacitor. Inside the cavity the signal tone is amplified through the parametric process of 3-wave mixing. Here, one photon of the pump tone at the frequency $2\omega_0$ is split into two photons, one at the signal frequency $\omega_s = \omega_0 + \Delta\omega$ and another one at the so-called idler frequency $\omega_i = \omega_0 - \Delta\omega$. The parameter $\Delta\omega$ is called detuning. In order to satisfy energy conservation, the following relation must be fulfilled

$$\omega_p = 2\omega_0 = \omega_s + \omega_i. \quad (1.17)$$
1.2 Superconducting Josephson parametric amplifiers

One has to distinguish between two cases. If the signal and idler frequencies coincide with \( \omega_0 \) (half the pump frequency), and hence, also coincide with each other \( \omega_s = \omega_i = \omega_p/2 \), the JPA works as a degenerate parametric amplifier. Because here signal and idler are at the same frequency, it is possible to amplify and deamplify signal field quadratures due to interference between the signal and idler modes. The consequences thereof are a squeezing of the output mode and the possibility to amplify the other quadrature without adding any noise. In the other case, \( \omega_s \neq \omega_p/2 \), the JPA works as a non-degenerate amplifier. Here, no squeezing occurs, both quadratures are amplified with the equal gain and, according to the Caves theorem \[14\], the JPA adds at least half a photon of noise to the signal, referenced to the JPA input. In the following discussion of the JPA we will focus on the non-degenerate case. The Hamiltonian describing this system is \[16\]

\[
\hat{H} = \hbar \omega_s \hat{a}_s^\dagger \hat{a}_s + \hbar \omega_i \hat{a}_i^\dagger \hat{a}_i + \hbar \chi \alpha_p \left( \hat{a}_s^\dagger \hat{a}_i^\dagger e^{-i\omega_p t} - \hat{a}_s \hat{a}_i e^{+i\omega_p t} \right),
\]

(1.18)

where \( \hat{a}^{(t)} \) with subscripts \( s,i \) are the annihilation (creation) operators of the signal and idler modes, respectively. As this is a reduced simplified Hamiltonian, zero-point energies were omitted here and the pump tone is assumed to be a quasi-classical signal. The parameter \( \chi \) is the nonlinearity, proportional to the \( \propto \varphi^2 \) term in the Josephson current-phase relation Eq.1.3. The term \( \alpha_p \) is the pump amplitude. The first two terms in the Hamiltonian describe signal and idler modes. The last term describes the parametric interaction where signal and idler photons are created by splitting a pump photon and vice versa.

In the following, we introduce an input-output formalism for field operators which describe quantum state evolution under parametric amplification. We base this discussion on previous works \[1\][14][17][18]. The input-output formalism shows how the input modes \( \hat{a}_s^{(t)}, \hat{a}_i^{(t)} \) are transformed into the output modes \( \hat{b}_s^{(t)}, \hat{b}_i^{(t)} \)

\[
\begin{pmatrix}
\hat{b}_s \\
\hat{b}_s^\dagger
\end{pmatrix} = \begin{pmatrix}
\cosh(r) & \sinh(r) \\
\sinh(r) & \cosh(r)
\end{pmatrix}
\begin{pmatrix}
\hat{a}_s \\
\hat{a}_i^\dagger
\end{pmatrix},
\]

(1.19)
here $r$ is the squeeze number which defines the amplifier gain. We can see that $\cosh(r)$ is the signal amplitude gain, $\sinh(r)$ is the idler amplitude gain. Therefore, the signal and idler power gains can be defined as

$$G^S = \cosh^2 r, G^I = \sinh^2 r.$$  

These gains fulfill the relation $G^S = G^I + 1$. Featuring high gains and quantum limited noise, JPAs have become indispensable tools for, among other things, state tomography of superconducting qubits and quantum signals detection in the microwave regime. However, the cavity, which is needed for the JPA, is detrimental to the amplifier bandwidth which is typically in the MHz-range. Frequency multiplexing is one method to scale superconducting qubit experiments [19–21]. This technique demands large bandwidths in the GHz range. Also, narrow bandwidths create the need to either tailor the JPA specifically to the used qubits frequency, or vice versa. Thus large bandwidths are desirable for flexibility concerns.

1.2.3 Josephson traveling wave parametric amplifiers (TWPA)

The Josephson traveling wave parametric amplifier is an evolution of the JPA concept. The main goal of this evolution is to solve the issue of narrow bandwidths, while trying to preserve gain and noise properties. The basic idea for the TWPA is to exchange the resonator cavity, which is the major cause of restricted bandwidths, with a transmission line, which inherently poses no constraints on the bandwidth, at least, up to a certain (cut-off) frequency. Whereas in a JPA the resonant frequency of a cavity is time-modulated, in a TWPA the per length inductance of a transmission line is time-modulated. A simple TWPA design is a chain of Josephson junctions (or, dc-SQUIDs) embedded into the transmission line (see Fig. 1.4).
Figure 1.4: Scheme of a simple TWPA, composed of a transmission line with Josephson junctions as nonlinear inductances. The power of the pump tone (blue) is converted to the power of the signal (green) and idler (red) tones. Symbols $\hat{a}$ and $\hat{b}$ are the field operators for the input and output fields respectively. Above, the evolution of the different tones while crossing the transmission line is shown schematically. Signal and idler tones grow exponentially by power transfer from the pump tone.

The main demand on the cells comprising the nonlinear transmission line is to be significantly smaller than the pump or signal wavelengths. This is the requirement for using the lumped-element model. For shorter wavelengths or larger cells, the breakdown of this model leads to considerable difficulties for engineering a TWPA.

In general, the modulation is provided by another microwave pump tone at the frequency $\omega_p$. The pump tone can either co-propagate with the signal and idler tones along the same transmission line or via a separate transmission line which couples to the signal line, e.g. through magnetic flux. Frequency dispersion leads to differing phase velocities for tones of different frequencies. This produces a shifting phase-relation between the pump, signal, and idler tones, a so-called phase-mismatch. The phase-mismatch of the three wavevectors

$$\Delta k = k_p - k_s - k_i$$  (1.21)
should be minimal, to ensure high gain throughout the TWPA. This leads to the greatest challenge for designing a TWPA - realizing the phase-matching between the phases of the signal, pump, and idler tones. Phase-mismatch $\Delta k$ arises because of several effects affecting the phase velocities of signal (and idler) and pump tones differently. The most prominent effects causing phase mismatch are chromatic dispersion, self-phase modulation (SPM), and cross-phase modulation (CPM).

**Chromatic dispersion**

Firstly, one has to consider chromatic dispersion which describes the frequency dependence of the phase velocity for a wave propagating along a transmission line.

**Self-phase modulation**

Self-phase modulation (SPM) is a nonlinear effect causing a microwave signal to experience an additional phase-shift proportional to its own power. It depends on the third-order nonlinearity of the Josephson phase-current relation Eq.1.3, i.e. the term $\propto \varphi^3$ in its expansion.

**Cross-phase modulation**

Cross-phase modulation (CPM) is also a nonlinear effect causing a phase-shift to a microwave signal. Like SPM it also depends on the the third-order nonlinearity of the Josephson phase-current relation. In contrast to SPM the phase-shift caused by CPM is dependent on the intensity of another wave co-propagating on the same transmission line \[ \phi_{NL} \propto k_1L(|I_{0,1}|^2 + 2|I_{0,2}|^2). \] (1.22)

Here, $I_{0,1}$ is the current amplitude of the wave experiencing the additional nonlinear phase-shift, $I_{0,2}$ is the current amplitude of the co-propagating wave, L is the length of the nonlinear transmission line and $k_1$ is the wavevector of the first
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wave. We point out, that, for a TWPA where the pump tone propagates on the same transmission line as the signal and idler tones, the dominating phase shifts will be caused by SPM for the pump tone and by CPM for the signal and idler tones. Apparently, the phase shifts depend mainly on the intensity of the pump. Therefore, this effect has to be counteracted, e.g. by dispersion engineering. Dispersion engineering tries to counteract effects that lead to phase mismatch by adjusting the dispersion. This can be accomplished, for example, by adding a resonator to the transmission line between cells. Dispersion engineering unfortunately adds complexity to the design. Another important design consideration is the characteristic impedance of the TWPA. Most microwave equipment is designed for $50\,\Omega$ characteristic impedance. Therefore it is desirable for the TWPA to comply with this convention, as an impedance mismatch leads to a reflection from the signal-in port of the amplifier [23]

$$\text{reflection} = -20\log \left| \frac{2Z_1}{Z_1 + Z_0} \right| \text{dB}. \quad (1.23)$$

Here $Z_0$ is the characteristic impedance of the transmission line leading to the TWPA and $Z_1$ that of the TWPA itself. TWPA's consisting of a nonlinear CPW tend to feature a high impedance of up to $1000\,\Omega$ due to the fact that Josephson inductance is rather high. We consider the characteristic impedance for a lossless CPW type transmission line [23]

$$Z_1 = \sqrt{\frac{L'}{C'}}. \quad (1.24)$$

where $L'$ and $C'$ are the inductance and capacitance per unit length, respectively. In order to remedy this problem of high impedance, one has to increase the Capacitance $C'$. This can for example be achieved by adding finger capacitors. Unfortunately it is not viable to reduce the characteristic impedance by increasing the inductance per unit length. The biggest contribution to the inductance per unit length is the Josephson inductance of the dc-SQUIDs. Therefore, reducing the inductance per unit length would require to either
place the dc-SQUIDs further apart or increase the critical currents of the use Josephson junctions. Both these options would weaken the parametric interaction between the pump and signal tones and reduce the gain of the TWPA.

\section*{1.2.4 Four-wave mixing}

In the 4-wave mixing parametric process four photons interact, two photons at the pump frequency are converted to one signal photon and one idler photon. To satisfy energy conservation the participating frequencies have to conform to the relation

\begin{equation}
\omega_p + \omega_p = \omega_s + \omega_i,
\end{equation}

where the subscripts \(p\), \(i\), and \(s\) stand for pump, idler, and signal, respectively. Assessing the Josephson current-phase relation Eq.1.3 one can expand it up to the third order around a dc-offset phase \(\varphi_{dc}\)

\begin{equation}
I_s(\varphi) \approx I_c \sin \varphi_{dc} \left( 1 - \frac{\varphi^2}{2} \right) + I_c \cos \varphi_{dc} \left( \varphi - \frac{\varphi^3}{6} \right).
\end{equation}

The process of 4-wave mixing is promoted by the third order nonlinearity in the Josephson inductance \(\propto \varphi^3\). Since this is equivalent to the Kerr nonlinearity in optical fibers, i.e. third order susceptibility \(\chi_3\), it is often referred to as Kerr-like nonlinearity. It is often utilized in TWPAs because it is readily available in Josephson junctions, without the need for an external phase bias leading to a nonzero \(\varphi_{dc}\). Apart from the desired 4-wave mixing this Kerr-like nonlinearity also gives rise to effects like SPM and CPM which are detrimental to a TWPA as they lead to a intensity dependent phase mismatch. A
1.2 Superconducting Josephson parametric amplifiers

Hamiltonian describing a 4-wave mixing TWPA is \[ \mathcal{H} = \int_0^\infty d\omega \hbar \omega \hat{a}^{\dagger}(\omega)\hat{a}(\omega) + \hbar \chi_3 \left( e^{-i2\omega_p t} \int_0^\infty d\omega \hat{a}^{\dagger}(\omega)\hat{a}^{\dagger}(2\omega_p - \omega) \int_0^z dx e^{-i\Delta kx} \right) + e^{i2\omega_p t} \int_0^\infty d\omega \hat{a}(\omega)\hat{a}(2\omega_p - \omega) \int_0^z dx e^{i\Delta kx} \right) \tag{1.27} \]

with the phase mismatch \[ \Delta k = 2k_p - k_s - k_i. \tag{1.28} \]

Here the pump tone is modeled as a classical signal which is a valid assumption as this tone is multiple orders of magnitude stronger than signal and idler tones. The main differences between this equation and the narrow-band JPA (Eq.1.18) are that (i) we have to integrate over multiple modes here, (ii) we must take into account the finite phase mismatch \( \Delta k \). The phase mismatch integral means, that after certain length of the TWPA, depending on the magnitude of the phase mismatch \( \Delta k \), the amplifier will begin to deamplify the signal again, i.e. when the exponent of the term \( e^{-i\Delta kx} \) becomes greater than \( \pi \). From this it is clear that minimal (better, strictly zero) phase mismatch is of paramount importance for a good TWPA.

1.2.5 Three-wave mixing

In contrast to 4-wave mixing, in the 3-wave mixing process satisfies the relation \[ \omega_p = \omega_s + \omega_i. \tag{1.29} \]

It is promoted by the second order nonlinearity \( \chi_2 \) in the Josephson inductance \( \propto \varphi^2 \). In order to utilize this nonlinearity we have to bias the Josephson junctions with a phase-offset, i.e. \( \varphi_{dc} \neq 0 \). This can be achieved by either current
biasing Josephson junctions with an additional dc-current or flux biasing the dc-SQUIDs. The Hamiltonian describing a 3-wave mixing-TWPA is

\[
\mathcal{H} = \int_0^\infty d\omega \hbar \omega \hat{a}^\dagger(\omega)\hat{a}(\omega) + \hbar \chi_2 \left( e^{-i\omega_p t} \int_0^\infty d\omega \hat{a}^\dagger(\omega)\hat{a}^\dagger(\omega_p - \omega) \int_0^z dx e^{-i\Delta k x} + e^{+i\omega_p t} \int_0^\infty d\omega \hat{a}(\omega)\hat{a}(\omega_p - \omega) \int_0^z dx e^{+i\Delta k x} \right),
\]

with the phase mismatch

\[
\Delta k = k_p - k_s - k_i.
\]

This is almost equivalent to the Hamiltonian for 4-wave mixing (Eq. 1.27), with the only notable difference of the change from \(2\omega_p\) and \(2k_p\) to \(\omega_p\) and \(k_p\), as for 3-wave mixing the pump is at double the signal frequency. As for 4-wave mixing, here it is also needed to minimize \(\Delta k\).

### 1.3 Flux-driven TWPA using 3-wave-mixing

The type of TWPA we chose to fabricate is a 3-wave mixing flux-driven TWPA. In the following, we will give an overview of the advantages of this design choice and introduce the most important equations describing it’s performance. For a more detailed description of this design we refer the reader to [6].

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1.3 Flux-driven TWPA using 3-wave-mixing

Figure 1.5: Scheme of a section of the 3-wave-mixing flux-driven TWPA. The inductances in the (upper) pump line couple inductively to the dc-SQUIDs in the signal line. Symbols \( \hat{a} \) and \( \hat{b} \) denote the field operators for the input and output fields, respectively. The indices \( s, i, \) and \( p \) denote the signal, idler, and pump tones, respectively. The mutual inductance of the signal and pump line cells are denoted by \( M \).

The principal structure of this design is shown in Fig. 1.5. It consists of two separate transmission lines. One transmission line is used for the pump tone, the second one is used for the signal and idler tones. The signal line is a nonlinear CPW made of dc-SQUIDs connected in series. The pump line is a conventional CPW. The signal and pump lines are placed sufficiently close to each other, so that the magnetic flux created by the pump tone can couple to the SQUID-loops in the signal line. It is advantageous to transmit the pump tone on a dedicated transmission line because the critical current of the Josephson junctions in the signal line may limit the maximum pump power. Additionally, as the pump tone propagates on a separate linear CPW it does not experience an additional phase shift due to SPM and does not induce XPM in the signal and idler tones. For the 3-wave mixing process the signal line is tuned to increase \( \varphi^2 \) nonlinearity and decrease the \( \varphi^3 \) nonlinearity. This additionally weakens the SPM and CPM effects for the signal and idler tones. As both, SPM and CPM can introduce phase mismatch this is
also desirable. Furthermore, this process can use smaller pump powers than
the 4-wave mixing process and has the advantage of a pump frequency lying
outside of the amplified frequency range which can easily be filtered from the
amplified signal. The equation describing the exponential gain of this type of
TWPA is
\[ G = \frac{|A_s|^2}{A_{g0}^2} = \cosh^2(gx) - \left( \frac{\Delta k}{2g} \right)^2 \sinh^2(gx). \] (1.32)
Here, \( \Delta k \) is the 3-wave mixing phase mismatch described by Eq. (1.31), \( x \) is the
position in the signal line and \( g \) is the gain factor described by
\[ g = \sqrt{(1 - \delta^2)g_0^2 - \left( \frac{\Delta k}{2} \right)^2}. \] (1.33)
Here \( g_0 \) denotes the maximum achievable gain factor, which is determined by
the pump power and coupling coefficient \( \kappa \). As SPM and CPM effects are
inherently small in a flux-driven 3-wave mixing TWPA phase mismatch \( \Delta k \) is
small. It is, however, not possible to reach zero phase mismatch in the entire
operating frequency range of the TWPA \[6\]. The detuning \( \delta \) is defined as
\[ \delta = \frac{2\omega_s - \omega_p}{\omega_p}, \] (1.34)
and it can additionally reduce the gain factor. We can conclude, that TWPAs
are potentially superior to the established JPAs, as they offer high gain and
low noise, and don’t suffer from limited bandwidths. The most important issue
in designing a TWPA is to minimize phase mismatch \( \Delta k \), as it sets a limit on
the length of the TWPA and therefore the possible gain. Secondly, to ensure
a sufficiently strong pump tone over the entire length of the TWPA one has to
use a as strong as possible pump input. This is limited in a Josephson TWPA
using a single transmission line for signal and idler and pump tones. TWPAs
utilizing separate lines for the signal and the pump are free of this limitation.
For these reasons the flux-driven TWPA using 3-wave mixing is a promising
concept for a high gain, low noise, and high bandwidth amplifier.
Chapter 2

TWPA fabrication

The main focus of this thesis is design and fabrication of a flux-driven TWPA. This chapter discusses design and fabrication challenges. As the result, we are going to present several promising TWPA samples and the corresponding fabrication routines.

2.1 Sample design

Here, we are going to present an experimental design of a flux-driven TWPA. We discuss general limitations due to our fabrication process and specific questions of the TWPA impedance matching.

2.1.1 Sample layout

In the following, we show a layout used in fabrication and comprising structures. These layout has been developed using the concept of a 3-wave mixing TWPA, as discussed in section 1.3 and adapted to our sample holder limitations.
We use conventional Coplanar Waveguide (CPW) geometry to form microwave transmission lines as required by the TWPA designs. These CPW lines are easy to design to match the 50 Ohm impedance and straightforward to fabricate using the electron beam lithography. The dimensions for the gap and center conductor width of the CPWs are calculated using the program txline with the aim to reach $50 \Omega$ impedance. This is needed in order to avoid signal reflections during the sample testing with external microwave equipment. In the signal line the massive center conductor is replaced by a chain of dc-SQUIDs (see Fig.2.1). The single dc-SQUIDs design has been taken from earlier fabrication runs in the framework of D. Arweiler’s master thesis [25]. This design is distinguished by the Josephson junctions made along the chain of dc-SQUIDs, as a more frequently used transversal junction design produced unwanted junctions when using the double-angle shadow evaporation technique [26]. The length of 20 $\mu$m of the dc-SQUID-loop along the CPW is chosen to keep the additional Josephson inductance relatively low, as it has an adverse effect on the impedance and increases the reflection losses. Also the width of 10 $\mu$m of the dc-SQUID-loop enables a geometrically seamless transition from the chain of dc-SQUIDs to the classical CPW. Additionally, the expected geometric inductance of the dc-SQUID-loop is sufficiently low, i.e. much smaller than the Josephson inductance. In order to inductively couple the pump tone to the dc-SQUIDs in the signal line, the these lines are placed in proximity to each other. In order to be able to connect the sample to a sample holder box and subsequently to the experimental setup, we use tapers at the end of the signal and pump lines. These tapers grow linearly in the width of the CPW gap and the width of the CPW central conductor, keeping their proportion constant in order to preserve the 50 $\Omega$ matching. The larger dimensions of the tapers are needed to allow for a sufficient area as required for bonding wires. Bonding connections have dimensions on the order of 100 $\mu$m, therefore we use square bonding pads of the size of 500 $\mu$m in order to allow for multiple bonds.
Figure 2.1: TWPA design (a). The CPWs consist of the ground plane (red) and a center conductor (green). The ground plane (red) does not fill the entire chip, but extends far enough from the transmission lines. Inside the functional part of the TWPA, shown in inset (b), the pump and signal transmission lines are in proximity of each other and the center conductor of the signal line is replaced with 100 dc-SQUIDs, see inset (c). The dc-SQUIDs consist of an aluminum superconducting loop and Josephson junctions which are fabricated in the area of the ghostlayer (yellow).

For a ground plane that extends over the total area of the chips the electron exposure during fabrication takes over 10 hours. A reduction of this time to around 3 hours is desirable for the purpose of writing a batch of 6 chips in one writing process. For this reason the ground plane does not extend over
the total area as only the ground plane near CPWs are expected to affect the function of the TWPA.

Figure 2.2: Part of the TWPA design. White squares are holes in the CPW ground plane and implement flux traps. The ground plane is shown in red, the ground plane edges in blue, and the CPW center conductor in green.

During fabrication optical microscope pictures are used for later evaluation of the applied fabrication process. Afterwards, a scanning electron microscope (SEM) is used to assess the structure of the Josephson junctions prior to measurements in a cryostat. Small test structures of 2 dc-SQUIDs are placed at each corner of the chip for in-situ testing purposes. Also, in later designs, bigger test structures of 10 dc-SQUIDs are added closer to the center, as the thickness, quality and electron sensitivity of the resist can vary greatly between the edge and center of the chip. We make holes in the ground plane in order to function as flux traps for pinning magnetic vortices (see Fig. 2.2). Otherwise, magnetic vortices moving across the superconductor can cause dissipative effects and disturb the local bias fields acting on the dc-SQUID chain. These flux traps have a size of 10\(\mu\)m by 10\(\mu\)m and are placed 10\(\mu\)m apart, with a clearance of 50\(\mu\)m from the transmission lines. The flux traps are omitted in some of the fabricated samples in order to troubleshoot problems with the lift-off process.
2.1 Sample design

2.1.2 Impedance estimation

As already mentioned in chapter 1, the characteristic impedance of the signal line, determines the proportion of the incident signal that will be reflected and transmitted. The so-called reflection loss described in equation \[1.23\] gives the loss of signal in dB. The transmission becomes maximal for equal characteristic impedances. Therefore, we have to match the impedance of the TWPA to the external one. In order to ensure a reasonable impedance matching for the signal line it’s geometry is simulated using the FastFieldSolvers suite and the results are used to estimate the characteristic impedance. We use FastHenry3 to simulate geometric inductance per cell for our structure and FastCap2 to extract capacitance per cell. FastModel was used to visualize the models for both programs. The geometry used in the simulations is chosen to be a section of the signal line of the length 200 µm which corresponds to 10 cells. The number of 10 cells was chosen to limit the elements in the simulation which strongly reduces the time and computation resources required and still yields a representative result for numerous dc-SQUIDs. The equation \[1.24\] yields the characteristic impedance in terms of the inductance and capacitance per unit length. In order to calculate the characteristic impedance the results from the simulations with FastHenry3 \(L_{l,FH3}\) and FastCap2 \(C_{l,FC2}\) are used. The Josephson inductance \(L_J\) for a single dc-SQUID is calculated from equation \[1.10\]. The terms \(L_l = L_{l,FH3}(200 \mu m) + 10L_J\) and \(C_l = C_{l,FC2}(200 \mu m)\) are inserted into equation \[1.24\] to obtain a result for the expected characteristic impedance.

Inductance estimation:

FastHenry3 is a program for calculating inductances from given designs. As this is not completely equivalent to a field analysis of the quasi-TEM mode in the CPW, we first verify the use of this program. For this, we simulate the CPW with FastHenry3 and compare the results to the expected value for \(L_l\)
from the equation \[27\]

\[ L_l = \frac{\mu_0}{4} \frac{K(k'_0)}{K(k_0)}, \]  \hspace{1cm} (2.1)

where \( K \) denotes the complete elliptic integral of the first order, \( k_0 \) and \( k'_0 \) are defined by

\[ k_0 = \frac{w}{w + 2s}, \]  \hspace{1cm} (2.2)

with the CPW center conductor width \( w \) and the gap width \( s \) and

\[ k'_0 = \sqrt{1 - k_0^2}. \]  \hspace{1cm} (2.3)

The result of the simulation in FastHenry3 yielded the result of \( 5.77 \times 10^{-11} \) H, equation 2.1 yields \( 1.16 \times 10^{-10} \) H. We conclude that FastHenry3 is useful to get a rough impedance estimation. For describing the geometry of a problem in FastHenry3, one uses segments connecting nodes. These segments represent a cuboid of a defined length, width, and height. A function for creating planes of conductors is also available. These planes consist of a two dimensional grid of nodes, connected by segments to the respectively neighboring nodes. Finally, the input and output points for the currents have to be defined. Fine tuning of models in FastHenry3 consists mainly in two tasks, namely, in defining points of equal potentials and in the discretization. Points of equal potential have to be used to short together certain parts of the ground plane which are not in contact. Discretization is dividing of segments and planes into smaller parts. This is usually done because too coarse elements lead to an additional discretization error during simulation. FastHenry3 natively offers options for discretization of segments and planes \[28\].
2.1 Sample design

Figure 2.3: Visualization of a part of our model used in FastHenry 3 for the simulation of a part of our signal line with 10 dc-SQUIDs. The visualization uses a wireframe depiction, so it shows the outlines of the defined segments in red and the segments itself in blue. The upper and lower rectangle represents the ground plane, the traces in the middle represent the dc-SQUIDs-loops of the signal line. It is noticeable here, that the model consist of rectangular segments which connect nodes, as the horizontal and vertical segments overlap at the nodes. Also it is noticeable, that the first node is put in the middle of the signal line, and therefore, the vertical trace at the beginning of the signal line is made up of two segments in our model.

FastHenry3 also offers support for simulating superconductors by considering the Meissner effect. For this functionality a London penetration depth of 100 nm is stated in the input file. Unfortunately, this does not include support for the simulation of Josephson junctions. We use loops of 20 µm by 10 µm for simulating geometric inductance of dc-SQUIDs and we do not consider the ghost structures created as an artifact through the double-angle shadow evaporation.
Simulating the structure shown in Fig. 2.3 provides the result of $2.24 \times 10^{-10}$ H which indicates a doubling of the inductance as compared to the usual CPW.

**Capacitance estimation**

For FastCap2 we follow a similar approach as for FastHenry3. We first verify the validity of using this program to simulate our structure by simulating a simple CPW structure and comparing the result to the value calculated from the equation

$$C_l = 4\epsilon_0\epsilon_{\text{eff}} \frac{K(k_0')}{K(k_0)}.$$  \hspace{1cm} (2.4)

Unfortunately, the input file format for FastHenry3 is not compatible to FastCap2.

FastCap2 input files describe quadrangles and triangles that comprise the surfaces of conductors and dielectrics. Unfortunately, FastCap2 does not feature automatic refinement of simulated structures and geometrical models with strong refinement can easily be using a few thousand triangle. For this reason a Python script was used to discretize a geometry and generate a input file from this geometry. We use only triangles and compose rectangles from four triangles sharing a vertex in the middle of the rectangle. The discretization algorithm divides the specified surfaces into rectangles smaller than the predefined maximum size and expresses these as four triangles. Geometrical models for capacitance simulation are also not completely interchangeable with the ones for inductance, as the (non-magnetic) substrate has no influence on inductance but must be taken into account for capacitance calculation. Also the background metallic plane on the back side of the substrate, i.e. the sample holder, has a big effect on the total capacitance. Substrates featuring different relative permittivities $\epsilon_r$ than air in FastCap2 are defined by surfaces. The
feature which allows a surface to be a conductor and a boundary between different relative permittivities $\epsilon_r$ simultaneously, contrary to the official documentation [29], is not implemented in the program.

![Visualization of a part of our model used in FastCap 2 for the simulation of the signal line of 10 dc-SQUIDs. The visualization uses a wireframe depiction, so it shows the outlines of the defined surfaces in red and the conducting surfaces in blue. The upper and lower conducting surfaces represent the CPW ground plane, the conducting surfaces in the middle represent the dc-SQUIDs of the signal line, the black surfaces are dielectric surfaces between the silicon substrate and the air. It is noticeable here that the model consists of rectangular segments subdivided into four triangles each.](image)

To work around this issue one can use 3-dimensional conductors, comprised of surfaces, together with a definition of the outside relative permittivity $\epsilon_r$ and a reference point on the inside of the conductor [30]. To build complicated geometrical models, using conducting surfaces as well as surfaces of dielectrics, FastCap2 features list files. In these files surfaces from multiple input files can be combined and defined as either a conductor surface or a dielectric surface.
Chapter 2 TWPA fabrication

For conducting surfaces one has to state a relative permittivity of the volume surrounding the conductor and for dielectric surfaces the relative permittivities inside and outside the dielectric. The simulation of the simple CPW structure yields a capacitance of $5.74 \times 10^{-14}$ F as compared to $3.19 \times 10^{-14}$ F, yielded by equation 2.4, which is on the same order of magnitude.

In Fig.2.4 we show the dc-SQUID chain model which we simulate in FastCap2. This simulation gives the capacitance of $5.46 \times 10^{-14}$ F.

**Impedance calculation**

The Josephson inductance for a dc-SQUID, is calculated to be $L_J = 4.11 \times 10^{-11}$ H using the value of $4.0 \mu$A for the critical current of the junctions. Using this result and $L_{1FH3}(200 \mu m) = 2.24 \times 10^{-10}$ H, which is the simulation result from FastHenry3, the total inductance of 200 μm of the signal line is calculated to be $6.35 \times 10^{-10}$ H. The result of the simulation with FastCap2 yields a capacitance of $C_l = C_{1FC2}(200 \mu m)$ for 200 μm of the signal line. Plugging this into equation 1.24 gives us the result of $107.9$ Ω for the characteristic impedance of our simple TWPA design. Considering inaccuracies in the simulation we can expect the characteristic impedance to not be higher than $200$ Ω which corresponds to a reflection of around 35% of the input power which is a tolerable value for proof-of-principle. For this reason we decided to use this design in fabrication and leave the impedance optimization for the future.

2.2 Fabrication

In this section, we will explain the process of double-angle shadow evaporation which was used to fabricate our TWPA. Furthermore, we give an insight into the iterative process of fine tuning the parameters in each fabrication step.
2.2 Fabrication

We demonstrate characterization photos during preliminary evaluation of our samples with the optical and electron microscopes.

2.2.1 Double-angle aluminum shadow evaporation

Double-angle shadow evaporation is an established process to fabricate Josephson junctions on chip. More specifically we use this process to create Al/AlO$_x$/Al junctions. The biggest advantage of this process over other established processes is low complexity in the fabrication of Josephson junctions, as only one iteration of this process is required to create junctions with a good control over the thickness of the Al layers. Also, it enables us to fabricate the ground plane, CPW center conductors, and dc-SQUIDs - all in one step using the same superconducting material (aluminum). The success of Double-angle shadow evaporation relies on multiple different parameters. In order to find suitable values for those good starting values for most of them are required. Then, these parameters can be iteratively optimized. The same technique has been used during multiple previous works at WMI (see Ref. [25,26,31]). We based our fabrication process on these works and iteratively improve them for our goals.

In Fig. 2.5 one can see main steps of the conventional shadow-evaporation process. It uses a double resist system to create a resist bridge in the upper resist. The Josephson junctions can then be created by depositing aluminum at two angles, with an oxidation step in between. Hereby, the pattern written into the resist by an electron beam beforehand appears twice on the substrate with a displacement upwards and downwards, respectively. The shadow of the resist bridge leads to a separation of traces in the design with the junction forming under this resist bridge. Junctions without this resist bridge would be short-circuited with a superconductor layer, and therefore, unusable.
Figure 2.5: Visualization of the double-angle shadow evaporation process. The depicted steps are: (a) spin-coating of PMMA 950 K, (b) exposure of the resist with an electron beam and writing a defined pattern, (c) development of the upper resist layer with AR-600-56, (d) further development of the lower layer resist using isopropanol and creating an undercut, (e) first evaporation step via depositing aluminum at the angle $-17^\circ$, (f) oxidation of the aluminum layer, (g) second evaporation step at angle $17^\circ$, (h) lift-off of the resist layers.
2.2 Fabrication

Two aluminum evaporation steps also lead to unused ghost structures next to the structure containing the Josephson junctions. These, however, do not influence the functioning of the junctions.

In the following, we give a description of the fabrication steps during the whole process of double-angle aluminum shadow evaporation.

**Cleaning and preparation of the substrate**

Our substrate is polished, high resistivity silicon wafers without a thermal oxide and with a thickness of 525 µm. The wafers are coated in resist to protect the surface during the cutting step. The size of our chips is 10 mm by 6 mm. This size of chips is compatible with our electron beam lithography, evaporation sample holders, and measurement sample holders. For this reason these dimensions define the boundaries for our TWPA designs. To proceed with spin-coating, we first had to remove the protective resist and clean the surfaces from dirt. For this we let the chips sit in room temperature acetone for around 10 minutes. Then, the chips are moved throughout two other beakers of warm (70°C) acetone and two beakers of warm isopropanol. In each step the beaker is subjected to strong ultrasound. During the transfer between the beakers it is important to keep the chips surface wet by rinsing with isopropanol or acetone, to prevent dirt from drying onto the surface. Rinsing means keeping a stream of fluid running over the surface of the chips. After that we use reactive ion etching (RIE) to remove remaining organic contaminations. Here, the chip is placed into a chamber where, after evacuation, a specified pressures of different gases can be added. Inside of the chamber, the chip is treated with oxygen plasma in a process called 'ashing'. We finish the preparation of the chips by heating them to 160°C for 10 minutes to evaporate remaining traces of solvents. After these procedures the substrate is ready for the fabrication.
Spin-coating

Electron beam lithography sets a stringent requirement on the homogeneity of the resist layer. In order to achieve this, in the process known as spin-coating, a few drops of liquid resist are dropped on the fast spinning chip. This results in the flattening of the resist drops under centrifugal forces and produces a rather flat resist layer. Our spin-coater is shown in Fig. 2.6. The resist dries while the chip is spun. Due to surface tension of the liquid resist, spin-coating produces a resist layer homogeneous in the middle. However, the resist gets notably thicker around the chip edges. Therefore, sensitive parts of the sample structure should be placed around the chip center. For double-angle shadow evaporation we need a double-layer resist system. The two layers consist of two different kinds of the same resist, namely polymethylmethacrylate (PMMA). For the first layer we used PMMA/MA33% and a rotation speed of 2000 rpm, the second layer uses PMMA 950K and a higher rotation speed of 4000 rpm to achieve smaller thickness. The resulting typical layer thicknesses are 700 µm and 70 µm for the lower and top layers, respectively, see Fig. 2.5 (a). After each spin-coating step, the chips are heated to 160°C for 10 min to evaporate solvents and bake the resist. The behavior of the resist during
spin-coating can be modeled in a simplified way by the following equation \[7\]

\[
\frac{\partial h}{\partial t} = -\frac{K \omega^2}{r} \frac{\partial}{\partial r} (r^2 h^3).
\] (2.5)

Here, \( h \) describes the height of the resist at a defined distance \( r \) from the chip center and at time \( t \), \( \omega \) is the angular velocity of the turntable, and \( K \) is a parameter which sums up the influence of density and viscosity of the resist. Equation \[2.5\] describes the flattening and smoothing of an initially arbitrary height profile \( h(r, 0) = h_0 \) like \[7\]

\[
h(t) = \frac{h_0}{\sqrt{1 + 4K \omega^2 h_0^2 t}}.
\] (2.6)

This differential equation, however, does not include the effect of the evaporation of solvents.

**Electron beam lithography**

In the process of electron beam lithography the pattern, previously designed as a .gds file is written into the resist with a high-voltage electron beam, see Fig.2.5 (b). The photons with high energy of 80 keV break the copolymer bonds in the resist and enable us to remove the exposed spots of the resist during the development step. A photo of our e-beam system is shown in Fig.2.7, a more detailed control scheme can be found in Fig.2.8. For writing sharp structures, the electron beam has to be focused on the surface of the resist. This is accomplished manually by focusing on previously placed gold nanoparticles. With a good focus, the electron beam can reach precision of under 10 nm. However, the resist also imposes limits on the resolution of the written pattern. In order to successfully expose the resist, one has to deposit a high enough dose, measured in C/m². A too high dose may, as well as a too low dose, cause problems later during the development. The dose is affected by an exposure time and beam current.
Although a shorter process duration is desirable, this has to be balanced, as a high beam current widens the electron beam, due to Coulomb repulsion of electrons. Therefore, the design is split and the narrow lines belonging to the chain of dc-SQUIDs are written with a low current option and the rest of the design, i.e. CPWs and the ground plane, with a high current option. The doses can be independently adjusted for each layer, see different colors in Fig. 2.1. The dose each layer is exposed to is calculated as the product of a base dose of the writing step times the partial dose for the specific layer. In order to obtain a resist bridge needed for double-angle shadow evaporation, we want to create a large undercut. To reach this we used a so-called ghost layer which uses a very small dose, compared to the junction dose, over a larger area.

**Development**

In the development process the parts of the resist exposed to the electron radiation are dissolved and removed. In order to create a resist bridge and form an undercut, we utilize a two step development process. In the first step we use the developer liquid AR-600-56 to remove the strongly exposed parts of the upper layer, see Fig. 2.5 (c).
The second part uses isopropanol at the temperature of 4°C to develop the weakly exposed lower layer resist. Cooling makes the development more reproducible at differing room temperatures, as temperature affects the reactivity of the isopropanol. The development time affects the undercut, as the isopropanol dissolves the lower resist, even around the exposed parts. To stop development the sample is blown dry with pure N₂ gas.

**Evaporation and oxidation**

After development the sample is transferred to the evaporation chamber which is pumped to a high vacuum lower than $10^{-8}$ mbar. Next, a pellet of aluminum is evaporated, by heating it with a high-power electron beam. As it reaches the boiling temperature of aluminum (2470°C), we start to detect evaporation of aluminum using quartz sensors. These sensors use the shift in resonance frequency of a quartz crystal to very accurately calculate the weight of a deposited material and from this estimate the thickness and deposition rate of the aluminum layer with an accuracy of 1 Å and 1 Å/sec, respectively. As soon as the deposition rate is stabilized at 10 Å/sec, the shutter opens and deposits a layer of 40 nm on the sample that is tilted by the angle of 17°, see Fig. 2.5(e). After the desired thickness of aluminum is reached, the shutter is closed, the electron beam power ramps down, and, after a certain cool down time, the process of oxidation begins.
Figure 2.9: Picture of the evaporation apparatus. The evaporation chamber (a) is evacuated with a turbomolecular pump (b). Samples are inserted through the airlock (c) which is separated from the process chamber by a flap valve (d).

For this oxygen is passed through the chamber at a predefined rate of 3 sscm (standard cubic centimeters per minute) and the flap valve between the chamber and the vacuum turbo pump partly closes to protect the pump. Due to the inflow of oxygen during the oxidation process the pressure is stable at a relatively high value of $3.3 \times 10^{-4}$ mbar. This oxidizes the aluminum layer in a reproducible way, see Fig. 2.5 (f). The thickness $d$ is on the square root of the product of oxygen pressure and oxidation time like $d^{3/2} = C_1 \ln(C_2 p^{1/2} t + C_3)$
2.2 Fabrication

Here, d denotes the oxide layer thickness, p the oxygen pressure and t the oxidation time and C stands for constants that depend on the units used. For this reason, the oxidation becomes increasingly unreproducible for thin oxide layers and unfeasible for thick ones. Additionally, the critical current density of the Josephson junctions depends exponentially on the thickness of the tunnel barrier formed by the oxide. After the oxidation the oxygen flow is shut off and the flap valve is fully opened again. Then, after a waiting period, the evaporation is repeated with the angle of $-17^\circ$ and a layer thickness of 70 nm, see Fig. 2.5 (g). Thus, a Al/AlO$_x$/Al-layer tunnel junction is produced.

Lift-off

In the last fabrication step the aluminum deposited on the resist is removed from the chip, i.e. lifted off. This leaves only the intended structures on the chip. First, we place the chip in a beaker with room temperature acetone for 2 minutes, afterwards we transfer it to a beaker of warm (70°C) acetone, while keeping the surface wet through rinsing it with acetone. During the consequent treatment with weak ultrasound for 2 minutes, most of the unwanted aluminum lifts off. For better results, the chip is moved to a second beaker of warm acetone and subjected to ultrasound again.

Finally, we move the chip to a beaker with isopropanol in order to prevent acetone stains. This step leaves us with the final chip holding our intended design and Al/AlO$_x$/Al Josephson junctions, see Fig. 2.5 (h).

In all fabrication steps, there are quite many parameters to consider. Some parameters do not influence the outcomes strongly while others have to be met very precisely. The main task in fabrication is to correctly differentiate between these two kinds of parameters, understand the influences of the design and the parameters themselves on the final outcomes, and adjust the important parameters.
2.2.2 Resulting samples and preliminary evaluation

In order to proceed in finding working parameters for all fabrication steps, we use microscope images from the optical and scanning electron microscope (SEM). Pictures with the optical microscope are taken from every chip after development and again after the lift-off step. Only a selection of samples after successful lift-off steps are examined using the SEM, as this procedure is very time intensive. These microscope pictures allowed us to determine which fabrication steps are faulty and fix these by adjusting various fabrication parameters. Microscopy also identifies chips where unsuccessful fabrication was not due to an error in the process or parameters but caused by an occasional contamination (e.g. dirt and dust particles). SEM pictures also help to find correct parameters for the partial and base doses during the design writing with the e-beam. We start the fabrication process by executing dose tests to find good parameters for the electron dose for the ground plane and CPW center conductor (ground plane base dose), and junctions and ghostlayers (junction base dose). This can shift from previously appropriate doses due to aging of the resist. To execute the dose test, we use a small test structure which features most critical parts of the TWPA design, i.e. CPWs, dc-SQUIDs and a bending in the CPWs. This structure is written 40 times on a single dose test chip, while increasing the ground plane base dose and junction base dose in steps of $0.05 \text{ C/m}^2$, starting from $7.0 \text{ C/m}^2$ and $4.5 \text{ C/m}^2$, respectively. Figure 2.10 shows typical photos of such dose tests.

Based on the evaluation of these dose tests with an optical microscope and the junctions with the SEM, we have found optimal doses for a ground plane base dose of $8.8 \text{ C/m}^2$ and junctions base dose of $6.3 \text{ C/m}^2$. After commencing the fabrication of TWPA chips, most chips were damaged in the step of lift-off, as the ultrasonic treatment ripped off parts of the intended ground plane.
2.2 Fabrication

Figure 2.10: Optical microscope image of the dose test sample DT 2. In (a) multiple test structures are visible. Each is written with a slightly different dose. In (b) the test structure, written with 8.8 C/m² for the ground plane and 6.3 C/m² for the junctions, is shown with a bigger magnification. The ground plane is written sufficiently cleanly. The quality of the junctions is not reliably recognizable in optical microscope photos.

Sometimes during lift-off, big chunks of the ground plane are getting detached from the substrate surface together with the aluminum on the remaining resist. This makes the chips unusable as it often damages the CPWs and, in some cases, the dc-SQUIDs, see Fig. 2.11 (a). As the other steps have been accomplished successfully according to the microscopy images, we have concentrated our efforts on this issue. Different procedures of lift-off have been tried. Unfortunately, it has been found that the application of ultrasound usually led to damaged chips.

For this reason, an alternative method has been tested. In this method no ultrasound is used but the samples are left to soak in acetone for longer. Then, a lift-off is attempted by pumping acetone over the sample surface with a pipette. This, unfortunately, often results samples with an incomplete lift-off or detached aluminum flakes sticking to other parts of the structure, see Fig. 2.11 (b).
Another procedure which we have tried is to hold the chips inside the acetone with tweezers while applying ultrasound, without touching the glass beaker itself. This has been later abandoned, as the ultrasound often leads to the chip slipping from the tweezers and the results are hardly reproducible. After different lift-off procedures have been tried, we considered the possibility of the development process being at fault for the failed lift-off, rather than the lift-off procedure itself. If during the second development step the lower PMMA/MA 33% resist is not fully removed at the exposed spot, during the evaporation, the aluminum can not properly adhere to the substrate surface. Therefore, this part of aluminum is prone to be torn off during lift-off. The main difference of the TWPA samples and dose test chips is that on the TWPA samples a much greater area of the ground plane. As the e-beam exposes a larger area of the resist, more of the resist dissolving during development. The larger quantities of resist dissolving in the developer fluid leads to contamination of the isopropanol, used as developer fluid. Due to that issue the contaminated isopropanol can leave small stains on the chip surface while drying the chip. As mentioned earlier, this can lead to a failed lift-off process. To wash off these stains of remaining resist and isopropanol, after the regular development routine, we shortly dip the samples into a beaker of clean isopropanol. The
isopropanol in this beaker is not used for a whole batch of 6 samples but renewed for every single sample, in order to avoid contamination. When the fabrication is carried out using this adjusted development routine, a lift-off using ultrasound most often is successful.

Figure 2.12: Optical microscope images of TWPA-65 after development (a) and after lift-off (b).

In Fig. 2.12 the sample TWPA-65 is shown under the optical microscope after the development and after lift-off. The lift-off step is notably successful. This sample is fabricated without the flux traps.

After successful lift-off we examine the Josephson junctions on the sample with the SEM. Most junctions exhibit similar problems to the ones in Fig. 2.13. From double-angle shadow evaporation one should expect to obtain junctions made up of three separated stripes of aluminum, single layer stripes up and down, and a single Al/AlO_x/Al stripe in the middle. In Fig. 2.13 it is clearly observable, that these stripes are not separated. The problem arising from these stripes being connected is that in this case there exists a superconducting layer shunting the junction and inhibiting its function.
Chapter 2  TWPA fabrication

Figure 2.13: SEM pictures of defect Josephson junctions. The aluminum traces leading up to the Josephson junction are overlapping which can lead to unintended shunting of the Josephson junction with a superconducting layer. The junction areas are not separated into three stripes as in Fig. 2.14 (b).

The traces leading up to the junction are also overlapping due to a misjudgement in design which has been corrected afterwards. This missing separation in the junction area most likely stems from either an overdevelopment or an overexposure in the junction area. In order to remedy this problem, we have conducted the second parameter test. In this test not only the dose is varied but also other parameters that we have suspected to have an influence on the junctions yield and quality. The varied parameters are the lengths of the overlap of the junction electrodes and partial dose of the ground plane edge, the base dose for the junctions and partial dose of the ghostlayers. The resulting samples and junctions are extensively studied using a SEM. The dose with which a layer is written is defined by the base dose times the partial dose. In the following, we define the specific doses and give typical values for those up to the parameter test. Junction stripes use the junction base dose $B_J$ (6.5 C/m²) and the junction partial dose $P_J$ (0.85), ghostlayers use the same base dose but the ghostlayer partial dose $P_{GL}$ (0.15). The ground plane and the CPW center conductors use the ground plane base dose $B_G$ (8.5 C/m²)
and the ground plane partial dose $P_{GP}$ (0.8), the edges of the ground plane use the same base dose but the edge partial dose $P_E$ (0.7).

Figure 2.14: Scanning electron microscope pictures of dc-SQUIDs and Josephson Junctions on a parameter test chip. The parameters that are varied within this test are the junction base dose $B_J$, ghostlayer partial dose $P_{GL}$, edge partial dose $P_E$ and the lengths of the junction overlap and ghostlayer, respectively. The best parameters yield dc-SQUIDs shown in (a) and the Josephson junction shown in (b). Image (c) shows a test with the higher ghostlayer partial dose $P_{GL}$ leading to a breakdown of the separation of the aluminum stripes. Image (d) shows a test with the insufficient partial dose for the edge $P_E$ of the ground plane for clean edges. In all cases we false-color the area of the Josephson junction in red.

Fig. 2.14 shows a selection of test structures with different parameters. The junction and ghostlayer lengths are found to have no influence on the yield
and quality of the junctions. The edge of the ground plane is written unevenly when using a partial dose $P_E$ of less than 0.7 (see Fig. 2.14 (d)).

Figure 2.15: Scanning electron microscope picture of a dc-SQUID in sample TWPA-65. A clear separation of three stripes of aluminum in the upper and lower branch of the dc-SQUID is visible. The middle stripes contain the overlap constituting the Josephson junctions false-colored with red.

In test structures with unfavorable parameters for the junctions, a negative influence of the ground plane due to the proximity effect is observable for the edge partial dose $P_E$ of 0.7 or higher. The junction base dose $B_J$ has no influence on the quality of the junctions inside the tested range of 5.0 - 6.5 C/m². The biggest influence on the junction yield and quality is due the ghostlayer partial dose $P_{GL}$. Junctions in the test structures with the partial dose $P_{GL}$ of 0.15 (as in TWPA samples up to the parameter test) are all
failed, see Fig.2.14 (c). Using the partial dose $P_{GL}$ of 0.1 for the ghostlayers only junctions also subjected to the proximity effect, i.e. the first and last junctions, were broken. For the partial dose $P_{GL}$ of 0.05 all junctions exhibited the correct separation of stripes, see Fig.2.14 (a) and (b). Using the results from this parameter test we proceed using a junction base dose $B_J$ of \(5.5 \text{ C/m}^2\) and a ground plane edge partial dose $P_E$ of 0.7. The previous ghostlayer partial dose $P_{GL}$ of 0.15 is dropped down to 0.05.

Multiple samples have been fabricated in the process of iteratively improving the fabrication procedure. Only for few samples a clean lift-off has been achieved. After evaluation of these with the SEM have decided to use the sample designated TWPA-65, i.e. the 65th sample fabricated, for actual cryogenic microwave measurements. The TWPA-65 chip is examined under the optical microscope, see Fig.2.12 and the SEM, see Fig.2.15. As, unfortunately, the SEM at ZNN, which features better image quality, was broken at the time, so we had to use the SEM at WMI where we have not been able to obtain a picture of better quality than Fig.2.15.
Chapter 3

Experiment and Results

This chapter discusses preparations for cryogenic microwaves measurements of the TWPA sample, and demonstrates preliminary experimental results.

3.1 Sample preparation

In order to use a sample in the experimental setup it is placed into a specific sample holder, then respective sample contact pads have to be connected to printed circuit board (PCB) lines of the sample holder by the means of the ultrasonic bonding technique. The sample is outfitted with a superconducting coil for magnetic field biasing of the dc-SQUIDs during the experiment. The sample holder is made of gold-plated oxygen-free copper. It is designed to hold a 10mm by 6mm chip. In order to connect external microwave cables, four SMA-compatible connectors are soldered to the sample holder. These connectors consist of a center pin and outside conductor. The outside connector is directly connected to the copper sample holder which acts as circuit ground. A so-called glass bead is soldered to the sample holder such that it connects the center pin to a pin above the center conductor on the PCB while isolating both pins from ground.
Center pins of the glass beads have to be soldered to central CPW electrodes on the PCB without creating a short to the ground. This is accomplished using Indium as a solder. This metal can be cut and formed correctly with a scalpel, afterwards this connection can be soldered by heating it with soldering iron. When applying the soldering iron one has to be careful not to pull the indium out of the designated area. This is done under microscope as the center conductor on the PCB is only 250 µm wide. In order to create a low inductance connection between the PCB and the chip, we place as many bonds as possible. Next, in order to apply external magnetic field to the TWPA, we require a magnetic coil coupled to the sample holder. For this, we fabricate a specific superconducting coil compatible with our sample holder. The coil is made using a coil turning machine at WMI using NbTi single-filament wire. To ensure good thermalization and mechanical stability of the superconducting coil at low temperatures, we cover every several layers with GE Varnish glue and let it solidify. Finally, the coil is fixed on top of the closed sample holder box. A silver wire is clamped between the coil and sample holder and connected to the silver plate of the cryostat, in order to provide reliable thermalization, see Fig. 3.2.

Figure 3.1: Picture of TWPA sample in the sample holder. Bonding wires connect the ground planes and center contact, respectively of the PCBs and the sample. To ensure low inductance as many as short as possible bond wires are used.
3.2 Experimental setup

To ensure proper function of the superconducting CPWs and the Josephson junctions comprising the TWPA, the experiment has to be conducted well below the critical temperature of aluminum $T_c = 1.2\, \text{K}$. The experiment is therefore set up inside a dilution cryostat that allows reaching temperatures of around 10 mK. The cryostat has several temperature stages which are thermally decoupled from each other and the outside world. Our dilution cryostat sits in a liquid He bath, surrounded by the liquid nitrogen jacket, which cools down the first cryostat stage to 4.2 K. The second stage relies on pumping on liquid He from a small pot inside the vacuum chamber which allows to bring the local temperature down to around 1.2 K. In order to reach even lower temperatures, one must use a so-called dilution process. During this process, the cooling effect takes place at a phase boundary between a $^3\text{He}$-rich and a $^3\text{He}$-poor phases of a $^3\text{He}/^4\text{He}$-mixture, when $^3\text{He}$ flows from the concentrated phase to the dilute phase. This process allows us to achieve temperatures of around 10 mK.

Figure 3.2: Picture of the coil mounted on top of the sample holder to the silver plate in the cryostat. Microwave cables with SMA connectors are connected to the sample holder. On the pump output port a $50\, \Omega$ terminator is connected and thermalized by connecting it to the silver mounting plate in the cryostat. The coil is additionally thermalized by clamping silver wire, which is also connected to the mounting plate, between the coil and sample holder.
Chapter 3 Experiment and Results

Figure 3.3: Scheme of the cryogenic experimental setup. The vector network analyzer (Keysight PNA-N5222A) is used to measure the a weak coherent signal transmission through the TWPA. An extra vector microwave generator (Rohde&Schwarz SMF 100A) is used to generate a strong coherent pump tone which is applied to the pump-in port of the TWPA. A current source (Keithley 6430) feeds a dc-current to the coil, mounted on top of the TWPA, to bias the dc-SQUIDs.

The signal and pump tones are generated outside of the fridge and fed into the fridge through all temperature stages via coaxial cables (see Fig. 3.3 for details). In order to suppress influence of room temperature thermal fluctuations on our measurements, we additionally attenuate the incoming signals using microwave attenuators. Microwave attenuators dissipate power of the microwaves passing through them. The dissipation of power is specified as negative gain in dB. The signal tone is generated by the Keysight PNA-N5222A vector network analyzer (VNA), attenuated at different temperature stages towards the TWPA, enters the signal line of the TWPA and, after amplification in by the
3.3 Microwave transmission measurements

After cooling the fridge to millikelvin temperatures, transmission measurements are conducted. We use the VNA to measure power and phase of the transmitted weak coherent signal through the TWPA as a function of magnetic field and pump signal properties (such as frequency and power). The coil current corresponding to one flux quantum $\Phi_0$ across the area of our dc-SQUIDs $200 \mu m^2$ is estimated to correspond to the coil current of roughly 300 $\mu$A. The frequency span is defined by the range of our HEMT amplifier which is 4 - 8 GHz.
Figure 3.4: Transmission measurements of a weak coherent tone with the power -100 dBm referred to the signal-in through the TWPA-65 sample calibrated against the response at zero coil current. Pump signal frequency is fixed to 12 GHz, its power is at -46 dBm referred to the pump-in port. Plot (a) shows the transmitted power. Plot (b) shows the phase of the transmitted signal. One can notice a broadband amplification for the coil currents $\simeq -50 \mu A$.

In the first measurement (see Fig. 3.4) the amplitude and phase of the transmitted signal tone were measured by the VNA in the range of 4 - 8 GHz for the coil current range of -50 to 50 $\mu A$. During this measurement the pump is set to 12 GHz was turned on. We are able to observe a rather weak but broadband gain around 0.5 dB over several GHz bandwidth (see Fig. 3.5).

The data observed in Fig. 3.4 hints that the coil is working as intended due to a smooth phase shift of the transmitted signal. Unfortunately, measurements in larger coil current ranges were not very conclusive due to irreproducibility problems. Most likely, the latter issues have been caused by the lack of proper magnetic shielding in this cryostat. This part can be improved in the future by using a solid superconducting shield around the sample holder.

Nevertheless, Fig. 3.5 clearly suggests that our first TWPA prototype might be on the right track as it shows a very broadband (up to several GHz) gain.
response. The latter is admittedly weak (around 0.5 dB) but can be improved in the future by fine tuning of the pump parameters (frequency and power), improved magnetic shielding, magnetic field tuning, and proper calibration of the input signals (this step might require use of by-pass cryogenic microwave switches).

![Figure 3.5: TWPA gain response at the coil current of $-50 \mu A$ over the frequency range of 4 - 8 GHz with the pump power turned on (12 GHz, power -46 dBm referred to the pump-in). The plotted values are calibrated against the response at 0 coil current.](image)

Furthermore, we have to note that this measurements have been performed with a rather basic TWPA design consisting of relatively low number of
dc-SQUIDs. Increasing this number to \( \simeq 1000 \) and optimizing further the impedance matching should increase further the performance of these devices.
Outlook

Superconducting Josephson traveling wave parametric amplifier (TWPA) already offer high gain and excellent noise properties in combination with comparatively large bandwidths. They have become an essential part of all state-of-the-art experiments with superconducting quantum circuits. Our main goal in this thesis was to design and fabricate a new type of these devices based on the flux-driven, 3-wave mixing TWPA design.

First, we have applied a new concept of a flux-driven, 3-wave mixing TWPA to create a design compatible with the double-angle aluminum shadow evaporation process. We have performed estimations of the device impedance using available theories and finite-element simulations in the FastHenry3 and FastCap2 environments. Next, we have adapted our aluminum shadow evaporation process to reliably fabricate multiple dc-SQUIDs on a single chip, as required for a TWPA. On the way to these devices, we have solved multiple fabrication issues specific to our designs, such as e-beam dose and lift-off problems. At the end, we have delivered a successful sample TWPA-65 with 100 dc-SQUIDs embedded into the CPW transmission line. Unfortunately, multiple downtimes among various fabrication devices, such as the electron beam lithographer, evaporation chamber, RIE, etc., have robbed us of much of valuable time to conduct detailed measurement tests of the produced samples. Therefore, the demonstrated measurements are quite limited. Nevertheless, these measurement results have already demonstrated a very promising behavior. As such, we have observed a gain of roughly 0.5 dB in the bandwidth.
Outlook

of several GHz which is a step in the right direction for the flux-driven TWPA development.

In the future, we could definitely improve the measurement results by making use of more careful microwave calibrations and better magnetic shielding. It is also important to note that the current TWPA design is the most basic one and can be improved much further and optimized in many ways. For example, we could enhance the impedance matching of the TWPA to the $50\,\Omega$ impedance standard by making use of extra capacitors in combination with the dc-SQUIDs. Also, the coupling between the pump and signal lines should be optimized and the total number of the dc-SQUIDs can be increased up to $\simeq 1000$ to achieve higher gains. In the end, making use of more sophisticated dispersion engineering techniques, to optimize the up-conversion and down-conversion rates, would be desirable.
Bibliography


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